FOUNDATION ENGINEERING

Theoretical: 3hrs/week; Tutorial: 1hrs/week

First semester

No	Title	hr
1	**Site investigation**: The purpose and method of the exploration program. **Bore holes**: Number, depth, the distance between bore holes, disturbed and undisturbed sample and the reasons of disturbance. **Field test**: Field van shear test of soil, standard penetration test (SPT), plate-load test.	12
2	**Settlement calculation**: Immediate settlement.	8
3	**Bearing capacity of soil**: Terzaghi equation for evalution bearing capacity of soil, effects of water and footing shapes on bearing capacity of soil, Skempton method for estimating the bearing capacity of clay soils and factor of safety.	18
4	**Footing design**: Unreinforced and reinforced spread footing design, wall footing, the effect of the moments on the dimensions of footing, rectangular combined footings, design of trapezoid-shaped footings, design of strap or cantilever footings and raft (mat) footing.	24

Second semester

No	Title	hr
5	**Piles**: Single pile in clay, single pile in sand, pile groups(the distribution of piles in groups), pile groups(the distribution of the loads on piles), efficiency of pile groups and negative skin friction.	24
6	**Lateral earth pressure**: Rankine's earth pressure theory-horizontal surface of soil, Rankine's theory-inclined surface of soil, Coulomb's earth pressure theory, stability of retaining walls and sheet piles.	20
7	**Slope stability**: Infinite slope, finite slope, Taylor method for estimating factor of safety, $Ø_u$=0 method of estimating factor of safety and method of slices.	14

FOUNDATION ENGINEERING

DIFINITIONS:

A foundation is defined as that part of the structure that supports the weight of the structure and transmits the load to underlying soil or rock. In general, foundation engineering applies the knowledge of geology, soil mechanics, rock mechanics, and structural engineering to the design and construction of foundations for buildings and other structures. The most basic aspect of foundation engineering deals with the selection of the type of foundation, such as using a shallow or deep foundation system. Another important aspect of foundation engineering involves the development of design parameters, such as the bearing capacity or estimated settlement of the foundation. Foundation engineering could also include the actual foundation design, such as determining the type and spacing of steel reinforcement in concrete footings. Foundation engineering often involves both geotechnical and structural engineers, with the geotechnical engineer providing the foundation design parameters such as the allowable bearing pressure and the structural engineer performing the actual foundation design.

Foundations are commonly divided into two categories: shallow and deep foundations. Table 1.1 presents a list of common types of foundations. In terms of geotechnical aspects, foundation engineering
often includes the following (Day, 1999a, 2000a):
• Determining the type of foundation for the structure, including the depth and dimensions.
• Calculating the potential settlement of the foundation
• Determining design parameters for the foundation, such as the bearing capacity and allowable soil bearing pressure.
• Determining the expansion potential of a site.
• Investigating the stability of slopes and their effect on adjacent foundations.
• Investigating the possibility of foundation movement due to seismic forces, which would also include the possibility of liquefaction.
• Performing studies and tests to determine the potential for deterioration of the foundation.
• Evaluating possible soil treatment to increase the foundation bearing capacity.
• Determining design parameters for retaining wall foundations.
• Providing recommendations for dewatering and drainage of excavations needed for the construction of the foundation.
• Investigating groundwater and seepage problems and developing mitigation measures during foundation construction.

• Site preparation, including compaction specifications and density testing during grading.
• Underpinning and field testing of foundations.

TABLE 1.1 Common Types of Foundations

Category	Common types	Comments
Shallow foundations	Spread footings	Spread footings (also called pad footings) are often square in plan view, are of uniform reinforced concrete thickness, and are used to support a single column load located directly in the center of the footing.
	Strip footings	Strip footings (also called wall footings) are often used for load-bearing walls. They are usually long reinforced concrete members of uniform width and shallow depth.
	Combined footings	Reinforced-concrete combined footings are often rectangular or trapezoidal in plan view, and carry more than one column load.
	Conventional slab-on-grade	A continuous reinforced-concrete foundation consisting of bearing wall footings and a slab-on-grade. Concrete reinforcement often consists of steel rebar in the footings and wire mesh in the concrete slab.
	Posttensioned slab-on-grade	A continuous posttensioned concrete foundation. The posttensioning effect is created by tensioning steel tendons or cables embedded within the concrete. Common posttensioned foundations are the ribbed foundation, California slab, and PTI foundation.
	Raised wood floor	Perimeter footings that support wood beams and a floor system. Interior support is provided by pad or strip footings. There is a crawl space below the wood floor.
	Mat foundation	A large and thick reinforced-concrete foundation, often of uniform thickness, that is continuous and supports the entire structure. A mat foundation is considered to be a shallow foundation if it is constructed at or near ground surface.
Deep foundations	Driven piles	Driven piles are slender members, made of wood, steel, or precast concrete, that are driven into place by pile-driving equipment.
	Other types of piles	There are many other types of piles, such as bored piles, cast-in-place piles, and composite piles.
	Piers	Similar to cast-in-place piles, piers are often of large diameter and contain reinforced concrete. Pier and grade beam support are often used for foundation support on expansive soil.
	Caissons	Large piers are sometimes referred to as caissons. A caisson can also be a watertight underground structure within which construction work is carried on.
	Mat or raft foundation	If a mat or raft foundation is constructed below ground surface or if the mat or raft foundation is supported by piles or piers, then it should be considered to be a deep foundation system.
	Floating foundation	A special foundation type where the weight of the structure is balanced by the removal of soil and construction of an underground basement.
	Basement-type foundation	A common foundation for houses and other buildings in frost-prone areas. The foundation consists of perimeter footings and basement walls that support a wood floor system. The basement floor is usually a concrete slab.

Note: The terms *shallow* and *deep* foundations in this table refer to the depth of the soil or rock support of the foundation.

SUBSURFACE EXPLORATIONS:

Purpose of Subsurface Explorations:

The process of identifying the layers of deposits that underlie a proposed structure and their physical characteristics is generally referred to as *subsurface exploration.* The purpose of subsurface exploration is to obtain information that will aid the geotechnical engineer in

1. Selecting the type and depth of foundation suitable for a given structure.
2. Evaluating the load-bearing capacity of the foundation.
3. Estimating the probable settlement of a structure.
4. Determining potential foundation problems (e.g., expansive soil, collapsible soil, sanitary landfill, and so on).
5. Determining the location of the water table.
6. Predicting the lateral earth pressure for structures such as retaining walls, sheet pile bulkheads, and braced cuts.
7. Establishing construction methods for changing subsoil conditions.

Subsurface exploration may also be necessary when additions and alterations to existing structures are contemplated.

PRELIMINARY INFORMATION AND PLANNING THE WORK

The first step in a foundation investigation is to obtain preliminary information, such as the following:

1. *Project location.* Basic information on the location of the project is required. The location of the project can be compared with known geologic hazards, such as active faults, landslides, or deposits of liquefaction prone sand.

2. *Type of project.* The geotechnical engineer could be involved with all types of foundation engineering construction projects, such as residential, commercial, or public works projects. It is important to obtain as much preliminary information about the project as possible. Such information could include the type of structure and use, size of the structure including the number of stories, type of construction and floor systems, preliminary foundation type (if known), and estimated structural loadings. Preliminary plans may even have been developed that show the proposed construction.

3. *Scope of work.* At the beginning of the foundation investigation, the scope of work must be determined. For example, the scope of work could include subsurface exploration and laboratory testing to determine the feasibility of the project, the preparation of foundation design parameters, and compaction testing during the grading of the site in order to prepare the building pad for foundation construction.

After the preliminary information is obtained, the next step is to plan the foundation investigation work. For a minor project, the planning effort may be minimal. But for large-scale projects, the plan can be quite extensive and could change as the design and construction progresses. The planning effort could include the following:

• Budget and scheduling considerations.
• Selection of the interdisciplinary team (such as geotechnical engineer, engineering geologist, structural engineer, hydrogeologist and the like) that will work on the project.
• Preliminary subsurface exploration plan, such as the number, location, and depth of borings.
• Document collection (Prior Development, Aerial Photographs and Geologic Maps, Topographic Maps, Building Code and Other Specifications, Documents at the Local Building Department, Forensic Engineering).
• Laboratory testing requirements.
• Types of engineering analyses that will be required for the design of the foundation.

Table 2.2 presents a summary of typical documents that may need to be reviewed prior to or during the construction of the project.

TABLE 2.2 Typical Documents that may Need to be Reviewed for the Project

Project phase	Type of documents
Design	Available design information, such as preliminary data on the type of project to be built at the site and typical foundation design loads If applicable, data on the history of the site, such as information on prior fill placement or construction at the site Data (if available) on the design and construction of adjacent property Local building code Special study data developed by the local building department or other governing agency Standard drawings issued by the local building department or other governing agency Standard specifications that may be applicable to the project, such as *Standard Specifications for Public Works Construction* or *Standard Specifications for Highway Bridges* Other reference material, such as seismic activity records, geologic and topographic maps, aerial photographs and the like.
Construction	Reports and plans developed during the design phase Construction specifications Field change orders Information bulletins used during construction Project correspondence between different parties Building department reports or permits

There are many different types of subsurface exploration, such as borings, test pits, or trenches. Table 2.3 presents general information on foundation investigations, samples and samplers, and subsurface exploration.

TABLE 2.3 Foundation Investigations, Samples, Samplers, and Subsurface Exploration

	Foundation investigations	
Three types of problems	Foundation problems	Such as the stability of subsurface materials, deformation and consolidation, and pressure on supporting structures
	Construction problems	Such as the excavation of subsurface material and use of the excavated material
	Groundwater problems	Such as the flow, action, and use of groundwater
Three phases of investigation	Subsurface investigation	Consisting of exploration, sampling, and identification in order to prepare rough or detailed boring logs and soil profiles
	Physical testing	Consisting of laboratory tests and field tests in order to develop rough or detailed data on the variations of physical soil or rock properties with depth
	Evaluation of data	Consisting of the use of soil mechanics and rock mechanics to prepare the final design recommendations based on the subsurface investigation and physical testing
	Samples and samplers	
Type of samples	Altered soil (nonrepresentative samples)	Soil from various strata that is mixed, has some soil constituents removed, or foreign materials have been added to the sample
	Disturbed soil (representative samples)	Soil structure is disturbed and there is a change in the void ratio but there is no change in the soil constituents
	Undisturbed samples	No disturbance in soil structure, with no change in water content, void ratio, or chemical composition
Types of samplers	Exploration samplers	Group name for drilling equipment such as augers used for both advancing the borehole and obtaining samples
	Drive samplers	Sampling tubes driven without rotation or chopping with displaced soil pushed aside. Examples include open drive samplers and piston samplers
	Core boring samplers	Rotation or chopping action of sampler where displaced material is ground up and removed by circulating water or drilling fluid
	Subsurface exploration	
Principal types of subsurface exploration	Indirect methods	Such as geophysical methods that may yield limited subsurface data. Also includes borings that are advanced without taking soil samples
	Semidirect methods	Such as borings that obtain disturbed soil samples
	Direct methods	Such as test pits, trenches, or borings that are used to obtain undisturbed soil samples
Three phases of subsurface exploration	Fact finding and geological survey	Gathering of data, document review, and site survey by engineer and geologist
	Reconnaissance explorations	Semidirect methods of subsurface exploration. Rough determination of groundwater levels
	Detailed explorations	Direct methods of subsurface exploration. Accurate measurements of groundwater levels or pore water pressure

Source: Adapted and updated from Hvorslev (1949).

SITE INVESTIGATIONS

The site investigation phase of the exploration program consists of planning, making test boreholes, and collecting soil samples at desired intervals for subsequent observation and laboratory tests. The approximate required minimum depth of the borings should be predetermined. The depth can be changed during the drilling operation, depending on the subsoil encountered. To determine the approximate minimum depth of boring, engineers may use the rules established by the American Society of Civil Engineers (1972):

1. Determine the net increase in the effective stress, $\Delta\sigma'$, under a foundation with depth as shown in Figure 2.9. (The general equations for estimating increases in stress are given in Chapter 5.)
2. Estimate the variation of the vertical effective stress, σ'_o, with depth.

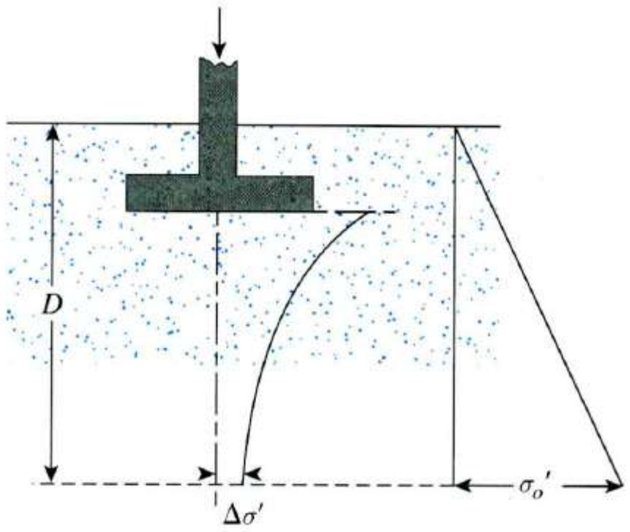


Figure 2.9 Determination of the minimum depth of boring

3. Determine the depth, $D = D_1$, at which the effective stress increase $\Delta\sigma'$ is equal to $(\frac{1}{10})q$ (q = estimated net stress on the foundation).
4. Determine the depth, $D = D_2$, at which $\Delta\sigma'/\sigma'_o = 0.05$.
5. Choose the smaller of the two depths, D_1 and D_2, just determined as the approximate minimum depth of boring required, unless bedrock is encountered.

If the preceding rules are used, the depths of boring for a building with a width of 30 m (100 ft) will be approximately the following, according to Sowers and Sowers (1970):

No. of stories	Boring depth	
1	3.5 m	(11 ft)
2	6 m	(20 ft)
3	10 m	(33 ft)
4	16 m	(53 ft)
5	24 m	(79 ft)

To determine the boring depth for hospitals and office buildings, Sowers and Sowers (1970) also used the following rules.

- For light steel or narrow concrete buildings,

$$\frac{D_b}{S^{0.7}} = a \tag{2.1}$$

where
D_b = depth of boring
S = number of stories

$$a = \begin{cases} \approx 3 \text{ if } D_b \text{ is in meters} \\ \approx 10 \text{ if } D_b \text{ is in feet} \end{cases}$$

- For heavy steel or wide concrete buildings,

$$\frac{D_b}{S^{0.7}} = b \tag{2.2}$$

where

$$b = \begin{cases} \approx 6 \text{ if } D_b \text{ is in meters} \\ \approx 20 \text{ if } D_b \text{ is in feet} \end{cases}$$

When deep excavations are anticipated, the depth of boring should be at least 1.5 times the depth of excavation.

Sometimes, subsoil conditions require that the foundation load be transmitted to bedrock. The minimum depth of core boring into the bedrock is about 3 m (10 ft). If the bedrock is irregular or weathered, the core borings may have to be deeper.

There are no hard-and-fast rules for borehole spacing. Table 2.4 gives some general guidelines. Spacing can be increased or decreased, depending on the condition of the subsoil. If various soil strata are more or less uniform and predictable, fewer boreholes are needed than in nonhomogeneous soil strata.

Table 2.4 Approximate Spacing of Boreholes

Type of project	Spacing (m)	Spacing (ft)
Multistory building	10–30	30–100
One-story industrial plants	20–60	60–200
Highways	250–500	800–1600
Residential subdivision	250–500	800–1600
Dams and dikes	40–80	130–260

The engineer should also take into account the ultimate cost of the structure when making decisions regarding the extent of field exploration. The exploration cost generally should be 0.1 to 0.5% of the cost of the structure. Soil borings can be made by several methods, including auger boring, wash boring, percussion drilling, and rotary drilling.

SOIL BORING

Exploratory holes into the soil may be made by hand tools, but more commonly truck- or trailer-mounted power tools are used.

Hand Tools

The earliest method of obtaining a test hole was to excavate a test pit using a pick and shovel. Because of economics, the current procedure is to use power excavation equipment such as a backhoe to excavate the pit and then to use hand tools to remove a block sample or shape the site for in situ testing. This is the best method at present for obtaining quality *undisturbed* samples or samples for testing at other than vertical orientation (see Figures below). For small jobs, where the sample disturbance is not critical, hand or powered augers held by one or two persons can be used.
Hand-augered holes can be drilled to depths of about 35 m, although depths greater than about 8 to 10 m are usually not practical. Commonly, depths are on the order of 2 to 5 m, as on roadways or airport runways, or investigations for small buildings.


Handle
Extension
Rod
150mm Gravel
Auger
150mm Soil
Auger
100mm Soil
Auger

Mounted Power Drills

*1)**Rotary drilling*** is another method of advancing test holes. This method uses rotation of the drill bit, with the simultaneous application of pressure to advance the hole. Rotary drilling is the most rapid method of advancing holes in rock unless it is badly fissured.

*2)**Continuous-flight augers*** with a rotary drill are probably the most popular method of soil exploration at present in North America, Europe, and Australia. The flights act as a screw conveyor to bring the soil to the surface. The method is applicable in all soils, although in saturated sand under several feet of hydrostatic pressure the sand tends to flow into the lead sections of the auger, requiring a washdown prior to sampling. Borings up to nearly 100 m can be made with these devices, depending on the driving equipment, soil, and auger diameter.

The augers may be *hollow-stem* or *solid* with the hollow-stem type generally preferred, as penetration testing or tube sampling may be done through the stem. For obvious reasons, borings do not have to be cased using continuous-flight augers, and this feature is a decided economic advantage over other boring methods.

Continuous-flight augers are available in nominal 1- to 1.5-m section lengths (with rapid attachment devices to produce the required boring depth) and in several diameters including the following:

Solid stem							
OD, mm	67	83	102	115	140	152	180
Hollow stem							
ID/OD, mm	64/160	70/180	75/205	90/230	100/250	127/250	152/305

SOIL SAMPLING

The most important engineering properties for foundation design are strength, compressibility, and permeability. Reasonably good estimates of these properties for cohesive soils can be made by laboratory tests on ***undisturbed*** samples, which can be obtained with moderate difficulty. It is nearly impossible to obtain a truly undisturbed sample of soil, so in general usage the term *undisturbed* means a sample where some precautions have been taken to minimize disturbance of the existing soil skeleton. In this context, the quality of an "undisturbed" sample varies widely between soil laboratories. The following represent some of the factors that make an undisturbed sample hard to obtain:

1. *The sample is always unloaded from the in situ confining pressures*, with some unknown resulting expansion. Lateral expansion occurs into the sides of the borehole, so in situ tests using the hole diameter as a reference are "disturbed" an unknown amount. This is the reason K_0 field tests are so difficult.

2. Samples collected from other than test pits are disturbed by volume displacement of the tube or other collection device. The presence of gravel greatly aggravates sample disturbance.

3. Sample friction on the sides of the collection device tends to compress the sample during recovery. Most sample tubes are (or should be) swaged so that the cutting edge is slightly smaller than the inside tube diameter to reduce the side friction.
4. There are unknown changes in water content depending on recovery method and the presence or absence of water in the ground or borehole.

5. Loss of hydrostatic pressure may cause gas bubble voids to form in the sample.

6. Handling and transporting a sample from the site to the laboratory and transferring the sample from sampler to testing machine disturb the sample more or less by definition.

7. The quality or attitude of drilling crew, laboratory technicians, and the supervising engineer may be poor.

8. On very hot or cold days, samples may dehydrate or freeze if not protected on-site. Furthermore, worker attitudes may deteriorate in temperature extremes.

Types of Soil Samples

Two Types of Soil Types Are Obtained:

- Disturbed Soil Samples.

- Undisturbed Soil Samples.

The degree of soil disturbance can be expressed as:

$$A_r = \frac{D_o^2 - D_i^2}{D_i^2} \times 100$$

Where:

A_r : Area ratio;

D_o, D_i: Outside and inside diameter of the sampler;

If $A_r \leq 10\%$ the sample is undisturbed.

Laboratory Soil Tests

- ***To determine the shear strength parameters (C and ϕ) and other strength and mechanical properties.***
- ***To classify the soil.***
- ***Performed on undisturbed and disturbed soil samples.***

Undisturbed and Disturbed Soil Samples

- ***Undisturbed soil samples** : Soils having the same structure ,properties, and water content of the original soil sample in the ground.*
- ***Disturbed soil samples** : Soils with structure, properties, and water content changed during the sampling or transportation process.*

Tests on Disturbed Samples

Disturbed Samples Are used in the Following Tests:

- **Grain size analysis.**
- **Liquid and plastic limit tests.**
- **Specific gravity test.**
- **Organic content test.**
- **Soil Classification.**
- **Compaction test.**
- **Direct shear test.**

Test on Undisturbed Samples

Undisturbed Samples Are used in the Following Tests:

- **Consolidation test.**
- **Permeability test.**
- **Direct shear test.**
- **Triaxial test.**

Methods of Soil Sampling

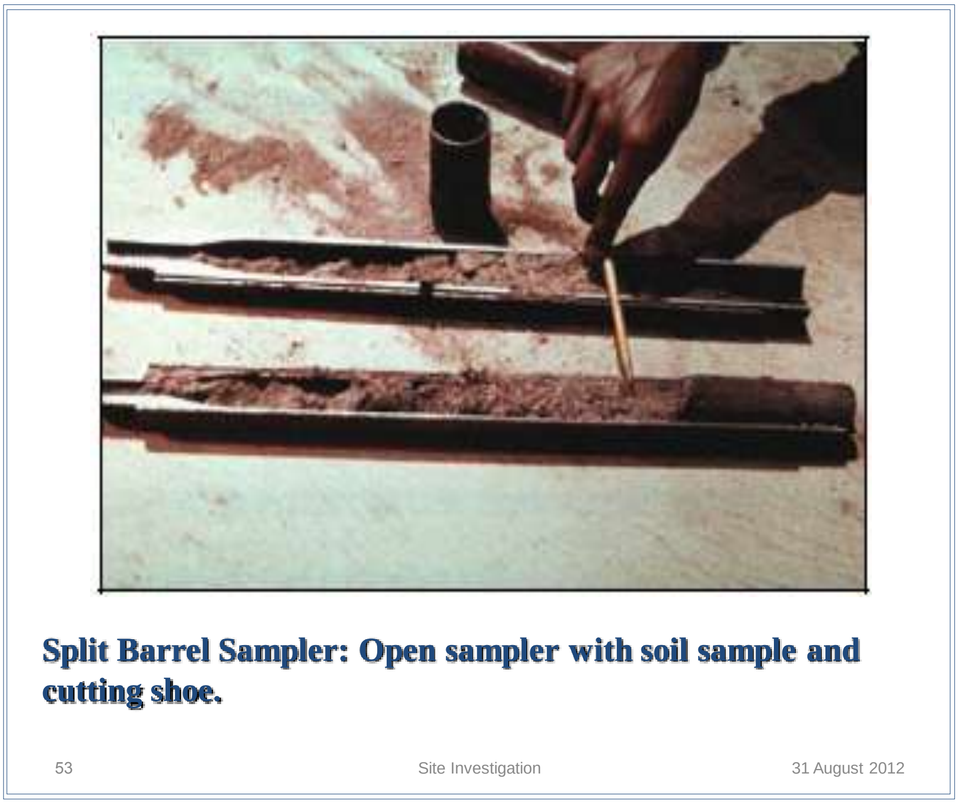
1- Split Spoon:

Undisturbed soil samples are obtained.

The drilling tools are replaced by such sampler when collecting the soil samples.

Sample recovery is difficult in sandy soils under the water table.

Can be used to perform the Standard Penetration Test (SPT).



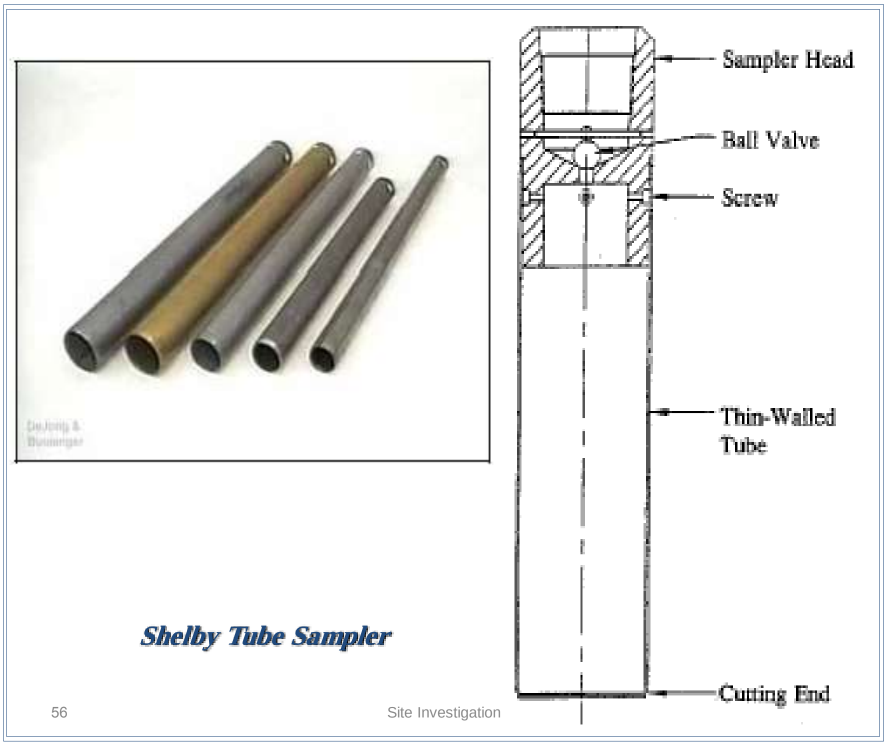
Split Barrel Sampler: Open sampler with soil sample and cutting shoe.

2- Shelby Tube (Thin Walled Tube):

Commonly used to obtain undisturbed clay samples.
The tube is attached to the end of the drilling rod.
The rod and sampler are lowered to the bottom of the hole, and the sampler is pushed into the soil.

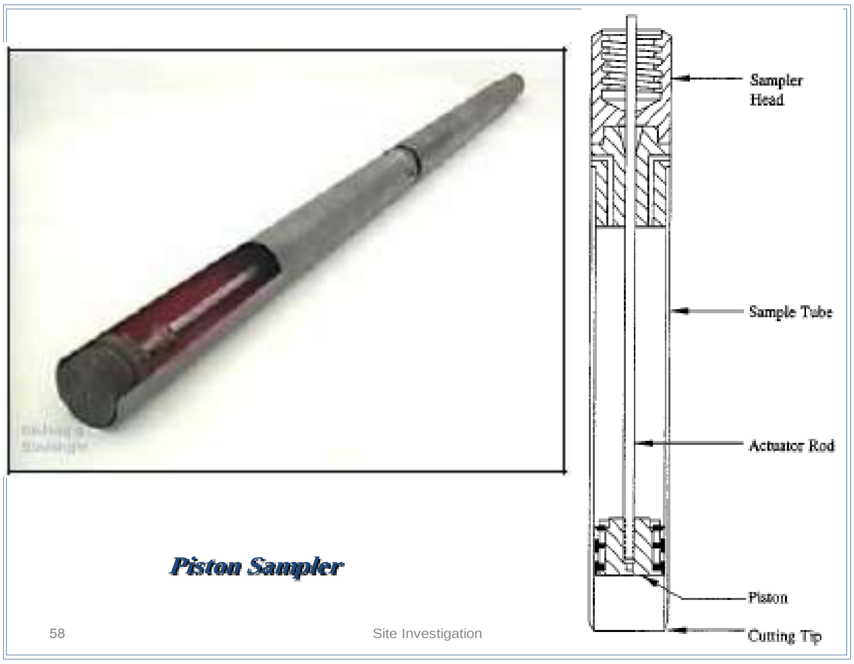
The sample inside the tube is then pulled out, trimmed, covered with hot wax, and sealed for transportation.
Shelby tube samples are used is consolidation, direct shear, and triaxial tests.

The following figure shows schematic representation of the Shelby tube sampler.



3- Piston Sampler (Thin Walled Tube with Piston):

-Used to obtain undisturbed samples with larger diameter
-The obtained samples are less disturbed than those obtained by the Shelby tube.
-Mainly used to prevent the soil from falling from the sampler.
-The following figure shows schematic representation of the piston sampler.



Field Soil Testing

1- Standard Penetration Test (SPT):
Performed with the borehole.
Reliable for cohesionless soils, especially in sand.
Unreliable for cohesive soils.

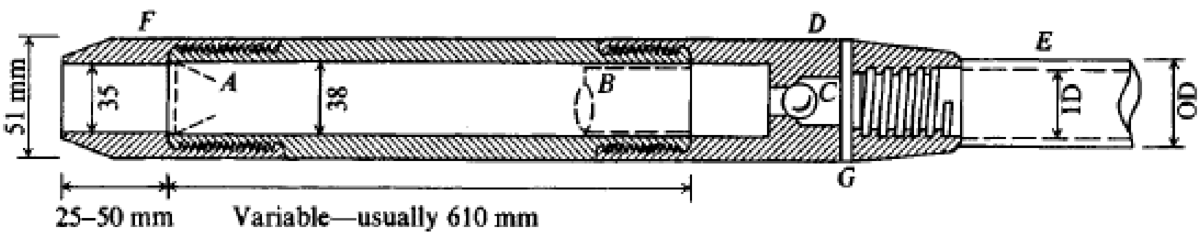
2- Cone Penetration Test (CPT):
Reliable for cohesive soils.
Unreliable for cohesionless soils.

3- Vane Shear Test:
Reliable for cohesive soils.
Unreliable for cohesionless soils.

THE STANDARD PENETRATION TEST (SPT)

The standard penetration test, developed around 1927, is currently the most popular and economical means to obtain subsurface information (both on land and offshore). It is estimated that 85 to 90 percent of conventional foundation design in North and South America is made using the SPT. This test is also widely used in other geographic regions. The method has been standardized as ASTM D 1586 since 1958 with periodic revisions to date. The test consists of the following:

1- Driving the standard split-barrel sampler of dimensions shown in Figure a distance of 460 mm into the soil at the bottom of the boring.



A—insert if used *B*—liner if used
C—ball check valve (provide suction on sample)
D—sampler-to-drill rod coupling
E—drill rod (A or AW)
F—drive shoe *G*—vent holes (used with *C*)

Drill rod sizes:
A: 41 OD × 29 ID mm 5.51 kg/m
AW: 44 OD × 32 ID mm 6.25 kg/m

(a) Standard split barrel sampler (also called a split spoon).
Specific sampler dimensions may vary by ± 0.1 to 1.0 mm.

2- Counting the number of blows to drive the sampler the last two 150 mm distances (total = 300 mm) to obtain the *N* number.

3- Using a 63.5-kg driving mass (or hammer) falling "free" from a height of 760 mm.

The exposed drill rod is referenced with three chalk marks 150 mm apart, and the guide rod (see Fig. 3-7) is marked at 760 mm (for manual hammers). The assemblage is then seated on the soil in the borehole (after cleaning it of loose cuttings). Next the sampler is driven a distance of 150 mm to seat it on undisturbed soil, with this blow count being recorded (unless the system mass sinks the sampler so no Af can be counted). The sum of the blow counts for the next two 150-mm increments is used as the penetration count *N* unless the last increment cannot be completed. In this case the sum of the first two 150-mm penetrations is recorded as *N*.

The boring log shows *refusal* and the test is halted if

1. 50 blows are required for any 150-mm increment.
2. 100 blows are obtained (to drive the required 300 mm).
3. 10 successive blows produce no advance.

From the several recent studies cited (and their reference lists) it has been suggested that the SPT be standardized to some energy ratio E_r which should be computed as:

$$E_r = \frac{\text{Actual hammer energy to sampler, } E_a}{\text{Input energy, } E_{\text{in}}} \times 100 \qquad (d)$$

There are several current suggestions for the value of the **standard energy ratio E_{rb}** as follows:

E_{rb}	Reference
50 to 55 (use 55)	Schmertmann [in Robertson et al. (1983)]
60	Seed et al. (1985); Skempton (1986)
70 to 80 (use 70)	Riggs (1986)

The author will use 70 since the more recent data using current drilling equipment with a safety or an automatic hammer and with driller attention to ASTM D 1586 details indicate this is close to the actual energy ratio E_r obtained in North American practice. If a different standard energy ratio E_{rb} is specified, however, it is a trivial exercise to convert to the different base, as will be shown next.

The standard blow count N'_{70} can be computed from the measured N as follows:

$$N'_{70} = C_N \times N \times \eta_1 \times \eta_2 \times \eta_3 \times \eta_4 \tag{3-3}$$

where η_i = adjustment factors from (and computed as shown) Table 3-3

N'_{70} = adjusted N using the subscript for the E_{rb} and the ′ to indicate it has been adjusted

C_N = adjustment for effective overburden pressure p'_o (kPa) computed [see Liao and Whitman (1986)][5] as

$$C_N = \left(\frac{95.76}{p'_o}\right)^{1/2}$$

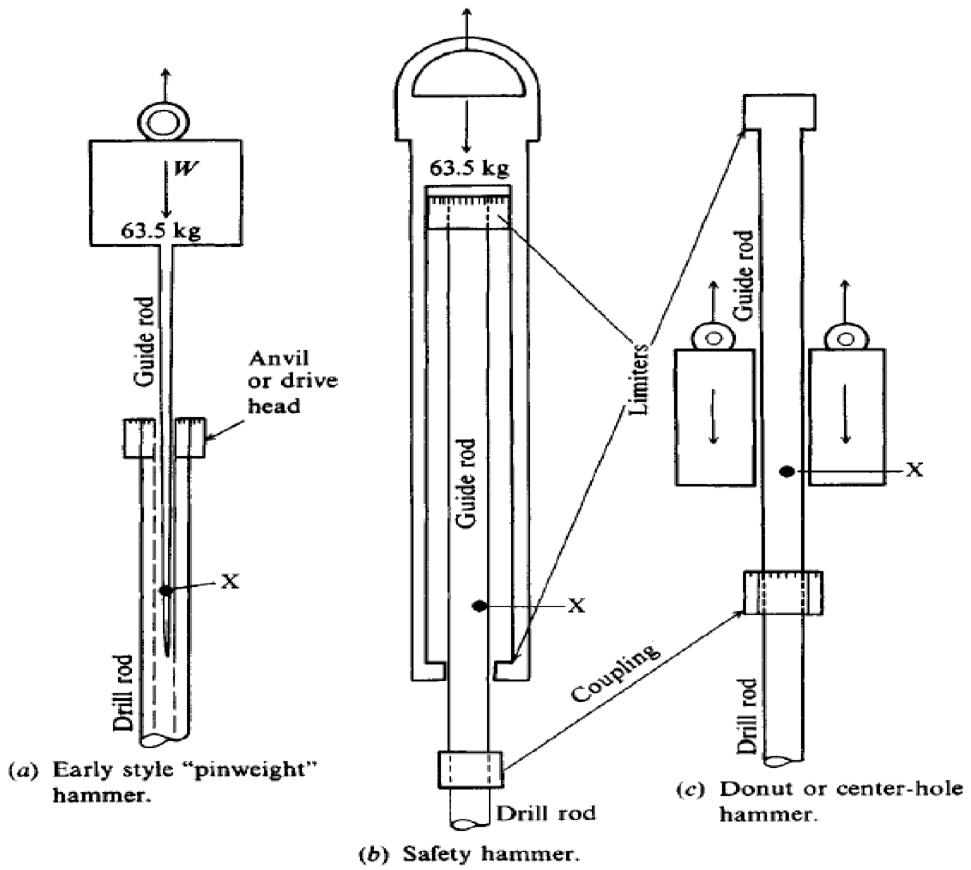


Figure 3-7 Schematic diagrams of the three commonly used hammers. Hammer (*b*) is used about 60 percent; (*a*) and (*c*) about 20 percent each in the United States. Hammer (*c*) is commonly used outside the United States. Note that the user must be careful with (*b*) and (*c*) not to contact the limiter and "pull" the sampler out of the soil. Guide rod *X* is marked with paint or chalk for visible height control when the hammer is lifted by rope off the cathead (power takeoff).

TABLE 3-3

Factors η_i For Eq. (3-3)*

Hammer for η_1					Remarks
	Average energy ratio E_r				
	Donut		Safety		
Country	R-P	Trip	R-P	Trip/Auto	R-P = Rope-pulley or cathead $\eta_1 = E_r/E_{rb} = E_r/70$ For U.S. trip/auto w/$E_r = 80$ $\eta_1 = 80/70 = 1.14$
United States/ North America	45	—	70–80	80–100	
Japan	67	78	—	—	
United Kingdom	—	—	50	60	
China	50	60	—	—	
Rod length correction η_2					
	Length	> 10 m		$\eta_2 = 1.00$	N is too high for $L < 10$ m
		6–10		= 0.95	
		4–6		= 0.85	
		0–4		= 0.75	
Sampler correction η_3					
		Without liner		$\eta_3 = 1.00$	Base value
	With liner:	Dense sand, clay		= 0.80	N is too high with liner
		Loose sand		= 0.90	
Borehole diameter correction η_4					
	Hole diameter:†	60–120 mm		$\eta_4 = 1.00$	Base value; N is too small when there is an oversize hole
		150 mm		= 1.05	
		200 mm		= 1.15	

* Data synthesized from Riggs (1986), Skempton (1986), Schmertmann (1978*a*) and Seed et al. (1985).

† $\eta_4 = 1.00$ for all diameter hollow-stem augers where SPT is taken through the stem.

Note that larger values of E_r *decrease* the blow count N nearly linearly, that is, E_{r45} gives $N = 20$ and E_{r90} gives $N = 10$; however, using the "standard" value of E_{r70} gives an N value for use in Eq. (3-3) of $N = 13$ for either drilling rig. We obtain this by noting that the energy ratio × blow count should be a constant for any soil, so

$$E_{r1} \times N_1 = E_{r2} \times N_2 \qquad (e)$$

or

$$N_2 = \frac{E_{r1}}{E_{r2}} \times N_1 \qquad (3\text{-}4)$$

For the arbitrarily chosen $E_{r1} = 70$, this gives, in general,

$$N_2 = \frac{70}{E_{r2}} \times N_1$$

For the previous example of N_2 for $E_{r45} = 20 = E_{r2}$ we obtain

$$20 = \frac{70}{45} \times N_1 \qquad \text{giving} \qquad N_1 = \frac{45}{70}(20) = 13 \quad \text{(use integers)}$$

If we convert N_{70} to N_{60} we have

$$N_2 = N_{60} = \frac{70}{60}(13) = 15 \quad \text{[which is larger as predicted by Eq. } (e)\text{]}$$

Using the relationship given by Eq. (e) we can readily convert any energy ratio to any other base, but we do have to know the energy ratio at which the blow count was initially obtained.

Example 3-2.

Given. $N = 20$; rod length $= 12$ m; hole diam. $= 150$ mm; $p'_o = 205$ kPa; use safety hammer with $E_r = 80$; dense sand; no liner

Required. What are the "standard" N'_i and N'_{60} based on the following?

$$E_{rb} = 70 \quad \text{and} \quad E_{rb} = 60 \qquad C_N = \left(\frac{95.76}{205}\right)^{1/2} = 0.68$$

$$\eta_i = 1.14 \quad \text{See sample computation shown in Table 3-3}$$

$$\eta_2 = 1.00 \quad L > 10 \text{ m}$$
$$\eta_3 = 1.00 \quad \text{usual United States practice of no liner}$$
$$\eta_4 = 1.05 \quad \text{slight oversize hole}$$

Use Eq. (3-3) and direct substitution in order:

$$N'_{70} = 0.68 \times 20 \times 1.14 \times 1 \times 1 \times 1.05$$
$$= \mathbf{16} \text{ (only use integers)}$$

for $E_{rb} = E_{r2}$ use Eq. (3-4), giving

$$N_2 = N'_{60} = \frac{70}{60} \times 16 = \mathbf{19}$$

Example 3-3. Same as Example 3-2 but with sample liner and $E_r = 60$.

$$C_N = 0.68 \text{ as before}$$

$$\eta_1 = \frac{60}{70} = 0.86 \qquad \eta_2 = 1$$

$$\eta_3 = 0.80 \quad \text{(dense sand given with liner)} \qquad \eta_4 = 1.05$$
$$N'_{60} = 0.68 \times 20 \times 0.86 \times 0.80 \times 1.05 = \mathbf{10}$$

$$N_2 = N'_{70} = \frac{60}{70} \times 10 = \mathbf{9} \text{ using [Eq. (3-4)]}$$

Example 3-4. Same as Example 3-2 but $E_r = 55$; $p'_c = 100$ kPa; 205 mm hollow stem auger, hole depth = 6 m.

$$C_N = \left(\frac{95.76}{100}\right)^{1/2} = 0.98 \quad (\text{using } p'_o = p'_c)$$

$$\eta_1 = 55/70 = 0.79 \qquad \eta_2 = 0.95 \quad (\text{since } 6 < 10 \text{ m})$$

$$\eta_3 = 1.0 \quad (\text{no liner}) \qquad \eta_4 = 1.0 \quad (\text{using hollow-stem auger})$$

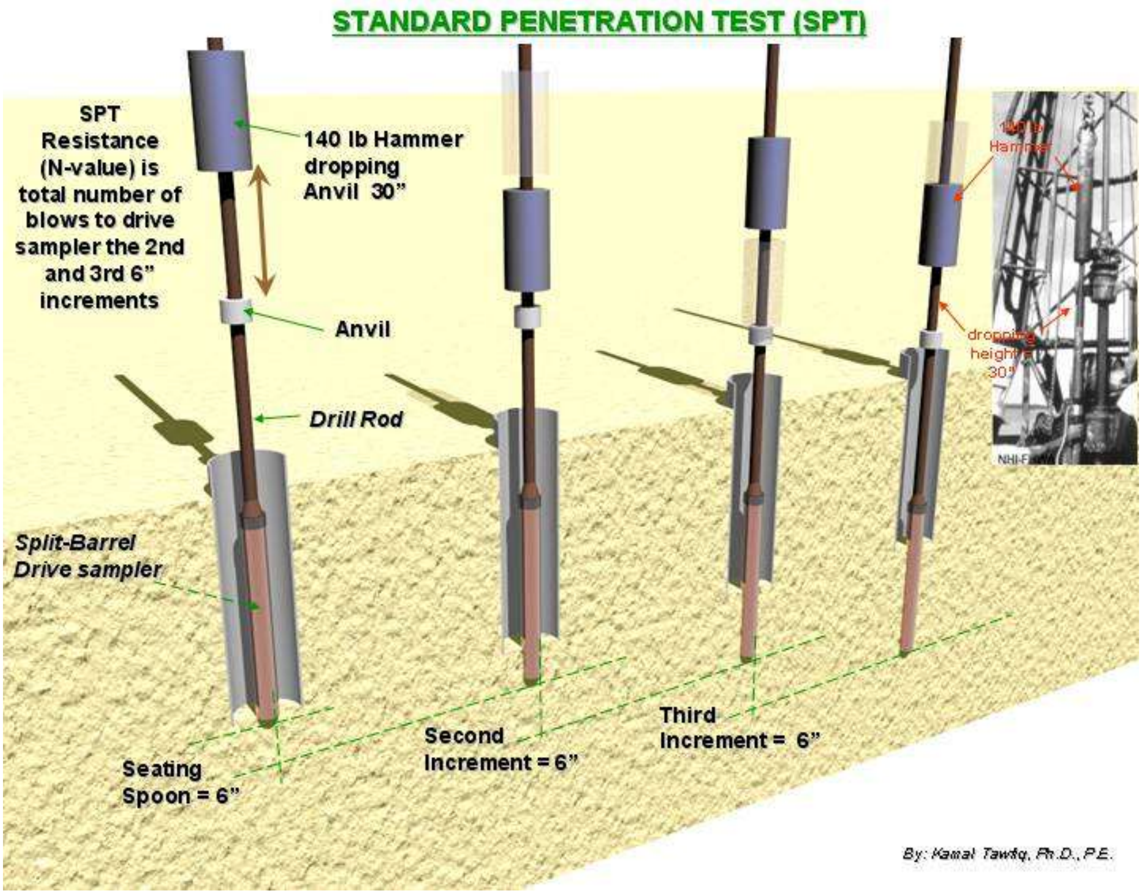
$$N'_{70} = 0.98 \times 20 \times 0.79 \times 0.95 \times 1.0 \times 1.0 = \mathbf{15}$$

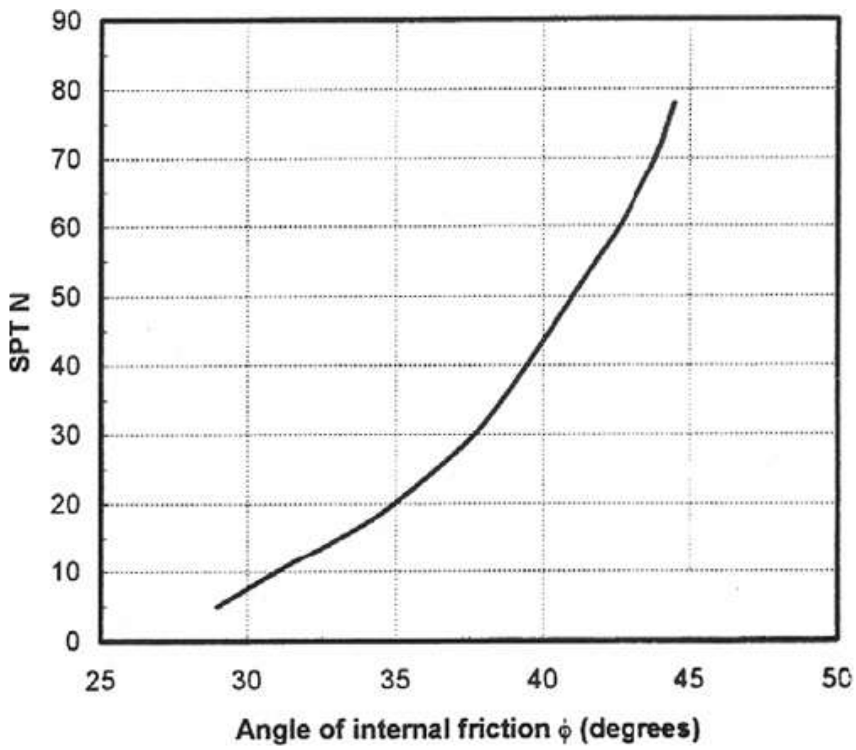
$$N_2 = N'_{60} = \frac{70}{60} \times 15 = \mathbf{17}$$

....

