Civinnovate

Discover, Learn, and Innovate in Civil Engineering

Engineering Hydrology
Selected Tutorial solutions

Course: B.E. (Civil)

Year/Part: 3rd /1st

Water balance

3. A small catchment of area 150 ha received a rainfall of 10.5 cm in 90 minutes due to a storm. At the outlet of the catchment, the stream draining the catchment was dry before the storm and experience a runoff lasting for 10 hours with an average discharge value of 2 m^3/s. The stream was again dry after the runoff event. (a) What is the amount of water which was not available to runoff due to combined effect of infiltration, evaporation and transpiration? (b) What is the ratio of runoff to precipitation?

Solution:
Catchment area = 150 ha = 150x10000 m^2=15000000 m^2
Precipitation = 10.5 cm=10.5/100 m =0.105m
Volume of precipitation (P) = 0.105x1500000=157500 m^3
Runoff = 2 m^3/s
Runoff volume in 10 hours (R) = 2x10x36000 = 72000 m^3
a) Amount of water which not available to runoff due to combined effect of infiltration, evaporation and transpiration = Loss (L) = ?
From Water balance equation

$$-(R+G+E+T)=$$

Here G+E+T = L and ΔS =0 as the stream was dry before and after the storm.
So, above equation reduces to
P-(R+L)=0
L=P-R = 157500-72000 = 85500 m^3

b) Runoff/Rainfall = 72000/157500 = 0.457

Precipitation

3. The annual precipitation at station Z and the average annual precipitation at 10 neighbouring station are as follows:

Year	Precipitation at Z (mm)	10 station average (mm)
1972	35	28
1973	37	29
1974	39	31
1975	35	27
1976	30	25
1977	25	21
1978	20	17
1979	24	21
1980	30	26
1981	31	31
1982	35	36
1983	38	39
1984	40	44
1985	28	32
1986	25	30
1987	21	23

Use double mass curve analysis to correct for any data inconsistencies at station Z.

Solution:

Year	Precipitation at Z (mm)	10 station average (mm)	Accumulated rainfall at Z (mm)	Accumulated rainfall of 10 stations (mm)	Corrected value
1987	21	23	21	23	
1986	25	30	46	53	
1985	28	32	74	85	
1984	40	44	114	129	
1983	38	39	152	168	
1982	35	36	187	204	
1981	31	31	218	235	
1980	30	26	248	261	
1979	24	21	272	282	17.76
1978	20	17	292	299	14.8

1977	25	21	317	320	18.5
1976	30	25	347	345	22.2
1975	35	27	382	372	25.9
1974	39	31	421	403	28.86
1973	37	29	458	432	27.38
1972	35	28	493	460	25.9

According to the plot, there is break in the slope of line from 1980. So the rainfall data at Z is inconsistent before 1980.

c = 200, a = 270, b = 220

Slope of original line (Mc) = c/b = 0.91

Slope of line after year 1980 (Ma) = a/b = 1.22