Empirical values for ϕ, D_r, and unit weight of granular soils based on the SPT at about 6 m depth and normally consolidated [approximately, $\phi = 28° + 15° D_r$ ($\pm 2°$)]

Description		Very loose	Loose	Medium	Dense	Very dense
Relative density D_r		0	0.15	0.35	0.65	0.85
SPT N'_{70}:	fine	1–2	3–6	7–15	16–30	?
	medium	2–3	4–7	8–20	21–40	> 40
	coarse	3–6	5–9	10–25	26–45	> 45
ϕ:	fine	26–28	28–30	30–34	33–38	
	medium	27–28	30–32	32–36	36–42	< 50
	coarse	28–30	30–34	33–40	40–50	
γ_{wet}, kN/m^3		11–16*	14–18	17–20	17–22	20–23


STANDARD PENETRATION TEST (SPT)
SPT Resistance (N-value) is total number of blows to drive sampler the 2nd and 3rd 6" increments
140 lb Hammer dropping Anvil 30"
Anvil
Drill Rod
Split-Barrel Drive sampler
Seating Spoon = 6"
Second Increment = 6"
Third Increment = 6"
By: Kamal Tawfiq, Ph.D., P.E.


SPT N
Angle of internal friction φ (degrees)
0
10
20
30
40
50
60
70
80
90
25
30
35
40
45
50

Depth, Number and Distribution of Boreholes

I- Depth of Boreholes:

1- Depth of boring ≈ 3 – 5, width of isolated footing.

2- Depth of boring ≈ 2 – 3, width of raft.

3- The boring should penetrate the sand layer (if exists) sufficiently to determine its continuity, (especially in pile foundations).

4- For deep excavation, depth of boring ≈ 1.5 excavation depth.

5- If rock is encountered, it should be penetrated 4 m, at least.

II- Distribution of Borings:

Structure	Spacing
Multistory Building	300 m^2, with 2 min. for each
One story industrial Building	300 – 500 m^2
Highways	250 – 500 linear meter
Residential Sub-Divisions	200 × 200 up to 400 × 400 m^2
Dams	50 up to 200 m for dam length

THE SOIL REPORT

When the borings or other field work has been done and any laboratory testing completed, the geotechnical engineer then assembles the data for a recommendation to the client. Computer analyses may be made where a parametric study of the engineering properties of the soil is necessary to make a "best" value(s) recommendation of the following:

1. Soil strength parameters of angle of internal friction Ø and cohesion *c*

2. Allowable bearing capacity (considering both strength and probable or tolerable settlements)

3. Engineering parameters such as E_s, μ, G, or k_s.

A plan and profile of the borings may be made as on Fig. 3-37, or the boring information may be compiled from the field and laboratory data sheets as shown on Fig. 3-38. Field and data summary sheets are far from standardized between different organizations, and further, the ASTM D 653 (Standard Terms and Symbols Relating to Soil and Rock) is seldom well followed.

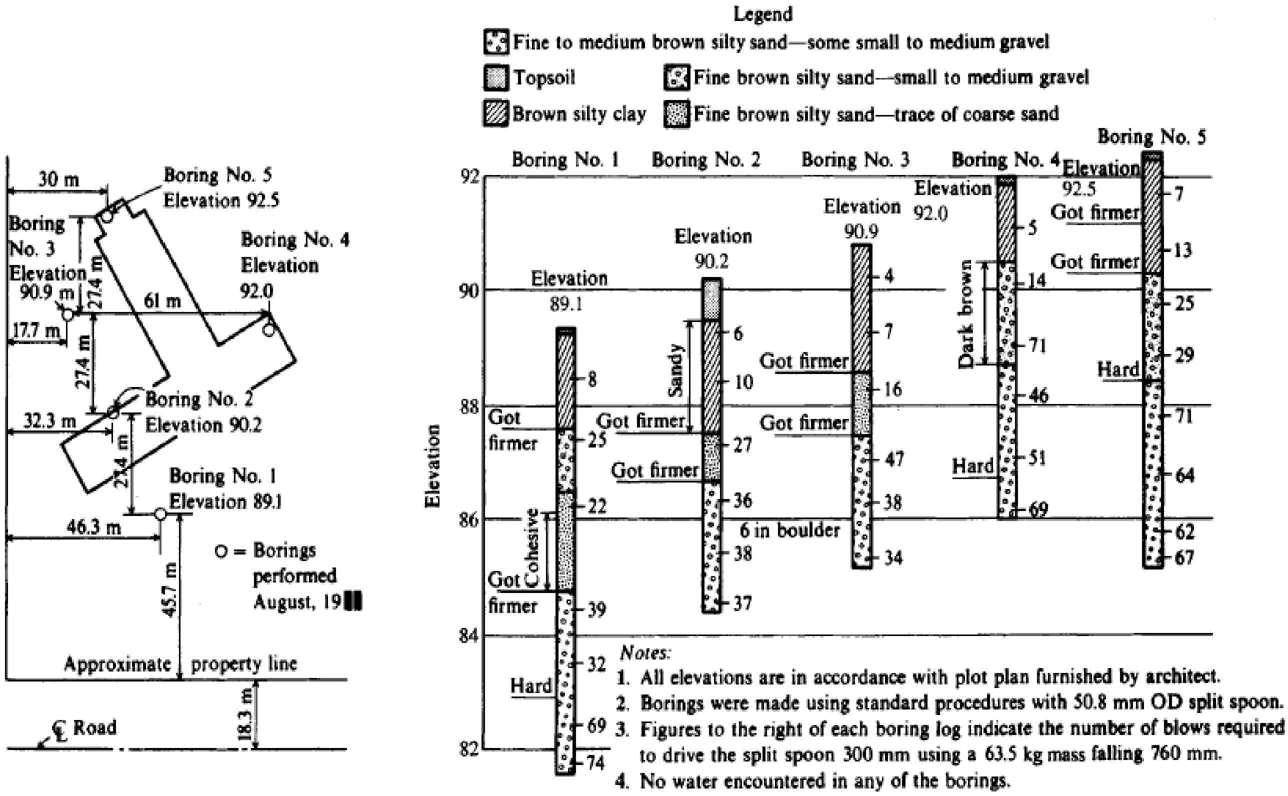


Figure 3-37 A method of presenting the boring information on a project. All dimensions are in meters unless shown otherwise.

BORING NO. B-04
DATE 12-03-92
W. & A. FILE NO. 55
SHEET 4 OF 7

WHITNEY & ASSOCIATES
INCORPORATED
2406 West Nebraska Avenue
PEORIA, ILLINOIS 61604

BORING LOG

PROJECT OHIO-AMERICAN ELEVATED WATER STORAGE TANK
LOCATION Ohio
BORING LOCATION See Plot Plan Sheet
DRILLED BY Winslow
BORING TYPE Hollow-Stem Auger
WEATHER CONDITIONS Partly Cloudy & Cool
SOIL CLASSIFICATION SYSTEM U.S.B.S.C.
SEEPAGE WATER ENCOUNTERED AT ELEVATION None
GROUND SURFACE ELEVATION 804.2
GROUND WATER ELEVATION AT 24+ HRS. 793.6
BORING DISCONTINUED AT ELEVATION 787.2
GROUND WATER ELEVATION AT COMPLETION 793.4

DESCRIPTION	DEPTH IN FEET	SAMPLE TYPE	N	Q_p	Q_u	D_d	M_c
Brown SILTY CLAY LOAM Organic Topsoil	6"						
Hard, Brown, Weathered GLACIAL SILTY CLAY TILL	03	SS	5 10 15(25)	4.5+	7.0	121	15
	06	SS	8 12 18(30)	4.5+	6.0	118	14
	09	SS	9 14 19(33)	4.5+	5.1	119	15
		SS	8 13 18(31)	4.5+	6.2	124	13
Hard, Gray, Unweathered GLACIAL SILTY CLAY TILL	12	SS	5 7 11(18)	4.5+	5.1	113	18
Very Stiff, Gray, Unweathered GLACIAL SILTY CLAY TILL	15	SS	5 5 8(13)	2.3	2.2	109	20
Hard, Gray LIMESTONE EXPLORATORY BORING DISCONTINUED	18						

N - BLOWS DELIVERED PER FOOT BY A 140 LB. HAMMER FALLING 30 INCHES
SS - SPLIT SPOON SAMPLE
ST - SHELBY TUBE SAMPLE

Q_p - CALIBRATED PENETROMETER READING - T.S.F.
Q_u - UNCONFINED COMPRESSIVE STRENGTH - T.S.F.
D_d - NATURAL DRY DENSITY - P.C.F.
M_c - NATURAL MOISTURE CONTENT - %

WHITNEY & ASSOCIATES
PEORIA, ILLINOIS

Figure 3-38 Boring log as furnished to client. N = SPT value; Q_p = pocket penetrometer; Q_u = unconfined compression test; D_d = estimated unit weight γ_s; M_c = natural water content w_N in percent.

IMMEDIATE SETTLEMENT

Foundation settlements must be estimated with great care for buildings, bridges, towers, power plants, and similar high-cost structures. For structures such as fills, earth dams, levees, braced sheeting, and retaining walls a greater margin of error in the settlements can usually be tolerated.

Except for occasional happy coincidences, soil settlement computations are only best estimates of the deformation to expect when a load is applied. During settlement the soil transitions from the current body (or self-weight) stress state to a new one under the additional applied load. The stress change Δq from this added load produces a time-dependent accumulation of particle rolling, sliding, crushing, and elastic distortions in a limited influence zone beneath the loaded area. *The statistical accumulation of movements in the direction of interest is the settlement.* In the vertical direction the settlement will be defined as ΔH.

Settlements are usually classified as follows:

1. Immediate, or those that take place as the load is applied or within a time period of about 7 days.

2. Consolidation, or those that are time-dependent and take months to years to develop. The Leaning Tower of Pisa in Italy has been undergoing consolidation settlement for over 700 years. The lean is caused by the consolidation settlement being greater on one side. This, however, is an extreme case with the principal settlements for most projects occurring in 3 to 10 years.

Immediate settlement analyses are used for all fine-grained soils including silts and clays with a degree of saturation $S \leq 90$ percent and for all coarse-grained soils with a large coefficient of permeability [say, above 10^{-3} m/s (see Table 2-3)].

TABLE 2-3
Order-of-magnitude values for permeability *k*, based on description of soil and by the Unified Soil Classification System, m/s

10^{0}		10^{-2}	10^{-5}	10^{-9}	10^{-11}
	Clean gravel GW, GP	Clean gravel and sand mixtures GW, GP SW, SP GM	Sand-silt mixtures SM, SL, SC	Clays	

Consolidation settlement analyses are used for all saturated, or nearly saturated, fine grained soils where the consolidation theory of Sec. 2-10 applies. For these soils we want estimates of both settlements ΔH and how long a time it will take for most of the settlement to occur.

IMMEDIATE SETTLEMENT COMPUTATIONS

The settlement of the corner of a rectangular base of dimensions $B' \times L'$ on the surface of an elastic half-space can be computed from an equation from the Theory of Elasticity [e.g., **Timoshenko and Goodier** (1951)] as follows:

$$\Delta H = q_o B' \frac{1-\mu^2}{E_s}\left(I_1 + \frac{1-2\mu}{1-\mu} I_2\right) I_F \qquad (5\text{-}16)$$

Where:

q_o = intensity of contact pressure in units of E_s
B' = least lateral dimension of contributing base area in units of ΔH
I_i = influence factors, which depend on L'/B' thickness of stratum H, Poisson's ratio μ, and base embedment depth D
E_s , μ = elastic soil parameters — (see Tables 2-7, 2-8, and 5-6)

The influence factors (see Fig. 5-7 for identification of terms) I_1 and I_2 can be computed using equations given by Steinbrenner (1934) as follows:

$$I_1 = \frac{1}{\pi}\left[M \ln \frac{(1+\sqrt{M^2+1})\sqrt{M^2+N^2}}{M(1+\sqrt{M^2+N^2+1})} + \ln \frac{(M+\sqrt{M^2+1})\sqrt{1+N^2}}{M+\sqrt{M^2+N^2+1}}\right] \qquad (a)$$

$$I_2 = \frac{N}{2\pi}\tan^{-1}\left(\frac{M}{N\sqrt{M^2+N^2+1}}\right) \qquad (\tan^{-1} \text{ in radians}) \qquad (b)$$

where $M = \dfrac{L'}{B'}$ $\qquad N = \dfrac{H}{B'}$

$$B' = \frac{B}{2} \text{ for center; } = B \text{ for corner } I_i$$

$$L' = L/2 \text{ for center; } = L \text{ for corner } I_i$$

The influence factor I_F is from the Fox (*1948b*) equations, which suggest that the settlement is reduced when it is placed at some depth in the ground, depending on Poisson's ratio and *L/B*. Figure 5-7 can be used to approximate I_F.

To compute the composite Steinbrenner influence factor I_s as

$$I_s = I_1 + \frac{1 - 2\mu}{1 - \mu} I_2 \qquad (c)$$

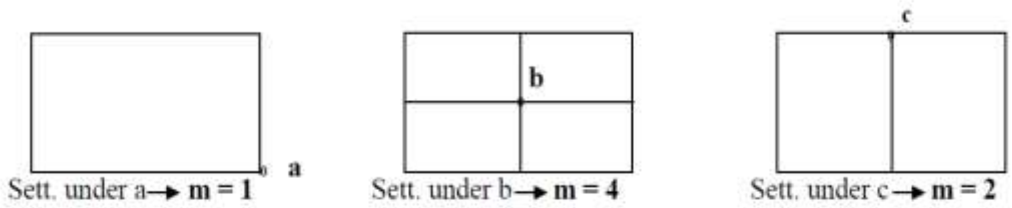
Equation (5-16) can be written more compactly as follows:

$$\Delta H = q_o B' \frac{1 - \mu^2}{E_s} m I_s I_F \qquad (5\text{-}16a)$$

This equation is strictly applicable to *flexible bases* on the half-space . In practice, most foundations are flexible. Even very thick ones deflect when loaded by the superstructure loads. Some theory indicates that if the base is rigid the settlement will be uniform (but may tilt), and the settlement factor *Is* will be about 7 percent less than computed by Eq. (c). On this basis if your base is "rigid" you should reduce the I_s factor by about 7 percent (that is, $I_{sr} = 0.93\, I_s$).

Equation (5-16a) is very widely used to compute immediate settlements. These estimates, however, have not agreed well with measured settlements. After analyzing a number of cases, the author concluded that the equation is adequate but the method of using it was incorrect. The equation should be used [see Bowles (1987)] as follows:

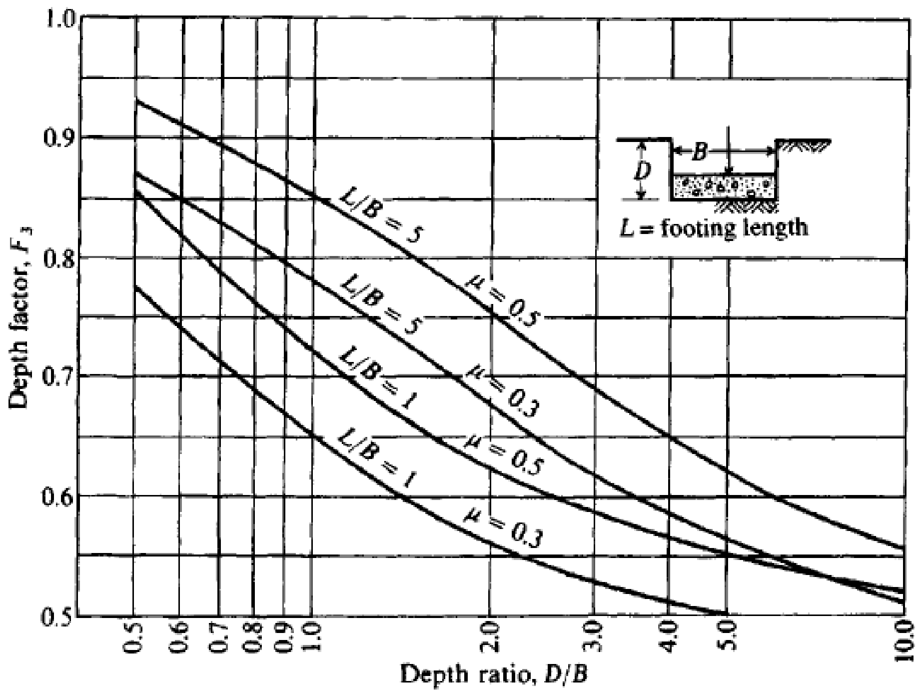
1. Make your best estimate of base contact pressure q_o.
2. For round bases, convert to an equivalent square.
3. Determine the point where the settlement is to be computed and divide the base (as in the Newmark stress method) so the point is at the corner or common corner of one or up to 4 contributing rectangles.



4. Note that the stratum depth actually causing settlement is not at $H/B \to \infty$, but is at either of the following:
a. Depth $z = 5B$ where B = least total lateral dimension of base.
b. Depth to where a hard stratum is encountered. Take "hard" as that where E_s in the hard layer is about $10E_s$ of the adjacent upper layer.
5. Compute the H/B' ratio. For a depth $H = z = 5B$ and for the center of the base we have $H/B' = 5B/0.5B = 10$. For a corner, using the same H, obtain $5B/B = 5$. This computation sets the depth $H = z$ = depth to use for all of the contributing rectangles. Do not use, say, $H = 5B = 15$ m for one rectangle and $H = 5B = 10$ m for two other contributing rectangles—use 15 m in this case for all.
6. Enter Table 5-2, obtain I_1 and I_2, with your best estimate for μ compute I_s, and obtain I_F from Fig. 5-7.
7. Obtain the weighted average E_s in the depth $z = H$. The weighted average can be computed
(where, for n layers, $H = \sum_i^n H_i$) as

$$E_{s,\mathrm{av}} = \frac{H_1E_{s1} + H_2E_{s2} + \cdots + H_nE_{sn}}{H} \qquad (d)$$

Figure 5-7 Influence factor I_F for footing at a depth D. Use actual footing width and depth dimension for this D/B ratio. Use program FFACTOR for values to avoid interpolation.



Immediate Settlement by Skempton's method :-

$$S_i = q \frac{B(1-\mu^2)}{E_s} I_w \quad \text{-------------- (3)}$$

The above equation used to calculate the S_i for foundation rest on the ***elastic homogenous soil***, the I_w is the influence factor taken from Table (5-4).

Table 5-4: Influence factor I_w

Shape		I_w		
		Flexible		Rigid
		Center	Corner	
Circle	-	1.00	0.64	0.79
Square	-	1.12	0.56	0.88
Rectangle	L/B 1.5	1.36	0.68	1.07
	2	1.53	0.77	1.21
	3	1.78	0.89	1.42
	5	2.10	1.05	1.70
	10	2.54	1.27	2.10
	20	2.99	1.49	2.46
	50	3.57	1.80	3.00
	100	4.01	2.00	3.43

TABLE 2-7

Values or value ranges for Poisson's ratio μ

Type of soil	μ
Clay, saturated	0.4–0.5
Clay, unsaturated	0.1–0.3
Sandy clay	0.2–0.3
Silt	0.3–0.35
Sand, gravelly sand	−0.1–1.00
commonly used	0.3–0.4
Rock	0.1–0.4 (depends somewhat on type of rock)
Loess	0.1–0.3
Ice	0.36
Concrete	0.15
Steel	0.33

It is very common to use the following values for soils:

μ	Soil type
0.4–0.5	Most clay soils
0.45–0.50	Saturated clay soils
0.3–0.4	Cohesionless—medium and dense
0.2–0.35	Cohesionless—loose to medium

TABLE 2-8

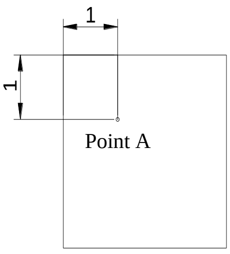
Value range* for the static stress-strain modulus E_s for selected soils (see also Table 5-6)

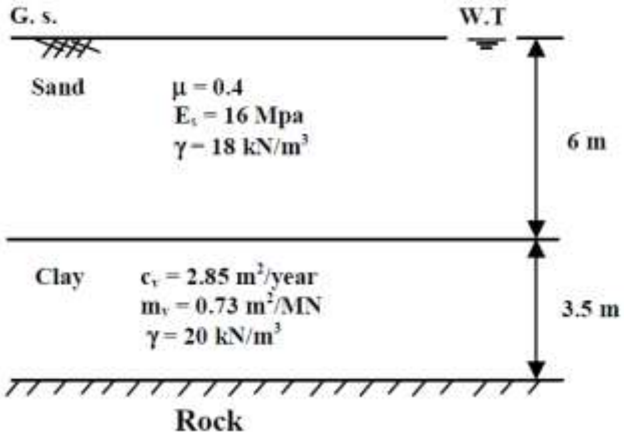
Field values depend on stress history, water content, density, and age of deposit

Soil	E_s, MPa
Clay	
Very soft	2–15
Soft	5–25
Medium	15–50
Hard	50–100
Sandy	25–250
Glacial till	
Loose	10–150
Dense	150–720
Very dense	500–1440
Loess	15–60
Sand	
Silty	5–20
Loose	10–25
Dense	50–81
Sand and gravel	
Loose	50–150
Dense	100–200
Shale	150–5000
Silt	2–20

*Value range is too large to use an "average" value for design.

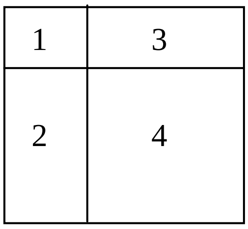
Ex. 1) A TV tower weighting (1000 kN) is constructed on a (3m * 3m) footing on ground surface on the site shown in Fig.
Calculate :
Immediate settlement at a point A of the footing by:
1-Skempton's method. (flexible)
2-Timoshenko and Goodier mehod. (rigid)





Es=16 N/mm^2 / 1000 N/kN *1000000 mm^2/m^2 = 16000 kN/m^2

q = 1000/(3×3)=111.11 kN/m^2



1- $S_i = q \dfrac{B(1-\mu^2)}{E_s} I_w$

Shape	B×L	L/B	I_w (flexible corner)	S_i (mm)	S_i (mm) (total)
1	1×1	1	0.56	3.26	18.7
2	1×2	2	0.77	4.49	
3	1×2	2	0.77	4.49	
4	2×2	1	0.56	6.53	

2- $S_i = qB' \dfrac{1-\mu^2}{E_s}\left(I_1 + \dfrac{1-2\mu}{1-\mu} I_2\right)$ H = 6 m

Shape	B×L	H/B N	L/B M	I_1	I_2	S_i (mm)	S_i (mm) (total)
1	1×1	6	1	0.457	0.026	2.71	13.9
2	1×2	6	2	0.563	0.050	3.38	
3	1×2	6	2	0.563	0.050	3.38	
4	2×2	3	1	0.363	0.048	4.42	

S_i rigid = 0.93 × 13.9 = 12.9 mm

Ex. 2) For the footing shown in Figure (1), estimate the immediate settlement at a point (A) by Skempton's method, assume rigid footing. E_s=18 Mpa, μ=0.35, q=120 kN/m^2.

Solution:

$$S_i = q \frac{B(1-\mu^2)}{E_s} I_w$$

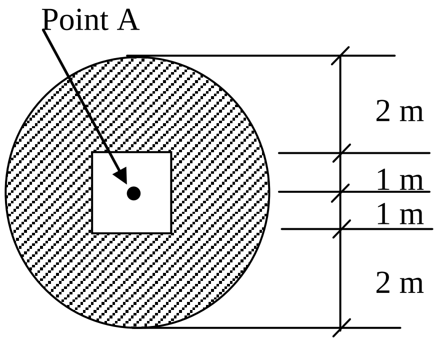


Figure (1)

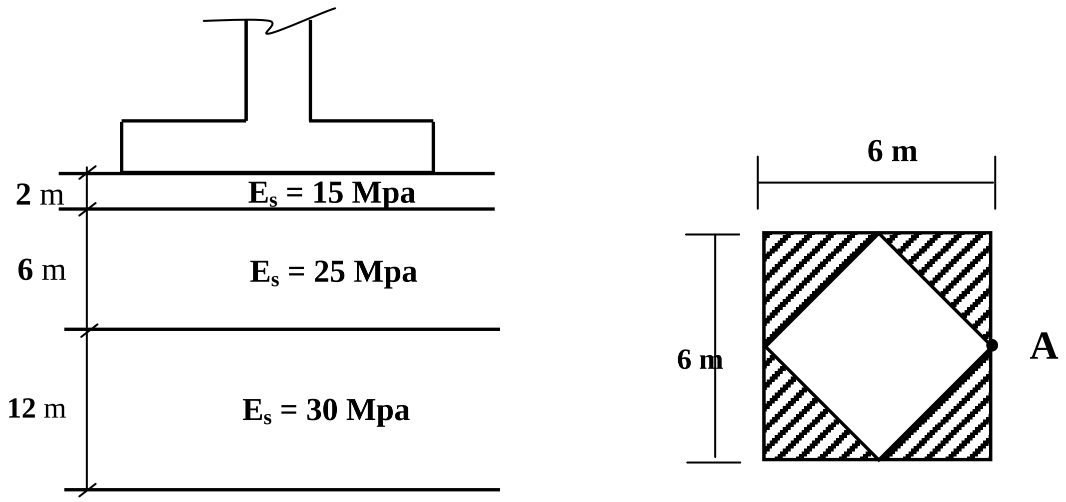
E_s=18 N/mm^2 / 1000 N/kN *1000000 mm^2/m^2 = 18000 kN/m^2

q= 120 kN/m^2

B for circle = D = 6 m
B for square = 2 m

Shape	I_w (rigid)	S_i (mm)	S_i (mm) (total)
circle	0.79	27.73	*27.73 – 10.296 = <u>17.43</u> mm*
square	0.88	10.296	

Ex. 3) Find the immediate settlement of point (**A**) for the flexible footing shown in **Fig.(2)**, q_{net} = 220 kN/m^2 , μ = 0.35. Use Timoshenko and Goodier mehod.



Solution:

$$\Delta H = q_o B' \frac{1-\mu^2}{E_s} m I_s I_F$$

$E_{s\ Average}$ = (2*15000 + 6*25000 + 12*30000)/20 = 27000 kPa.

S_i = S1 + S2 – S3 H = 5B = 5 * 6 = 30 m (include all layers)
Use H = 2 + 6 + 12 = 20 m

For shape S1 and S2
M = L'/B' = 6/3 = 2
N = H/B' = 20/3 = 6.67

I_1 = 0.58 , I_2 = 0.04522

$$I_s = I_1 + \frac{1-2\mu}{1-\mu} I_2 \quad = 0.6$$

$$\Delta H = q_o B' \frac{1-\mu^2}{E_s} m I_s I_F \quad = 220 * 3 * (1-0.35^2)*2*0.6* 1/27000 = 0.02574 \text{ m}$$

= 25.74 mm

For shape S3 M = 1 N = 20/ 4.24 = 4.7 I_1 = 0.429 I_2 = 0.0352 I_s = 0.445

$$\Delta H = q_o B' \frac{1-\mu^2}{E_s} m I_s I_F \quad = (220 * 4.24 *(1-0.35^2)*1*0.445 * 1) / 27000 = 0.01349 \text{ m}$$

= 13.49 mm

$S_{i\ total}$ = 25.74 – 13.49 = 12.25 mm

BEARING CAPACITY

Introduction:

The soil must be capable of carrying the loads from any engineered structure placed upon it without a shear failure and with the resulting settlements being tolerable for that structure.

The recommendation for the allowable bearing capacity q_a to be used for design is based on the *minimum* of either

1. Limiting the settlement to a tolerable amount (see Chap. 5)

2, The ultimate bearing capacity, which considers soil strength, as computed in the following sections The allowable bearing capacity based on shear control q_a is obtained by reducing (or dividing) the ultimate bearing capacity q_{ult} (based on soil strength) by a safety factor SF that is deemed adequate to avoid a base shear failure to obtain

$$q_a = \frac{q_{\text{ult}}}{\text{SF}} \qquad (4\text{-}1)$$

The safety factor is based on the type of soil (cohesive or cohesionless), reliability of the soil parameters, structural information (importance, use, etc.), and consultant caution.

Bearing Capacity

From Fig. 4-1*a* and Fig. 4-2 it is evident we have two potential failure modes, where the footing, when loaded to produce the maximum bearing pressure q_{ult}, will do one or both of the following:

a. Rotate as in Fig. 4-1*a* about some center of rotation (probably along the vertical line *Oa)* with shear resistance developed along the perimeter of the slip zone shown as a circle.

b. Punch into the ground as the wedge *agb* of Fig. 4-2 or the approximate wedge *ObO'* of Fig. *4-la*.

Bearing Capacity Equations

1- The Terzaghi Bearing-Capacity Equation:-

One of the early sets of bearing-capacity equations was proposed by Terzaghi (1943) as shown in Table 4-1. Terzaghi used shape factors noted when the limitations of the equation were discussed.

Terzaghi's bearing-capacity equations were intended for "shallow" foundations where **D ≤ B.**

2- Meyerhof 's Bearing-Capacity Equation

Meyerhof (1951, 1963) proposed a bearing-capacity equation similar to that of Terzaghi but included a shape factor s_q with the depth term N_q. He also included depth factors d_i and inclination factors i_i [both noted in discussion of Eq. (j)] for cases where the footing load is inclined from the vertical. These additions produce equations of the general form shown in

Table 4-1, with select N factors computed in Table 4-4. Program BEARING is provided on disk for other N_i values.

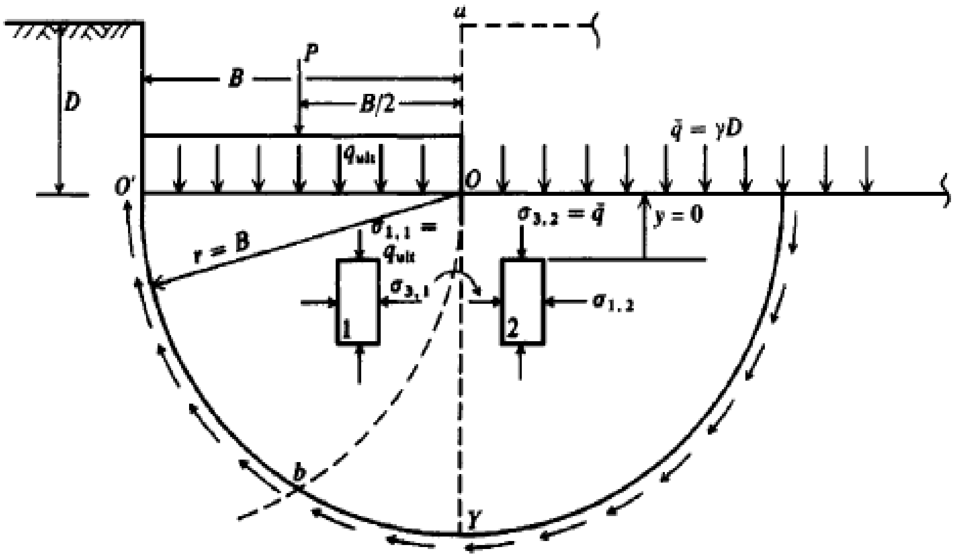
3- Hansen's Bearing-Capacity Method

Hansen (1970) proposed the general bearing-capacity case and N factor equations shown in Table 4-1. This equation is readily seen to be a further extension of the earlier Meyerhof (1951) work. Hansen's shape, depth, and other factors making up the general bearing capacity equation are given in Table 4-5. These represent revisions and extensions from earlier

proposals in 1957 and 1961. The extensions include base factors for situations in which the footing is tilted from the horizontal b_i and for the possibility of a slope β of the ground supporting the footing to give ground factors g_i. Table 4-4 gives selected N values for the Hansen equations together with computation aids for the more difficult shape and depth factor terms. Use program BEARING for intermediate N_i factors, because interpolation is not recommended, especially for $\phi \geq 35°$.

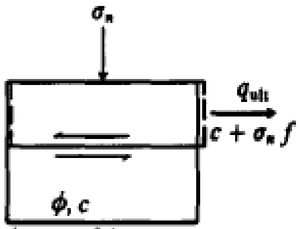
4- Vesic's Bearing-Capacity Equations

The Vesic (1973, *1915b)* procedure is essentially the same as the method of Hansen (1961) with select changes. The N_c and N_q terms are those of Hansen but N_γ is slightly different (see Table 4-4). There are also differences in the i_i, b_i, and g_i terms as in Table 4-5c. The Vesic equation is somewhat easier to use than Hansen's because Hansen uses the *i* terms in computing shape factors s_i whereas Vesic does not (refer to Examples 4-6 and 4-7 following).



(a) Footing on $\phi = 0°$ soil.

Note: $\bar{q} = p'_o = \gamma' D$, but use $\bar{q}$, since this is the accepted symbol for bearing capacity computations.

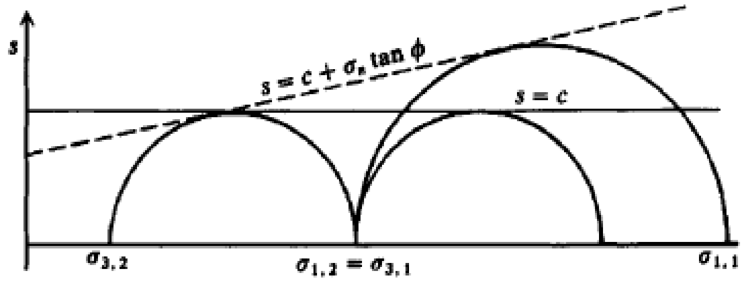


Area = dA

Friction = $\sigma_n \tan\phi = \sigma_n f$

Cohesion = $\frac{c\,dA}{dA}$

(b) Physical meaning of Eq. (2-52) for shear strength.



(c) Mohr's circle for (a) and for a ϕ–c soil.

Figure 4-1 Bearing capacity approximation on a $\phi = 0$ soil.

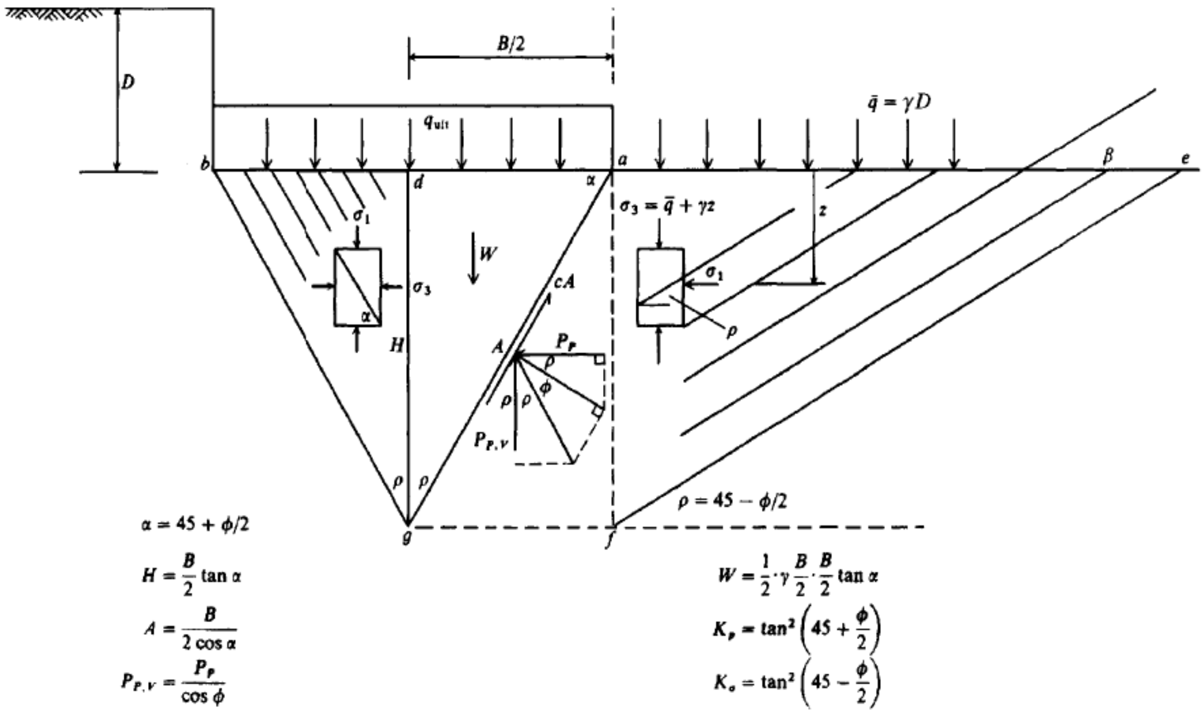


Figure 4-2 Simplified bearing capacity for a ϕ–c soil.

TABLE 4-1

Bearing-capacity equations by the several authors indicated

Terzaghi (1943). See Table 4-2 for typical values and for $K_{p\gamma}$ values.

$$q_{ult} = cN_c s_c + \bar{q}N_q + 0.5\gamma B N_\gamma s_\gamma \qquad N_q = \frac{a^2}{a\cos^2(45+\phi/2)}$$

$$a = e^{(0.75\pi - \phi/2)\tan\phi}$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = \frac{\tan\phi}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi} - 1\right)$$

For:	strip	round	square
s_c =	1.0	1.3	1.3
s_γ =	1.0	0.6	0.8

Meyerhof (1963).* See Table 4-3 for shape, depth, and inclination factors.

$$q_{ult} = cN_c s_c d_c i_c + \bar{q}N_q s_q d_q i_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma i_\gamma \qquad (s_c = 1 \text{ for inclined load})$$

$$N_q = e^{\pi\tan\phi}\tan^2\left(45 + \frac{\phi}{2}\right)$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = (N_q - 1)\tan(1.4\phi)$$

Hansen (1970).* See Table 4-5 for shape, depth, and other factors.

General:† $q_{ult} = cN_c s_c d_c i_c g_c b_c + \bar{q}N_q s_q d_q i_q g_q b_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma i_\gamma g_\gamma b_\gamma$

when $\phi = 0$

use $q_{ult} = 5.14 s_u(1 + s'_c + d'_c - i'_c - b'_c - g'_c) + \bar{q}$

N_q = same as Meyerhof above

N_c = same as Meyerhof above

$$N_\gamma = 1.5(N_q - 1)\tan\phi$$

Vesić (1973, 1975).* See Table 4-5 for shape, depth, and other factors.
Use Hansen's equations above.

N_q = same as Meyerhof above

N_c = same as Meyerhof above

$$N_\gamma = 2(N_q + 1)\tan\phi$$

*These methods require a trial process to obtain design base dimensions since width B and length L are needed to compute shape, depth, and influence factors.

†See Sec. 4-6 when $i_i < 1$.

TABLE 4-2

Bearing-capacity factors for the Terzaghi equations

Values of N_γ for ϕ of 0, 34, and 48° are original Terzaghi values and used to back-compute $K_{p\gamma}$

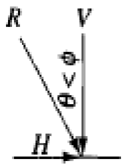
ϕ, deg	N_c	N_q	N_γ	$K_{p\gamma}$
0	5.7*	1.0	0.0	10.8
5	7.3	1.6	0.5	12.2
10	9.6	2.7	1.2	14.7
15	12.9	4.4	2.5	18.6
20	17.7	7.4	5.0	25.0
25	25.1	12.7	9.7	35.0
30	37.2	22.5	19.7	52.0
34	52.6	36.5	36.0	
35	57.8	41.4	42.4	82.0
40	95.7	81.3	100.4	141.0
45	172.3	173.3	297.5	298.0
48	258.3	287.9	780.1	
50	347.5	415.1	1153.2	800.0

*$N_c = 1.5\pi + 1$. [See Terzaghi (1943), p. 127.]

TABLE 4-3

Shape, depth, and inclination factors for the Meyerhof bearing-capacity equations of Table 4-1

Factors	Value	For
Shape:	$s_c = 1 + 0.2K_p\dfrac{B}{L}$	Any ϕ
	$s_q = s_\gamma = 1 + 0.1K_p\dfrac{B}{L}$	$\phi > 10°$
	$s_q = s_\gamma = 1$	$\phi = 0$
Depth:	$d_c = 1 + 0.2\sqrt{K_p}\dfrac{D}{B}$	Any ϕ
	$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\dfrac{D}{B}$	$\phi > 10$
	$d_q = d_\gamma = 1$	$\phi = 0$
Inclination:	$i_c = i_q = \left(1 - \dfrac{\theta°}{90°}\right)^2$	Any ϕ
	$i_\gamma = \left(1 - \dfrac{\theta°}{\phi°}\right)^2$	$\phi > 0$
	$i_\gamma = 0$ for $\theta > 0$	$\phi = 0$



Where $K_p = \tan^2(45 + \phi/2)$ as in Fig. 4-2

θ = angle of resultant R measured from vertical without

TABLE 4-4

Bearing-capacity factors for the Meyerhof, Hansen, and Vesić bearing-capacity equations

Note that N_c and N_q are the same for all three methods; subscripts identify author for N_γ

ϕ	N_c	N_q	$N_{\gamma(H)}$	$N_{\gamma(M)}$	$N_{\gamma(V)}$	N_q/N_c	$2\tan\phi(1-\sin\phi)^2$
0	5.14*	1.0	0.0	0.0	0.0	0.195	0.000
5	6.49	1.6	0.1	0.1	0.4	0.242	0.146
10	8.34	2.5	0.4	0.4	1.2	0.296	0.241
15	10.97	3.9	1.2	1.1	2.6	0.359	0.294
20	14.83	6.4	2.9	2.9	5.4	0.431	0.315
25	20.71	10.7	6.8	6.8	10.9	0.514	0.311
26	22.25	11.8	7.9	8.0	12.5	0.533	0.308
28	25.79	14.7	10.9	11.2	16.7	0.570	0.299
30	30.13	18.4	15.1	15.7	22.4	0.610	0.289
32	35.47	23.2	20.8	22.0	30.2	0.653	0.276
34	42.14	29.4	28.7	31.1	41.0	0.698	0.262
36	50.55	37.7	40.0	44.4	56.2	0.746	0.247
38	61.31	48.9	56.1	64.0	77.9	0.797	0.231
40	75.25	64.1	79.4	93.6	109.3	0.852	0.214
45	133.73	134.7	200.5	262.3	271.3	1.007	0.172
50	266.50	318.5	567.4	871.7	761.3	1.195	0.131

* $= \pi + 2$ as limit when $\phi \to 0°$.

Slight differences in above table can be obtained using program BEARING.EXE on diskette depending on computer used and whether or not it has floating point.

TABLE 4-5a

Shape and depth factors for use in either the Hansen (1970) or Vesić (1973, 1975b) bearing-capacity equations of Table 4-1. Use s'_c, d'_c when $\phi = 0$ only for Hansen equations. Subscripts H, V for Hansen, Vesić, respectively.

Shape factors	Depth factors
$s'_{c(H)} = 0.2\dfrac{B'}{L'} \quad (\phi = 0°)$ $s_{c(H)} = 1.0 + \dfrac{N_q}{N_c} \cdot \dfrac{B'}{L'}$ $s_{c(V)} = 1.0 + \dfrac{N_q}{N_c} \cdot \dfrac{B}{L}$ $s_c = 1.0$ for strip	$d'_c = 0.4k \quad (\phi = 0°)$ $d_c = 1.0 + 0.4k$ $k = D/B$ for $D/B \le 1$ $k = \tan^{-1}(D/B)$ for $D/B > 1$ k in radians
$s_{q(H)} = 1.0 + \dfrac{B'}{L'} \sin\phi$ $s_{q(V)} = 1.0 + \dfrac{B}{L} \tan\phi$ for all ϕ	$d_q = 1 + 2\tan\phi(1 - \sin\phi)^2 k$ k defined above
$s_{\gamma(H)} = 1.0 - 0.4\dfrac{B'}{L'} \ge 0.6$ $s_{\gamma(V)} = 1.0 - 0.4\dfrac{B}{L} \ge 0.6$	$d_\gamma = 1.00$ for all ϕ

Notes:

1. Note use of "effective" base dimensions B', L' by Hansen but not by Vesić.
2. The values above are consistent with either a vertical load or a vertical load accompanied by a horizontal load H_B.
3. With a vertical load and a load H_L (and either $H_B = 0$ or $H_B > 0$) you may have to compute two sets of shape s_i and d_i as $s_{i,B}$, $s_{i,L}$ and $d_{i,B}$, $d_{i,L}$. For i, L subscripts of Eq. (4-2), presented in Sec. 4-6, use ratio L'/B' or D/L'.

TABLE 4-5b

Table of inclination, ground, and base factors for the Hansen (1970) equations. See Table 4-5c for equivalent Vesić equations.

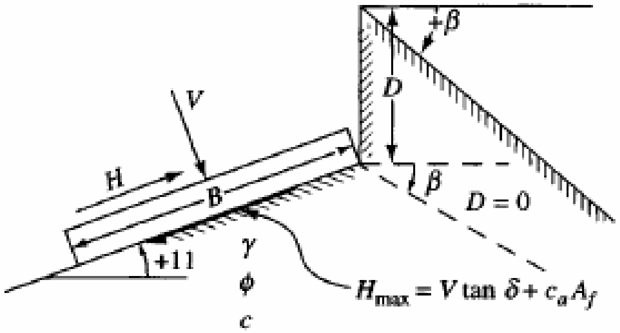
Inclination factors	Ground factors (base on slope)
$i'_c = 0.5 - \sqrt{1 - \dfrac{H_i}{A_f C_a}}$	$g'_c = \dfrac{\beta^\circ}{147^\circ}$
$i_c = i_q - \dfrac{1 - i_q}{N_q - 1}$	$g_c = 1.0 - \dfrac{\beta^\circ}{147^\circ}$
$i_q = \left[1 - \dfrac{0.5H_i}{V + A_f c_a \cot\phi}\right]^{\alpha_1}$ $2 \leq \alpha_1 \leq 5$	$g_q = g_\gamma = (1 - 0.5\tan\beta)^5$
	Base factors (tilted base)
$i_\gamma = \left[1 - \dfrac{0.7H_i}{V + A_f c_a \cot\phi}\right]^{\alpha_2}$	$b'_c = \dfrac{\eta^\circ}{147^\circ} \quad (\phi = 0)$
$i_\gamma = \left[1 - \dfrac{(0.7 - \eta^\circ/450^\circ)H_i}{V + A_f c_a \cot\phi}\right]^{\alpha_2}$ $2 \leq \alpha_2 \leq 5$	$b_c = 1 - \dfrac{\eta^\circ}{147^\circ} \quad (\phi > 0)$ $b_q = \exp(-2\eta\tan\phi)$ $b_\gamma = \exp(-2.7\eta\tan\phi)$ η in radians

Notes:

1. Use H_i as either H_B or H_L, or both if $H_L > 0$.
2. Hansen (1970) did not give an i_c for $\phi > 0$. The value above is from Hansen (1961) and also used by Vesić.
3. Variable c_a = base adhesion, on the order of 0.6 to 1.0 × base cohesion.
4. Refer to sketch for identification of angles η and β, footing depth D, location of H_i (parallel and at top of base slab; usually also produces eccentricity). Especially note V = force *normal* to base and is not the resultant R from combining V and H_i.

Notes: $\beta + \eta$ 90° (Both β and η have signs (+) shown.)

β ϕ



For: $L/B \le 2$ use ϕ_{tr}

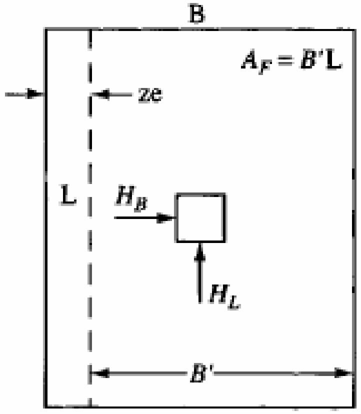
$L/B > 2$ use $\phi_{ps} = 1.5\,\phi_{tr} - 17°$

$\phi_{tr} \le 34°$ use $\phi_{tr} = \phi_{ps}$

δ = friction angle between base and soil ($.5\phi \le \delta \le \phi$)

$A_f = B'L'$ (effective area)

c_a = base adhesion (0.6 to 1.0c)



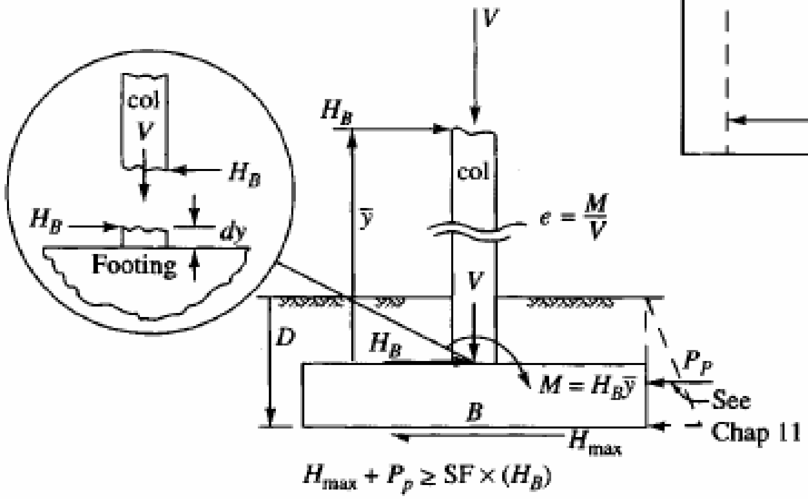


TABLE 4-5c

Table of inclination, ground, and base factors for the Vesić (1973, 1975b) bearing-capacity equations. See notes below and refer to sketch for identification of terms.

Inclination factors	Ground factors (base on slope)
$i_c' = 1 - \dfrac{mH_i}{A_f c_a N_c}$ ($\phi = 0$)	$g_c' = \dfrac{\beta}{5.14}$ β in radians
$i_c = i_q - \dfrac{1 - i_q}{N_q - 1}$ ($\phi > 0$)	$g_c = i_q - \dfrac{1 - i_q}{5.14 \tan\phi}$ $\phi > 0$
i_q, and m defined below	i_q defined with i_c
$i_q = \left[1.0 - \dfrac{H_i}{V + A_f c_a \cot\phi}\right]^m$	$g_q = g_\gamma = (1.0 - \tan\beta)^2$

Inclination factors	Base factors (tilted base)
$i_\gamma = \left[1.0 - \dfrac{H_i}{V + A_f c_a \cot\phi}\right]^{m+1}$	$b_c' = g_c'$ ($\phi = 0$)
$m = m_B = \dfrac{2 + B/L}{1 + B/L}$	$b_c = 1 - \dfrac{2\beta}{5.14\tan\phi}$
$m = m_L = \dfrac{2 + L/B}{1 + L/B}$	$b_q = b_\gamma = (1.0 - \eta\tan\phi)^2$

Notes:

1. When $\phi = 0$ (and $\beta \neq 0$) use $N_\gamma = -2\sin(\pm\beta)$ in N_γ term.
2. Compute $m = m_B$ when $H_i = H_B$ (H parallel to B) and $m = m_L$ when $H_i = H_L$ (H parallel to L). If you have both H_B and H_L use $m = \sqrt{m_B^2 + m_L^2}$. Note use of B and L, not B', L'.
3. Refer to Table sketch and Tables 4-5a,b for term identification.
4. Terms N_c, N_q, and N_γ are identified in Table 4-1.
5. Vesić always uses the bearing-capacity equation given in Table 4-1 (uses B' in the N_γ term even when $H_i = H_L$).
6. H_i term $\le$ 1.0 for computing i_q, i_γ (always).

Which Equations to Use

Use	Best for
Terzaghi	Very cohesive soils where $D/B \leq 1$ or for a quick estimate of q_{ult} to compare with other methods. *Do not use* for footings with moments and/or horizontal forces or for tilted bases and/or sloping ground.
Hansen, Meyerhof, Vesić	Any situation that applies, depending on user preference or familiarity with a particular method.
Hansen, Vesić	When base is tilted; when footing is on a slope or when $D/B > 1$.

Notes for some design considerations

1- Influence depth of footing:

The influnce depth of loading at which the failure surface occurd take the foolowing :-

$$d = \frac{B}{2}\tan\left(45 + \frac{\phi}{2}\right) \quad \text{or} \quad d \cong B$$

2- For " Local shear failure" mode soil, i. e. if foundation constructed on this kind of soil, the strength parameters (C , ϕ) must be modified to use it in B. C. Equation, where:

$$\bar{c} = \frac{2}{3}c \qquad \text{and} \qquad \acute{\phi} \approx \tan^{-1}\left(\frac{2}{3}\tan\phi\right)$$

* using $\bar{c}$ Instead of C in first contribution .

* using $\acute{\phi}$ instead of ϕ in table (4-2) to calculate B. C. factors (N_c , N_γ , N_q)

3- Factor of safety:

The actual pressure (stress) from the structure to the soil must be not exceed the " allowable bearing capacity" of the soil (q_{all}), where:

$$q_{des.} \quad \text{or} \quad q_{all} = \frac{q_{ult}}{F.S}$$

The factor of safety ranged between (1.5 – 6) depends on :

1- Type of structure (permenant or temporary)
2- Sensitivity of structure .
3- Extent of soil exploration .
4- Conditions of construction .
5- Load condition (static , dynamic)

It is recommended that F. S. is (2 – 4) .

4- Gross and Net Soil Pressure: ضغط التربة الصافي والأجمالي

Gross (total) pressure : is the pressure duo to the total loads above foundation level . It includes the load above ground , self-weight of foundation and the weight of soil above foundation .

$$q_{gross} = (W_{D+L} + W_F + W_s) / A$$

Net pressure : is the intensity of pressure at the base of foundation excluding the existing pressure duo to the soil above foundation .

$$(q_{ult})_{net} = (q_{ult})_{gross} - \gamma D$$

Notes :

1- The bearing capacity equations (q_{ult}) are based on gross soil pressure which is every thing a bove the foundation level .

2- Settlements are caused by net increases in soil pressure over the existing overburden pressure.

Where : $(q_{ult})_{net} = q_{ult} - \gamma_2 D$

So that the B. C. Eq. will be :

$$(q_{ult})_{net} = C N_c + 0.5 B \gamma_1 N\gamma + \gamma_2 D (N_q - 1)$$

5- Bearing capacity for footing on layered soils :

A possible alternative for c - ϕ soils with anumber of thin layers is to use average values of c and ϕ in B. C. Eq. obtained as :

$$c_{av} = \frac{c_1 H_1 + c_2 H_2 + c_3 H_3 + \cdots + c_n H_n}{d}$$

$$\phi_{av} = \tan^{-1} \frac{H_1 \tan\phi_1 + H_2 \tan\phi_2 + \cdots + H_n \tan\phi_n}{d}$$

If $d = \frac{B}{2} \tan (45 + \frac{\emptyset}{2})$ one or more iterations may be required to obtain the best average (c and ϕ) values, or if $d \cong B$, using the number of layers within this depth as shown :

6- Effect of water table:

Bowles suggested the following equation to calculate effective unit weight for water table:

$$\gamma_e = (2H - d_w)\frac{d_w}{H^2}\gamma_{wet} + \frac{\gamma'}{H^2}(H - d_w)^2$$

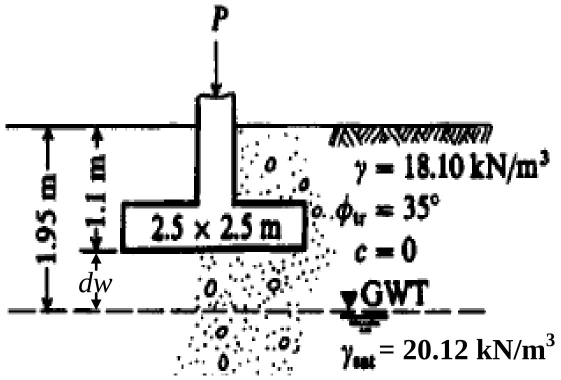
where $H = 0.5B\tan(45° + \phi/2)$

d_w = depth to water table below base of footing

γ_{wet} = wet unit weight of soil in depth d_w

γ' = submerged unit weight below water table $= \gamma_{sat} - \gamma_w$

Example 4-8. A square footing that is vertically and concentrically loaded is to be placed on a cohesionless soil as shown in Fig. E4-8. The soil and other data are as shown.



From Fig. E4-8 we obtain $d_w = 0.85$ m and $H = 0.5B\tan(45° + \phi/2) = 2.40$ m. Substituting into Eq. (4-4), we have

$$\gamma_e = (2 \times 2.4 - 0.85)\frac{0.85 \times 18.10}{2.4^2} + \frac{20.12 - 9.807}{2.4^2}(2.40 - 0.85)^2$$
$$= 14.85 \text{ kN/m}^3$$

Note

Since the soil wedge beneath round and square bases is much closer to a triaxial than plane strain state, the adjustment of ϕ_{tr} to ϕ_{ps} is recommended only when $L/B > 2$.

$$\phi_{ps} = 1.5\phi_{triaxial} - 17$$

Example 4-2. A footing load test made by H. Muhs in Berlin [reported by Hansen (1970)] produced the following data:

$$D = 0.5 \text{ m} \qquad B = 0.5 \text{ m} \qquad L = 2.0 \text{ m}$$

$$\gamma' = 9.31 \text{ kN/m}^3 \qquad \phi_{\text{triaxial}} = 42.7° \qquad \text{Cohesion } c = 0$$

$$P_{\text{ult}} = 1863 \text{ kN (measured)} \qquad q_{\text{ult}} = \frac{P_{\text{ult}}}{BL} = \frac{1863}{0.5 \times 2} = 1863 \text{ kPa (computed)}$$

Required. Compute the ultimate bearing capacity by both Hansen and Meyerhof equations and compare these values with the measured value.

Solution.

a. Since $c = 0$, any factors with subscript c do not need computing. All g_i and b_i factors are 1.00; with these factors identified, the Hansen equation simplifies to

$$q_{\text{ult}} = \gamma' D N_q s_q d_q + 0.5\gamma' B N_\gamma s_\gamma d_\gamma$$

$$L/B = \frac{2}{0.5} = 4 \rightarrow \phi_{ps} = 1.5(42.5) - 17 = 46.75°$$

$$\text{Use } \phi = 47°$$

From a table of ϕ in 1° increments (table not shown) obtain

$$N_q = 187 \qquad N_\gamma = 299$$

Using linear interpolation of Table 4-4 gives 208.2 and 347.2. Using Table 4-5*a* one obtains [ge the $2\tan\phi(1 - \sin\phi)^2$ part of d_q term from Table 4-4] the following:

$$s_{q(H)} = 1 + \frac{B'}{L'}\sin\phi = 1.18 \qquad s_{\gamma(H)} = 1 - 0.4\frac{B'}{L'} = 0.9$$

$$d_q = 1 + 2\tan\phi(1 - \sin\phi)^2\frac{D}{B'} = 1 + 0.155\frac{D}{B'}$$

$$= 1 + 0.155\left(\frac{0.5}{0.5}\right) = 1.155 \qquad d_\gamma = 1.0$$

With these values we obtain

$$q_{\text{ult}} = 9.31(0.5)(187)(1.18)(1.155) + 0.5(9.31)(0.5)(299)(0.9)(1)$$
$$= \mathbf{1812} \text{ kPa vs. 1863 kPa measured}$$

b. By the Meyerhof equations of Table 4-1 and 4-3, and $\phi_{ps} = 47°$, we can proceed as follows:

Step 1. Obtain $N_q = 187$

$$N_\gamma = (N_q - 1)\tan(1.4\phi) = 413.6 \rightarrow 414$$

$$K_p = \tan^2\left(45 + \frac{\phi}{2}\right) = 6.44 \rightarrow \sqrt{K_p} = 2.54$$

$$s_q = s_\gamma = 1 + 0.1K_p\frac{B}{L} = 1 + 0.1(6.44)\frac{0.5}{2.0} = 1.16$$

$$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\frac{D}{B} = 1 + 0.1(2.54)\frac{0.5}{0.5} = 1.25$$

Step 2. Substitute into the Meyerhof equation (ignoring any c subscripts):

$$
\begin{aligned}
q_{\text{ult}} &= \gamma' D N_q s_q d_q + 0.5\gamma B N_\gamma s_\gamma d_\gamma \\
&= 9.31(0.5)(187)(1.16)(1.25) + 0.5(9.31)(0.5)(414)(1.16)(1.25) \\
&= 1262 + 1397 = \mathbf{2659}\ \text{kPa}
\end{aligned}
$$

Footing with Eccentric loads:

A footing may be loaded eccentrically, eccentric loading of shallow foundations occurs when a vertical load Q is applied at a location other than the centroid of the foundation (Fig. 1.5a), or when a foundation is subjected to a centric load of magnitude Q and moment M (Fig. 1.5b) and from a concentric column with an axial load and moment a bout one or both axes. In such cases, the load eccentricities may be given as .

$$e_L = \frac{M_B}{Q}$$

and

$$e_B = \frac{M_L}{Q}$$

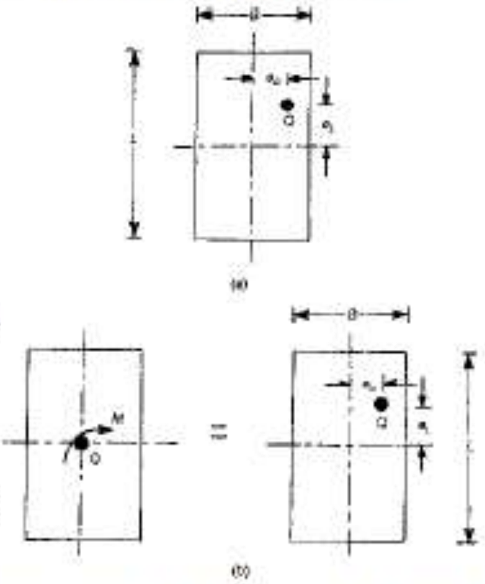


Figure 1-5 Eccentric load on rectangular foundation

Where:

e_L , e_B = load eccentricities, respectively, in the direction of *long* and *short* axes of the foundation

M_B , M_L = moment components about the *short* and *long* axes of the foundation, respectively

The effective dimensions and then effective area should be used in the bearing capacity equation , as :

$A_f = L' * B'$

Where : A_f : effective area of footing

L' : effective of length ; $L' = L - 2e_L$

B' : effective width ; $B' = B - 2e_B$

To estimate the bearing capacity for the footing with eccentricity use one of the following :

Method 1: Use Meyerhof's equation with original dimensions of footing, then :

$$q_{ult} = (q_{ult})_{computed} * Re$$

where:

Re : reduction factor ; Re = Re$_B$ * Re$_L$

$$Re_B = 1 - \frac{2e}{B} \quad \text{or} \quad Re_L = 1 - \frac{2e}{L} \qquad \text{(cohesive soil)}$$

$$Re_B = 1 - \sqrt{\frac{e}{B}} \quad \text{or} \quad Re_L = 1 - \sqrt{\frac{e}{L}} \qquad \text{(cohesionless soil)}$$

And

$$q_{act} = \frac{V}{A} = \frac{V}{BL}$$

In practice the e/B ratio is seldom greater than 0.2 and is usually limited to $e < B/6$. In these reduction factor equations the dimensions B and L are referenced to the axis about which the base moment occurs.

Method 2: Use either the Hansen or ' bearing-capacity equation given in Table 4-1 with the following adjustments:

a. **Use *B′* in the base width ($0.5\gamma\ B\ N_\gamma$) term.**
b. **Use *B′* and *L′* in computing the shape factors.**
c. **Use actual *B* and L for all depth factors.**

$$q_{act} = \frac{V}{A_f} = \frac{V}{B'\,L'}$$

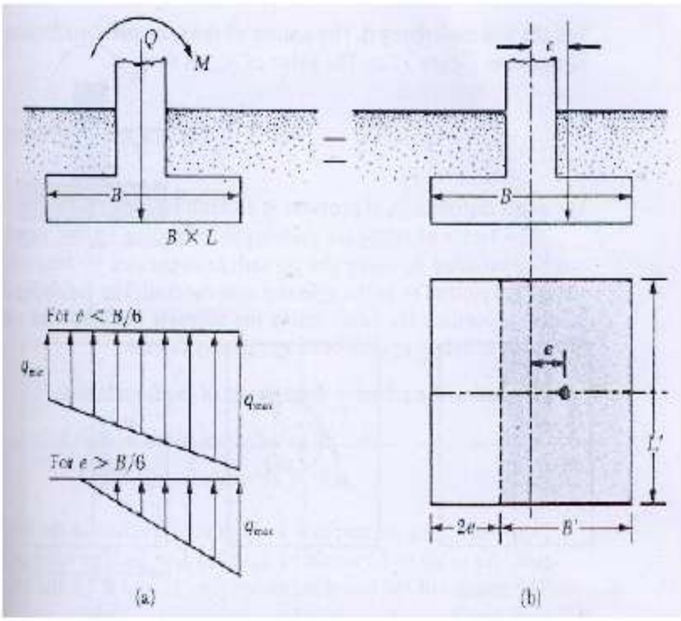
ملاحظة : في الطريقتين التي تم ذكرهما اعلاه لايجاد قيمة q_{ult} لاساس يتعرض الى عزم او حمل لامركزي يمكن مقارنة q_{all} الناتجة في حالة اعطاء معامل الامان F.S مع $q_{\substack{max\\min}}$ الناتجة من توزيع الضغط تحت الاساس نتيجة هذا التحميل :

$$\boxed{q_{\substack{max\\min}} = \frac{Q}{BL}\left(1 \pm \frac{6e_B}{B} \pm \frac{6e_L}{L}\right)}$$

حيث يجب ان يكون :

$q_{max} < q_{all}$

$q_{min} > 0$



ملاحظة مهمة :

$$\boxed{q_{max} = \frac{Q}{BL}\left(1 + \frac{6e_B}{B} + \frac{6e_L}{L}\right)}$$

And

$$\boxed{q_{min} = \frac{Q}{BL}\left(1 - \frac{6e_B}{B} - \frac{6e_L}{L}\right)}$$

Note : in these equations, when the eccentricity e becomes equal B/6 , q_{min} is zero and for e > B/6 , q_{min} will be negative , which means that tension will develop

Because soil cannot take any tension , there will then be a separation between the foundation and the soil underlying it . The nature of the pressure distribution on the soil as shown in Figure above . The value of q_{max} is then $q_{max} = 4\,Q/\,3L(B-2e)$

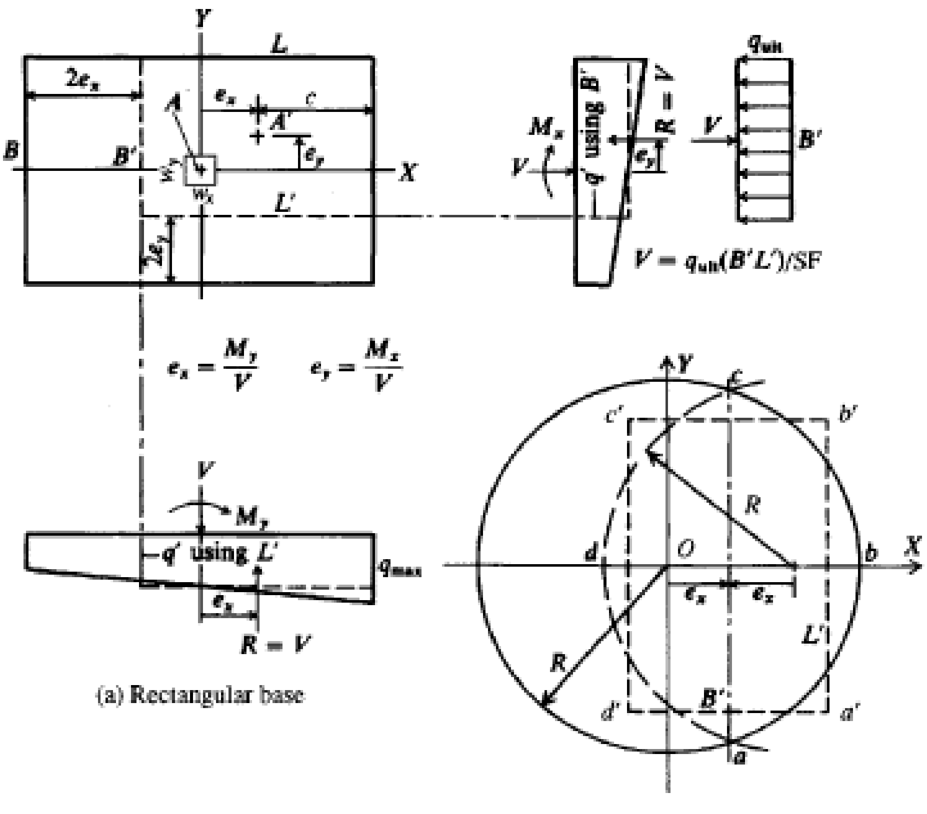
For design (considered in Chap. 8) the minimum dimensions (to satisfy ACI 318-) of a rectangular footing with a central column of dimensions $w_x \times w_y$ are required to be

$$B_{\min} = 4e_y + w_y \qquad B' = 2e_y + w_y$$
$$L_{\min} = 4e_x + w_x \qquad L' = 2e_x + w_x$$

Final dimensions may be larger than $B_{\min}$ or $L_{\min}$ based on obtaining the required allowable bearing capacity.

The ultimate bearing capacity for footings with eccentricity, using either the Meyerhof or Hansen/Vesić equations, is found in *either* of two ways:

Method 1. Use either the Hansen or Vesić bearing-capacity equation given in Table 4-1 with the following adjustments:



(a) Rectangular base

(b) Round base

Example 4-5. A square footing is 1.8 × 1.8 m with a 0.4 × 0.4 m square column. It is loaded with an axial load of 1800 kN and M_x = 450 kN · m; M_y = 360 kN · m. Undrained triaxial tests (soil not saturated) give ϕ = 36° and c = 20 kPa. The footing depth D = 1.8 m; the soil unit weight γ = 18.00 kN/m³; the water table is at a depth of 6.1 m from the ground surface.

Required. What is the allowable soil pressure, if SF = 3.0, using the Hansen bearing-capacity equation with B', L'; Meyerhof's equation; and the reduction factor R_e?

Solution. See Fig. E4-5.

$$e_y = 450/1800 = 0.25 \text{ m} \qquad e_x = 360/1800 = 0.20 \text{ m}$$

Both values of e are $< B/6 = 1.8/6 = 0.30$ m. Also

$$B_{\min} = 4(0.25) + 0.4 = 1.4 < 1.8 \text{ m given}$$
$$L_{\min} = 4(0.20) + 0.4 = 1.2 < 1.8 \text{ m given}$$

Now find

$$B' = B - 2e_y = 1.8 - 2(0.25) = 1.3 \text{ m} \qquad (B' < L')$$
$$L' = L - 2e_x = 1.8 - 2(0.20) = 1.4 \text{ m} \qquad (L' > B')$$

By Hansen's equation. From Table 4-4 at ϕ = 36° and rounding to integers, we obtain

$$N_c = 51 \qquad N_q = 38 \qquad N_\gamma = 40$$
$$N_q/N_c = 0.746 \qquad 2\tan\phi(1 - \sin\phi)^2 = 0.247$$
$$\text{Compute } D/B = 1.8/1.8 = 1.0$$

Now compute

$$s_c = 1 + (N_q/N_c)(B'/L') = 1 + 0.746(1.3/1.4) = 1.69$$
$$d_c = 1 + 0.4D/B = 1 + 0.4(1.8/1.8) = 1.40$$
$$s_q = 1 + (B'/L')\sin\phi = 1 + (1.3/1.4)\sin 36° = 1.55$$
$$d_q = 1 + 2\tan\phi(1 - \sin\phi)^2 D/B = 1 + 0.247(1.0) = 1.25$$
$$s_\gamma = 1 - 0.4\frac{B'}{L'} = 1 - 0.4\frac{1.3}{1.4} = 0.62 > 0.60 \quad \text{(O.K.)}$$
$$d_\gamma = 1.0$$
$$\text{All } i_i = g_i = b_i = 1.0 \text{ (not 0.0)}$$

The Hansen equation is given in Table 4-1 as

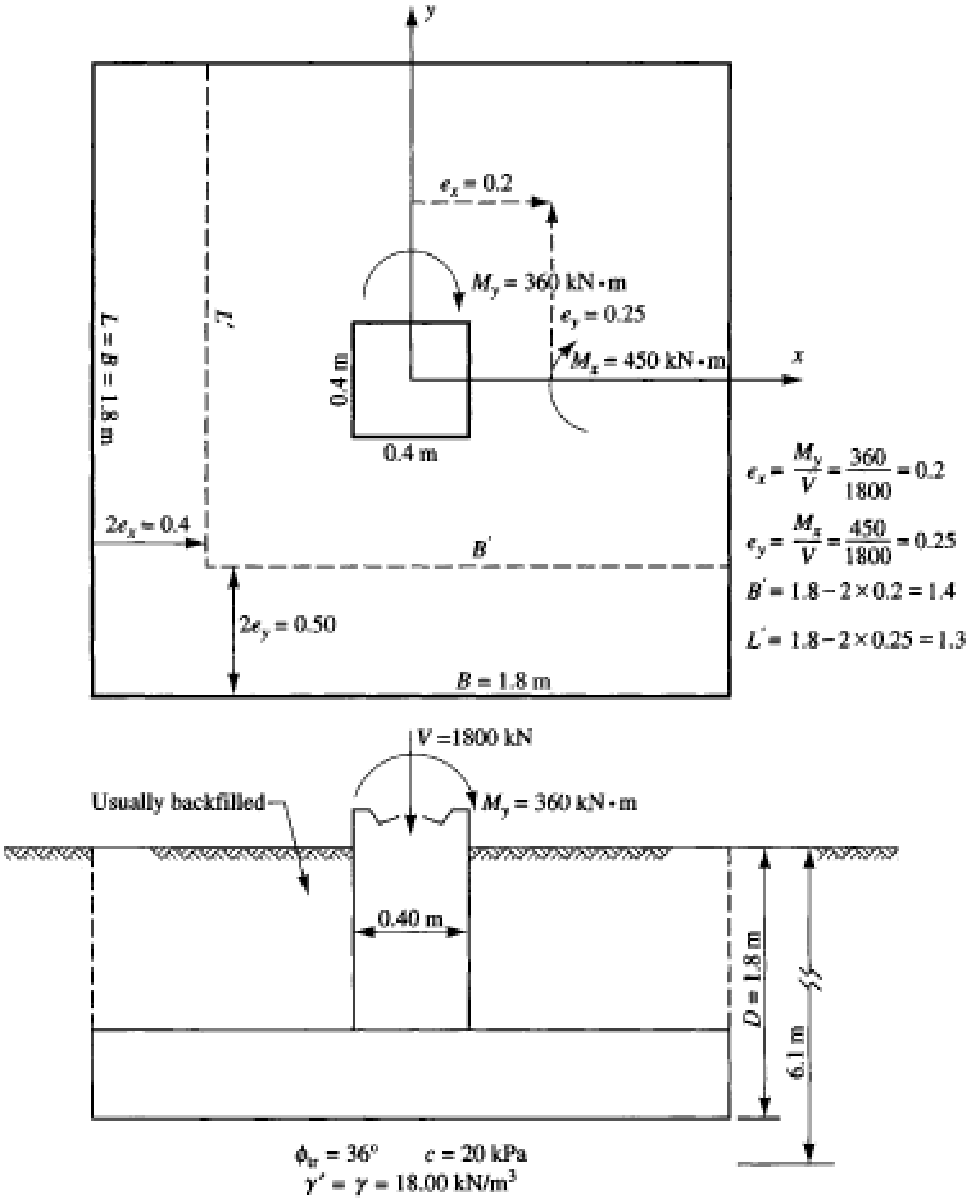
$$q_{\text{ult}} = cN_c s_c d_c + \bar{q}N_q s_q d_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma$$

Inserting values computed above with terms of value 1.0 not shown (except d_γ) and using B' = 1.3, we obtain

$$q_{\text{ult}} = 20(51)(1.69)(1.4) + 1.8(18.0)(38)(1.55)(1.25) + 0.5(18.0)(1.3)(40)(0.62)(1.0)$$
$$= 2413 + 2385 + 290 = 5088 \text{ kPa}$$

For SF = 3.0 the allowable soil pressure q_a is

$$q_a = 5088/3 = 1696 \rightarrow \mathbf{1700} \textbf{ kPa}$$



y
$e_x = 0.2$
$M_y = 360$ kN·m
$e_y = 0.25$
$M_x = 450$ kN·m
x
L′
L = B = 1.8 m
0.4 m
0.4 m
$2e_x = 0.4$
B′
$2e_y = 0.50$
B = 1.8 m
$e_x = \frac{M_y}{V} = \frac{360}{1800} = 0.2$
$e_y = \frac{M_x}{V} = \frac{450}{1800} = 0.25$
$B' = 1.8 - 2 \times 0.2 = 1.4$
$L' = 1.8 - 2 \times 0.25 = 1.3$
V = 1800 kN
$M_y = 360$ kN·m
Usually backfilled
0.40 m
D = 1.8 m
6.1 m
$\phi_{tr} = 36°$ c = 20 kPa
$\gamma' = \gamma = 18.00$ kN/m³

Figure E4-5

The actual soil pressure is

$$q_{act} = \frac{1800}{B'L'} = \frac{1800}{1.3 \times 1.4} = \mathbf{989}\ \text{kPa}$$

Note that the allowable pressure q_a is very large, and the actual soil pressure q_{act} is also large. With this large actual soil pressure, settlement may be the limiting factor. Some geotechnical consultants routinely limit the maximum allowable soil pressure to around 500 kPa in recommendations to clients for design whether settlement is a factor or not. Small footings with large column loads are visually not very confidence-inspiring during construction, and with such a large load involved this is certainly not the location to be excessively conservative.

By Meyerhof's method and R_e. This method uses actual base dimensions $B \times L$:

$$K_p = \tan^2(45 + \phi/2) = \tan^2(45 + 36/2) = 3.85$$
$$\sqrt{K_p} = 1.96$$

From Table 4-3,

$$N_c = 51 \qquad N_q = 38 \text{ (same as Hansen values)} \qquad N_\gamma = 44.4 \rightarrow 44$$

Also

$$s_c = 1 + 0.2K_p\frac{B}{L} = 1 + 0.2(3.85)\frac{1.8}{1.8} = 1.77$$
$$s_q = s_\gamma = 1 + 0.1K_p\frac{B}{L} = 1.39$$
$$d_c = 1 + 0.2\sqrt{K_p}\frac{D}{B} = 1 + 0.2(1.96)\frac{1.8}{1.8} = 1.39$$
$$d_q = d_\gamma = 1 + 0.1(1.96)(1.0) = 1.20$$

Now direct substitution into the Meyerhof equation of Table 4-1 for the vertical load case gives

$$\begin{aligned} q_{ult} &= 20(51)(1.77)(1.39) + 1.8(18.0)(38)(1.39)(1.20) \\ &\qquad + 0.5(18.0)(1.8)(44)(1.39)(1.20) \\ &= 2510 + 2054 + 1189 = 5752\ \text{kPa} \end{aligned}$$

There will be two reduction factors since there is two-way eccentricity. Use the equation for cohesionless soils since the cohesion is small (only 20 kPa):

$$R_{eB} = 1 - \left(\frac{e_y}{B}\right)^{0.5} = 1 - \sqrt{0.25/1.8} = 1 - 0.37 = 0.63$$
$$R_{eL} = 1 - \left(\frac{e_x}{L}\right)^{0.5} = 1 - \sqrt{0.2/1.8} = 0.67$$

The reduced $q_{ult} = 5752(R_{eB}R_{eL}) = 5752(0.63 \times 0.67) = 2428$ kPa. The allowable (SF = 3) soil pressure = 2428/3 = 809 → **810** kPa. The actual soil pressure = $1800/(BL) = 1800/(1.8 \times 1.8) = 555$ kPa.

Meyerhof's reduction factors were based on using small model footings (B on the order of 50 mm), but a series of tests using a 0.5 × 2 m concrete base, reported by Muhs and Weiss (1969), indicated that the Meyerhof reduction method is not unreasonable.

Given. A 2 × 2 m square footing has the ground slope of $\beta = 0$ for the given direction of H_B, but we would use $\beta \approx -80°$ (could use $-90°$) if H_B were reversed along with passive pressure P_P to resist sliding and base geometry shown in Fig. E4-7.

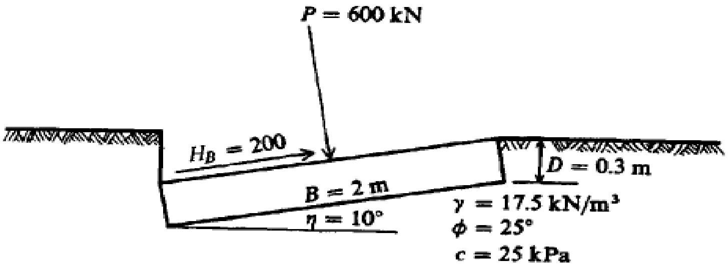


Figure E4-7

Required. Are the footing dimensions adequate for the given loads if we use a safety (or stability) factor SF = 3?

Solution. We may use any of Hansen's, Meyerhof's, or Vesić's equations.

Hansen's method. Initially let us use Hansen's equations (to illustrate further the interrelationship between the i_i and s_i factors).

$$\text{Assumptions:} \quad \delta = \phi \qquad c_a = c \qquad D = 0.3\text{ m (smallest value)}$$
$$A_f = B \times L = 2 \times 2 = \mathbf{4}\text{ m}^2$$

First check sliding safety (and neglect the passive pressure P_P for $D = 0.3$ m on right side)

$$F_{\max} = A_f c_a + V \tan\phi = (4)(25) + 600 \tan 25 = 379.8\text{ kN}$$
$$\text{Sliding stability (or SF)} = F_{\max}/H = 379.8/200 = \mathbf{1.90} \qquad \text{(probably O.K.)}$$

From Table 4-4 (or computer program) obtain the *Hansen* bearing capacity and other factors (for $\phi = 25°$) as

$$N_c = 20.7 \qquad N_q = 10.7 \qquad N_\gamma = 6.8 \qquad N_q/N_c = 0.514$$
$$2\tan\phi(1-\sin\phi)^2 = 0.311$$

Compute $D/B = D/B' = D/L' = 0.3/2 = \mathbf{0.15}$.

Next compute depth factors d_i:

$$d_\gamma = \mathbf{1.00}$$
$$d_c = 1 + 0.4D/B = 1 + 0.4(0.3/2) = \mathbf{1.06}$$
$$d_q = 1 + 2\tan\phi(1-\sin\phi)^2(D/B) = 1 + 0.311(0.15) = 1.046 \rightarrow \mathbf{1.05}$$

Compute the inclination factors i_i so we can compute the shape factors:

$$V + A_f c_a \cot\phi = 600 + (2 \times 2)(25)\tan 25 = 600 + 214.4 = 814.4$$

We will use exponents $\alpha_1 = 3$ and $\alpha_2 = 4$ (instead of 5 for both—see text):

$$i_{q,B} = \left[1 - \frac{0.5H_B}{V + A_f c_a \cot\phi}\right]^3 = [1 - 0.5(200)/814.4]^3 = \mathbf{0.675}$$

$$i_{\gamma,B} = \left[1 - \frac{(0.7 - \eta/450°)H_B}{V + A_f c_a \cot\phi}\right]^4 = [1 - (0.7 - 10/450)(200)/814.4]^4$$
$$= [1 - 0.68(200)/814.4]^4 = \mathbf{0.481}$$
$$i_{\gamma,L} = 1.00 \text{ (since } H_L = 0.0)$$

We can now compute $i_{c,B}$ as

$$i_{c,B} = i_q - \frac{1 - i_q}{N_q - 1} = 0.675 - (1 - 0.675)/(10.7 - 1) = \mathbf{0.641}$$

Using the just-computed i factors, we can compute shape factors $s_{i,B}$ as follows. With $H_L = 0.0$ and a square base it is really not necessary to use double subscripts for the several shape and inclination factors, but we will do it here to improve clarity:

$$s_{c,B} = 1 + \frac{N_q}{N_c} \cdot \frac{B' i_{c,B}}{L} = 1 + 0.514[2(0.641)]/2 = \mathbf{1.329}$$

$$s_{q,B} = 1 + \sin\phi\left(\frac{B' i_{q,B}}{L}\right) = 1 + \sin 25°[2(0.675)/2.0] = \mathbf{1.285}$$

$$s_{\gamma,B} = 1 - 0.4\left(\frac{B' i_{\gamma,B}}{L i_{\gamma,L}}\right) = 1 - 0.4[(2 \times 0.481)/(2 \times 1)] = \mathbf{0.808} > 0.60$$

Next we will compute the b_i factors:

$$\begin{aligned} \eta° &= 10° = 0.175 \text{ radians} \\ b_{c,B} &= 1 - \eta°/147° = 1 - 10/147 = \mathbf{0.93} \\ b_{q,B} &= \exp(-2\eta\tan\phi) = \exp[-2(.175)(\tan 25)] = \mathbf{0.849} \\ b_{\gamma,B} &= \exp(-2.7\eta\tan\phi) = e^{-2.7\times0.175\times0.466} = \mathbf{0.802} \end{aligned}$$

We can now substitute into the Hansen equation, noting that with a horizontal ground surface all $g_i = 1$ (not 0):

$$\begin{aligned} q_{\text{ult}} = {} & cN_c s_{c,B} d_{c,B} i_{c,B} b_{c,B} + \bar{q} N_q s_{q,B} d_{q,B} i_{q,B} b_{q,B} + \\ & \tfrac{1}{2}\gamma B' N_\gamma s_{\gamma,B} d_{\gamma,B} i_{\gamma,B} b_{\gamma,B} \end{aligned}$$

Directly substituting, we have

$$\begin{aligned} q_{\text{ult}} &= 25(20.7)(1.329)(1.06)(0.641)(0.93) + \\ &\quad 0.3(17.5)(10.7)(1.285)(1.05)(0.675)(0.849) + \\ &\quad \tfrac{1}{2}(17.5)(2.0)(6.8)(0.808)(1.0)(0.481)(0.802) \\ &= 434.6 + 43.4 + 37.1 = 515.1 \text{ kPa} \end{aligned}$$

For a stability number, or SF, of 3.0,

$$\begin{aligned} q_a &= q_{\text{ult}}/3 = 515.1/3 = 171.7 \rightarrow \mathbf{170} \text{ kPa} \qquad \text{(rounding)} \\ P_{\text{allow}} &= A_f \times q_a = (B \times L)q_a = (2 \times 2 \times 170) = 680 \text{ kPa} > 600 \qquad \text{(O.K.)} \end{aligned}$$

Vesić method. In using this method note that, with $H_L = 0.0$ and a square footing, it is only necessary to investigate the B direction without double subscripts for the shape, depth, and inclination terms. We may write

$$\begin{aligned} N_c &= 20.7; \qquad N_q = 10.7 \text{ as before but } N_\gamma = 10.9 \\ N_q/N_c &= 0.514 \qquad 2\tan\phi(1 - \sin\phi)^2 = 0.311 \end{aligned}$$

The Vesić shape factors are

$$s_c = 1 + \frac{N_q}{N_c} \cdot \frac{B'}{L'} = 1 + 0.514(2/2) = \mathbf{1.514}$$

$$s_q = 1 + \frac{B'}{L'}\tan\phi = 1 + (2/2)\tan 25° = \mathbf{1.466}$$

$$s_\gamma = 1 - 0.4\frac{B'}{L'} = 1 - 0.4(1.0) = \mathbf{0.60}$$

All d_i factors are the same as Hansen's, or

$$d_c = \mathbf{1.06} \qquad d_q = \mathbf{1.05} \qquad d_\gamma = \mathbf{1.00}$$

For the Vesić i_i factors, we compute m as

$$m = \frac{2 + B'/L'}{1 + B'/L'}$$
$$= \frac{2 + 2/2}{1 + 2/2} = \frac{3}{2} = \mathbf{1.5}$$
$$V + A_f c_a \cot\phi = 814.4 \text{ kN}; \ H = 200 \text{ kN}$$
$$i_q = \left[1 - \frac{H}{V + A_f c_a \cot\phi}\right]^m = (1 - 200/814.4)^{1.5} = \mathbf{0.655}$$
$$i_\gamma = \left[1 - \frac{H}{V + A_f c_a \cot\phi}\right]^{m+1} = (1 - 200/814.4)^{1.5+1} = \mathbf{0.494}$$
$$i_c = i_q - \frac{1 - i_q}{N_q - 1} = 0.655 - \frac{1 - 0.655}{10.7 - 1} = \mathbf{0.619}$$

The b_i factors are

$$b_c = 1 - \frac{2\beta}{5.14\tan\phi} = \mathbf{1.0} \qquad \text{(since ground slope } \beta = 0)$$
$$b_q = b_\gamma = (1 - \eta\tan\phi)^2 = (1 - 0.175\tan 25)^2 = \mathbf{0.843}$$

The Vesić equation is

$$q_{\text{ult}} = cN_c s_c d_c i_c b_c + \bar{q}N_q s_q d_q i_q b_q + \tfrac{1}{2}\gamma B N_\gamma s_\gamma d_\gamma i_\gamma b_\gamma$$

Directly substituting ($B = 2.0$ m, $\gamma = 17.5$ kN/m^3, and $D = 0.3$ m), we have

$$q_{\text{ult}} = 25(20.7)(1.514)(1.06)(0.619)(1.0) + 0.3(17.5)(10.7)(1.466)(1.05)(0.655)(0.843) + \tfrac{1}{2}(17.5)(2.0)(10.9)(0.60)(1.0)(0.494)(0.843)$$
$$= 514.1 + 47.7 + 47.7 = 609.5 \text{ kPa}$$
$$q_a = q_{\text{ult}}/3 = 609.5/3 = 203.2 \rightarrow \mathbf{200} \text{ kPa}$$

There is little difference between the Hansen (170 kPa) and the Vesić (200 kPa) equations. Nevertheless, let us do a confidence check using the Meyerhof equation/method.

Meyerhof method. Note Meyerhof does not have ground g_i or tilted base factors b_i.

$$\phi = 25° > 10° \text{ O.K.} \qquad D/B' = 0.3/2.0 = \mathbf{0.15}$$
$$\sqrt{K_p} = \tan(45° + \phi/2) = \tan 57.5° = \mathbf{1.57};\ K_p = \mathbf{2.464}$$

See Meyerhof's equation in Table 4-1 and factors in Table 4-3:

$$s_c = 1.0,\ s_q = s_\gamma = 1 + 0.1K_p\frac{B}{L} = 1 + 0.1(2.464)(2/2) = \mathbf{1.25}$$
$$d_c = 1 + 0.2\sqrt{K_p}\cdot\frac{D}{B'} = 1 + 0.2(1.57)(0.15) = \mathbf{1.05}$$
$$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\cdot\frac{D}{B'} = 1 + 0.1(1.57)(0.15) = \mathbf{1.02}$$

Let us define the angle of resultant θ as

$$\theta = \tan^{-1}(H/V) = \tan^{-1}(200/600) = \mathbf{18.4^\circ}$$

Use θ to compute Meyerhof's inclination factors:

$$i_c = i_q = (1 - \theta^\circ/90^\circ)^2 = (1 - 18.4/90)^2 = \mathbf{0.633}$$
$$i_\gamma = (1 - \theta/\phi)^2 = (1 - 18.4/25)^2 = 0.0696 \rightarrow \mathbf{0.07}$$

Using Meyerhof's equation for an inclined load from Table 4-1, we have

$$q_{\text{ult}} = cN_c s_c d_c i_c s + \bar{q}N_q s_q d_q i_q + \tfrac{1}{2}\gamma N_\gamma s_\gamma d_\gamma i_\gamma$$

Making a direct substitution (Meyerhof's N_i factors are the same as Hansen's), we write

$$\begin{aligned} q_{\text{ult}} &= 25(20.7)(1)(1.05)(0.633) + 0.3(17.5)(10.7)(1.25)(1.02)(0.633) + \\ &\quad \tfrac{1}{2}(17.5)(2.0)(6.8)(1.25)(1.02)(0.07) \\ &= 344.0 + 45.3 + 10.6 = 399.9 \text{ kPa} \end{aligned}$$

The allowable $q_a = q_{\text{ult}}/3 = 399.9/3 = 133.3 \rightarrow \mathbf{130}$ kPa.

Terzaghi equation. As an exercise let us also use the Terzaghi equation:

$$N_c = 25.1 \qquad N_q = 12.7 \qquad N_\gamma = 9.7 \qquad (\text{from Table 4-2 at } \phi = 25^\circ)$$

Also, $s_c = 1.3 \qquad s_\gamma = 0.8 \qquad$ (square base).

$$\begin{aligned} q_{\text{ult}} &= cN_c s_c + \bar{q}N_q + \tfrac{1}{2}\gamma B N_\gamma s_\gamma \\ &= (25)(25.1)(1.3) + 0.3(17.5)(12.7) + \tfrac{1}{2}(17.5)(2.0)(9.7)(0.8) \\ &= 815.8 + 66.7 + 135.8 = 1018.3 \rightarrow \mathbf{1018} \text{ kPa} \\ q_a &= q_{\text{ult}}/3 = 1018/3 = 339 \rightarrow \mathbf{340} \text{ kPa} \end{aligned}$$

Summary. We can summarize the results of the various methods as follows:

Hansen	170 kPa
Vesić	225
Meyerhof	130
Terzaghi	340

The question is, what to use for q_a? The Hansen-Vesić-Meyerhof average seems most promising and is $q_{a,\text{av}} = (170 + 225 + 130)/3 = \mathbf{175}$ kPa. The author would probably recommend using q_a = **175** kPa. This is between the Hansen and Vesić values; Meyerhof's equations tend to be conservative and in many cases may be overly so. Here the Terzaghi and Meyerhof equations are not appropriate, because they were developed for horizontal bases vertically loaded. It is useful to make the Terzaghi computation so that a comparison can be made, particularly since the computations are not difficult.[4]

FOOTING DESIGN

Foundation

Foundations are usually divided into:

1) **Shallow Foundations are used when the top layers of soil can support the applied loads with accepted settlement. They can take any form of the followings:**

   - Spread (isolated) Footing, - Combined Footing,
   - Strap – Beam Footing - Wall Footing,
   - Strip Footing, - Raft Foundation

2) **Deep Foundations are used if the top soil is weak and cannot support the structure loads. They used to transmit the loads to the stronger deeper soil layers. Forms of deep foundations are piles, pillars, caissons, ….etc.**

Foundation Safety

Foundation should be safe against:

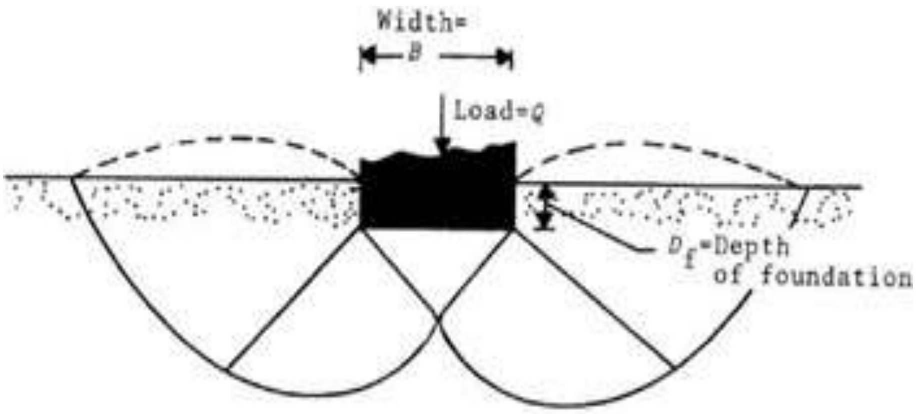
1- Shear failure in soil.

2- Excessive total or differential settlements.

3 -Depression settlement due to excessive dewatering.

4 -Uplift during construction due to high G.W.T.

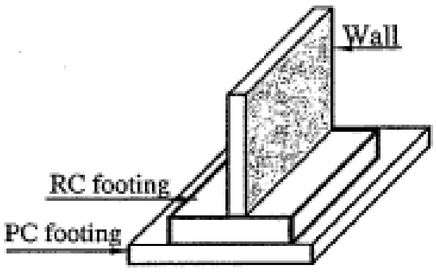
5 -Sliding or overturning due to large horizontal loads.



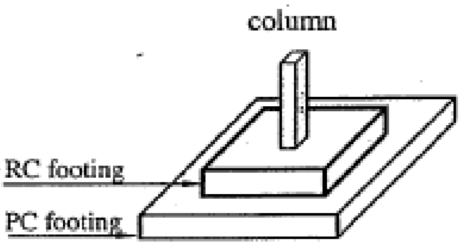
Shear failure

Deferential settlement

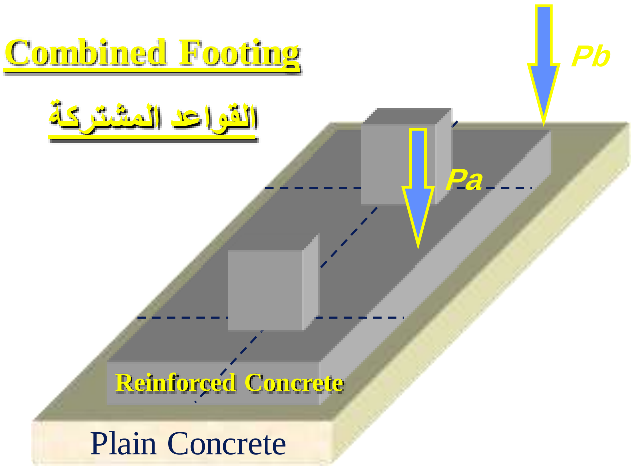
Overturning

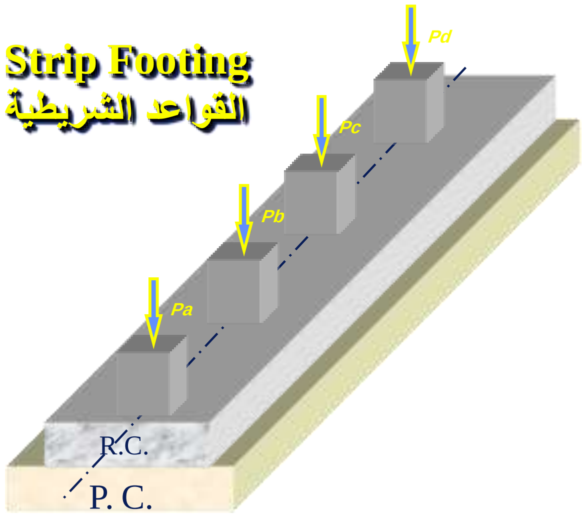


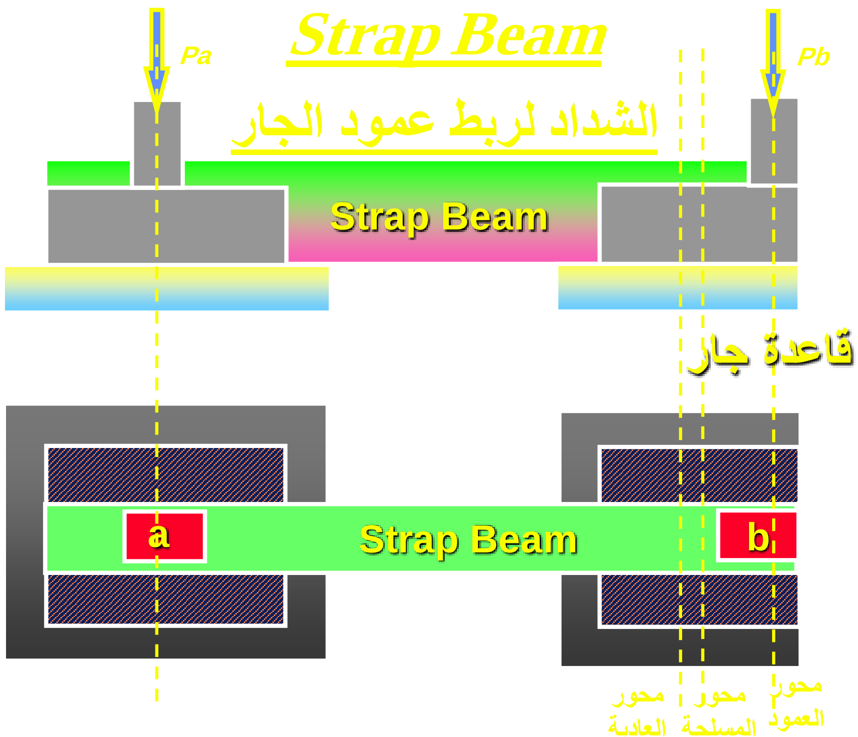
(a) Wall footing



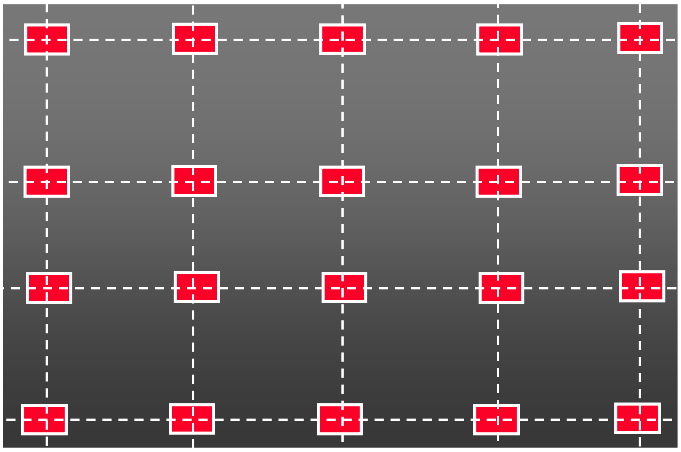
(b)Isolated footing

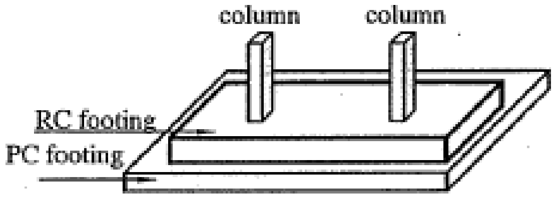




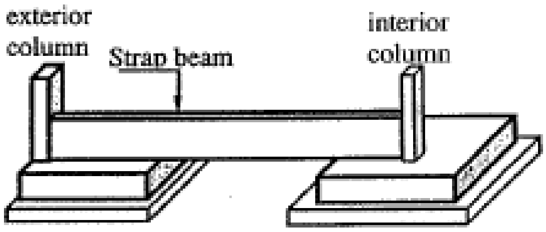


Raft Foundation

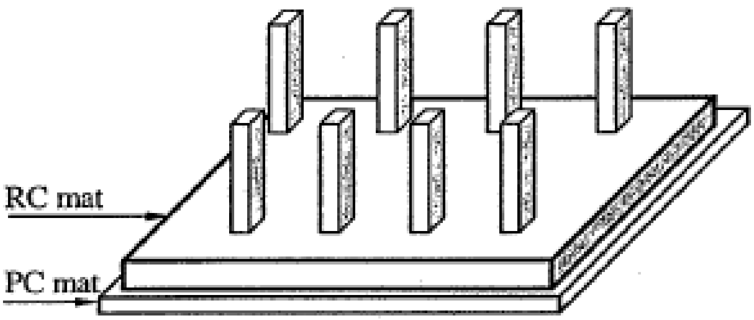




(c) Combined footing

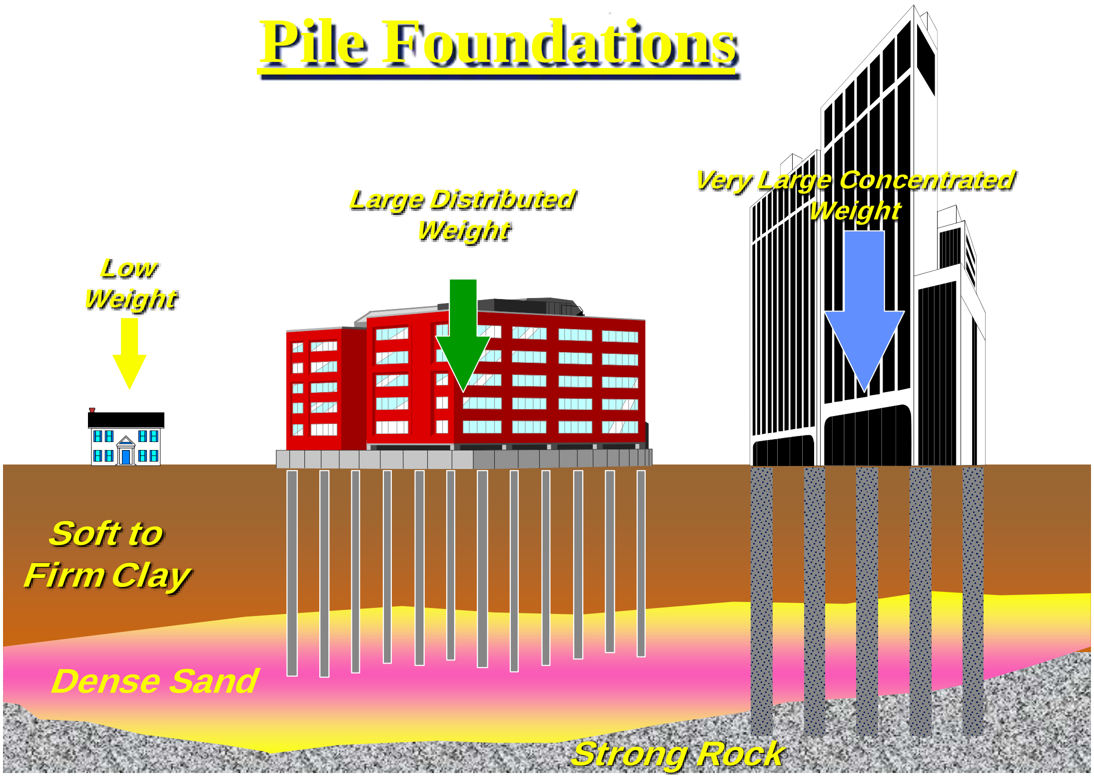


(d) Strap footing



(e) Raft foundation

Pile Foundations



Selection of Type of Foundations

Approximate Loads of Structures:

-Residential and Housing, 1.0 up to 1.2 t/m^2 per floor
-Commercial and Office, 1.2 up to 1.5 t/m^2 per floor
-Schools and Hospitals, 1.5 to 2.0 t/m^2 per floor
-These loads are multiplied by the number of floors, then divided by the foundation area to determine the soil stresses, then the type of foundation.
Three options will be available:
1- Stress on soil < q_{all} soil, R.C foundation area ≤ 2/3 foundation area ➔ Isolated footings.
2- Stress on soil < q_{all} soil, R.C foundation area > 2/3 foundation area ➔ Raft foundation.
3- Stress on soil > q_{all} soil ➔ Pile foundation.

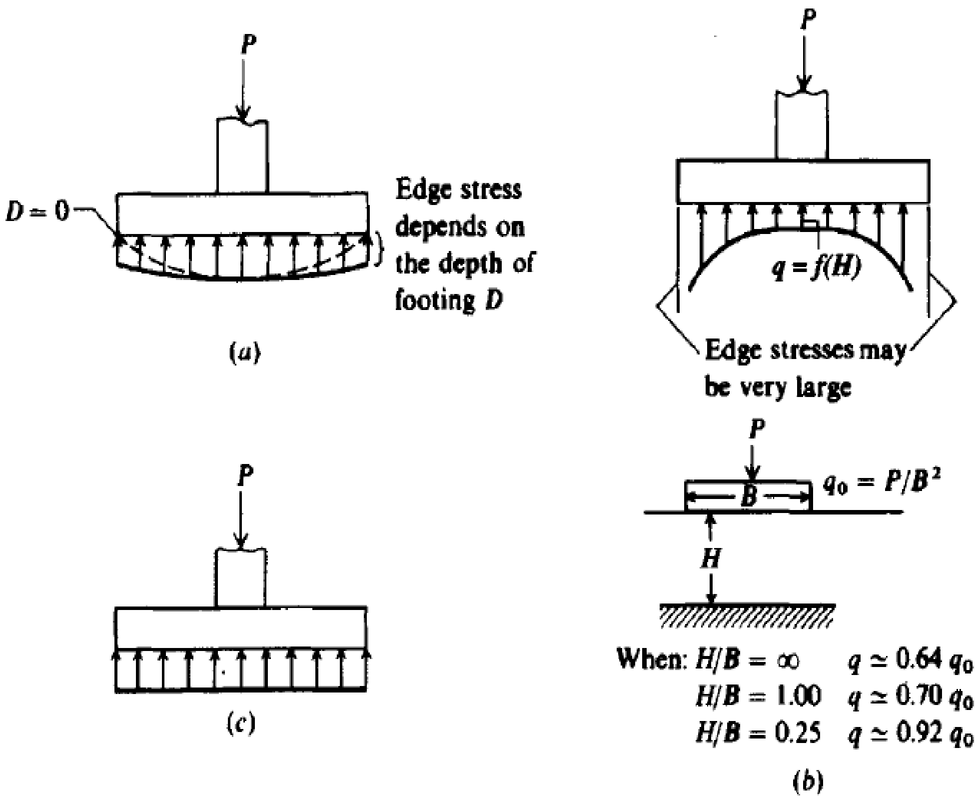


Figure 8-2 Probable pressure distribution beneath a rigid footing. (*a*) On a cohesionless soil; (*b*) generally for cohesive soils; (*c*) usual assumed linear distribution.

DESIGN OF SHALLOW FOUNDATION

Design of Shallow Foundation :

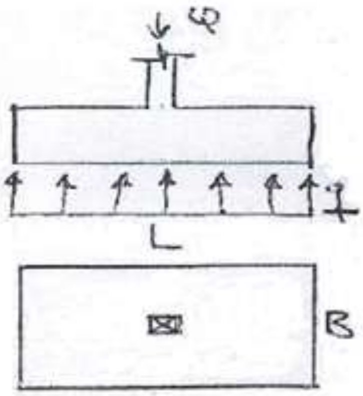
Each foundation with $D/B \leq 4$ is referred to it as " shallow foundation ".
Shallow foundation can be classified into

1- Spread footing (support one column).
2- Combined footing (support more than one column).

Spread Footing:

1) With concentric load only

The applied axial load acts at the center of gravity footing the pressure under the footing is uniform .



$q_{act} = Q/A \leq q_{all}$

$A = Q/q_{all}$

q_{all} = is given uniformly pressure under footing

Example: Design the following footing Q = 1000 kN , q_{all} = 200 kPa

Solution:

$A = Q/q_{all} = 1000/200 = 5\ m^2$

Square : $B = \sqrt{5} = 2.24$ m

Rectangle with B = 2 m , L = A/B = 5/2 = 2.5 m

Circle : $D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4*5}{\pi}} = 2.53$ m

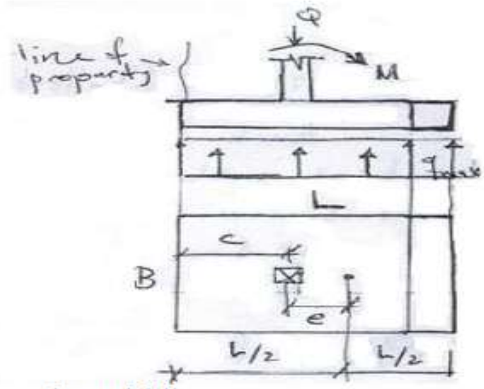
2) With eccentric load

To find the dimension of the footing that supports a column with axial load and moment , There are two methods :

A) Uniform pressure :

Location of the resultant of loads must be act at the center of gravity of the footing

$q_{act} = Q/A \le q_{all}$
$e = M / Q$
$A = Q/q_{all}$
$L/2 = c + e$
$L = 2 (c + e)$
$B = A / L$

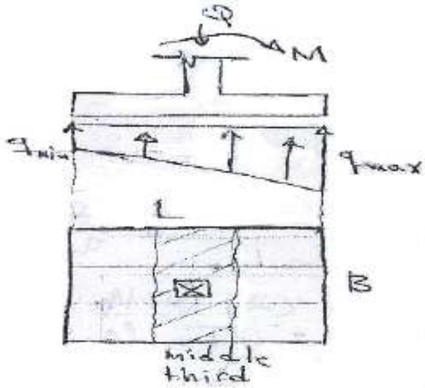


B) Varied Pressure :

Location of the resultant of loads must be act at the middle Third of the base .

$$q_{\substack{max\\min}} = \frac{Q}{BL}\left(1 \pm \frac{6\,e_L}{L}\right)$$

$q_{max} \le q_{all}$
$q_{min} \ge 0$



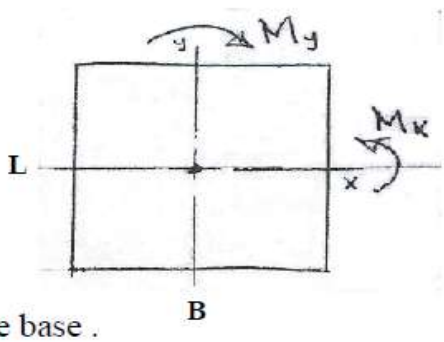
Note : A footing with moments about both axes :

$$q_{\substack{max\\min}} = \frac{Q}{BL}\left(1 \pm \frac{6\,e_B}{B} \pm \frac{6\,e_L}{L}\right)$$

$$e_L = \frac{Mx}{Q}$$

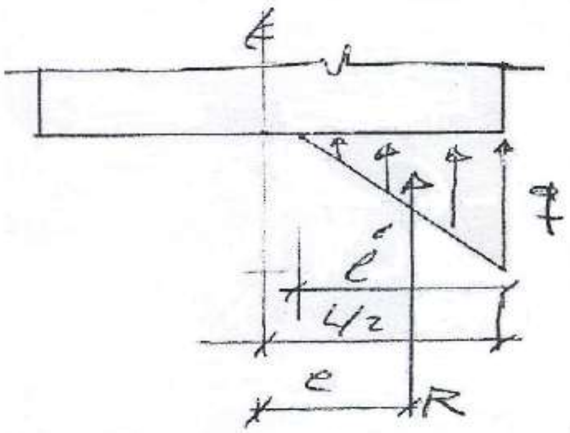
and

$$e_B = \frac{My}{Q}$$



Note : if e > L/6 then , there is a tension force on the base .
So , the compression stress (pressure) computed as :-
$\sum Fy = 0$
$R = q\,(l'\,B/2)$
$q = 2R / l'B$ --------- (1)
$L/2 - e = l'/3$
$l' = 3\,(L/2 - e)$ ------- (2)
sub. (2) in (1) :-

$$\mathbf{q = 4R / 3B\,(L - 2e_L)}$$



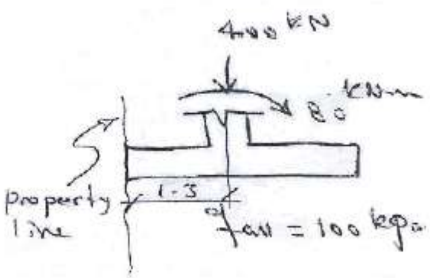
Example: Design the following footing for the cases:

a) Uniform pressure

b) resultant within middle third.

Solution:

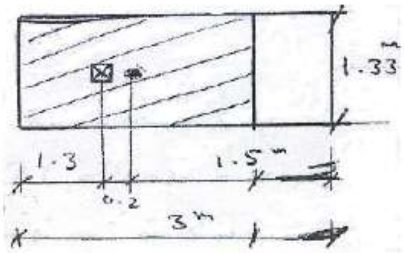
a) $e = M/Q = 80/400 = 0.2$ m



$L = 2(e + 1.3) = 2(0.2 + 1.3) = 3$ m

$A = Q/q_{all} = 400/100 = 4\ m^2$

$B = A/L = 4/3 = 1.33$ m



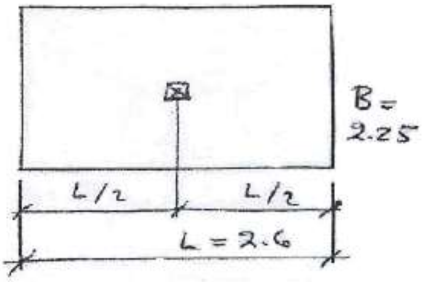
b) $L = 2 * 1.3 = 2.6$ m $(2.6/6 > e)$

$$q_{max} = \frac{400}{2.6 * B}\left(1 + \frac{6 * 0.2}{2.6}\right) = 100$$

$B = 2.25$ m

$$q_{min} = \frac{400}{2.6 * 2.25}\left(1 - \frac{6 * 0.2}{2.6}\right)$$

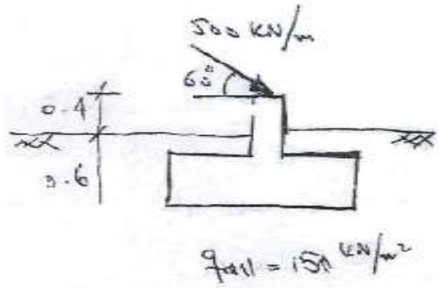
$= 36.8\ kN/m^2 \quad > 0$ O.K



Example: Design the strip footing shown:

Solution: the footing is strip $L = 1$

1) Uniform pressure



$A = Q/q_{all} = 433/150 = 2.89\ m^2$

$B = A/L = 2.89/1 = 2.89$ m

2) Non-uniform pressure:

e= (M/P) = (500 cos 60 * 1)/(500 sin 60)

= 0.577 m

$$q_{max} = q_{all} = 150 = \frac{433}{1 * B}\left(1 + \frac{6 * 0.577}{B}\right)$$

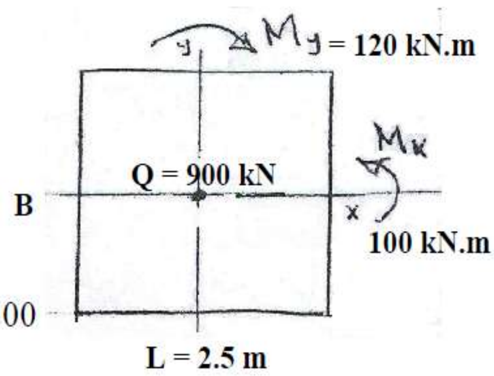
$0.346\ B^2 - B - 3.46 = 0$

$B = 4.92 \approx 5$ m $(B/6 = 5/6 = 0.833 > 0.577)$ O.K

$$q_{min} = \frac{433}{5 * 1}\left(1 - \frac{6 * 0.577}{5}\right) = 26.64\ kN/m^2 > 0$$ O.K

Example: Find B for the footing shown ,if $q_{all} = 200$ kPa

Solution:



$$e_L = \frac{My}{Q} = \frac{120}{900} = 0.133\,m$$

$$e_B = \frac{Mx}{Q} = \frac{100}{900} = 0.111\,m$$

$$q_{max} = \frac{900}{2.5 * B}\left(1 + \frac{6 * 0.111}{B} + \frac{6 * 0.133}{2.5}\right) = 200$$

$0.556\,B^2 - 1.319B - 0.666 = 0$

$B = 2.8$ m

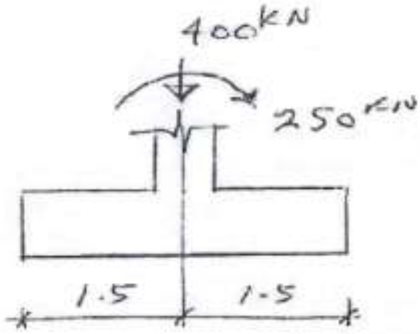
$e_B = 0.111 < 2.8/6 = 0.47$

$e_L = 0.133 < 2.5/6 = 0.42$

$$q_{min} = \frac{900}{2.5 * 2.8}\left(1 - \frac{6 * 0.111}{2.8} - \frac{6 * 0.133}{2.5}\right) = 57\,kN/m^2 > 0 \quad \text{O.K}$$

Example: Find B for the footing shown ,if $q_{all} = 200$ kPa

Solution:



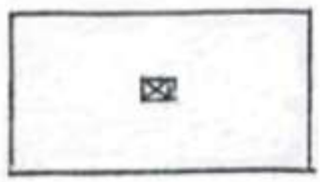
$e = M/Q = 250 / 400 = 0.625$ m

$L/6 = 3/6 = 0.5 < e$ Not Good

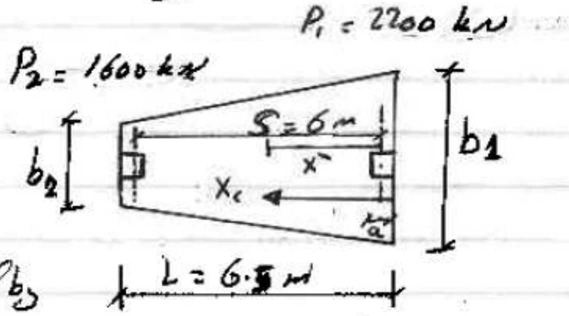
$q = 4\,R / 3\,B\,(L - 2\,e_L)$

$= 4 * 400 / 3\,B\,(3-2*0.625) = 200$

$B = 1.52$ m



EX: Redesign trapezoidal combined footing for a uniform soil pressure, shown in Fig below



$q_{all} = 200$ kPa.

Col size = 0.5×0.5 m

The following procedure should be followed.

1- Find $\bar{x}$ by taking moment at external col.

2- Calculate area of trapizoidal by

$$A = \frac{b_1 + b_2}{2} * L \quad \text{------} (1)$$

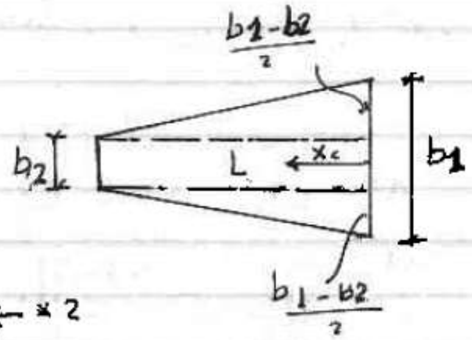
This area should be equal.

$$A = \frac{P_1 + P_2}{q_{all}} \quad \text{----} (2)$$

From eq (1) & eq (2) can be obtained a relationship between b_1 & b_2.

$$X_c = \bar{X} + \frac{a_1}{2}$$

$$X_c = \frac{L}{3}\left(\frac{2b_2 + b_1}{b_2 + b_1}\right)$$



This is comes from

$$x_c = \frac{(b_2 \cdot L)\left(\frac{L}{2}\right) + \left(\frac{b_1 - b_2}{2} \cdot \frac{L}{2}\right)\frac{L}{3} * 2}{\frac{b_1 + b_2}{2} \cdot L}$$

So X in Trapizoidal $\frac{L}{3} < X_c < \frac{L}{2}$

Sol. Find center of forces.

$$\bar{X} = \frac{1600\ (6)}{2200 + 1600} = 2.53 \text{ m} \quad \text{(From cent of ext. col.)}$$

$$X_c = 2.53 + 0.25 = 2.78 \quad \text{(From Age of ext. Col.)}$$

$$A = \left(\frac{P_1 + P_2}{q_{all}}\right) = \frac{2200 + 1600}{200} = 19 \text{ m}^2$$

$$A = \left(\frac{b_1 + b_2}{2}\right) L = 19 \ m^2 \qquad (L = 6m + 0.5 = 6.5m)$$

$$\therefore (b_1 + b_2) \times 6.5 = 38$$

$$\therefore b_1 + b_2 = 5.846 \text{ ------ } (1)$$

$$X_c = \frac{L}{3}\left(\frac{2b_2 + b_1}{b_1 + b_2}\right)$$

$$2.78 = \frac{6.5}{3}\left(\frac{2b_2 + b_1}{b_1 + b_2}\right)$$

$$b_1 = 2.57 \ b_2 \text{ ------ } (2)$$

$$2.57 \ b_2 + b_2 = 5.846$$

$$b_2 = 1.64 \ m$$

$$b_1 = 4.2$$

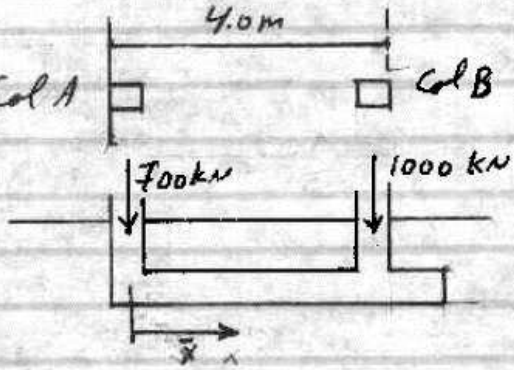
EX: Design Rectangular Combined Footing shown in Fig.
For uniform soil pressure

Sol.

1- Find Center of forces.

$$\bar{x} = \frac{1000(3.7)}{1700} = 2.171 \ m$$

For center of col A



Col size = 0.3 x 0.3 m

$q_{all} = 150 \ kN/m^2$

2- Find the length of footing such that center of forces and centroid coincide.

$$L = 2(2.171 + 0.15) = 4.642 m$$

Use $L = 4.65 \ m$.

3- Find the width of footing.

$$q_{all} \geq q_{act} = \frac{P}{BL}$$

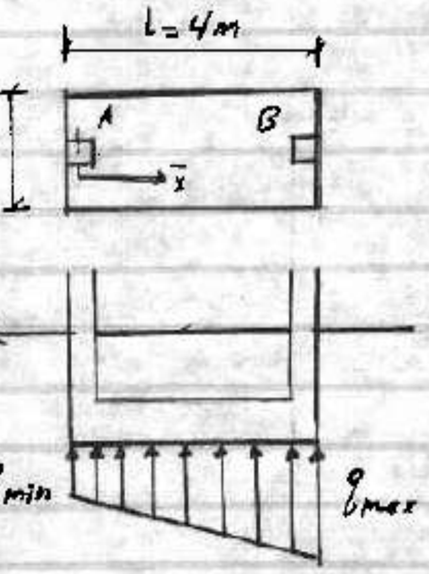
$$150 = \frac{1700}{B(4.65)}$$

$$B = 2.45 \ m$$

$$\therefore Area = 4.65 \times 2.45 = 11.4 \ m^2$$

EX: If the length of footing for the last example is 4.0 m. Redesign the width of footing.

sol:



1- Find Center of forces.

$$\bar{X} = \frac{1000\,(3.7)}{1700} = 2.17\ m$$

2- Find the eccentricity

$$e = \bar{X} - \frac{L}{2} = (2.17 + 0.15) - 2 = 0.32\ m$$

∴ no uniform pressure.

3- Find width such that $q_{max} \le q_{all}$

$$q_{max} = \frac{P}{BL}\left(1 + \frac{6e}{L}\right) = \frac{1700}{B(4)}\left(1 + \frac{6\,(0.32)}{4}\right) = 150$$

$B = 4.19\ m$ use $B = 4.2\ m$

4- Check minimum pressure

$$q_{min} = \frac{P}{BL}\left(1 - \frac{6e}{L}\right) = \frac{1700}{4.2(4)}\left(1 - \frac{6(0.32)}{4}\right)$$

$$= 52.6\ kPa > 0 \quad \therefore o.k.$$

$$\therefore Area = 4\,(4.2) = 16.8\ m^2$$

Example 9-1. Design a rectangular combined footing using the conventional method.

Given. f'_c = 21 MPa (column and footing) f_y = Grade 400 q_a = 100 kPa

Column number	Working loads						
	DL	LL, kN	M_D	M_L	P	P_u, kN	M_u, kN · m
1	270	270	28	28	540	837	86.8
2	490	400	408	40	890	1366	124
				Total	1430	2203	

Ultimate values = $1.4DL + 1.7LL$, etc.

$$\text{Soil}: \quad q_{\text{ult}} = \frac{\sum P_u}{\sum P} q_a = \frac{2203}{1430}(100) = \mathbf{154.1} \text{ kPa}$$

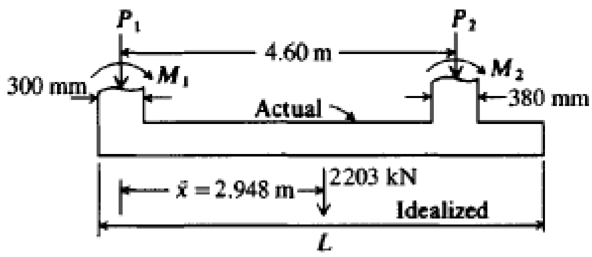


Figure E9-1a

It is necessary to use q_{ult} so base eccentricity is not introduced between computing L using q_a and L using q_{ult}.

Solution.

Step 1. Find footing dimensions.

$$\sum M_{\text{col.1}} = R\bar{x} \qquad \text{where} \qquad R = \sum P_u = 837 + 1366 = 2203 \text{ kN}$$

For uniform soil pressure R must be at the centroid of the base area (problem in elementary statics), so we compute

$$R\bar{x} = M_1 + M_2 + SP_{\text{ult},2}$$

$$2203\bar{x} = 86.8 + 124.0 + 4.60(1366)$$

$$\bar{x} = \frac{6494.4}{2203} = 2.948 \text{ m}$$

It is evident that if $\bar{x}$ locates the center of pressure the footing length is

$$L = 2 \times (\tfrac{1}{2} \text{ width of col. } 1 + \bar{x}) = 2 \times (0.150 + 2.948) = \mathbf{6.196} \text{ m}$$

Also for a uniform soil pressure q_{ult} = 154.1 kPa, the footing width B is computed as

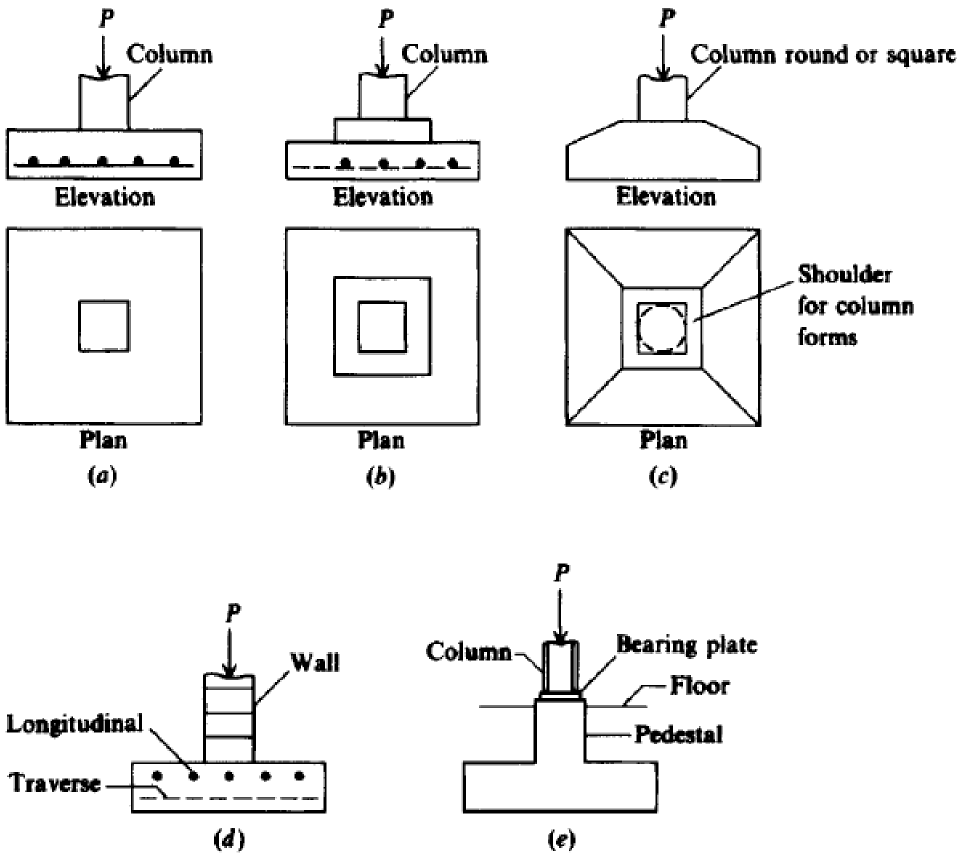
$$BLq_{\text{ult}} = P_{\text{ult}}$$

$$B = \frac{2203}{6.196 \times 154.1} = \mathbf{2.307} \text{ m}$$

We will have to use these somewhat odd dimensions in subsequent computations so that shear and moment diagrams will close. We would, however, round the dimensions for site use to

$$L = \mathbf{6.200} \text{ m} \qquad B = \mathbf{2.310} \text{ m}$$

STRUCTURAL DESIN OF SPREAD FOOTING



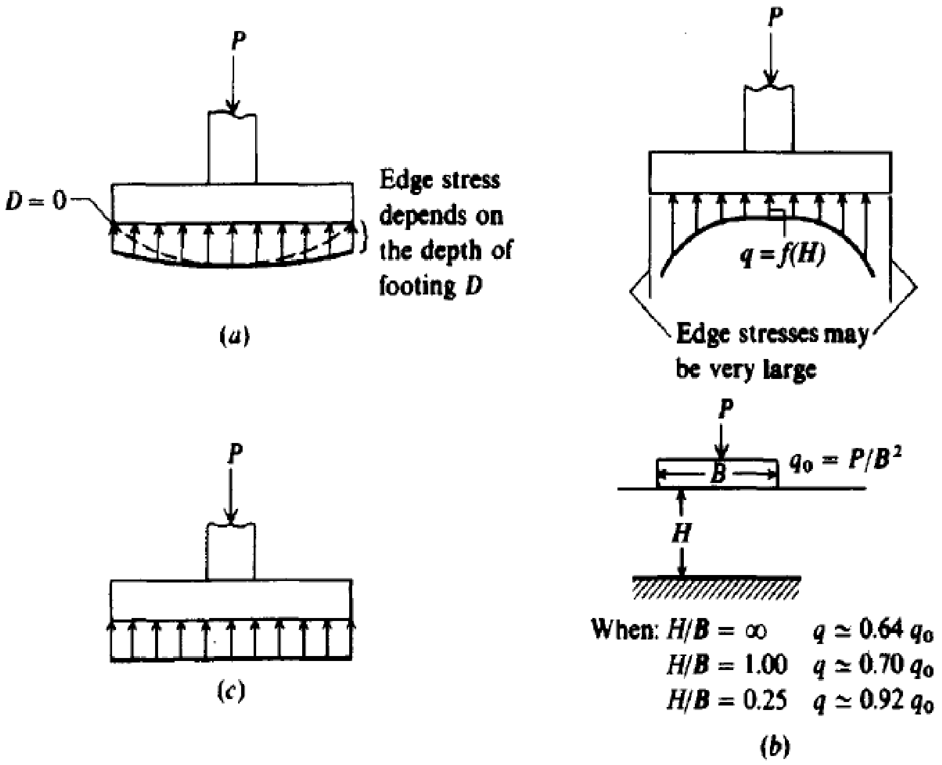
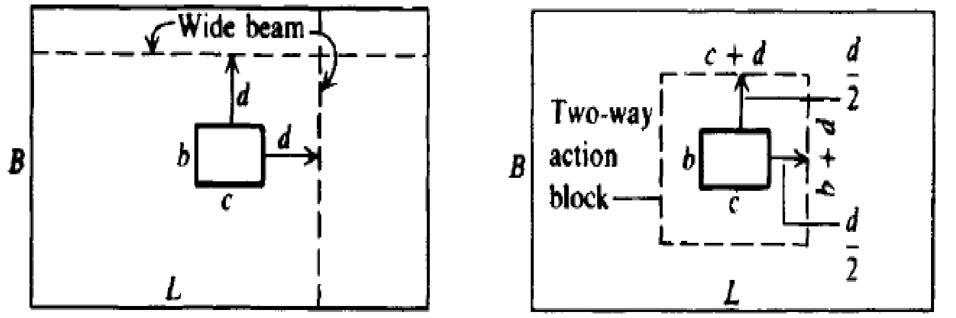


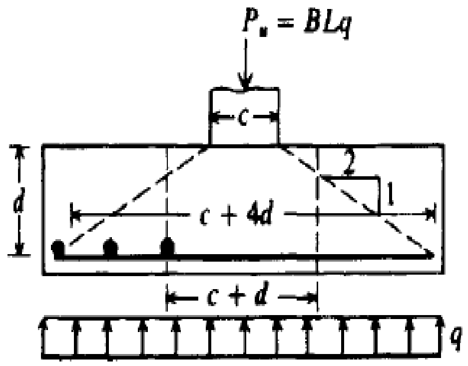
Figure 8-2 Probable pressure distribution beneath a rigid footing. (*a*) On a cohesionless soil; (*b*) generally for cohesive soils; (*c*) usual assumed linear distribution.

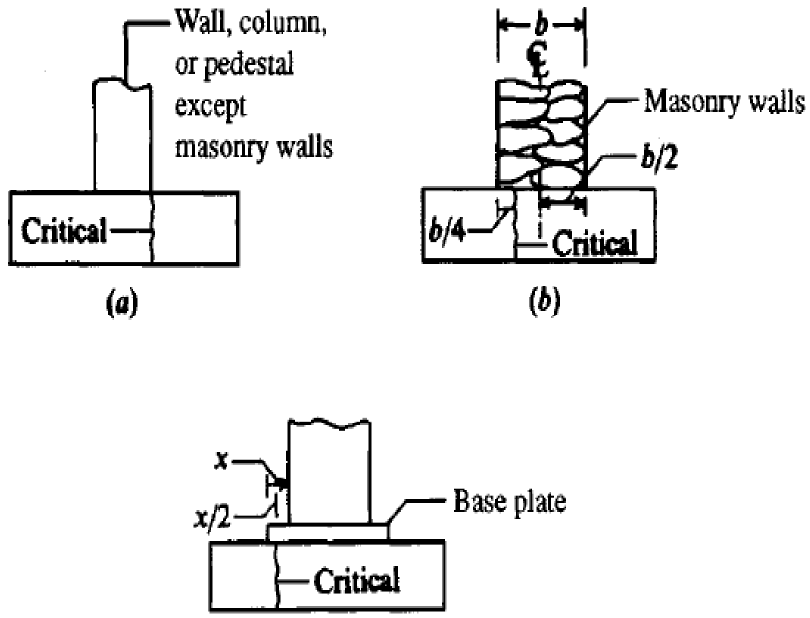


(a) See Fig. E8-1a for round column.

(b)

$A_2 = (b + 4d)(c + 4d)$
$A_1 = b \times c$





(a)

(b)

(c)

Figure 8-5 Sections for computing bending moment. Bond is computed for section indicated in (*a*) for all cases; however, for convenience use bond at same section as moment.

Example: Design a spread footing for the given data:

B×B size, q_{all}=200 kPa, DL=350 kN, LL=450 kN, $f'c$=21 Mpa, fy=400 Mpa.

Column size=0.35 × 0.35 m, use Ø 16 mm bars.

Solution:

Step 1: find the dimensions

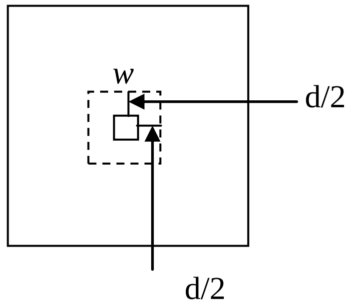
$q_{act.} \le q_{all} = P/A$ $\quad 200 = (350+450)/A \quad A = 4\ m^2 \quad B = 2\ \ m$

step 2: find the effective depth (d) of footing using

- **two-way action (punching shear):**

$$d^2\left(v_c + \frac{q}{4}\right) + d\left(v_c + \frac{q}{2}\right)w = (BL - w^2)\frac{q}{4}$$

$$v_c = \frac{1}{3}\phi\sqrt{f'c} = \frac{1}{3}\times 0.85\times\sqrt{21} = 1.298 N/mm^2 = 1298 kN/m^2$$



$$q = q_{ult} = \frac{1.2\times(350)+1.6\times(450)}{2^2} = 285 kN/m^2$$

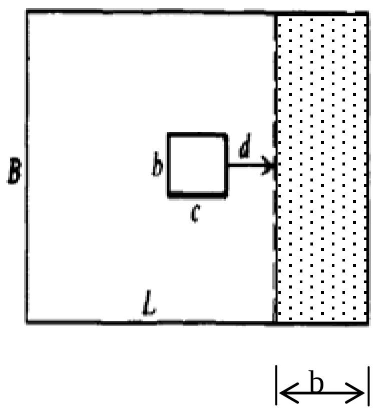
$$w = 0.35m$$

$$\therefore d^2(1298+\frac{285}{4}) + d(1298+\frac{285}{2})0.35 = (2\times2-0.35^2)(\frac{285}{4})$$

$$1369.25d^2 + 504.2d - 276.3 = 0$$

$\therefore$

$$d = 0.3m = 300mm$$



- **For wide beam action**

B = L = 2 m

$$v_c = \frac{1}{6}\phi\sqrt{f'c} = \frac{1}{6}\times 0.85\times\sqrt{21} = 0.649 N/mm^2 = 649 kN/m^2$$

b = (L/2 – w/2 – d)

$A_{wide} = b \times B$

$v_c \times d \times B = q_{ult} \times b \times B$

649 × d = 285 × (2/2 – 0.35/2 – d)

649 d = 235.125 – 285 d

d = 0.25 m < d = 0.3 m (punching shear)

then,

use d = 0.3 m = 300 mm (punching shear controlled).

$$H = d + cover = 300 + 70 + \frac{1}{2} d_{bar} \cong 378mm$$

Use

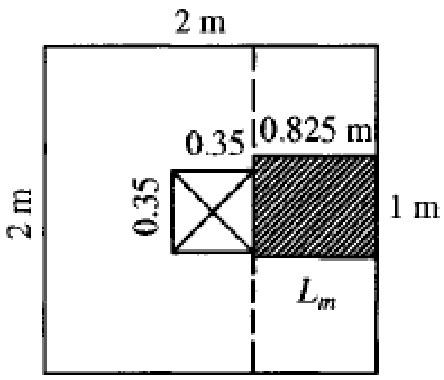
$$H = 400mm$$

Step 3: find required reinforcement:-
The arm of moment is to the face of column,

$$L_m = \frac{B}{2} - \frac{w}{2} = \frac{(B-w)}{2} = \frac{(2-0.35)}{2} = 0.825m$$

$$M_u = \frac{q_u \times L_m{}^2}{2} = \frac{285 \times (0.825)^2}{2} = 97kN.m$$

$$\rho = \frac{\left(1 \mp \sqrt{1 - \frac{2.6222 \times M_u \times 10^6}{bd^2 f'c}}\right)}{\left(\frac{1.18 fy}{f'c}\right)}$$



$$b = 1000mm,;;; d = 300mm;;;$$

$$\rho = 0.0031$$

$$\rho_{\min.} = \frac{1.4}{fy} = \frac{1.4}{400} = 0.0035.........then...used$$

$$A_s = \rho \times b \times d$$

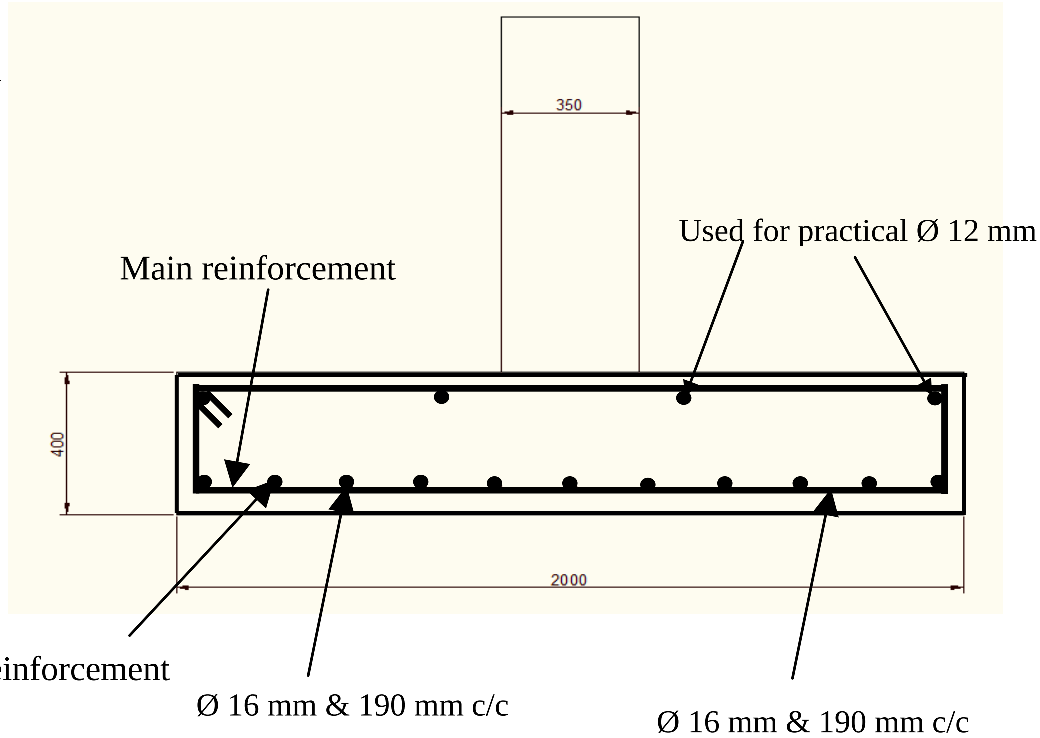
$$A_s = 0.0035 \times 1000 \times 300 = 1050mm^2$$

No. of bars=1050/201= 6 bars

Spacing = 1000/(6-1) = 200 mm c/c

Then, use Ø 16 mm bars @ 190 mm c/c in two directions for equal distribution of bars.

Step 4: sketch



DEEP FOUNDATION (PILES)

SINGLE PILES-STATIC CAPACITY:

Piles are structural members of timber, concrete, and/or steel that are used to transmit surface loads to lower levels in the soil mass. This transfer may be by vertical distribution of the load along the pile shaft or a direct application of load to a lower stratum through the pile point

- Piles are commonly used (refer to Fig. 16-1) for the following purposes:

1. To carry the superstructure loads into or through a soil stratum. Both vertical and lateral loads may be involved.
2. To resist uplift, or overturning forces, such as for basement mats below the water table or to support tower legs subjected to overturning from lateral loads such as wind.
3. To compact loose, cohesionless deposits through a combination of pile volume displacement and driving vibrations. These piles may be later pulled.
4. To control settlements when spread footings or a mat is on a marginal soil or is underlain by a highly compressible stratum.
5. To stiffen the soil beneath machine foundations to control both amplitudes of vibration and the natural frequency of the system.
6. As an additional safety factor beneath bridge abutments and/or piers, particularly if scour is a potential problem.
7. In offshore construction to transmit loads above the water surface through the water and into the underlying soil. This case is one in which partially embedded piling is subjected to vertical (and buckling) as well as lateral loads.

CONCRETE PILES:

Table 16-1 (Bowles) indicates that concrete piles may be precast, prestressed, cast in place, or of composite construction.

Precast Concrete Piles:

Piles in this category are formed in a central casting yard to the specified length, cured, and then shipped to the construction site. If space is available and a sufficient quantity of piles needed, a casting yard may be provided at the site to reduce transportation costs.

Cast-in-Place Piles:

A cast-in-place pile is formed by drilling a hole in the ground and filling it with concrete. The hole may be drilled (as in caissons), or formed by driving a shell or casing into the ground. The casing may be driven using a mandrel, after which withdrawal of the mandrel empties the casing. The casing may also be driven with a driving tip on the point, providing a shell that is ready for filling with concrete immediately, or the casing may be driven open-end, the soil entrapped in the casing being jetted out after the driving is completed.

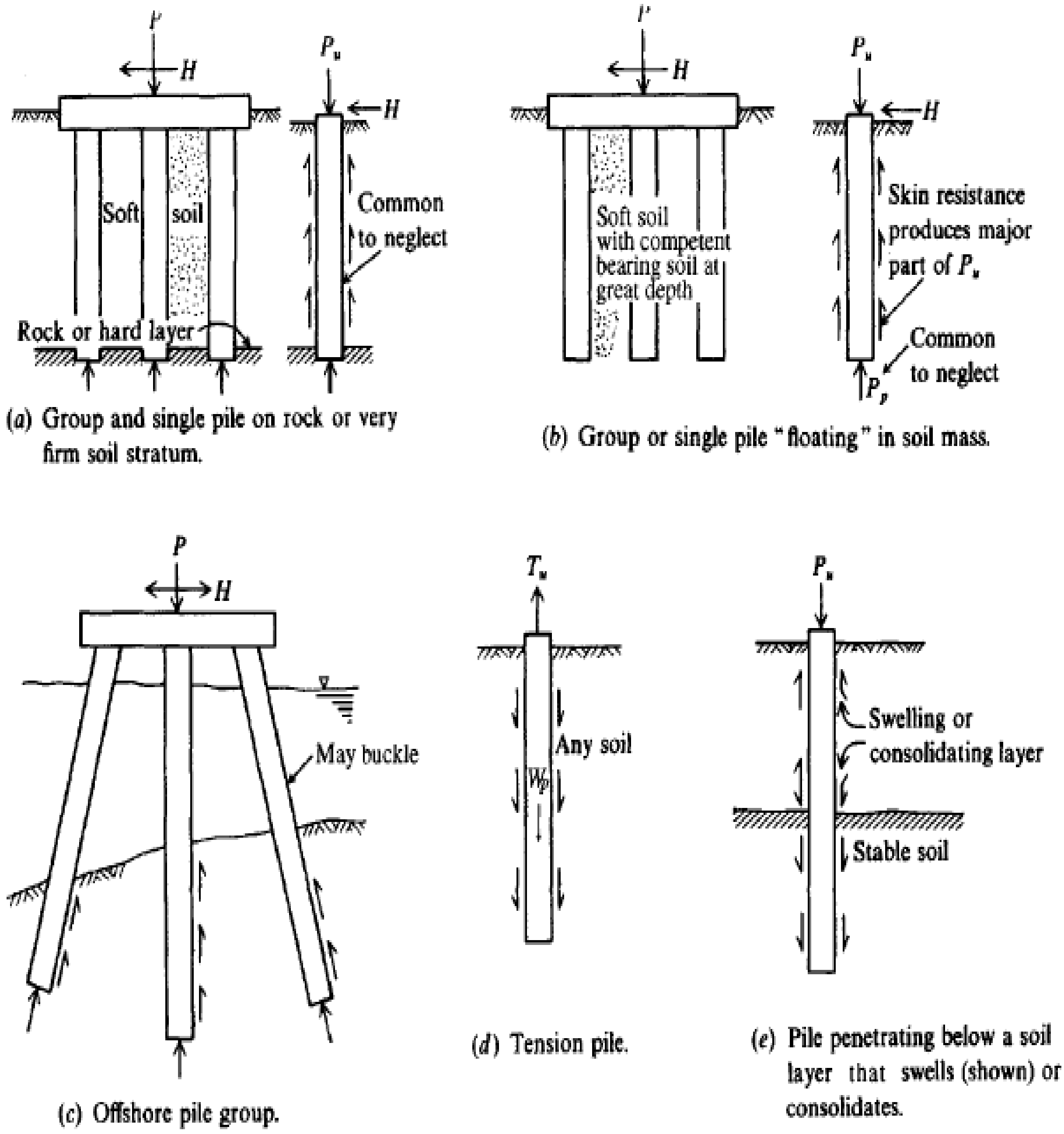
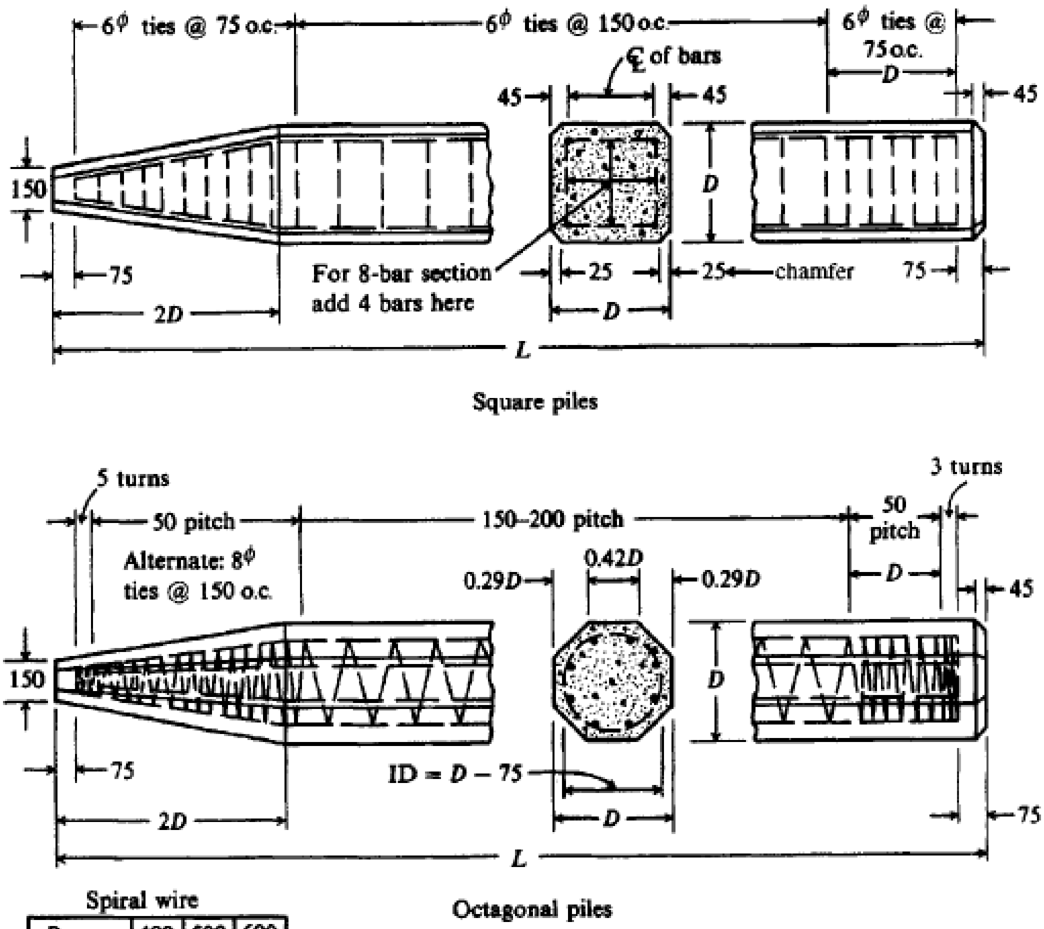


Figure 16-1 Typical pile configurations. Note that, whereas analysis is often for a single pile, there are usually three or more in a group. Typical assumptions for analysis are shown. Lateral load H may not be present in (a) or (b).



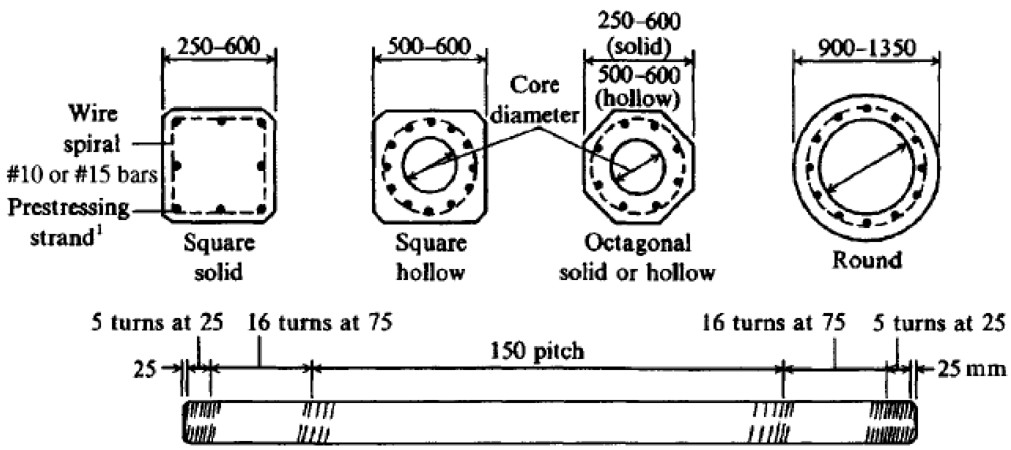
Spiral wire

D, mm	400	500	600
US bar	#5	#4	#3
SI bar	15	10	10

mm

6φ = 6 mm diameter (not standard diameter SI bar)

Figure 16-4 Typical details of precast piles. Note all dimensions in millimeters. [*After PCA (1951).*]



[1] Strand: 9.5–12.7 mm ($\frac{3}{8}$ to $\frac{1}{2}$ in.) nominal diam., f_u = 1860 MPa

Figure 16-5 Typical prestressed concrete piles (see also App. A, Table A-5); dimensions in millimeters.

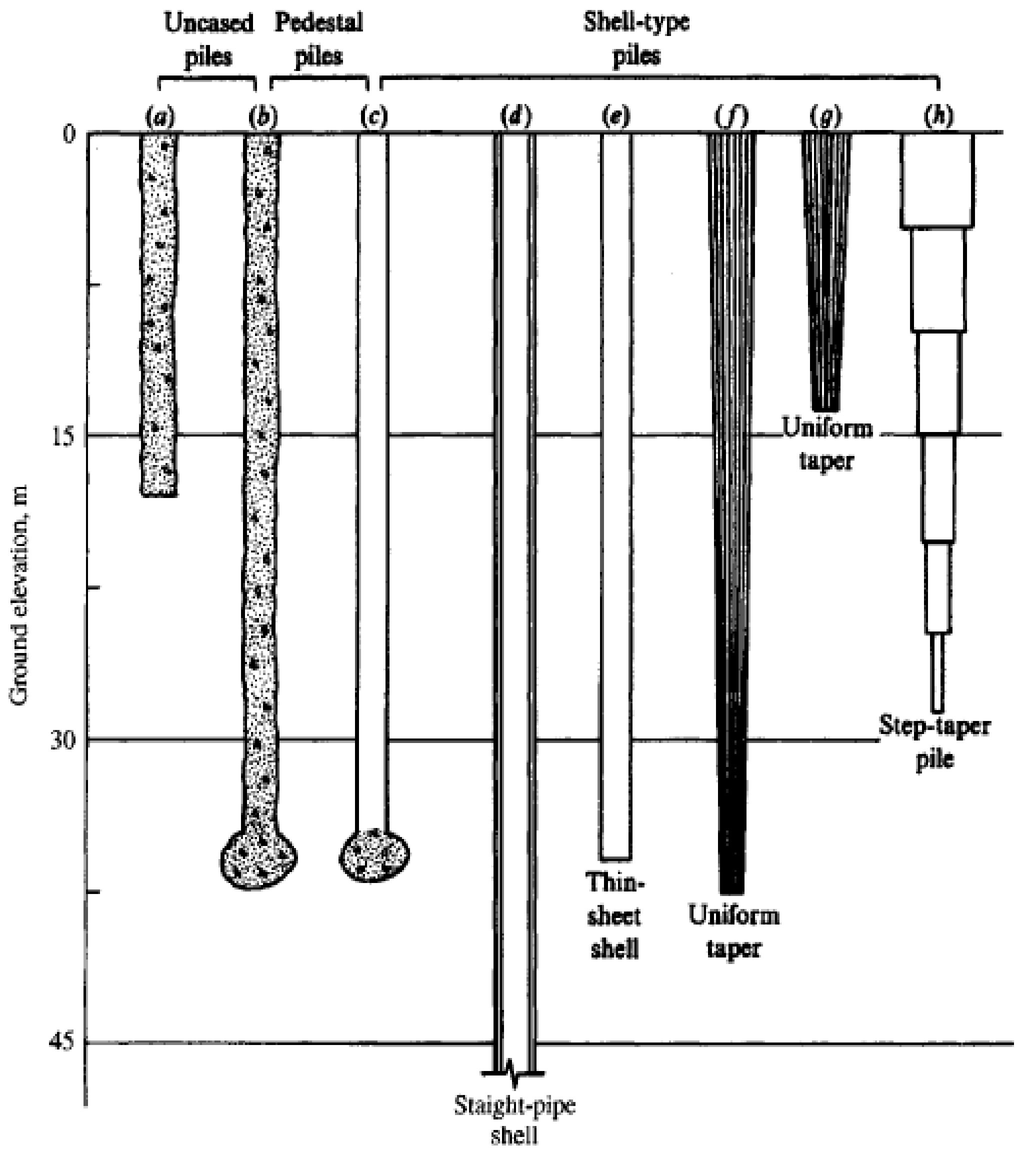


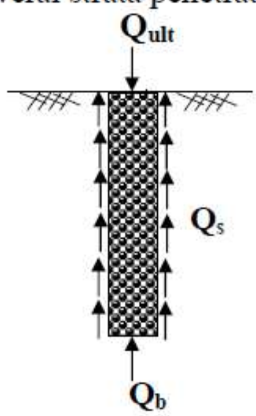
Figure 16-7 Some common types of cast-in-place (patented) piles: (*a*) Commonly used uncased pile; (*b*) Franki uncased pedestal pile; (*c*) Franki cased pedestal pile; (*d*) welded or seamless pipe; (*e*) Western cased pile; (*f*) Union or Monotube pile; (*g*) Raymond standard; (*h*) Raymond step-taper pile. Depths shown indicate usual ranges for the various piles. Current literature from the various foundation equipment companies should be consulted for design data.

Static Pile Capacity

In this approach the pile capacity can be estimated based on soil properties. The soil parameters needed are the angle of internal friction (ϕ) and the cohesion (**c**). So, the ultimate static pile capacity can be computed as:-

$$Q_{ult} = Q_b + \Sigma Q_s$$

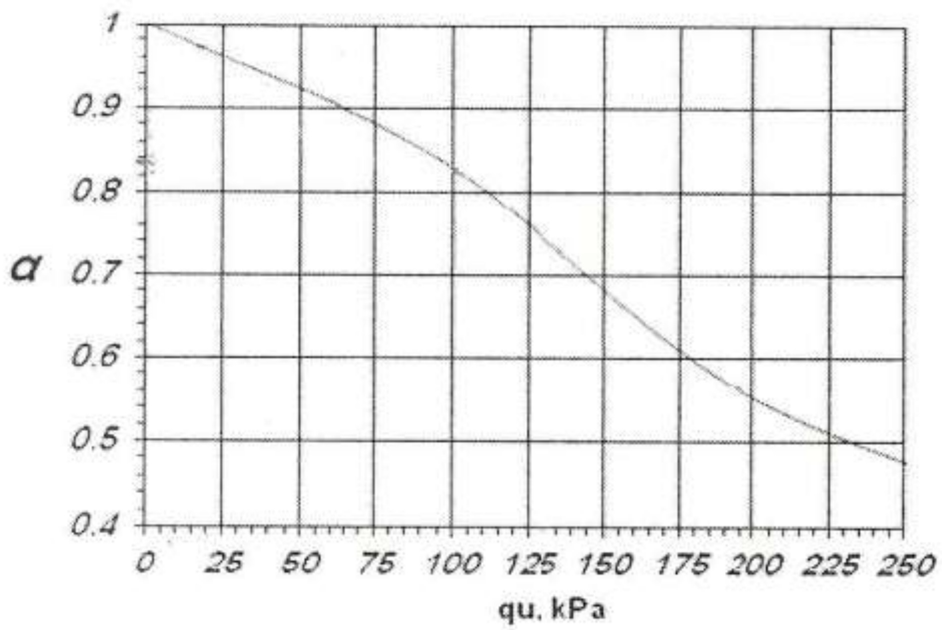
Where: Q_b : end bearing (end resistance).
Q_s : Skin resistance (shaft friction) contribution from several strata penetrated by pile.



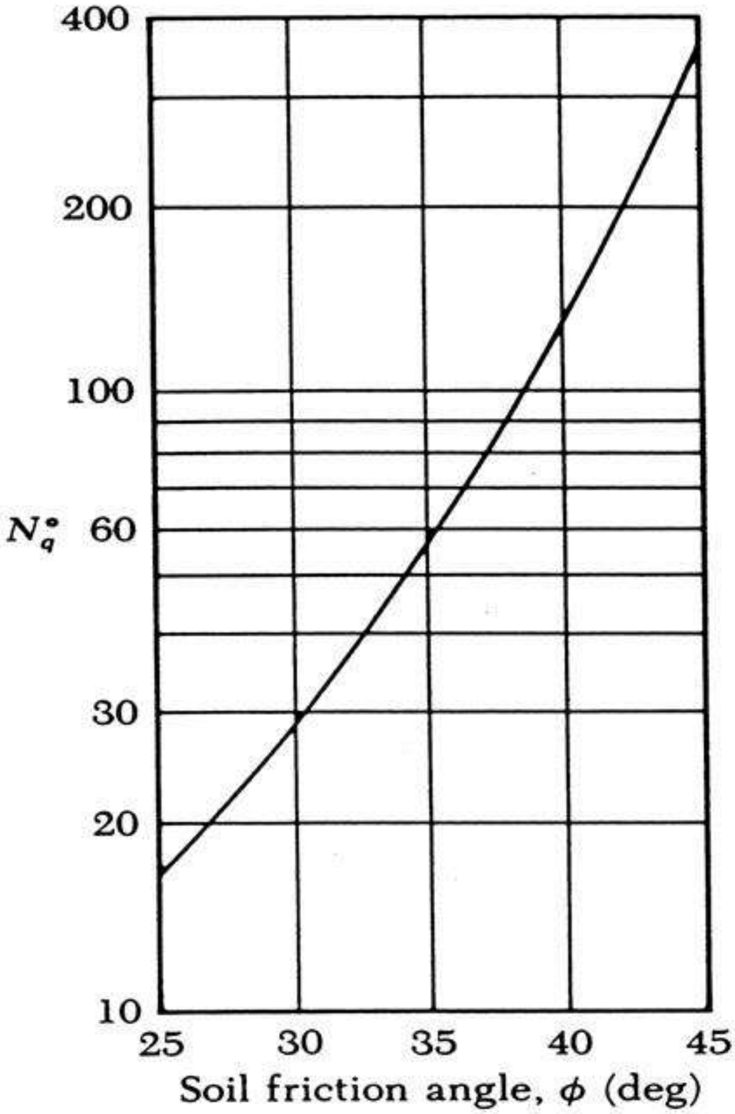
And the allowable pile capacity is:-

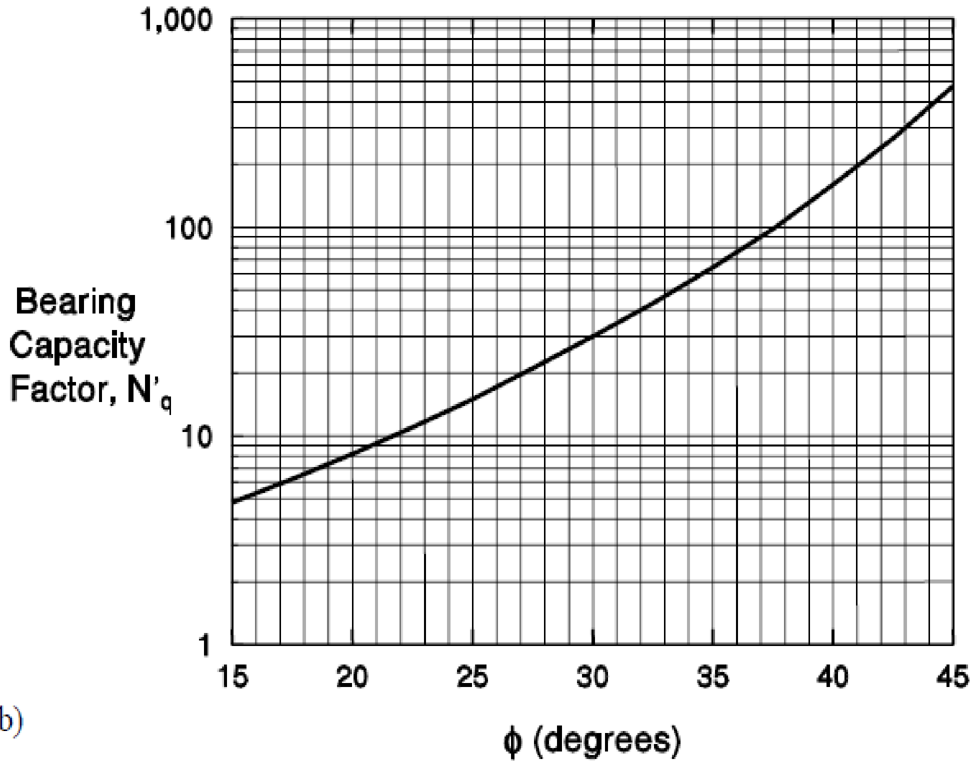
$$Q_{all} = \frac{Q_{ult}}{FS} \quad \text{or} \quad \frac{Q_b}{FS1} + \frac{Q_s}{FS2}$$

Which one is small.



Values of reduction factor α for calculation of static capacity of friction piles


400
200
100
N_q^* 60
30
20
10
25
30
35
40
45
Soil friction angle, ϕ (deg)



9-13(b)

Figure 9-13. Chart for estimating α_t coefficient and bearing capacity factor N'_q (FHWA, 2006a).

TABLE 4-4

ϕ	N_c
0	5.14*
5	6.49
10	8.34
15	10.97
20	14.83
25	20.71
26	22.25
28	25.79
30	30.13
32	35.47
34	42.14
36	50.55
38	61.31
40	75.25
45	133.73
50	266.50

$\mathbf{Q_{ult} = Q_b + \sum Q_s}$

$\mathbf{Q_b \le Q_{b\,max}}$ $\mathbf{Q_{b\,max}}$ **for sand (C=0) =** $\mathbf{(50\ N_q \tan Ø)\ A_b}$
$\mathbf{Q_{b\,max}}$ **for** $\mathbf{(C \ne 0) = (C_b\ N_c + 50\ N_q \tan Ø)\ A_b}$

$$\mathbf{Q_{ult\,(s.p)} = (\ C_b\ N_c + q\ N_q\ )\ A_b\ + \sum(\ \alpha\ C_s + \sigma_v\ K \tan \delta\ )A_s}$$

$\mathbf{C_b}$ **= cohesion of the layer of the end of pile.**

-N_c = 9 for piles ($Ø_b$ = 0).
-For $Ø_b$ > 0 is given from special chart, OR:
N_c = {N_c from table 4-4 above (Bowles) × 1.6 } (Bowles page 892) .
1.6 ≈ d_c (suggested by Bowles).
q = soil pressure from the ground surface to the end of pile.
N_q = Using figure above (meyerhof). We can neglect the end bearing resistance for Ø ≤ 15 degrees.
α = Using figure. (α = 0.45 for bored pile).
C_s = cohesion of each layer surrounding the pile.
σ_v = pressure of soil from the ground surface to the mid- height of pile within the layer.

$\mathbf{K} = (1-\sin\phi)\sqrt{OCR} \le 1.75$ for driven pile .
$\mathbf{K} = 1-\sin\phi / 1+\sin\phi$ for bored pile .
$\delta = \frac{2}{3}\phi$

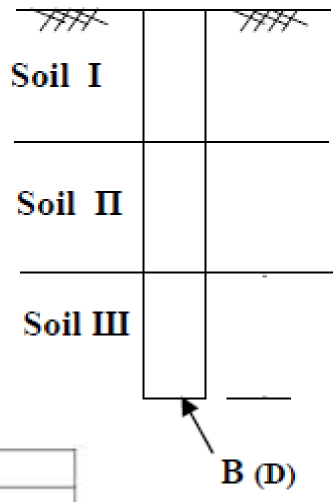


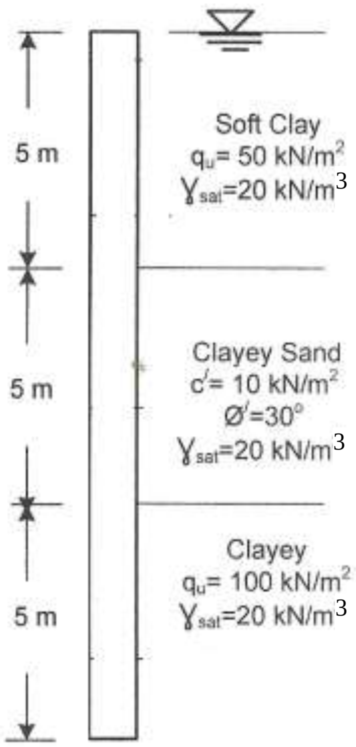
Table (P2): Values of δ and k for various type of piles

Pile type	$\delta°$	k		
		Loose	Medium	Dense
Steel	20	0.5	0.75	1.0
Concrete	0.75 ϕ	1.0	1.5	2.0
Timber	0.67 ϕ	1.5	2.75	4.0

Example: for the pile shown in Figure below, Find the ultimate capacity: pile size D = 0.3 m.

Solution:

$Q_{sp} = \sum Q_s + Q_b$



1-Skin resistance:

Layer 1

$\mathbf{Q_{s1} = (\alpha\, C_s + \sigma_v\, K \tan\delta\,)A_s}$

α for q_u = 50 kPa = 0.92 from figure

$C_s = q_u/2 = 50/2 = 25$ kPa.

$\delta = 0$

$\mathbf{Q_{s1} = (0.92 \times 25\ kN/m^2 + 0\,) \times (\,\pi(0.3\ m) \times 5\ m)}$

= <u>108.33</u> kN.

Layer 2

$\mathbf{Q_{s2} = (\alpha\, C_s + \sigma_v\, K \tan\delta\,)A_s}$

α for $q_u = 2 \times C = 2 \times 10 = 20$ kPa = 0.96 from figure

C_s= 10 kPa.

$\delta = 2/3\,(30) = 20$

$K = 1 - \sin 30 = 0.5$

$\sigma_v = (20 - 9.81) \times 7.5\ m = 76.425\ kN/m^2$

$\mathbf{Q_{s2} = (0.96 \times 10\ kN/m^2 + 76.425\ kN/m^2 \times 0.5 \tan 20) \times (\,\pi(0.3\ m) \times 5\ m)}$

= <u>110.7</u> kN.

Layer 3

α for q_u = 100 kPa = 0.83 from figure

$C_s = q_u/2 = 100/2 = 50$ kPa.

$\delta = 0$

$\mathbf{Q_{s3} = (0.83 \times 50\ kN/m^2 + 0\,) \times (\,\pi(0.3\ m) \times 5\ m)}$

= <u>195.47</u> kN.

2-End resistance:

$Q_b = (C_b\, N_c + q\, N_q)\, A_b$

For Ø = 0 $\quad N_q = 1$ (not found in figure) (given)

$Q_b = [\,(100/2) \times 9 + (20\text{-}9.81) \times 5\text{ m} \times 3 \times 1\,] \times (\pi(0.3\text{ m})^2 /4)$
$= 42.6\ \text{kN}$

$Q_{b\,max}$ **for** $(C \neq 0) = (C_b\, N_c + 50\, N_q \tan Ø)\, A_b = (50\times9 + 50\times1\times\tan 0) \times (\pi(0.3\text{ m})^2 /4) = 31.8$ **kN Use it**

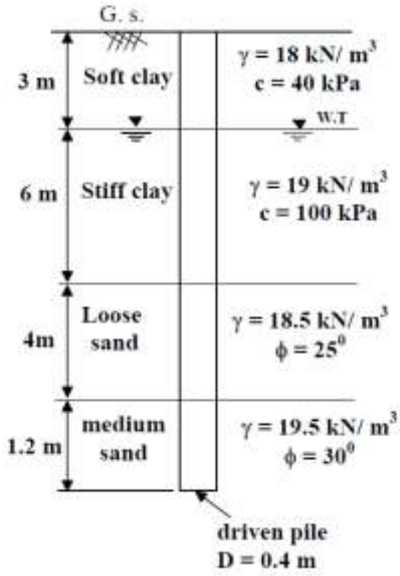
$Q_{sp} = \sum Q_s + Q_b = (108.33 + 110.7 + 195.47 + 31.8)$
$= 446.3\ \text{kN}$

****

Q1:

For the soil – pile system shown in Fig. Compute allowable pile capacity,
F.S = 2.5

Solution:



$Q_{sp} = \sum Q_s + Q_b$

1-Skin resistance:

Layer 1

$Q_{s1} = (\alpha\, C_s + \sigma_v\, K \tan \delta\,) A_s$

α for $q_u = 2C_s = 40 \times 2 = 80$ kPa $= 0.865$ from figure

$\delta = 0$

$Q_{s1} = (0.865 \times 40\text{ kN/m}^2 + 0\,) \times (\,\pi(0.4\text{ m}) \times 3\text{ m})$
$= \underline{130.4}$ **kN.**

Layer 2

$Q_{s2} = (\alpha\, C_s + \sigma_v\, K \tan \delta\,) A_s$

α for $q_u = 2 \times C = 2 \times 100 = 200$ kPa $= 0.54$ from figure

$Q_{s2} = (0.54 \times 100\text{ kN/m}^2 + 0\,) \times (\,\pi(0.4\text{ m}) \times 6\text{ m}) = \underline{406.9}$ **kN.**

Layer 3

$Q_{s3} = (\alpha\ C_s + \sigma_v\ K \tan\delta\ )A_s$

$C_s = 0$

$\delta = 2/3\ (25) = 16.67$

$K = 1 – \sin 25 = 0.577$

$\sigma_v = 18 \times 3\ m + (19\text{-}9.81) \times 6\ m + (18.5 – 9.81) \times 2\ m$

$= 126.52\ kN/m^2$

$Q_{s3} = (\ 0\ + 126.52\ kN/m^2 \times 0.577 \tan 16.67) \times (\ \pi(0.4\ m) \times 4\ m)$

$= \underline{109.82}\ kN.$

Layer 4

$Q_{s4} = (\alpha\ C_s + \sigma_v\ K \tan\delta\ )A_s$

$C_s = 0$

$\delta = 2/3\ (30) = 20$

$K = 1 – \sin 30 = 0.5$

$\sigma_v = 18 \times 3\ m + (19\text{-}9.81) \times 6\ m + (18.5 – 9.81) \times 4\ m + (19.5 – 9.81) \times 0.6\ m = 149.71\ kN/m^2$

$Q_{s4} = (\ 0\ + 149.71\ kN/m^2 \times 0.5 \tan 20) \times (\ \pi(0.4\ m) \times 1.2\ m)$

$= \underline{41.1}\ kN.$

2-End resistance:

$Q_b = (\ C_b\ N_c + q\ N_q\ )\ A_b$

$Q_b = \{(0+18*3+(19-9.81)*6+(18.5-9.81)*4+(19.5-9.81)*1.2*30\}\ 3.14*(0.4)^2/4 = 586\ kN$

For $Ø = 30$ $N_q = 30$

$Q_{bmax} = 50*30\tan 30*3.14*(0.4)^2/4 = 108.83\ kN$ USE IT

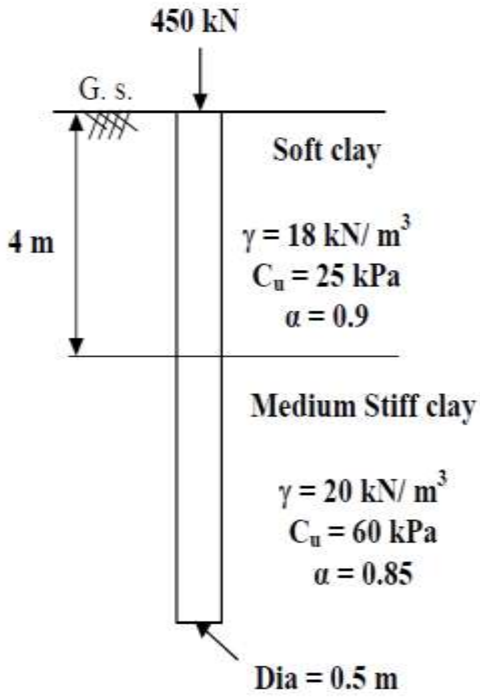
$Q_{ult.sp} = \sum Q_s + Q_b = (130.4 + 406.9 + 109.82 + 41.1 + 108.83)$

$= 797.9\ kN$

$Q_{all.sp} = 797.9/2.5 = 319.2\ kN$

Q2:

Estimate the pile length required to carry 450 kN axial load, use SF=2.5. (neglect q*Nq).



Solution:

$Q_{ult} = Q_b + \Sigma Q_s$

$Q_b = 9\, c_u\, A_b = 9 * 60 * \frac{\pi}{4} * (0.5)^2 = 105.975 \text{ kN}$

$\Sigma Q_s = Q_{s1} + Q_{s2}$

$Q_{s1} = \alpha\, C_u\, A_{s1}$

$Q_{s1} = 0.9 * 25 * \pi\,(0.5) * 4 = 141.3 \text{ kN}$

$Q_{s2} = \alpha\, C_u\, A_{s2}$

$Q_{s2} = 0.85 * 60 * \pi\,(0.5) * L = 80.07\, L$

$\Sigma Q_s = 141.3 + 80.07\, L$

$Q_{ult} = 105.975 + 141.3 + 80.07\, L$

$= 247.275 + 80.07\, L = 450 * 2.5 = 1125$

$\longrightarrow L = 10.96 \text{ m} \approx 11 \text{ m}$

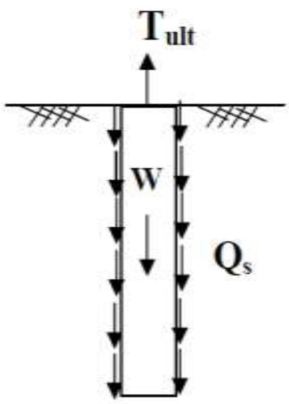
Total length of pile is 15 m .

Tension Pile

It is used to resist uplift force which can be developed from hydrostatic pressure, expansion soil, overturning due to wind, …. etc.

Ultimate tension resistance is:-

$$T_{ult} = \sum Q_s + w$$



And the allowable tension resistance is :

$$T_{all} = \frac{T_{ult}}{FS}$$

Where : Q_s : skin friction

W: weight of pile

The weight of pile may be neglected for more safety.

Example 6: Estimate the length required to sustain an uplift force of 400 kN for the pile Shown , Use F.S = 3 .

Solution:

$$T_{ult} = \sum Q_s + w$$

Neglect the pile weight (w)

$$\sum Q_s = Q_{s1} + Q_{s2}$$

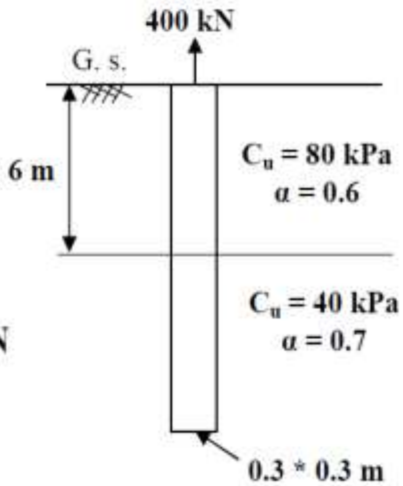
$$Q_{s1} = \alpha\, C_u\, A_{s1} = 0.6 * 80 * 4*0.3 * 6 = 345.6 \text{ kN}$$

$$Q_{s2} = \alpha\, C_u\, A_{s2} = 0.7 * 40 * 4*0.3 * L = 33.6\, L$$

$$T_{ult} = 345.6 + 33.6\, L = FS * T_{all} = 3*400 = 1200 \text{ kN}$$

$$\longrightarrow L = 25.4 \approx 26 \text{ m}$$

Total length of pile is 32 m .



Pile Groups:

To allow for misalignment and bending moments.

No. of piles ≥ 3 to support a major column.
No. of piles ≥ 2 to support a foundation wall.

- Suggested minimum pile spacing according to building codes:

* friction piles min. spacing is 2D or 1.75H $\geq$ 75 cm.
D = pile diameter . H = diagonal of a rectangular shape or H-pile.

* point bearing piles min. spacing is 2D or 1.75H $\geq$ 60 cm.

Optimal spacing S = (2.5 to 3)D or (2 to 3)H for vertical load.

Pile Group Capacity:

Solid block method:

Assuming the pile cap is perfectly rigid and the soil continued within the periphery of the piles behaves as a solid block, the entire block may then be visualized as one deep footing.

$$\mathbf{Q_{pg} = (S\ L\ \rho + q_{ult}\ A - \gamma\ L\ A) \leq nQ_{sp}}$$

Q_{pg} = pile group capacity.
S L ρ = block shear.

$S = (q_u/2) + (\sigma_v\ K \tan Ø)$

L = length of pile embedded in soil.
ρ = perimeter of area enclosing all the piles in the group.
$\mathbf{q_{ult}}$ = C Nc + q Nq
A = area enclosing all the piles in the group.
$\boldsymbol{\gamma}$ = unit weight of soil within the block (L * A).
n = No. of piles.
$\mathbf{Q_{sp}}$ = ultimate capacity of an individual pile.

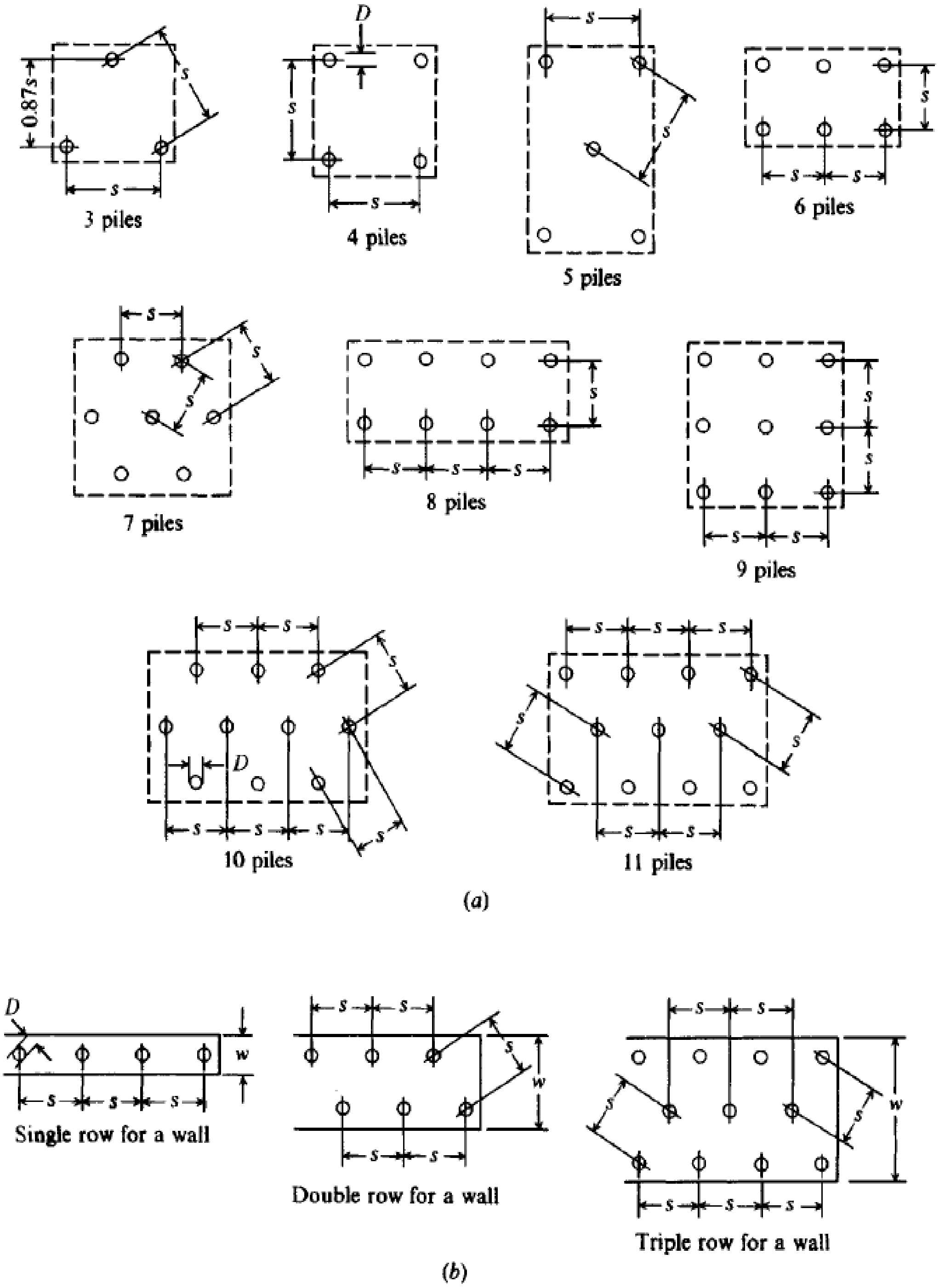
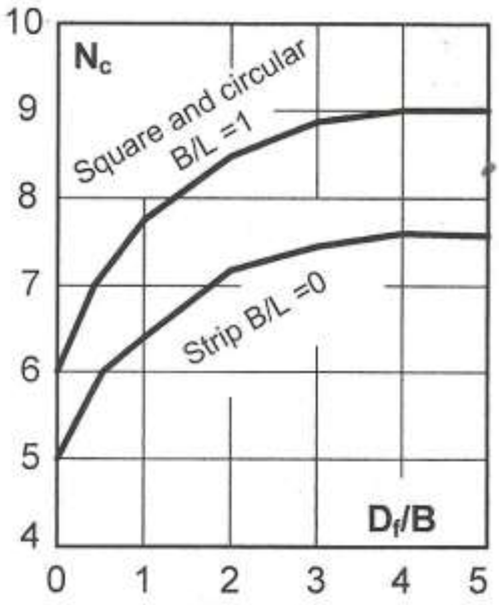


Figure 18-1 Typical pile-group patterns: (*a*) for isolated pile caps; (*b*) for foundation walls.

Suggested minimum center-to-center pile spacings by several building codes are as follows:

Pile type	BOCA, 1993 (Sec. 1013.8)	NBC, 1976 (Sec. 912.1*l*)	Chicago, 1994 (Sec. 13-132-120)
Friction	$2D$ or $1.75H \geq 760$ mm	$2D$ or $1.75H \geq 760$ mm	$2D$ or $2H \geq 760$ mm
Point bearing	$2D$ or $1.75H \geq 610$ mm	$2D$ or $1.75H \geq 610$ mm	

Example: The total load on a pile group is 1690 kN. Boring indicates a very deep layer of fairly uniform clay. The clay has an average unconfined compressive strength q_u = 86 kN/m^2. A factor of safety of 3 is desired using 12m piles having an average diameter of 30cm. Assuming an adhesion factor of 0.87 and neglecting the end bearing of an individual pile, design a rectangular pattern of pile group, suggest the spacing and check the group capacity, use figure to find Nc. **γ = 18 kN/m^3**



Bearing capacity factor N_c vs relative foundation depth for ø = 0 condition

Solution:

$$\mathbf{Q_{sp\ ult} = (\alpha\ C_s + \sigma_v\ K \tan\delta\ )A_s}$$

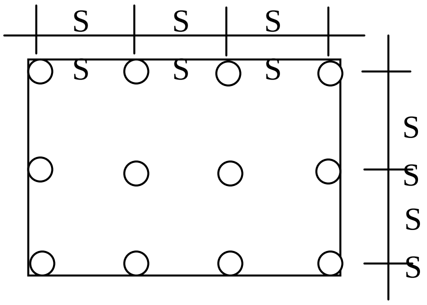
$$\mathbf{\delta = 0}$$

$$\mathbf{Q_{sp\ ult} = (0.87 \times (86/2)\ kN/m^2 + 0\ ) \times (\ \pi(0.3\ m) \times 12\ m)}$$
$$\mathbf{= \underline{423}\ kN.}$$

$\mathbf{Q_{sp\ all}}$ = 423/ SF = 423/3 = 141 kN.

Then,

No. of piles = 1690 / 141 = 11.9 pile. Use 12 pile 3 * 4 pattern as shown,



Let S = 3D = 3 * 0.3 = 0.9 m > 75 cm OK.
L = 3 S + D = 3 * 0.9 + 0.3 = 3 m
B = 2 S + D = 2 * 0.9 + 0.3 = 2.1 m

Now, find pile group capacity:

$$Q_{pg} = (S\ L\ \rho + q_{ult}\ A - \gamma\ L\ A) \le nQ_{sp}$$

$S = (q_u/2) + (\sigma_v\ K \tan Ø) = C = 86/2=43\ kN/m^2$

$\rho = (2.1 + 3)*2= 10.2$ m

$q_{ult} = C\ Nc + q\ Nq = 43 * (8.55) + 12*18*1$

$= 583.6$ kPa.

From figure
Df/B=12/2.1= 5.71
B/L=2.1/3=0.7
Nc = 8.55

$A = 3 * 2.1 = 6.3\ m^2$

$Q_{pg\ ult} = 43 * 12 * 10.2 + 583.6 * 6.3 – 18 * 12 * 6.3 = 7579$ kN

$Q_{pg\ all} = 7579/3 = 2526.3$ kN. Then, the pile group system gives allowable capacity larger than **nQ_{sp}** $= 12 * 141 = 1692$ kN.

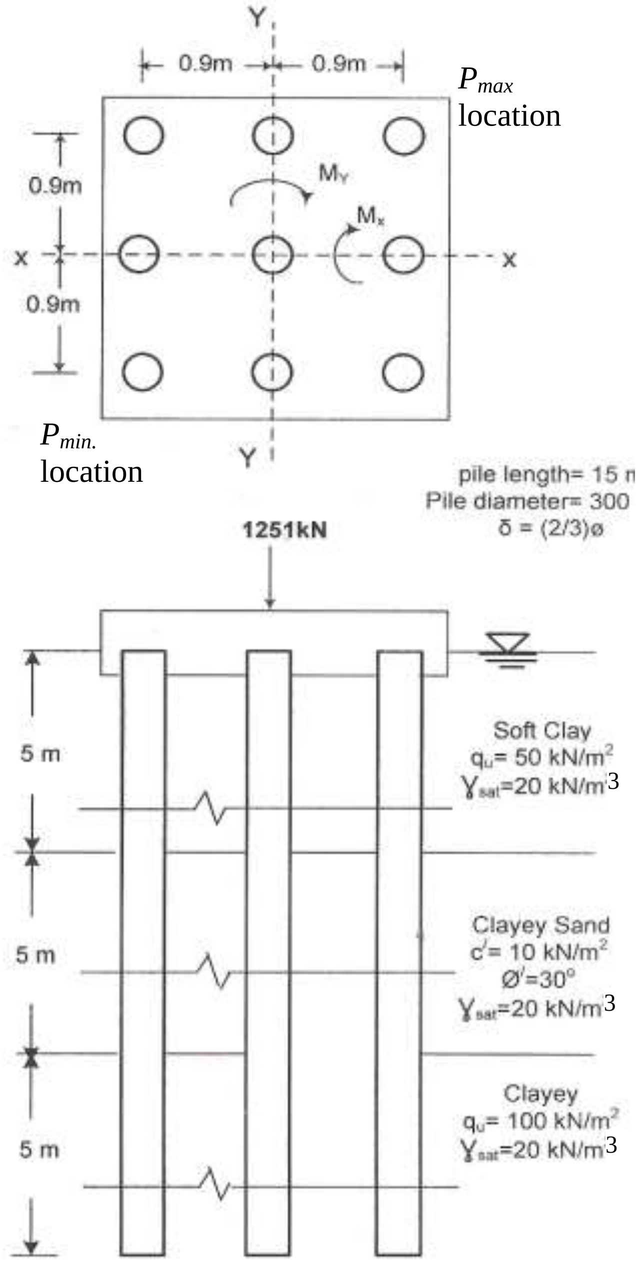
Then the value of capacity 1692 kN is govern.

Example:

Considering the applied concentric load and the moments of $M_x = 110$ kN.m and $M_y = 120$ kN.m as indicated in the figure, calculate the maximum and minimum pile reactions.

Solution:

$$P_{\substack{Max.\\Min.}} = \frac{\sum V}{n} \pm \frac{\sum M_x \times x}{\sum d_x^{\ 2}} \pm \frac{\sum M_y \times y}{\sum d_y^{\ 2}}$$



x = distance in y-direction from external row to the x-axis.

y = distance in x-direction from external row to the y-axis.

d_x = distance in y-direction from each pile to the x-axis.

d_y = distance in x-direction from each pile to the y-axis.

$x = 0.9$ m , $y = 0.9$ m

$\sum (d_x)^2 = 6 * (0.9)^2$ m^2 = 4.86 m^2

$\sum (d_y)^2 = 6 * (0.9)^2$ m^2 = 4.86 m^2

$$P_{\substack{Max.\\Min.}} = \frac{1251}{9} \pm \frac{110 \times 0.9}{4.86} \pm \frac{120 \times 0.9}{4.86}$$

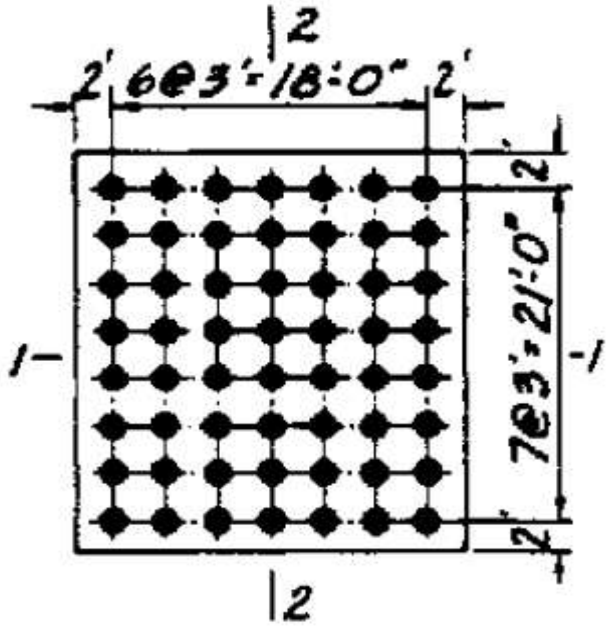
$$P_{Max.} = 181.6 kN$$

$$P_{Min.} = 96.4 kN$$

HW: Find P_{max} and P_{min}, if the maximum load is 1724 kips,

M_{1-1} = 3306 kips.ft ; M_{2-2} = 3726 kips.ft is the foundation safe or not.

Solution:



$$P_{\substack{Max.\\Min.}} = \frac{\sum V}{n} \pm \frac{\sum M_x \times x}{\sum d_x^{\,2}} \pm \frac{\sum M_y \times y}{\sum d_y^{\,2}}$$

$M_x = M_{1-1}$, $M_y = M_{2-2}$

$x = 10.5\ ft$, $y = 9\ ft$

$$\sum d_x^{\,2} = 14*(1.5^2 + 4.5^2 + 7.5^2 + 10.5^2)$$
$$= 2646\ ft^2$$

$$\sum d_y^{\,2} = 16*(3^2 + 6^2 + 9^2)$$
$$= 2016\ ft^2$$

$$P_{\substack{Max.\\Min.}} = \frac{1724}{56} \pm \frac{3306 \times 10.5}{2646} \pm \frac{3726 \times 9}{2016}$$

$P_{\substack{Max.\\Min.}}$ = *60.5 kips.ft , 1.03 kips.ft respectively.*

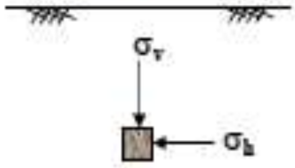
Then, the foundation is safe. (there is no tension)

LATERAL EARTH PRESSURE

Lateral Earth Pressure

Lateral earth pressure is a significant design element in a number of foundation engineering problems. Retaining and sheet-pile walls, both braced and unbraced excavations, and earth or rock contacting tunnel walls and other underground structures require a quantitative estimate of the lateral pressure on a structural member for either a design or stability analysis.

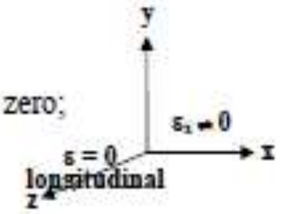
For any element in a soil mass as shown in Fig. , there are two types of normal stresses. Normal stress due to gravity is called " vertical stress " or " overburden pressure" (σ_v) , while the perpendicular normal stress to σ_v is called " horizontal stress" or " lateral Pressure " (σ_h) .



The value of lateral pressure σ_h is proportional to the value of σ_v as :-

$$\sigma_h = k \,.\, \sigma_v$$

Where: k: the coefficient of lateral earth pressure depending on soil type and the state of Soil pressure .

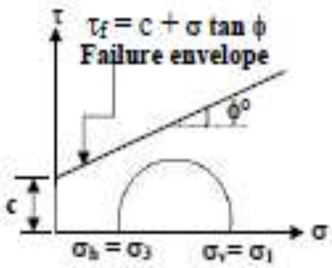


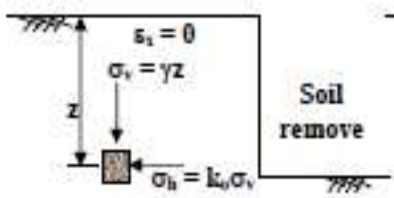
- It is assume that the strain in longitudinal direction will be zero; thus it taken in to consideration the strain in plane .
- There are three kinds of lateral stressor strain

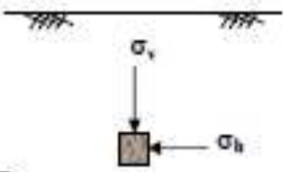
Types of soil pressure

1- At rest condition

- The lateral strain is zero ($\varepsilon_x = 0$) .
- The vertical and horizontal stresses on any element of soil are the principal stresses .
- The Mohr circle don't touch the failure envelope (elastic equilibrium).($\tau < \tau_f$)
- The value of effective horizontal stress is : $\sigma_h = k_o\, \sigma_v$
- The value of k_o is coefficient of at rest lateral earth pressure, which is computed as:-
  $k_o = 1 - \sin\phi$ (for normally consolidated soils).
  $k_o = (1\text{-}\sin\phi)\sqrt{OCR}$ (for over consolidated soils).
- The cut in this kind (condition) don't need any retained structure.







If we have insert a wall of zero thickness into a normally consolidated , isotropic , cohesionless soil mass as shown in **Fig.** below and then excavate the soil from the left side of the wall to a depth H , if the wall is allowed to moves ($\varepsilon_x \neq 0$), there are two types of stress conditions :

2- Active condition

The wall is moves toward the excavation with a lateral strain $\varepsilon_x \neq 0$, then the lateral stress is decreased until it reach a min. value of σ_h (termed active pressure case) and the Mohr circle touch the failure envelope (Plastic equilibrium), So;

$$\sigma_h = \sigma_a = k_a\,\sigma_v \quad (\text{when } c = 0)$$

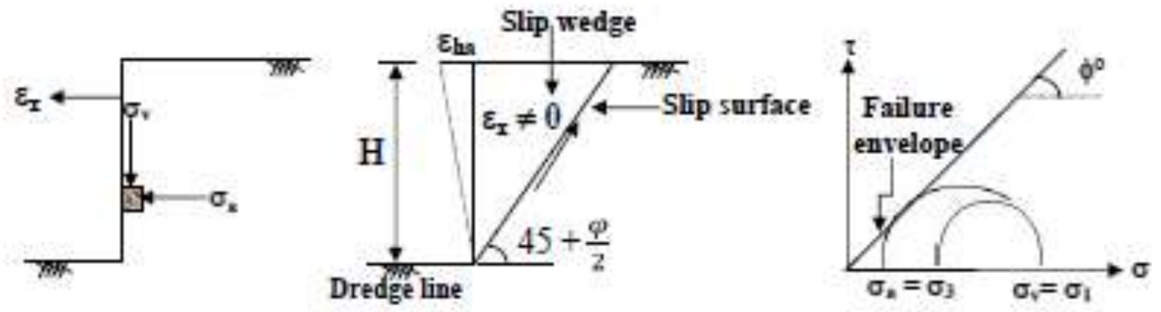
Where:

σ_v : vertical stress and the major stress σ_1 .

σ_a : active lateral earth pressure and the minor stress σ_3 .

K_a: coefficient of active lateral earth pressure .

The slip wedge is at min. volume and the slip surface at ($45+\frac{\varphi}{2}$) with horizontal



3- Passive condition

If The wall is moves toward the soil with a lateral strain $\varepsilon_x \neq 0$, then the lateral stress is increases until it reach a max. value of σ_h (termed passive pressure case) and the Mohr circle touch the failure envelope (Plastic equilibrium), So;

$$\sigma_h = \sigma_p = k_p\,\sigma_v \quad (\text{when } c = 0)$$

Where:

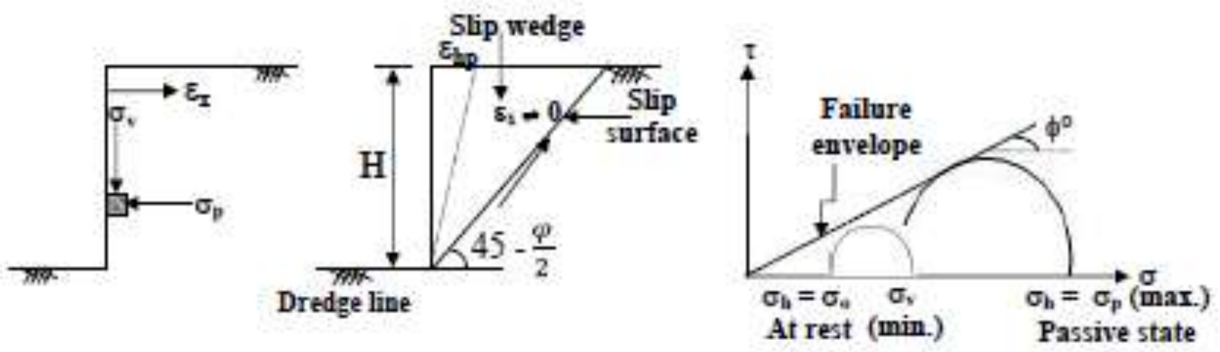
σ_v : vertical stress and the minor stress σ_3 .

σ_p : passive lateral earth pressure and the major stress σ_1.

K_p: coefficient of passive lateral earth pressure .

and the slip surface make angles ($45-\frac{\varphi}{2}$) with horizontal. The displacement of wall which need to reach a plastic equilibrium will be higher than this for active state i.e.

$$\varepsilon_{hp} >> \varepsilon_{ha}$$



Summary

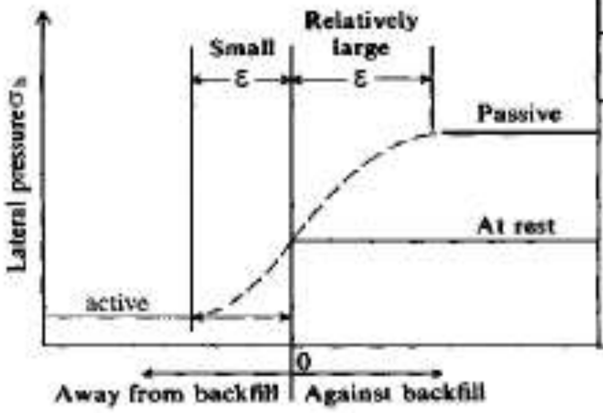
The summary for all the above will be as follows :

1- $\varepsilon_x = 0 \longrightarrow$ at rest condition ; $\sigma_h = \sigma_o = k_o\,\sigma_v$.

2- $\varepsilon_x \neq 0$ $\begin{cases} \sigma_h \text{ decreases} \longrightarrow \text{active condition ; } \sigma_h = \sigma_a = k_a\,\sigma_v . \\ \sigma_h \text{ increases} \longrightarrow \text{passive condition ; } \sigma_h = \sigma_p = k_p\,\sigma_v \end{cases}$

3- $\sigma_a < \sigma_o < \sigma_p$

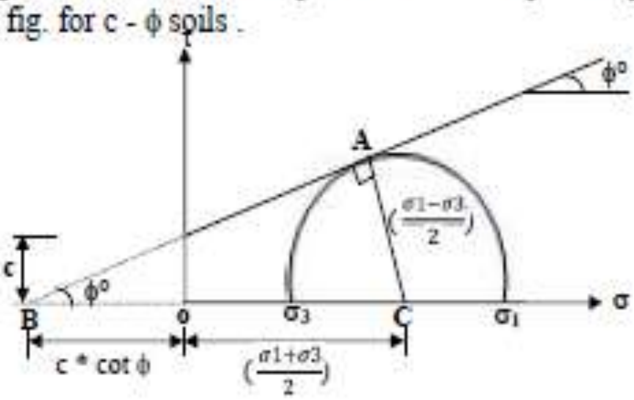
4- Passive stress occurs at high strain if it is compared with the strain of active stress. (see Fig. below)



Usual range of earth pressure coefficients	
Cohesionless soils	Cohesive soils
3–14	1–2
0.4–0.6	0.4–0.8
0.33–0.22	1–0.5

Active and passive pressure (for c - φ soils)

To derive a general formula to compute the active or passive pressure use Mohr's Circle as shown in fig. for c - φ soils .



$$\text{Sin}\,\phi = \frac{AC}{OB+OC} = \frac{(\sigma1-\sigma3)/2}{\left(\frac{\sigma1+\sigma3}{2}\right)+c\cot\varphi}$$

$$1/2\,\sigma_1 \sin\phi + 1/2\,\sigma_3 \sin\phi + c\cot\phi = 1/2\,\sigma_1 - 1/2\,\sigma_3$$

$$\sigma_1\,(1-\sin\phi) = \sigma_3\,(1-\sin\phi) + 2c\cot\phi$$

<u>Active state:</u> $\sigma_1 = \sigma_v$ and $\sigma_3 = \sigma_a$

$$\sigma_a = \sigma_v \left(\frac{1-\sin\varphi}{1+\sin\varphi}\right) - 2c\left(\frac{\cos\varphi}{1+\sin\varphi}\right)$$

$$= \sigma_v \left(\frac{1-\sin\varphi}{1+\sin\varphi}\right) - 2c\left(\sqrt{\frac{1-\sin\varphi}{1+\sin\varphi}}\right)$$

Let , $k_a = \frac{1-\sin\varphi}{1+\sin\varphi} = \tan^2(45 - \frac{\varphi}{2})$

$$\rightarrow \boxed{\sigma_a = \sigma_v k_a - 2c\sqrt{k_a}}$$

<u>passive state:</u> $\sigma_1 = \sigma_p$ and $\sigma_3 = \sigma_v$

$$\sigma_p = \sigma_v \left(\frac{1+\sin\varphi}{1-\sin\varphi}\right) + 2c\left(\frac{\cos\varphi}{1-\sin\varphi}\right)$$

$$= \sigma_v \left(\frac{1+\sin\varphi}{1-\sin\varphi}\right) + 2c\left(\sqrt{\frac{1+\sin\varphi}{1-\sin\varphi}}\right)$$

Let , $k_p = \frac{1+\sin\varphi}{1-\sin\varphi} = \tan^2(45 + \frac{\varphi}{2})$

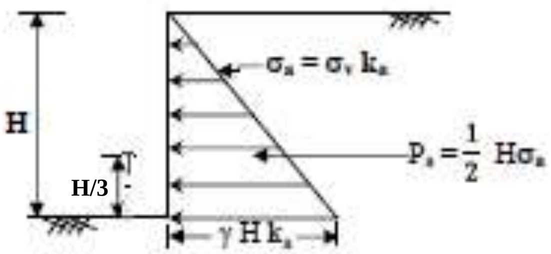
$$\rightarrow \boxed{\sigma_p = \sigma_v k_p + 2c\sqrt{k_p}}$$

Distribution of lateral stress

<u>Active state:</u>

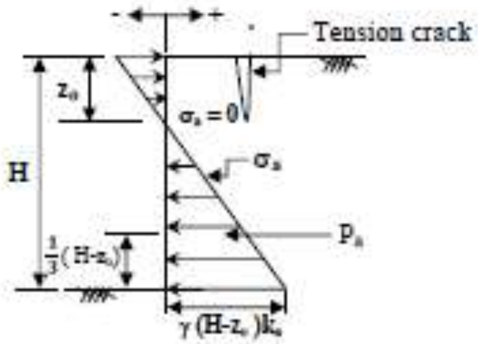
a) **For cohesionless soil (c = 0)**

When the cut occur in homogenous cohesionless soil the distribution of lateral stress and total lateral pressure force as shown in Fig. :-



b) **For cohesive soil ($c \neq 0$)**

When $c \neq 0$, the tension zone occur at surface of soil within a depth Z_o, the distribution of lateral active stress and total active force will be as shown in Fig.:-



The stress distribution of active state for c- ϕ soil is :-

$\sigma_a = \sigma_v k_a - 2c\sqrt{k_a}$

When $z = 0 \rightarrow \sigma_a = -2c\sqrt{k_a}$

When $z = z_o \rightarrow \sigma_a = 0$, so that:-

$\sigma_a = 0 = \gamma z_o k_a - 2c\sqrt{k_a}$

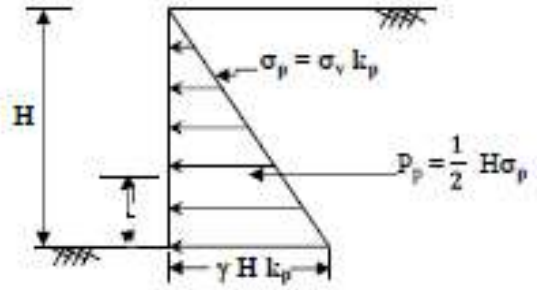
$\gamma z_o k_a = 2c\sqrt{k_a} \rightarrow Z_0 = \frac{2c}{\gamma\sqrt{k_a}}$

$$P_a = \int_{z_0}^{H} \sigma_a \, dz = \int_{z_0}^{H} (\gamma z\, ka - 2c\sqrt{k_a})\, dz = \frac{\gamma}{2} z^2 k_a - 2c\, z\sqrt{k_a} \Big]_{z_o}^{H}$$

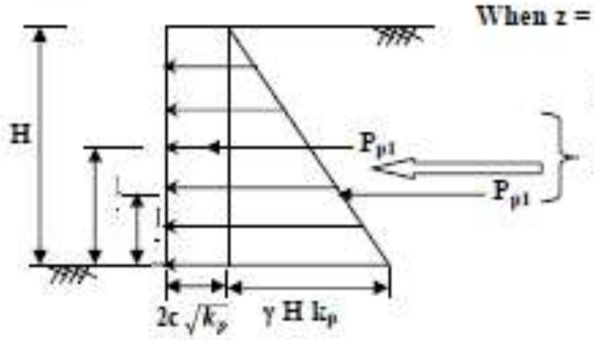
$$\boxed{P_a = \frac{1}{2}\gamma(H^2 - z_o^2)k_a - 2c(H - z_o)\sqrt{k_a}}$$

<u>passive state:</u>

a) **For cohesionless soil ($c = 0$)**



b) **For cohesive soil ($c \neq 0$)**



When $z = 0 \rightarrow \sigma_p = 2c\sqrt{k_p}$

$P_p = \int_0^H \sigma_p dz$

$P_p = \frac{1}{2}\gamma H^2 k_p + 2c\sqrt{k_p}\, H$

Surcharge and Cut in Non homogenous Soils

When a soil surface exerted by surcharge (fill of soil, buildings, live loads,...etc), the vertical stress will be increased by this surcharge on any point within a soil mass where :-

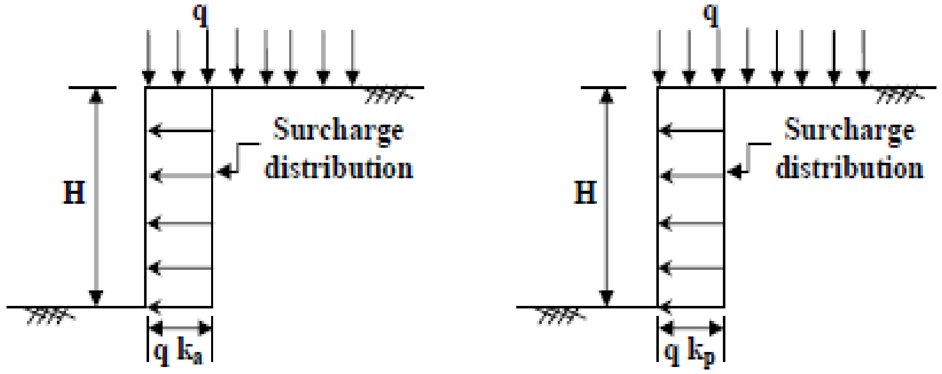
$$\sigma_v = \gamma z + q$$

q : Surcharge pressure

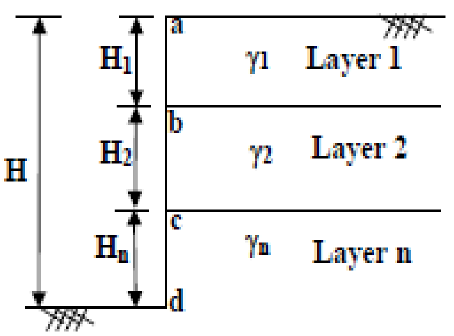
at this situation the lateral stress will be increased by multiplying the surcharge with the lateral earth pressure coefficient at active or passive state , where :-

$$\sigma_a = (\gamma z + q)\, k_a \quad \text{or} \quad \sigma_p = (\gamma z + q)\, k_p$$

From this situation the distribution of surcharge contribution in lateral pressure will be uniform a long cut as shown in Fig. :-



On the other hand if cut excavated in layered soil (nonhomogenous) , the distribution of pressure will be treated for each layer a lone (as homogenous) , and consider the above layer as a surcharge as shown :-



From a – b (homogenous soil and surcharge above is zero)
From b – c (homogenous soil and surcharge above is $\gamma_1 H_1$)
From c – d (homogenous soil and surcharge above is $\gamma_1 H_1 + \gamma_2 H_2$)
And so on ....

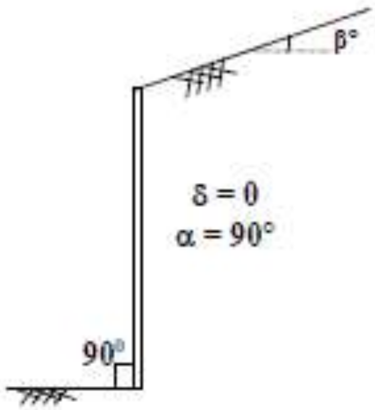
Theories of Lateral Earth Pressure

There are two main theories to compute the lateral earth pressure (i.e. coefficient of lateral earth pressure):

1- Rankine theory (1857):

This theory is deals with calculating active and passive earth pressure coefficient , the assumptions of this theory are :-

- **Plane slip surface of soil failure .**
- **No friction between the soil and the wall .**
- **Vertical wall .**
- **Homogenous and isotropic soil .**
- **Backfill soil may be inclined by angle β .**

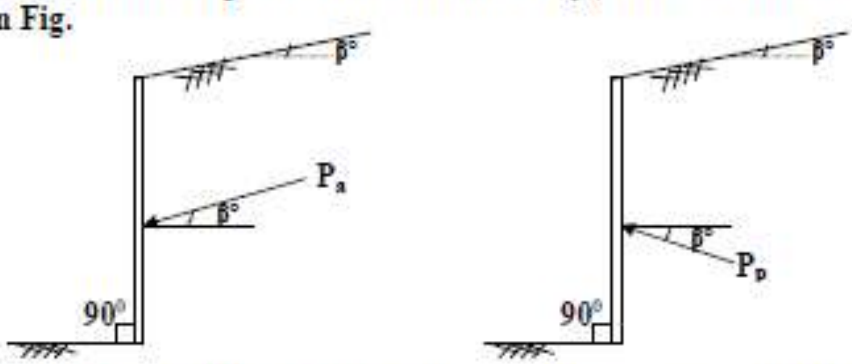


$$K_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$

$$K_p = \cos\beta \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}$$

If $\beta = 0$: $k_a = \frac{1-\sin\varphi}{1+\sin\varphi}$, $k_p = \frac{1+\sin\varphi}{1-\sin\varphi}$

- **If β = 0 , the total active or passive force is horizontal (normal to the wall).**
- **If β > 0 , the total active or passive force is inclined by β from the horizontal as shown in Fig.**



It must be noted here , that the lateral stress (active or passive) distribution will be inclined at β angle if the designer need a horizontal and vertical components one should compute :-

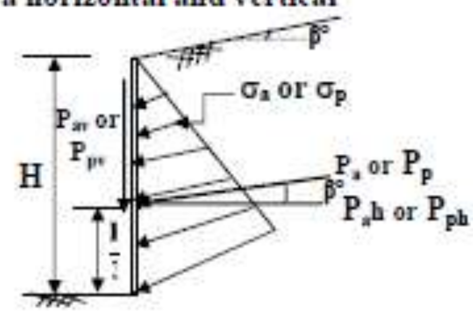
$\sigma_{ah} = \sigma_a \cos\beta$; $\sigma_{av} = \sigma_a \sin\beta$

$\sigma_{ph} = \sigma_p \cos\beta$; $\sigma_{pv} = \sigma_p \sin\beta$

And hence :-

$P_{ah} = P_a \cos\beta$; $P_{av} = P_a \sin\beta$

$P_{ph} = P_p \cos\beta$; $P_{pv} = P_p \sin\beta$



It can be use Tables (E1 , E2) instead of formulas to find the values of k_a and k_p for different values of ϕ and β .

Table (E1): Coefficient of active earth pressure (k_a) based on Rankine equation.

β \ ϕ	26	28	30	32	34	36	38	40
0	0.39046	0.36103	0.33333	0.30726	0.28271	0.25962	0.23788	0.21744
5	0.39586	0.36559	0.33720	0.31055	0.28552	0.26202	0.23994	0.21921
10	0.41335	0.38023	0.34952	0.32097	0.29437	0.26955	0.24637	0.22471
15	0.44801	0.40857	0.37295	0.34050	0.31076	0.28337	0.25807	0.23463
20	0.51516	0.46049	0.41421	0.37388	0.33811	0.30600	0.27692	0.25042
25	0.69991	0.57268	0.49359	0.43364	0.38469	0.34313	0.30697	0.27502

Table (E2): Coefficient of passive earth pressure (k_p) based on Rankine equation.

β \ ϕ	26	28	30	32	34	36	38	40
0	2.56107	2.76983	3.00000	3.25459	3.53713	3.85184	4.20375	4.59891
5	2.50697	2.71453	2.94309	3.19566	3.47575	3.78755	4.13604	4.52724
10	2.34630	2.55070	2.77480	3.02160	3.29462	3.59796	3.93649	4.31606
15	2.08256	2.28362	2.50171	2.74010	3.00236	3.29255	3.61541	3.97656
20	1.71409	1.91755	2.13185	2.36179	2.61164	2.88572	3.18877	3.52620
25	1.17357	1.43430	1.66412	1.89418	2.13519	2.39384	2.67579	2.98670

2- Coulomb Theory (1776):

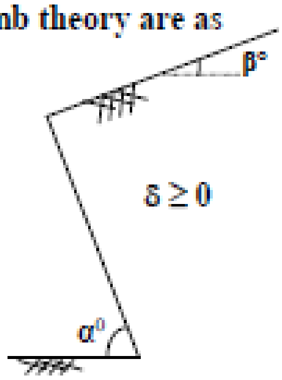
It is one of the earliest methods (1776), for estimating the lateral earth pressure coefficients . Coulomb made a number of assumptions as follows:-

- Plane slip surface of soil failure .
- The wall can be friction or frictionless .
- the wall may be inclined by an angle α from the vertical .
- Homogenous and isotropic soil .
- Backfill soil may be inclined by angle β .
- the failure wedge is rigid body .

The final form of k_a and k_p formulas based on the coulomb theory are as follows:-

$$K_a = \frac{\sin^2(\alpha+\phi)}{\sin^2\alpha\,\sin(\alpha-\delta)\left[1+\sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi-\beta)}{\sin(\alpha-\delta)\sin(\alpha+\beta)}}\right]^2}$$

$$K_p = \frac{\sin^2(\alpha-\phi)}{\sin^2\alpha\,\sin(\alpha+\delta)\left[1-\sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi+\beta)}{\sin(\alpha+\delta)\sin(\alpha+\beta)}}\right]^2}$$



If β = δ = 0 and α = 90° (a smooth vertical wall with horizontal backfill):

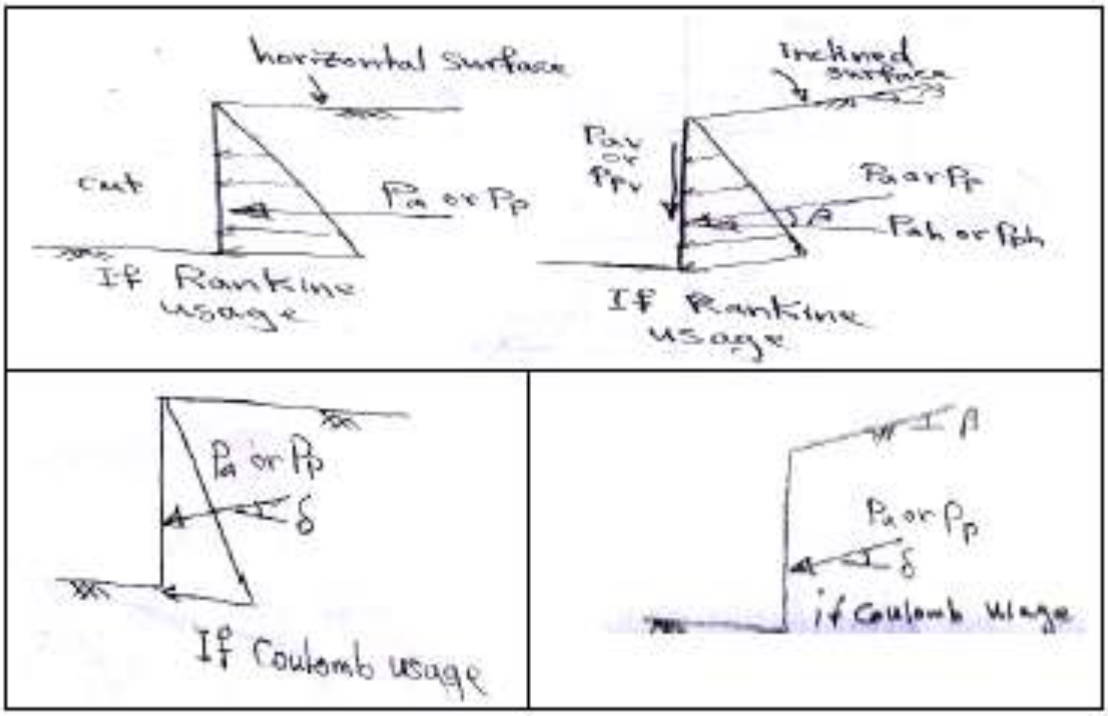
$$k_a = \frac{1-\sin\varphi}{1+\sin\varphi} \quad , \quad k_p = \frac{1+\sin\varphi}{1-\sin\varphi}$$

- Active force or passive inclined by an angle δ from the normal line on the wall.
- It can be taken the values of k_a and k_p from Tables (E3 , E4), instead of using the above formulas .

Notes:

1- When using Rankine' s theory for estimating active or passive pressure , the influence line of this pressure will act horizontally against retaining structure for horizontal surface of soil , and for inclined surface at β angle , this force acting at the same inclination with respect to retaining structure as shown in Fig.

2- When using Coulomb theory for estimating active or passive pressure , this force will acting with angle of adhesion (δ) with respect retaining structure in spite of inclination of soil surface as shown in Fig.



ملاحظات مهمة:

١- نستخدم الكثافة المغمورة في الطبقات الواقعة ضمن المياه الجوفية لان حساب الاجهادات دائما يكون بالاجهادات الفعالة . هناك ضغط هيدروستاتيكي للماء ضمن هذه الطبقات يجب ان يؤخذ بنظر الاعتبار عند التصميم .

٢- اذا لم يذكر النظرية في السؤال نستخدم Rankine .

Table (E3): Coefficient of active earth pressure (k_a) based on Coulomb equation.

	δ \ ϕ	26	28	30	32	34	36	38	40
$\beta = 0$ and $\alpha = 90$	0	0.3905	0.3610	0.3333	0.3073	0.2827	0.2596	0.2379	0.2174
	5	0.3722	0.3448	0.3189	0.2945	0.2714	0.2497	0.2292	0.2098
	10	0.3590	0.3330	0.3085	0.2852	0.2633	0.2426	0.2230	0.2045
	12	0.3550	0.3294	0.3053	0.2824	0.2608	0.2404	0.2211	0.2029
	14	0.3516	0.3264	0.3026	0.2801	0.2588	0.2387	0.2196	0.2016
	16	0.3487	0.3239	0.3004	0.2782	0.2571	0.2372	0.2184	0.2006
	18	0.3464	0.3219	0.2986	0.2766	0.2558	0.2361	0.2175	0.1999
	20	0.3447	0.3203	0.2973	0.2755	0.2549	0.2354	0.2169	0.1994
	22	0.3434	0.3193	0.2964	0.2748	0.2544	0.2350	0.2166	0.1992

	δ \ ϕ	26	28	30	32	34	36	38	40
$\beta = 5$ and $\alpha = 90$	0	0.4139	0.3817	0.3516	0.3233	0.2968	0.2720	0.2487	0.2269
	5	0.3959	0.3656	0.3372	0.3105	0.2855	0.2620	0.2399	0.2192
	10	0.3831	0.3541	0.3269	0.3015	0.2775	0.2550	0.2338	0.2139
	12	0.3792	0.3506	0.3238	0.2987	0.2751	0.2529	0.2320	0.2123
	14	0.3759	0.3477	0.3213	0.2964	0.2731	0.2511	0.2305	0.2110
	16	0.3733	0.3454	0.3192	0.2946	0.2715	0.2498	0.2293	0.2101
	18	0.3713	0.3436	0.3176	0.2932	0.2703	0.2488	0.2285	0.2094
	20	0.3698	0.3422	0.3165	0.2923	0.2695	0.2481	0.2280	0.2090
	22	0.3688	0.3414	0.3158	0.2917	0.2691	0.2478	0.2278	0.2089
	δ \ ϕ	26	28	30	32	34	36	38	40
$\beta = 10$ and $\alpha = 90$	0	0.4431	0.4071	0.3737	0.3425	0.3135	0.2865	0.2612	0.2377
	5	0.4256	0.3913	0.3595	0.3299	0.3023	0.2765	0.2525	0.2300
	10	0.4134	0.3802	0.3495	0.3210	0.2944	0.2696	0.2464	0.2247
	12	0.4097	0.3769	0.3466	0.3183	0.2920	0.2675	0.2446	0.2232
	14	0.4068	0.3743	0.3442	0.3162	0.2902	0.2659	0.2432	0.2220
	16	0.4045	0.3722	0.3423	0.3145	0.2887	0.2646	0.2421	0.2211
	18	0.4028	0.3706	0.3409	0.3133	0.2877	0.2637	0.2414	0.2205
	20	0.4017	0.3696	0.3400	0.3126	0.2870	0.2632	0.2410	0.2202
	22	0.4012	0.3692	0.3396	0.3122	0.2868	0.2631	0.2409	0.2202

Table (E4): Coefficient of passive earth pressure (k_p) based on Coulomb equation.

$\beta = 0$ and $\alpha = 90$

δ \ ϕ	26	28	30	32	34	36	38	40
0	2.5611	2.7698	3.0000	3.2546	3.5371	3.8518	4.2037	4.5989
5	2.9541	3.2149	3.5052	3.8293	4.1928	4.6023	5.0658	5.5930
10	3.4376	3.7698	4.1433	4.5653	5.0445	5.5915	6.2198	6.9460
12	3.6647	4.0328	4.4487	4.9210	5.4604	6.0801	6.7966	7.6310
14	3.9157	4.3251	4.7902	5.3214	5.9317	6.6377	7.4600	8.4257
16	4.1947	4.6520	5.1744	5.7748	6.4694	7.2788	8.2295	9.3560
18	4.5065	5.0196	5.6095	6.2920	7.0875	8.0221	9.1300	10.4561
20	4.8570	5.4356	6.1054	6.8861	7.8037	8.8916	10.1943	11.7715
22	5.2534	5.9096	6.6748	7.5743	8.6410	9.9187	11.4663	13.3644

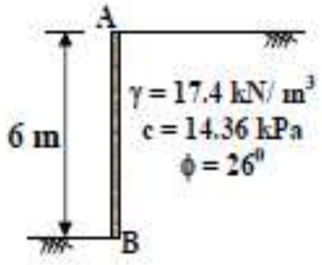
$\beta = 5$ and $\alpha = 90$

δ \ ϕ	26	28	30	32	34	36	38	40
0	2.9429	3.2027	3.4918	3.8147	4.1769	4.5848	5.0465	5.5717
5	3.4761	3.8088	4.1832	4.6063	5.0870	5.6358	6.2662	6.9951
10	4.1516	4.5882	5.0857	5.6561	6.3140	7.0779	7.9715	9.0257
12	4.4760	4.9663	5.5286	6.1774	6.9310	7.8129	8.8536	10.0930
14	4.8396	5.3932	6.0321	6.7746	7.6437	8.6697	9.8921	11.3637
16	5.2499	5.8783	6.6087	7.4641	8.4742	9.6781	11.1279	12.8945
18	5.7160	6.4336	7.2743	8.2673	9.4513	10.8776	12.6162	14.7640
20	6.2492	7.0742	8.0491	9.2117	10.6129	12.3215	14.4330	17.0828
22	6.8642	7.8197	8.9597	10.3341	12.0106	14.0833	16.6854	20.0111

$\beta = 10$ and $\alpha = 90$

δ \ ϕ	26	28	30	32	34	36	38	40
0	3.3854	3.7125	4.0804	4.4959	4.9678	5.5066	6.1253	6.8405
5	4.1042	4.5357	5.0276	5.5915	6.2418	6.9970	7.8804	8.9225
10	5.0471	5.6341	6.3141	7.1073	8.0400	9.1462	10.4712	12.0756
12	5.5120	6.1825	6.9650	7.8855	8.9776	10.2861	11.8714	13.8159
14	6.0424	6.8133	7.7204	8.7971	10.0877	11.6518	13.5711	15.9603
16	6.6522	7.5449	8.6049	9.8761	11.4171	13.3089	15.6647	18.6472
18	7.3591	8.4013	9.6514	11.1676	13.0296	15.3492	18.2869	22.0803
20	8.1862	9.4139	10.9034	12.7334	15.0140	17.9035	21.6360	26.5688
22	9.1637	10.6248	12.4206	14.6595	17.4973	21.1643	26.0127	32.6018

Example 1: A 6 m high retaining wall is to support a soil as shown . Determine the Rankine active and passive force per unit length of the wall and its line Of action .



Solution:

Active state:

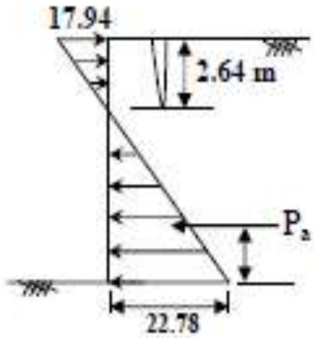
$$k_a = \frac{1-\sin 26}{1+\sin 26} = 0.39$$

$$Z_0 = \frac{2c}{\gamma\sqrt{k_a}} = \frac{2 * 14.36}{17.4\sqrt{0.39}} = 2.64 \text{ m}$$

Point	z	σ_v	σ_a
A	0	0	$-2 * 14.36\sqrt{0.39} = -17.94$
B	6	$17.4 * 6 = 104.4$	$104.4 * 0.39 - 2 * 14.36\sqrt{0.39} = 22.78$

$$P_a = \frac{1}{2}\,\sigma_a\,(H - z_o) = \frac{1}{2} * 22.78\,(6 - 2.64) = 38.27 \text{ kN/m}$$

$$\bar{y} = \frac{1}{3}\,(H - z_o) = \frac{1}{3}\,(6 - 2.64) = 1.12 \text{ m from bottom}$$



Passive state :

$K_p = 1/k_a = 1/0.39 = 2.56$

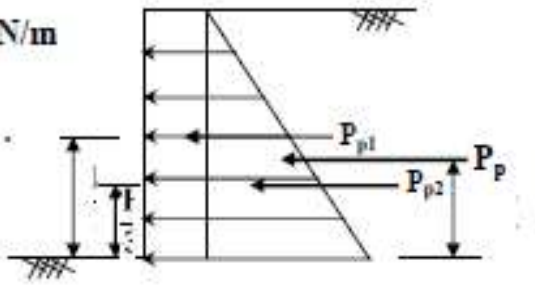
Point	z	σ_v	σ_p
A	0	0	$2 * 14.36\sqrt{2.56} = 45.95$
B	6	$17.4 * 6 = 104.4$	$104.4 * 2.56 + 2 * 14.36\sqrt{2.56} = 313.22$

$$P_{p1} = 45.95 * 6 = 275.7 \text{ kN/m at } \frac{H}{2} = 3 \text{ m from bottom.}$$

$$P_{p2} = \frac{1}{2} * 6\,(313.22 - 45.95) = 801.81 \text{ kN/m at } \frac{H}{3} = 2 \text{ m from bottom}$$

$$P_p = P_{p1} + P_{p2} = 275.7 + 801.81 = 1077.51 \text{ kN/m}$$

$$\bar{y} = \frac{801.81 * 2 + 275.7 * 3}{1077.51} = 2.26 \text{ m from bottom .}$$

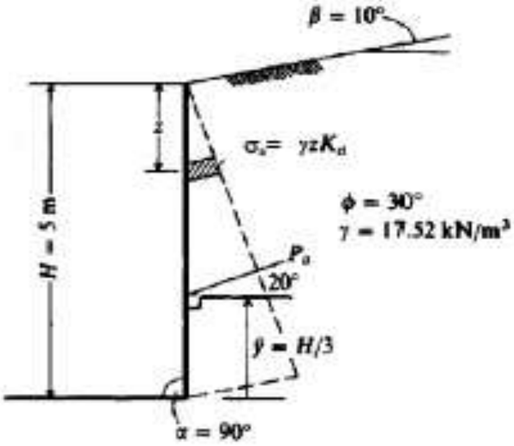


Example 2: what is the total active force per meter of wall for the soil –wall system Shown in Fig. using the Coulomb equation and show the point of its action.

Solution:

Take the wall friction angle

$\delta = \frac{2}{3}\phi = 20°$ (a common estimate)



$k_a = 0.34$ (from Table E3)

$\sigma_a = \sigma_v k_a = \gamma z k_a$

$P_a = \int_0^H \gamma z \, ka \, dz = \frac{1}{2}\gamma H^2 ka$

$P_a = \frac{1}{2}\ 17.52 * 5^2 * 0.34 = 74.5$ kN/m

Summing moment about the top , we have

$P_a \bar{y} = \int_0^H \gamma z \, ka \, z \, dz = \frac{1}{3}\gamma H^3 ka$

$\bar{y} = \frac{2\gamma H^3 ka}{3\gamma H^2 ka} = \frac{2}{3} H$ from top

Or $\bar{y} = H - \frac{2}{3}H = \frac{1}{3}H$ from bottom (value usually used)

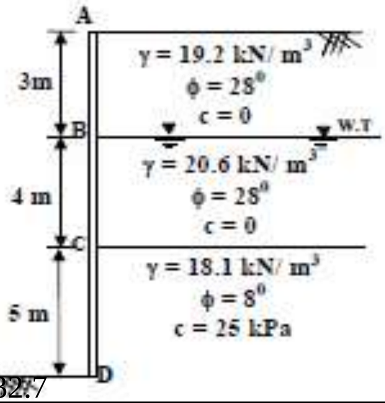
Example 3: For cut shown in fig. find the total active force: Neglect the pore pressure

Solution:

$k_a(A-B) = \frac{1-\sin 28}{1+\sin 28} = 0.361$

$k_a(B-C) = 0.361$

$k_a(C-D) = \frac{1-\sin 8}{1+\sin 8} = 0.756$



$\sigma_A = 0$

$\sigma_B = 19.2 \times 3 \times 0.361 = 20.79$

$\sigma_{c(top)} = 20.79 + [(20.6 - 9.81) \times 4] \times 0.361 = 36.37$

$\sigma_{c(bottom)} = [(19.2 * 3 + 4 * (20.6 - 9.81)]0.756 - 2 * 25\sqrt{0.756} = 32.7$

$\sigma_D = 32.7 + [(18.1 - 9.81) \times 5] \times 0.756 = 64$

$then,,, P_{total} = P1 + p2 + p3 + p4 + p5$

$p1 = \frac{1}{2} \times 3 \times 20.79 = 31.18$

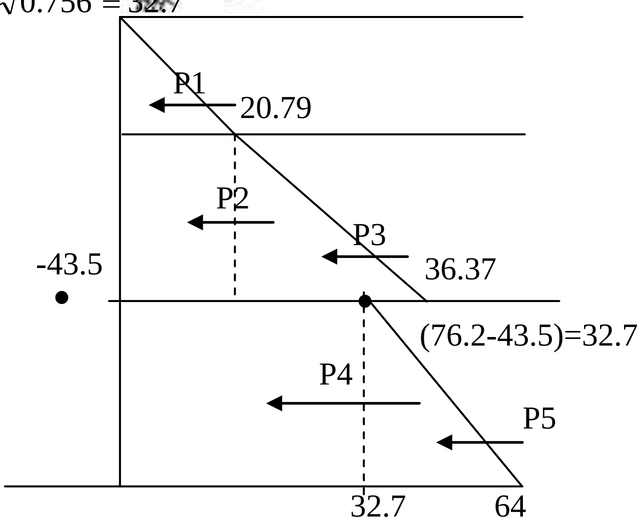
$p2 = 20.79 \times 4 = 83.16$

$p3 = \frac{1}{2} \times (36.37 - 20.79) \times 4 = 31.16$

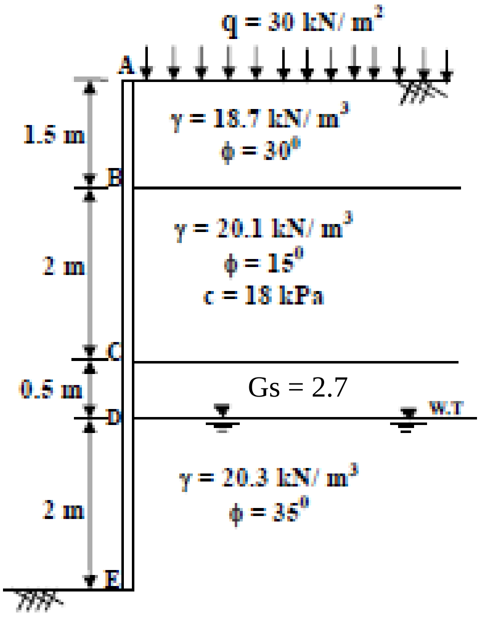
$p4 = 32.7 \times 5 = 163.5$

$p5 = \frac{1}{2} \times (64 - 32.7) \times 5 = 78.25$

$P_{total} = 387 kN$



Example 4: Determine the total horizontal active thrust on the vertical wall shown. Angle of friction between wall and soil in each layer is zero .



Solution:

Soil (A - B)

$$k_a = \frac{1-\sin 30}{1+\sin 30} = 0.33$$

Soil (B - C)

$$k_a = \frac{1-\sin 15}{1+\sin 15} = 0.59$$

Soil (C - D)

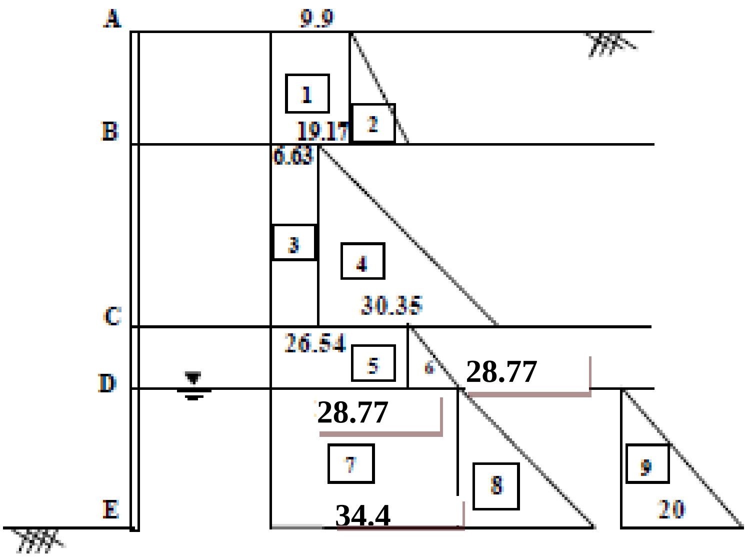
$$k_a = \frac{1-\sin 35}{1+\sin 35} = 0.27$$

$20.3 = [(2.7+e)/(1+e)]\times 9.81$

$e = 0.589$

$\gamma_d = [(2.7/(1+0.589)] \times 9.81 = 16.66\ kN/m^3$

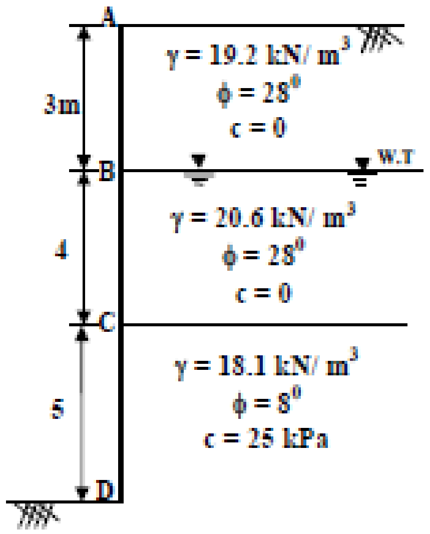
Point	depth	σ_v	σ_a
A	0	30	9.9
B_1	1.5	30 +1.5 * 18.7 = 58.1	19.17
B_2	1.5	58.1	6.63
C_1	3.5	58.1 +20.1 * 2 = 98.3	30.35
C_2	3.5	98.3	26.54
D_1	4	98.3 + 16.67 *0.5= 106.6	28.77
D_2	4	106.6	28.77
E	6	106.6 +(20.3-10)*2= 127.6	34.4



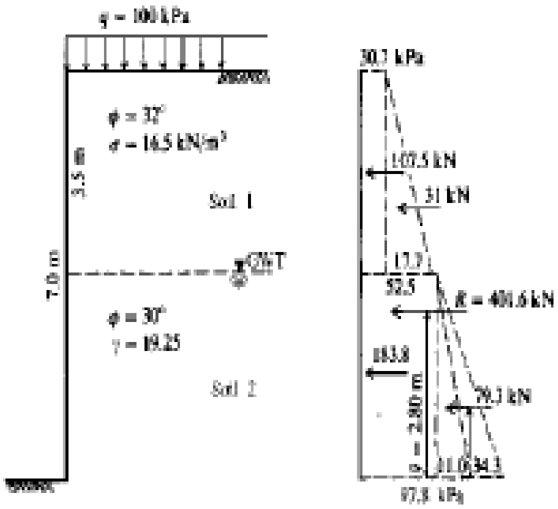
$P_1 = 9.9 * 1.5 = 14.85$ kN/m
$P_2 = 1/2(\ 19.17 - 9.9\ ) * 1.5 = 6.95$ kN/m
$P_3 = 6.63 * 2 = 13.26$ kN/m
$P_4 = 1/2(\ 30.35 - 6.63\ ) * 2 = 23.72$ kN/m
$P_5 = 26.54 * 0.5 = 13.27$ kN/m
$P_6 = 1/2(\ 29.28 - 26.54\ ) * 0.5 = 0.69$ kN/m
$P_7 = 29.28 * 2 = 58.56$ kN/m
$P_8 = 1/2(\ 34.86 - 29.28\ ) * 2 = 5.58$ kN/m
$P_9 = 1/2(\ 20 * 2\ ) = 20$ kN/m

$P_a = \sum P = 156.88$ kN/m

__H.W. 1:__ For cut shown in fig. find the total active force:



__H.W. 2__: What is the total active force/unit width of wall and what is the location of the resultant for the system shown in Fig.? Use the Coulomb equations and take a smooth wall so $\delta = 0°$.



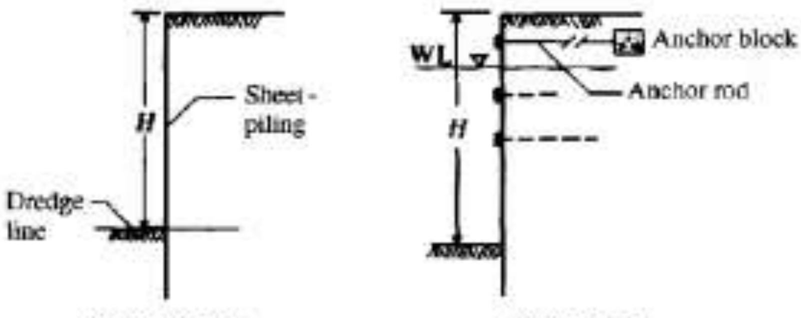
Sheet Pile Walls

Sheet pile walls are widely used for:-

1- Large and small waterfront structures .
2- Beach erosion protection .
3- Stabilizing ground slopes .
4- Trenches , and different supported excavations .
5- Cofferdams .

There are two types of sheet piles as shown in Figure :-

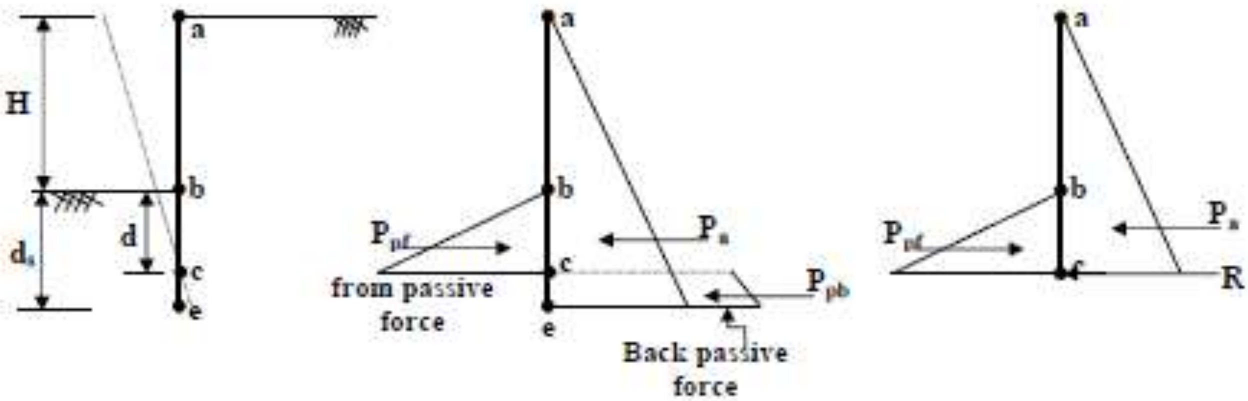
a- Cantilever sheet pile: It is used to support cuts under about 3 m in height . Also , the kind of soil is cohesion less . This kind of S.P. may be as a temporary Structure .
b- Anchored sheet pile : mainly the cantilever S.P. wall is unsuitable in clay soils And/or in excavations exceeds 3 m height . So by providing an anchor in the form of tie near the top of the wall , the required depth of penetration is reduced together with reduction of lateral deflection .



Stability Analysis

a- Cantilever sheet pile walls

At this kind of sheeting , the stability depends entirely on the passive resistance developed in front of the wall and the wall will fail by rotating about point (c) as shown below :-



Assume that the passive force at back of wall (P_{pb}) acts as a point load at (c) Depends as (R) . for equilibrium take moment about c , and use a suitable Factor of safety at passive side:-

$\Sigma M_c = 0$; where:-

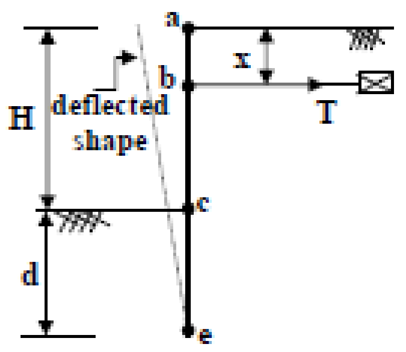
$$P_a * \frac{H+d}{3} = P_{pf} * \frac{d}{3} * \frac{1}{FS} \quad \text{............. (1)}$$

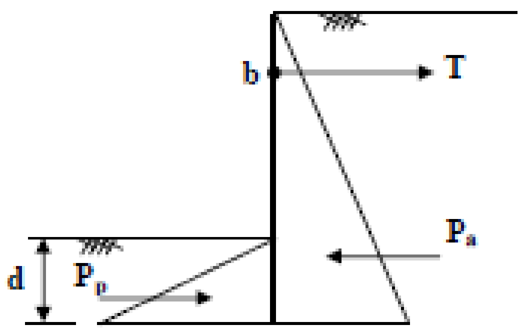
The F.S = 2 – 3 ; and from eq . 1 it can be calculate a suitable depth of penetration (d) . Since the length (c – e) is ignored in analysis, so the final embedment length of cantilever S.P. wall will be :-

$d_s = 1.2$ d (d will increased 20 % of its length)

b - Anchored sheet pile walls

The anchor rod is providing in from of tie near the top of wall at backfill side. The stability of this kind of S.P. can be analyzed , using " Free- earth support method " as follows :-





Here, two unknown :-

1- Tension force (T) required for anchor rod .
2- Total penetration depth of S.P. (d) .

By taking moment about point (b) :-

$\Sigma M_b = 0$;

$$P_a*\{(2/3)(H+d) - x\} = (P_p/FS)\{(H+d) - (d/3) - x\} \quad \text{............. (2)}$$

Where F.S = 2 – 3

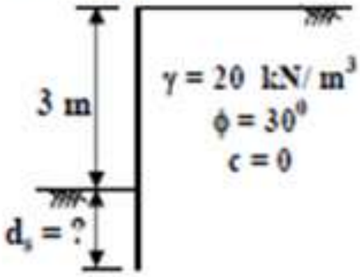
From eq. (2) it can be find the penetration depth (d) By sum the forces horizontally :-

$T = P_a - P_p$ .... (3)

From eq. (3) it can be find the tension force (T in kN/m length)

The anchor ties are usually spaced at intervals of 2 – 3 m .

<u>Example 6:</u> A cantilever sheet pile wall is to be support the side of an excavation 3 m . Determine the safe driving depth . Use F.S = 2 .



<u>Solution:</u>

$K_a = \tan^2(45 + \frac{\varphi}{2}) = \tan^2(45 + \frac{30}{2}) = 0.33$

$K_p = \frac{1}{k_a} = 3$

(or take the above values from Tables E1 and E2)

$P_a = \frac{1}{2}\ \gamma\ (H + d)^2\ k_a = \frac{20}{2}\ (3 + d)^2 * 0.33 = 3.3\ (3 + d)^2$

$P_p = \frac{1}{2}\ \gamma\ d^2 k_p\ = \frac{20}{2} d^2 * 3 = 30\ d^2$

$P_a\ (\frac{H+d}{3}) = P_{pf} * \frac{d}{3} * \frac{1}{FS}$

$3.3(3+d)^2\ \{(3+d)/3)\} = 30\ d^2 * (d/3) * (1/2)$

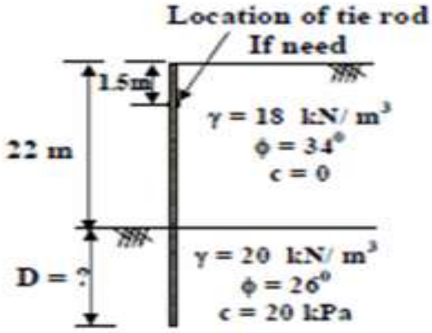
$-1.1817\ d^3 + 3\ d^2 + 9\ d + 9 = 0$

Then,

$d = 4.56$ m

$d_s = 1.2\ d = 1.2 * 4.56 = 5.47$ m

Example 8: For cut shown in Fig. Find a suitable penetration depth of sheet pile. Is sheet pile cantilever or anchored ? if it anchored , calculate how much it need for tie rod force (T) where it located at 1.5 m under G.S.



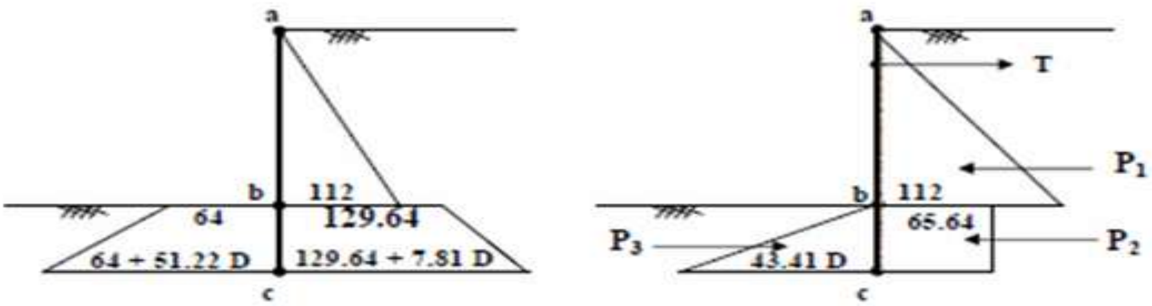
Solution:

Due to a deep cut it expected to be anchored S.P.

$K_{a(a-b)} = 0.2827$

$K_{a(b-c)} = 0.3905$ — From Tables E1 & E2

$K_{p(b-c)} = 2.5611$



Point	$\sigma_a = \gamma z\, k_a - 2c\sqrt{k_a}$ (kPa)	$\sigma_p = \gamma z\, k_p + 2c\sqrt{k_p}$ (kPa)
a	0	0
$b_{(upper)}$	18 * 22* 0.2827 = 112	0
$b_{(lower)}$	18*22*0.3905- 2*20 √0.3905 = 129.64	0 + 2 *20 √2.5611 = 64
c	129.64 + 20 D* 0.3905 = 129.64 + 7.81 D	64 + 20 D* 2.5611 = 64 + 51.22 D

$P_1 = \frac{112 \cdot 22}{2} = 1232$ kN

$P_2 = 65.64D$ kN

$P_3 = 21.7D^2$ kN

Take moment about Tie rod :-

$$P_1\left(\frac{2}{3} * 22 - 1.5\right) + P_2\left(\frac{D}{2} + 20.5\right) - P_3\left(\frac{2}{3}D + 20.5\right) = 0$$

$$D^3 + 28.47\,D^2 - 93\,D = 1121$$

$D \approx 7.1$ m (by trial & error)

$T = (P_1 + P_2) - P_3$

$= (1232 + 65.64 * 7.1) - 21.7 * 7.1^2$

$= 1698 - 1094 = 604$ kN/ m length (tension force)

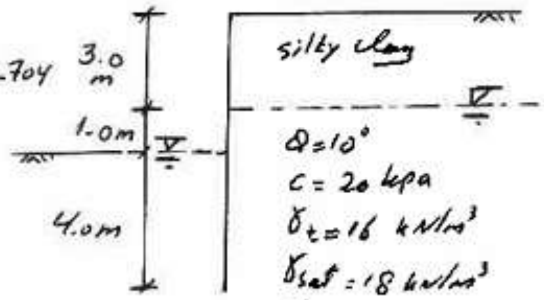
The assumption of anchored pile is suitable .

Q_2: Check the stability of the Cantilever sheet pile shown below,

Sol.

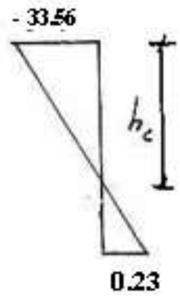
$k_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 10}{1+\sin 10} = 0.704$

$k_p = \frac{1}{k_a} = \frac{1}{0.704} = 1.42$



$P_{a(0)} = k_a \gamma h - 2c\sqrt{k_a} = 16(0)(0.704) - 2(20)\sqrt{0.704} = -33.56$ kN/m²

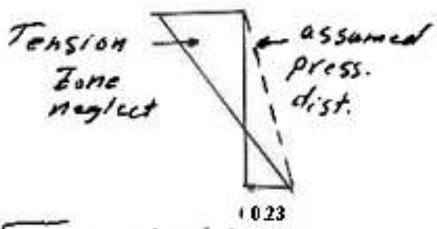
$P_{a(3)} = 0.704 \times 16 \times 3 - 2(20)\sqrt{0.704} = 0.23$ kN/m²



So, There will be a negative tension cracks (tension Zone).

$h_c = \frac{2c\sqrt{k_a}}{\gamma k_a} = \frac{2c}{\gamma\sqrt{k_a}} = \frac{2 \times 20}{16\sqrt{.704}} = 2.98\,m$

Note: For design purposes neglect the tension zone to be on the safe side and the pressure distribution will be as shown below



$P_{a(8)} = (q + \gamma_1 h_1 + \gamma_2 h_2) k_a - 2c\sqrt{k_a} =$
$= (0 + 16 \times 3 + (18-10)5)\,0.704 - 2 \times 20\sqrt{.704} = 28.13 \frac{kN}{m^2}$

$P_w = \gamma_w h_w = 10 \times 5 = 50$ kN/m²

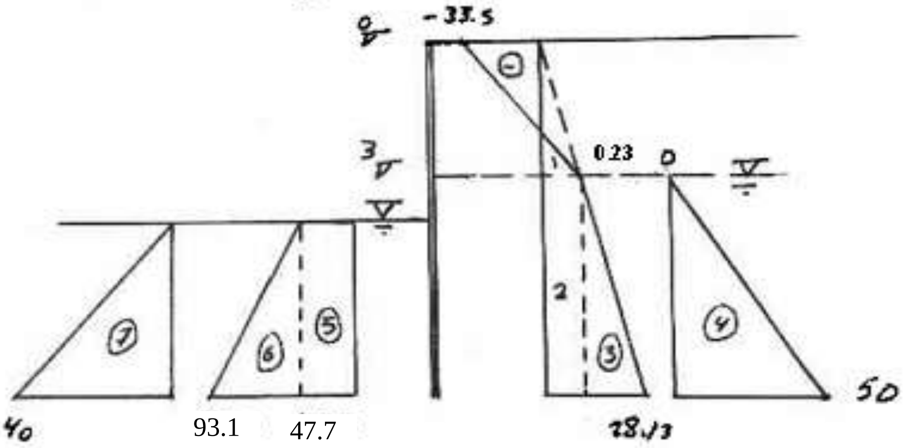
Passive Pressure

$P_{p(4)} = (q + \gamma h) k_p + 2c\sqrt{k_p} = (0)1.42 + 2 \times 20\sqrt{1.42} = 47.8 \frac{kN}{m^2}$

$$P_{P(8)} = (0 + ((8-10)4))1.42 + 2 \times 20\sqrt{1.42} = 93.1 \quad kN/m^2$$

$$P_{(w)} = 10 \times 4 = 40 \; kN/m^2$$

The pressure diagram will be as shown below.



Total active area (1+2+3+4).

$$\Sigma A_{act} = \frac{1}{2}(0.23)(3) + 0.23 \times 5 + \frac{1}{2}(28.13 - 0.23)(5) + \frac{1}{2}(50)(5) = 196.2 \quad kN/m^2$$

Total Passive area (5+6+7).

$$47.7\,(4) + \frac{1}{2}(93.1 - 47.7)(4) + \frac{1}{2}(40)(4) = 361.6 \quad kN/m^2$$

ΣM_{act} about bottom.

$$0.345 \times 6 + 1.15\,(2.5) + 69.75 \times \left(\frac{5}{3}\right) + 125\left(\frac{5}{3}\right) = 329.5 \quad kN.m$$

Σ passive about bottom.

$$190.8\,(2) + 90.8\left(\frac{4}{3}\right) + 80\left(\frac{4}{3}\right) = 609.3 \quad kN.m$$

$$F.S \text{ against sliding} = \frac{P_{P_T}}{P_{a_T}} = \frac{361.6}{196.2} = 1.84 < 2 \quad \text{not o.k.}$$

$$F.S \text{ against overturning} = \frac{\Sigma MP}{\Sigma M_a} = \frac{609.3}{329.5} = 1.85 < 2 \quad \text{not o.k.}$$

CONCRETE RETAINING WALLS

INTRODUCTION

Retaining walls are used to prevent retained material from assuming its natural slope. Wall structures are commonly used to support earth, coal, ore piles, and water. Most retaining structures are vertical or nearly so; however, if the *a* angle in the Coulomb earth-pressure coefficient of Eq. (11-3) is larger than 90°, there is a reduction in lateral pressure that can be of substantial importance where the wall is high and a wall tilt into the backfill is acceptable.
Retaining walls may be classified according to how they produce stability:

1. Mechanically reinforced earth—also sometimes called a "gravity" wall
2. Gravity—either reinforced earth, masonry, or concrete
3. Cantilever—concrete or sheet-pile
4. Anchored—sheet-pile and certain configurations of reinforced earth

$$P_a = \frac{\gamma H^2}{2} K_a$$

where

$$K_a = \frac{\sin^2(\alpha + \phi)}{\sin^2 \alpha \sin(\alpha - \delta)\left[1 + \sqrt{\dfrac{\sin(\phi + \delta)\sin(\phi - \beta)}{\sin(\alpha - \delta)\sin(\alpha + \beta)}}\right]^2} \qquad (11\text{-}3)$$

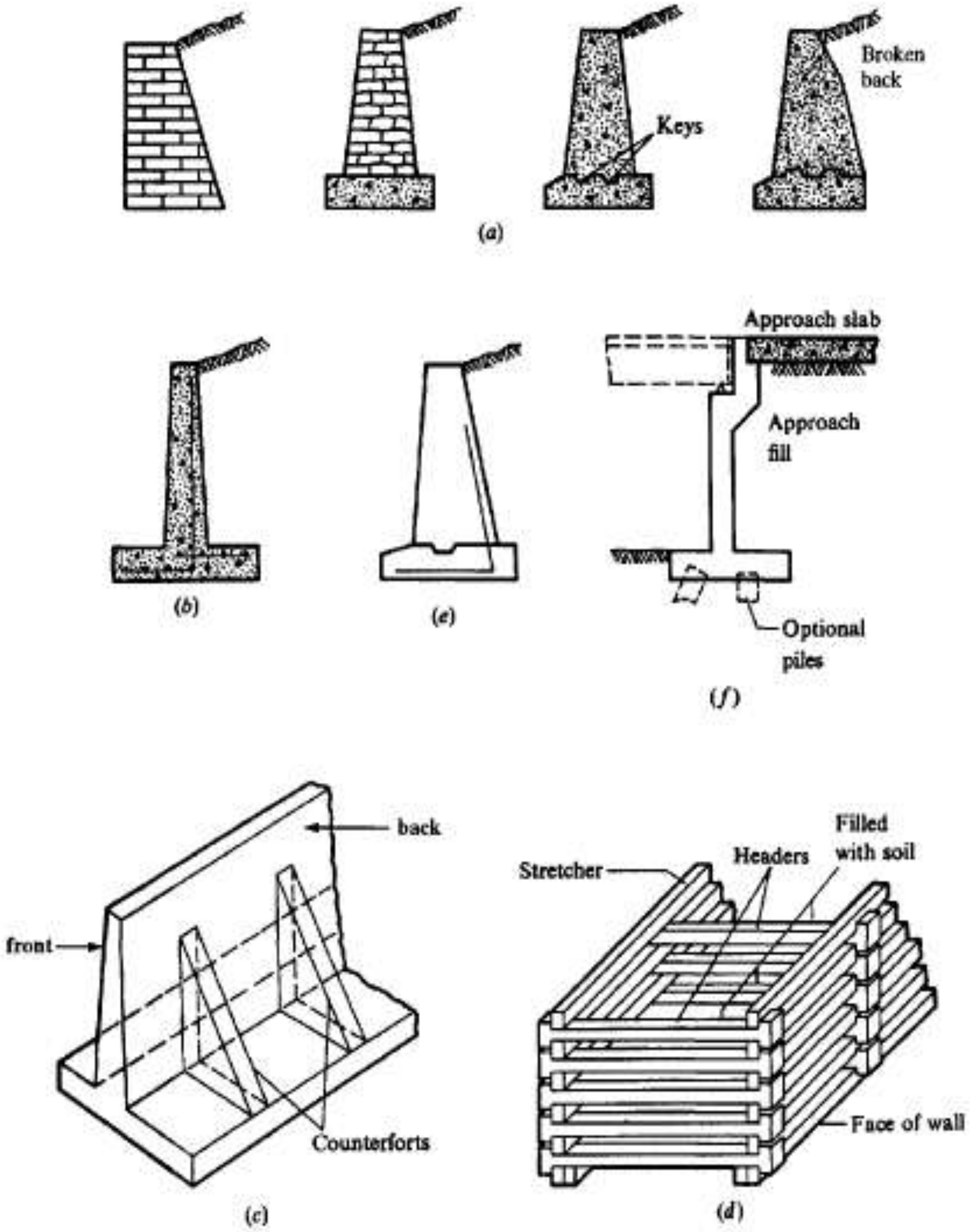


Figure 12-9 Types of retaining walls. (*a*) Gravity walls of stone masonry, brick, or plain concrete—weight provides stability against overturning and sliding; (*b*) Cantilever wall; (*c*) Counterfort, or buttressed wall—if backfill covers the counterforts the wall is termed a counterfort; (*d*) Crib wall; (*e*) Semigravity wall (uses small amount of steel reinforcement); (*f*) Bridge abutment.

CANTILEVER RETAINING WALLS

Figure 12-10 identifies the parts and terms used in retaining wall design. Cantilever walls have these principal uses at present:

1. For low walls of fairly short length, "low" being in terms of an exposed height on the order of 1 to 3.0 m and lengths on the order of 100 m or less.

2. Where the backfill zone is limited and/or it is necessary to use the existing soil as backfill. This restriction usually produces the condition of Fig. 11-12b, where the principal wall pressures are from compaction of the backfill in the limited zone defined primarily by the heel dimension.

3. In urban areas where appearance and durability justify the increased cost.

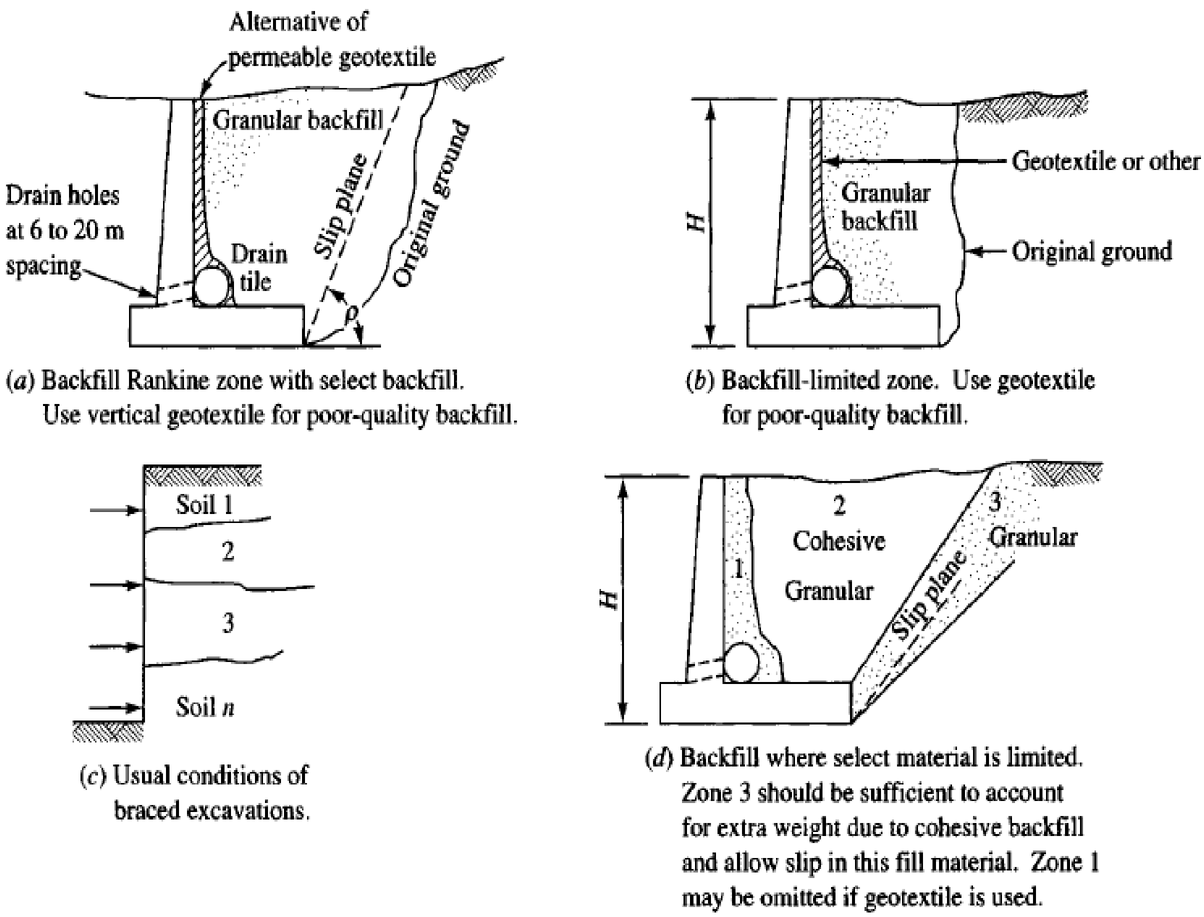


Figure 11-12 Various backfill conditions. The longitudinal collector (or drain) pipe is optional.

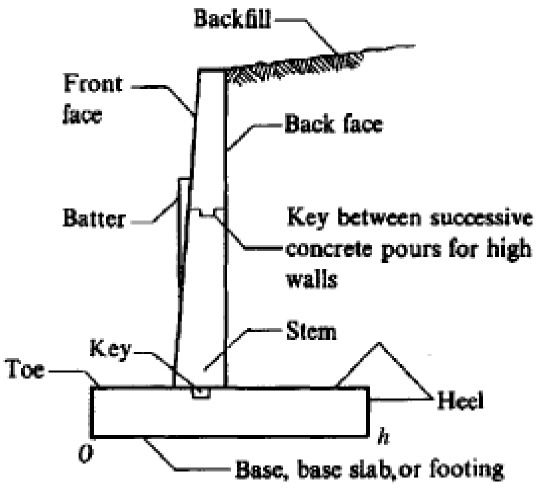


Figure 12-10 Principal terms used with retaining walls. Note that "toe" refers to both point O and the distance from front face of stem; similarly "heel" is point h or distance from backface of stem to h.

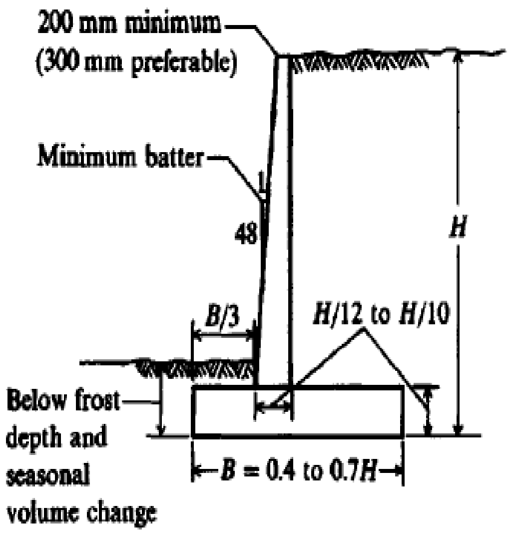
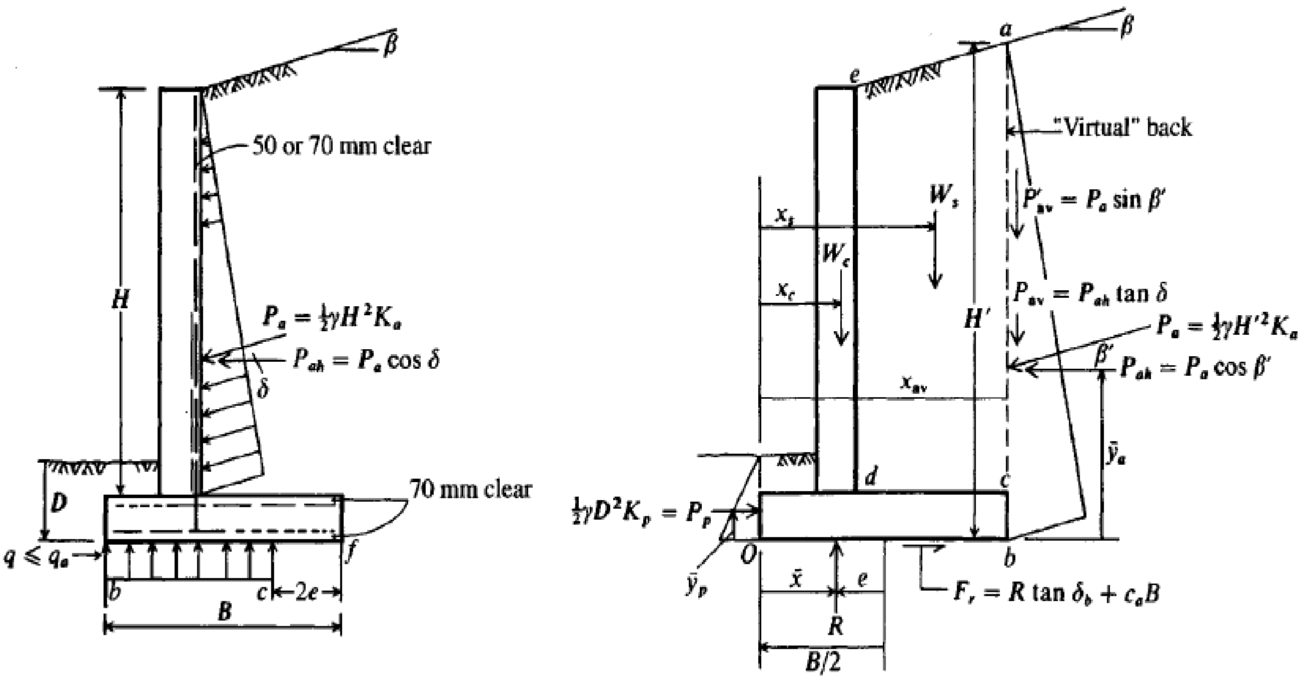


Figure 12-11 Tentative design dimensions for a cantilever retaining wall. Batter shown is optional.



(*a*) Wall pressure to use for shear and bending moment in stem design. Also shown is bearing capacity pressure diagram based on Fig. 4-4 using $B' = b - 2e$ and $L = L' = 1$ unit.

(*b*) Wall pressure for overall stability against overturning and sliding. W_c = weight of all concrete (stem and base); W_s = weight of soil in zone *acde*. Find moment arms x_i any way practical – usually using parts of known geometry. Use this lateral pressure for base design and bearing capacity.

Figure 12-12 General wall stability. It is common to use the Rankine K_a and $\delta = \beta$ in (*a*). For β' in (*b*) you may use β or ϕ since the "slip" along *ab* is soil-to-soil. In any case compute $P_{av} = P_{ah} \tan\phi$ as being most nearly correct.

Sliding and Overturning Wall Stability

The wall must be safe against sliding. That is, sufficient friction F_r must be developed between the base slab and the base soil that a safety factor SF or stability number N_s(see Fig. 12-12b) is

$$SF = N_s = \frac{F_r + P_p}{P_{ah}} \geq 1.25 \text{ to } 2.0 \qquad (12\text{-}4)$$

All terms are illustrated in Fig. *12-12b.* Note that for this computation the total vertical force R is

$$R = W_c + W_s + P'_{av}$$

These several vertical forces are shown on Fig. 12-12b. The heel force P'_{av} is sometimes not included for a more conservative stability number. The friction angle δ between base slab and soil can be taken as $\varnothing$ where the concrete is poured directly onto the compacted base soil. The base-to-soil adhesion is usually a fraction of the cohesion—values of 0.6 to 0.8 are commonly used. Use a passive force P_p if the base soil is in close contact with the face of the toe. One may choose not to use the full depth of D in computing the toe P_p if it is possible a portion may erode. For example, if a sidewalk or roadway is in front of the wall, use the full depth (but not the surcharge from the sidewalk or roadway, as that may be removed for replacement); for other cases one must make a site assessment.

The wall must be safe against overturning about the toe. If we define these terms:

$\bar{x}$ = location of R on the base slab from the toe or point O. It is usual to require this distance be within the middle 1/3 of distance Ob—that is, $\bar{x} > B/3$ from the toe.

P_{ah} = horizontal component of the Rankine or Coulomb lateral earth pressure against the vertical line ab of Fig. 12-12b (the "virtual" back).

$\bar{y}$ = distance above the base Ob to P_{ah}.

P_{av} = vertical shear resistance on virtual back that develops as the wall tends to turn over. This is the only computation that should use P_{av}. The δ angle used for P_{av} should be on the order of the residual angle $\varnothing_r$ since the Rankine wedge soil is in the state of Fig. 11-lc and "follows" the wall as it tends to rotate.

We can compute a stability number N_0 against overturning as

$$N_o = \frac{M_r}{M_o} = \frac{\sum W_i \bar{x} + P_{av} B}{P_{ah} \bar{y}} \geq 1.5 \text{ to } 2.0 \tag{12-5}$$

In both Eqs. (12-4) and (12-5) the stability number in the given range should reflect the importance factor and site location. That is, if a wall failure can result in danger to human life or extensive damage to a major structure, values closer to 2.0 should be used. Equation (12-5) is a substantial simplification used to estimate overturning resistance. On-site overturning is accompanied by passive resistances at (1) the top region of the base slab at the toe, (2) a zone along the heel at cb that tends to lift a soil column along the virtual back face line ab, and (3) the slip of the Rankine wedge on both sides of ab. Few walls have ever overturned—failure is usually by sliding or by shearoff of the stem. The $\sum(W_c+W_s)$ and location $\bar{x}$ are best determined by dividing the wall and soil over the heel into rectangles and triangles so the areas (and masses) can be easily computed and the centroidal locations identified. Then it becomes a simple matter to obtain

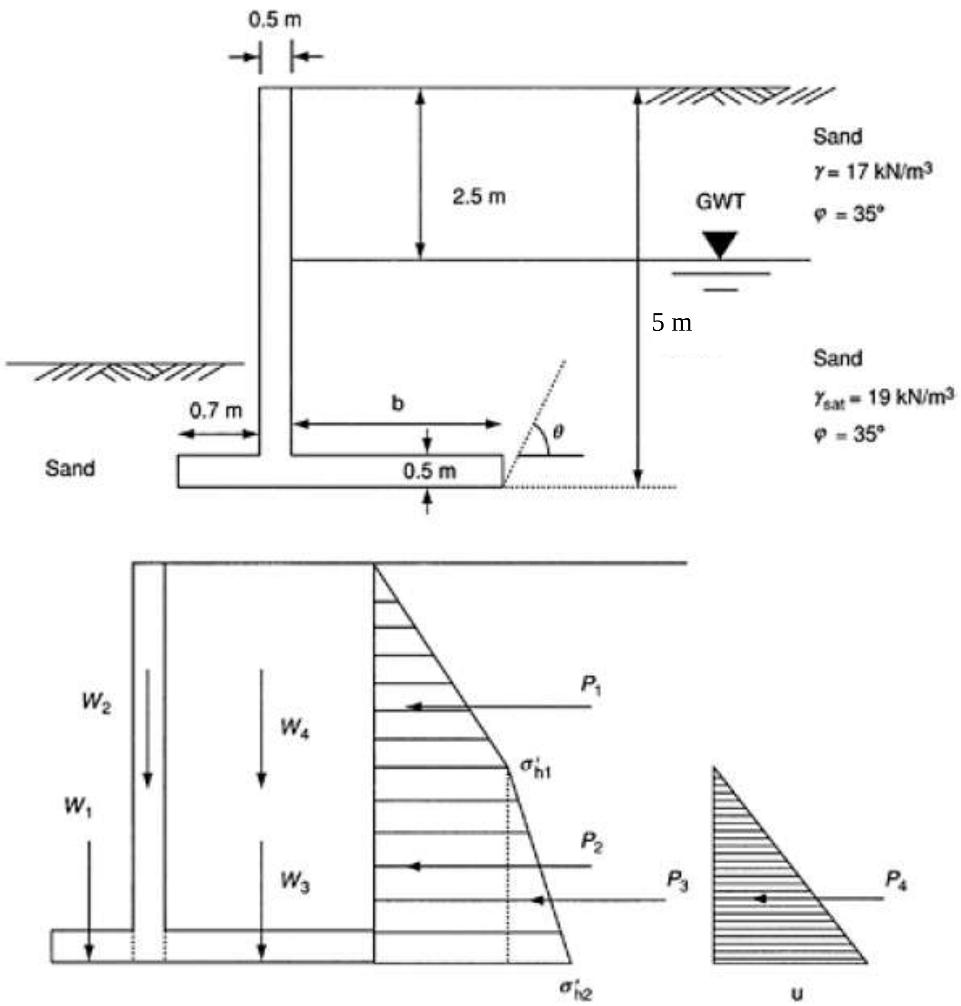
$$(W_c + W_s + P'_{av})\bar{x} = P_{ah}\bar{y} - P_p\bar{y}_p$$

$$\bar{x} = \frac{M_o - P_p\bar{y}_p}{W_c + W_s + P'_{av}}$$

If there is no passive toe resistance (and/or P'_{av} is ignored) the preceding equations are somewhat simplified.

Example:

For the cantilever retaining wall shown in figure, calculate the width of the heel, *b*, required to ensure stability of the wall against overturning. In addition, determine the angle, *θ,* of the potential active shear plane with respect to horizontal. Then, Calculate the factor of safety against sliding. (neglect the passive action, use b=2m)



In order to facilitate the calculation process, we divide the cantilever wall into sections. We then calculate the weight per unit width (W_i) and moment arm *(xi)* for each block:

W_1=23.5×0.5×0.7=8.23 kN (per meter)
W_2=23.5×5×0.5=58.75 kN
W_3=23.5×0.5×b=11.75b kN
W_4=(17×2.5 +19×2)b=80.5b kN

x_1=0.35 m
x_2=0.95 m
x_3=1.20+b/2
x_4=1.20+b/2

We then calculate the active earth pressure and the water pressure on the wall. For lateral earth pressure calculations, we use $K_a=\tan^2(45–35/2)=0.271$

$$\sigma'_{h1} = 17 \times 2.5 \times 0.271 = 11.52 \text{ kPa}$$
$$\sigma'_{h2} = (17 \times 2.5 + \{19 - 9.8\} \times 2.5) \times 0.271 = 17.75 \text{ kPa}$$
$$u = 9.8 \times 2.5 = 24.5 \text{ kPa}$$

The corresponding forces (per meter), P_1 to P_4, together with their moment arms, y_1 to y_4, are calculated as follows:

P_1=0.5×11.52×2.5=14.4 kN
P_2=11.52×2.5= 28.8 kN
P_3=0.5×(17.75−11.52)×2.5=7.79 kN
P_4=0.5×24.5×2.5=30.63 kN

y_1=2.5+2.5/3=3.33 m
y_2=2.5/2=1.25 m
y_3=y_4=2.5/3=0.83 m

The factor of safety against overturning is calculated from

$$FS_{overturning} = \frac{\sum_{i=1}^{4} W_i x_i}{\sum_{i=1}^{4} P_i y_i}$$
$$= \frac{8.23 \times 0.35 + 58.75 \times 0.95 + 11.75b \times (1.2 + b/2) + 80.5b \times (1.2 + b/2)}{14.4 \times 3.33 + 28.8 \times 1.25 + 7.79 \times 0.83 + 30.63 \times 0.83}$$

In order to ensure stability, the factor of safety must be at least equal to **1.5**. Accordingly, we solve the equation above for b and obtain:
***b*= 0.78 m**

The angle, θ, that the potential active failure surface makes with respect to horizontal is simply equal to $45+\varphi/2=45+35/2=62.5^{o}$.

To calculate the factor of safety against sliding:

SF = N_o = [(summation of vertical forces) × tan δ + C_b × B] / (summation of lateral forces)

C_b = cohesion of base soil = (0.6 – 0.8)$C_{back\ soil}$

C = 0 in this example

SF = [($W_1+W_2+W_3+W_4$) × tan δ] / ($P_1+P_2+P_3+P_4$)

$W_1=23.5\times0.5\times0.7=8.23$ kN (per meter)
$W_2=23.5\times5\times0.5=58.75$ kN
$W_3=23.5\times0.5\times2= 23.5$ kN
$W_4=(17\times2.5 +19\times2) \times 2 =161$ kN

$P_1=0.5\times11.52\times2.5=14.4$ kN
$P_2=11.52\times2.5= 28.8$ kN
$P_3=0.5\times(17.75-11.52)\times2.5=7.79$ kN
$P_4=0.5\times24.5\times2.5=30.63$ kN

SF = [(8.23+58.75+23.5+161) × tan 35] / (14.4+28.8+7.79+30.63)

SF = 2.157

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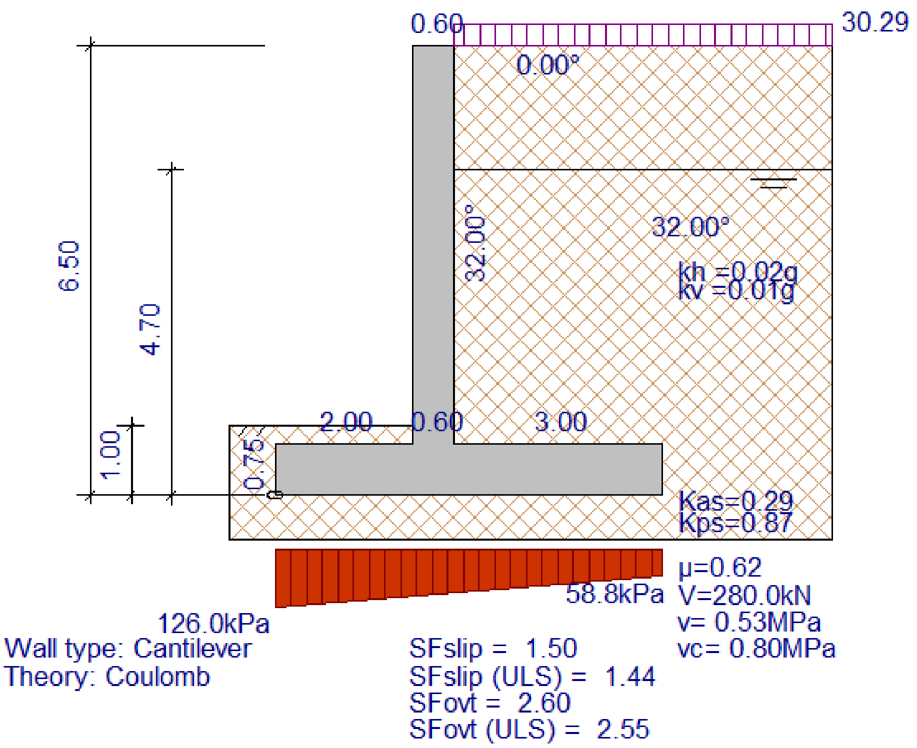
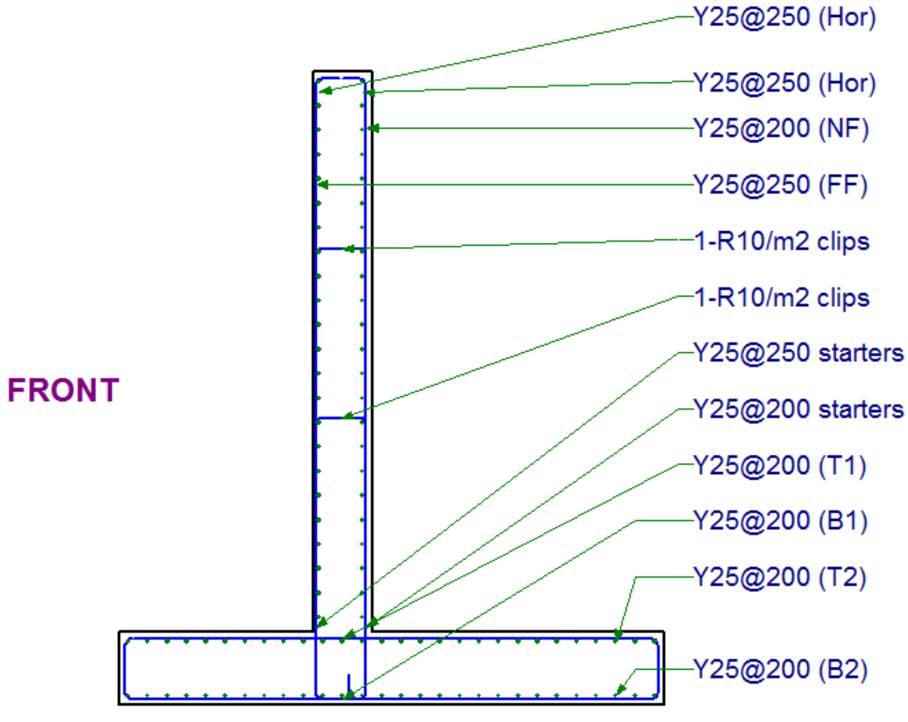




SECTION

STABILITY OF SLOPES

Stability of Slopes

The slopes in soils are artificial or natural. The artificial slopes (non-made), the canel side slopes, the gradiant of roads, embankments & earth fill dams.

· Cutting and unsupported excavation, in all slopes, there is a tendency to degrade to more stable towards horizontal. The forces which cause instability are these of gravity & seepage, while resisting to failure is mainly from combination of slope geometry and the shear strength of soil. The shape of failure surface depend on the characteristics of soil and the slope geometry, its shape be parallel with the slope surface for sandy soils, while it be the arc of circle for the clay soils and infinite slopes.

Kinds of Slopes Failure

1) Finite Slopes (Rotational): These occure in homogeneous chahesion soils. The movement taking place usually along a curved rupture surface, it occure in the non-made slopes, such that the side slopes of earth channel & earth fill dams.

2) Infinite slopes (Translation):-

These may occur where the weak layer lies near a parallel to surface, the movement at failure taking place usually along plane parallel to the slope surface.

Factor of Safety: The failure occure when the shear stress equal to or greater than shear strength of soil, so the factor of safety is

$$F.S = \frac{\text{shear strength } (S)}{\text{shear stress } (\tau)} \qquad (S > \tau)$$

$$= \frac{c + \sigma \tan \phi}{C_m + \sigma \tan \phi_m} \longrightarrow (1)$$

where:-

S = shear strengt

τ = shear stress

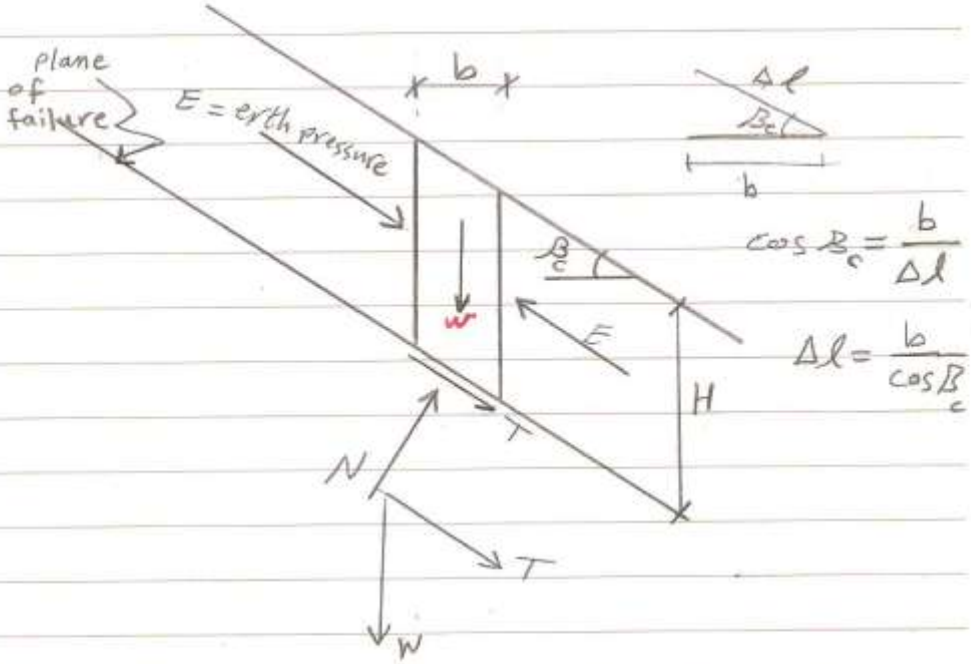
ϕ & c = angle of repose & cohesion of slope.

ϕ_m & C_m = The mobilized angle of repose & cohesion. القيم تمثل حالة على وشك الفشل.

$$F.S = \frac{c}{C_m} \qquad \text{if} \quad \phi_m = \phi = 0 \,. \ (\text{cohesion soil})$$

$$F.S = \frac{\tan \phi}{\tan \phi_m} \qquad (\text{Cohesion less Soil}).$$

Infinite Slopes :-



The forces acting on element are :-

$W = \gamma H b$ (weight of element)

$T = W \sin\beta_c$ (tangential reaction on failure plane)

$N = W \cos\beta_c$ (normal reaction on failure plane)

$$\sigma = \frac{N}{(b/\cos\beta_c)} = \frac{\gamma H b \cos\beta_c}{\frac{b}{\cos\beta_c}} = \gamma H \cos^2\beta_c \longrightarrow (2)$$

$$\tau = \frac{T}{b/\cos\beta_c} = \frac{\gamma H b \sin\beta_c}{b/\cos\beta_c}$$

$$= \gamma H \sin\beta_c \cos\beta_c \longrightarrow (3)$$

For the mobilized shear stress will be:

$$\tau' = c_m + \sigma' \tan \phi_m$$

from 2 & 3

$$\gamma H \cos^2 \beta \tan \phi_m + c_m = \gamma H \sin \beta_c \cos \beta_c$$

$$\frac{c_m}{\gamma H} = \sin \beta_c \cos \beta_c - \cos^2 \beta_c \tan \phi_m$$

$$\frac{c_m}{\gamma H} = \cos^2 \beta \left(\tan \beta_c - \tan \phi_m \right)$$

For any β:

$$\frac{c_m}{\gamma H} = \cos^2 \beta \left(\tan \beta - \tan \phi_m \right)$$

For infinite slopes

$\frac{c_m}{\gamma H}$ is the Taylor's stability No.

لذا فان أقصى زاوية للميل قبل الانزلاق هي :

$$\tan \beta = \tan \phi$$

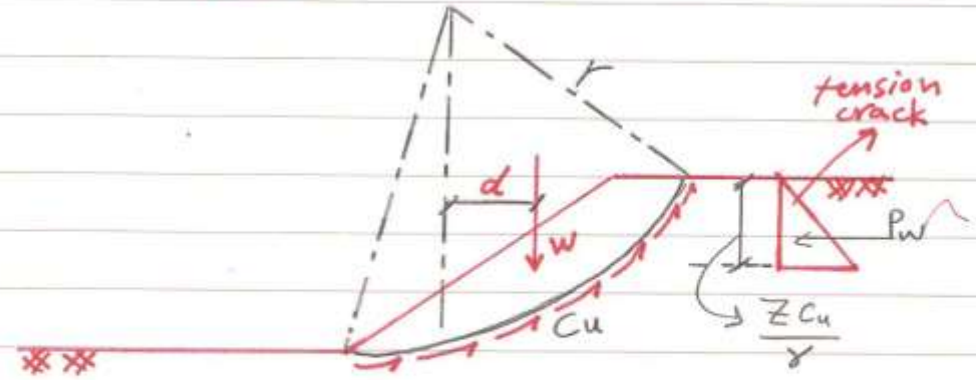
$\beta = \phi$ (for dry cohesionless soil).

then, $F.S = 1$

$\beta < \phi \quad F.S > 1$

For cohesive soil $\phi = 0$

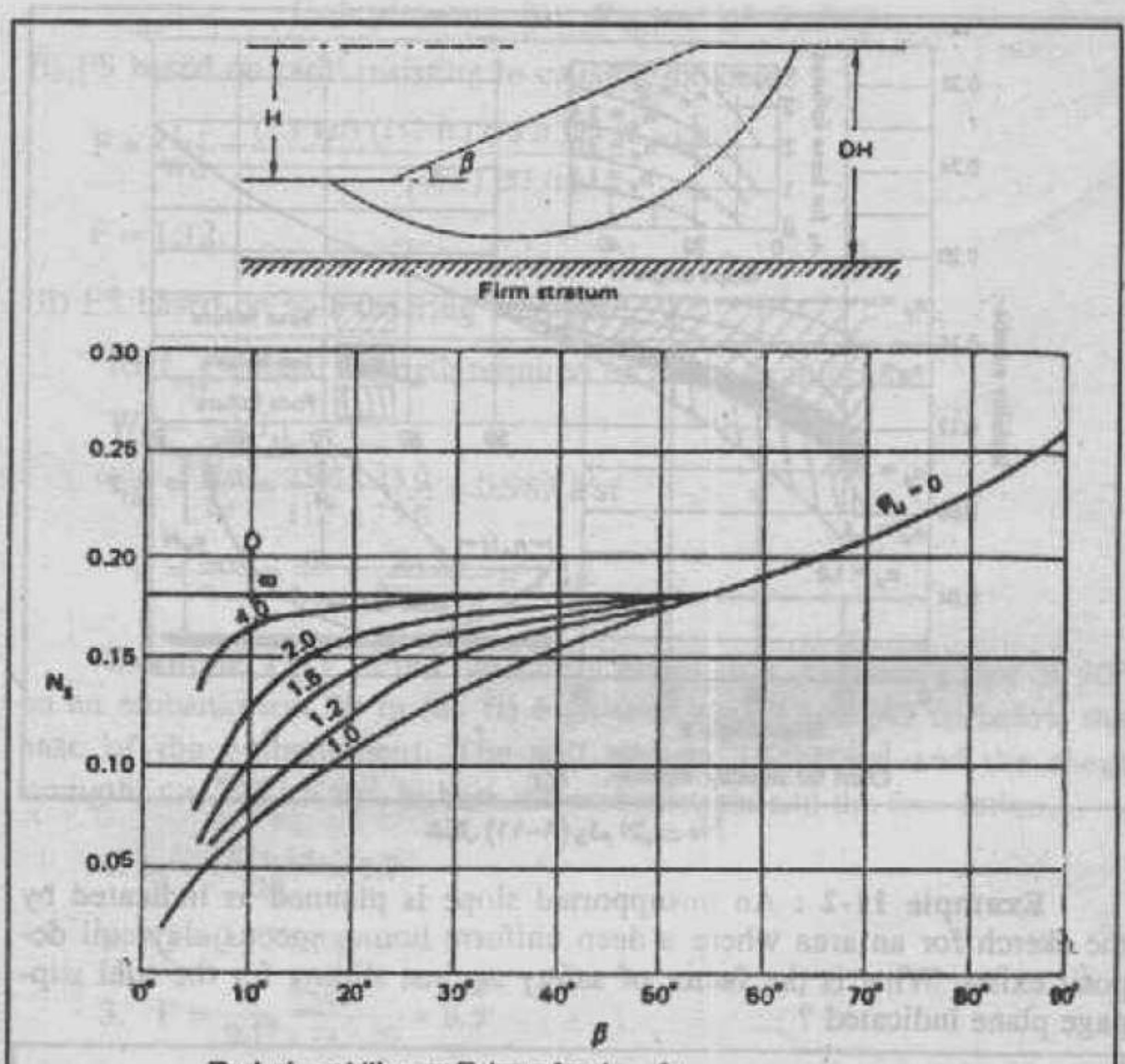
$$F = \frac{C_u L_u r}{W \cdot d}$$



$\phi_u = 0$ analysis

ولقد قدم Taylor معامل الاتزان Stability Coefficient لتحليل الميول المتجانسة ، ولميل ارتفاعه H فان معامل الثبات لسطح الانهيار حيث معامل الأمان أقل ما يمكن يعبر عنه بالعلاقة ،

$$N_s = \frac{C_u}{F.s \; \gamma H}$$; N_s (Figure) For $\phi = 0$.



Taylor's stability coefficients for $\phi_u = 0$.

شكل (١١-٧) معامل تيلور للثبات

Example 11-1 : A 45° slope is excavated to a depth of 8 m in a deep layer of saturated clay of unit weight 19 kN/m^3: the relevant shear strength parameters are $C_u = 65\ kN/m^2$ and $\phi_u = 0$. Determine the factor of safety for the trial failure surface specified in Fig. 11-7.

In Fig. 11-7 the cross-sectional area ABCD is 70 m^2.

Weight of soil mass = 70 x 19 = 1330 kN/m.

The centroid of ABCD is 4.5 m from O. The angle AOC is $89\frac{1}{2}°$ and radius OC is 12.1 m. The arc length ABC is calculated as 18.9 m. The factor of safety is given by :

$$F = \frac{C_u L_a r}{Wd}$$

$$= \frac{65 \times 18.9 \times 12.1}{1330 \times 4.5} = 2.48$$

This is the factor of safety for the trial failure surface selected and is not necessarily the minimum factor of safety.

The minimum factor of sactor of safety can be estimated by the following relation.

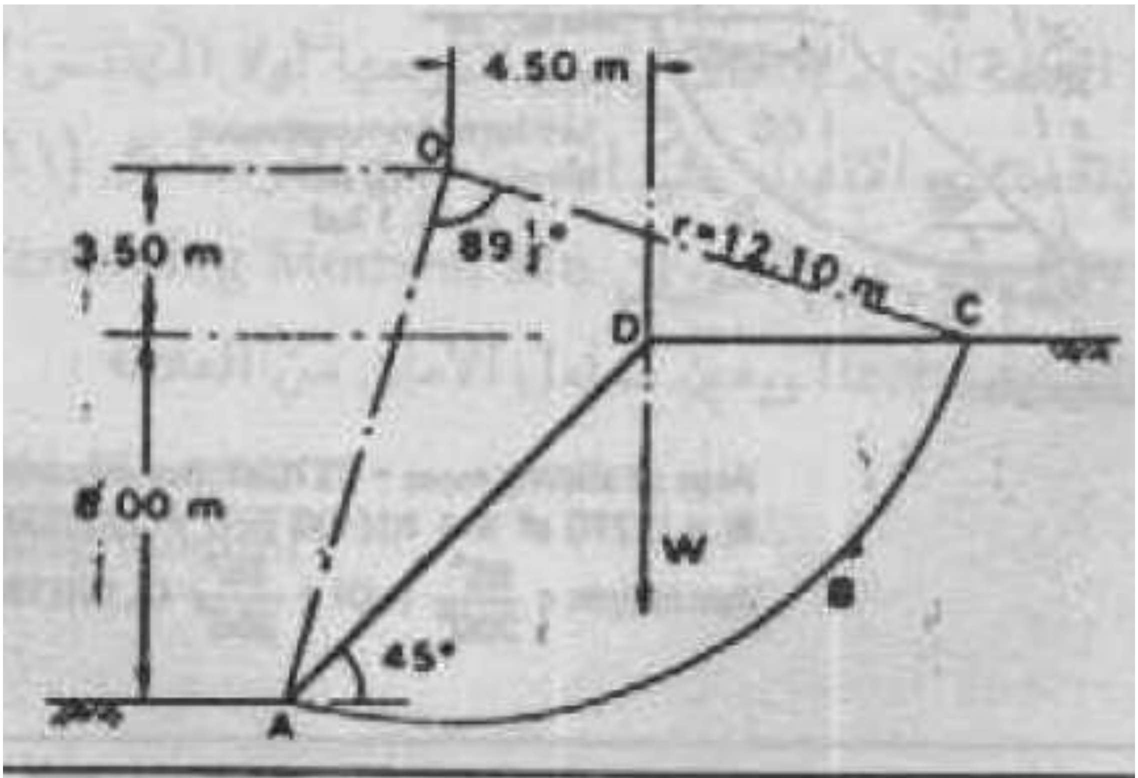
From Fig. 11-6, $\beta = 45°$ and assuming that D is large, the value of N_s is 0.18. Then

$$F = \frac{C_u}{N_s \gamma H}$$

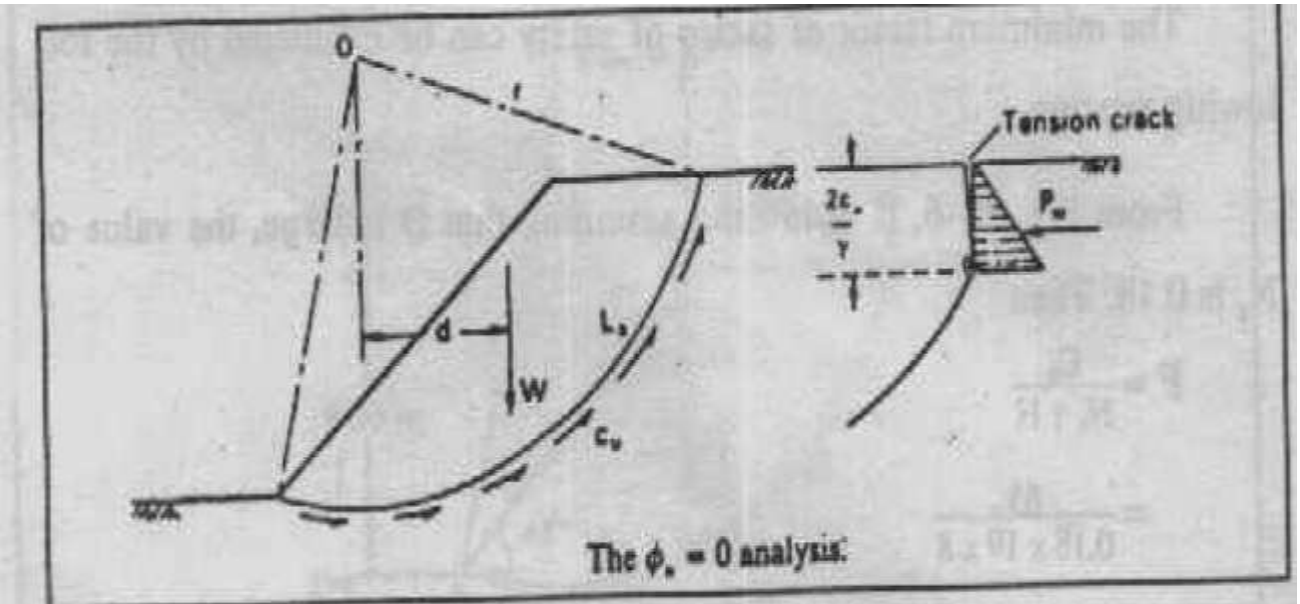
$$= \frac{65}{0.18 \times 19 \times 8}$$

$$= 2.37$$

S = (π/180)*θ* r = (π/180) 89.5 * 12.1 = 18.9 m



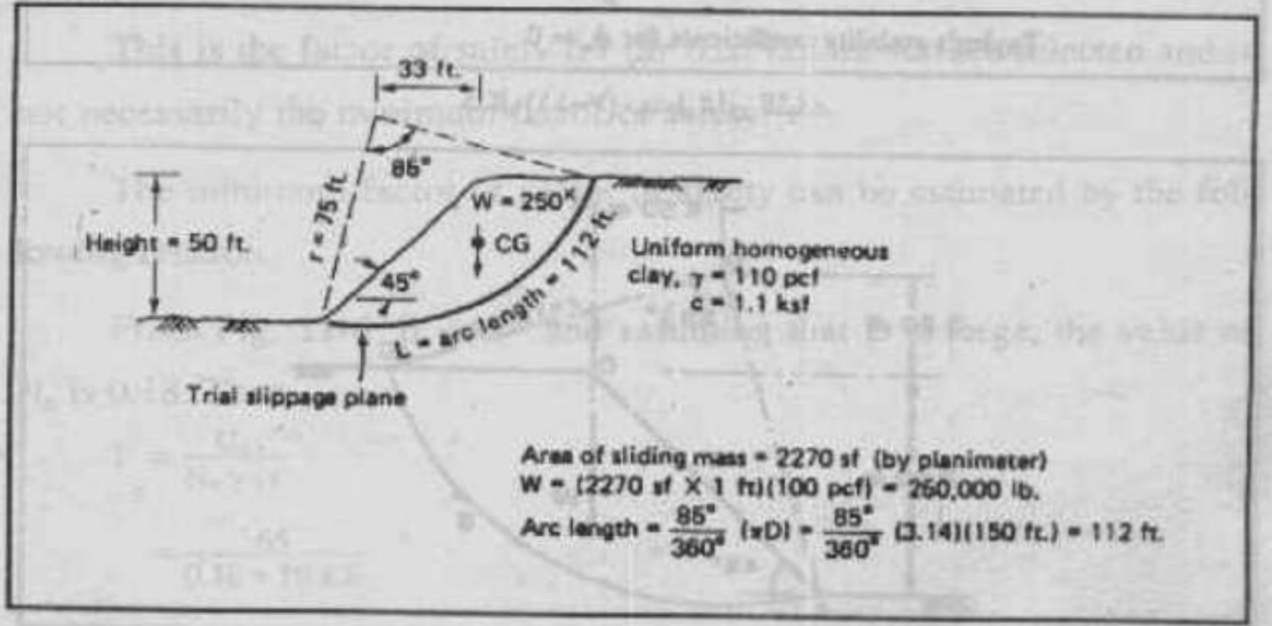
شكل (١١-٨) مثال (١١-١)



The $\phi_u = 0$ analysis.

شكل (١١-٦) دوران الميول في التربة المتماسكة

Example 11-2 : An unsupported slope is planned as indicated by the sketch for an area where a deep uniform homogeneous clay-soil deposit exists. What is the factor of safety against sliding for the trial slippage plane indicated ?



شكل (١١-١٠) مثال (١١-٢)

Calculations for Factor of Safety

(i) FS based on ratio resisting to causing moments :

$$F = \frac{c\,L\,r}{W d} = \frac{(1.1 \text{ ksf})\,(112 \text{ ft.})\,(75 \text{ ft.})\,(1 \text{ ft. width})}{(250^k)\,(33 \text{ ft.})}$$

$$F = 1.12$$

(ii) FS based on soil shearing strength :

let τ_{req} = shear strength required for slope equilibrium.

$$Wd = \tau_{req}\,Lr$$

$$\tau_{req} = \frac{Wd}{Lr} = \frac{250^k \times 33 \text{ ft}}{112^k \times 75 \text{ ft}} = 0.985 \text{ k'st}$$

$$F = \frac{\tau_{max}}{\tau_{req}} = \frac{C}{\tau_{req}} = \frac{1.1 \text{ ksf}}{0.985 \text{ ksf}} = 1.12$$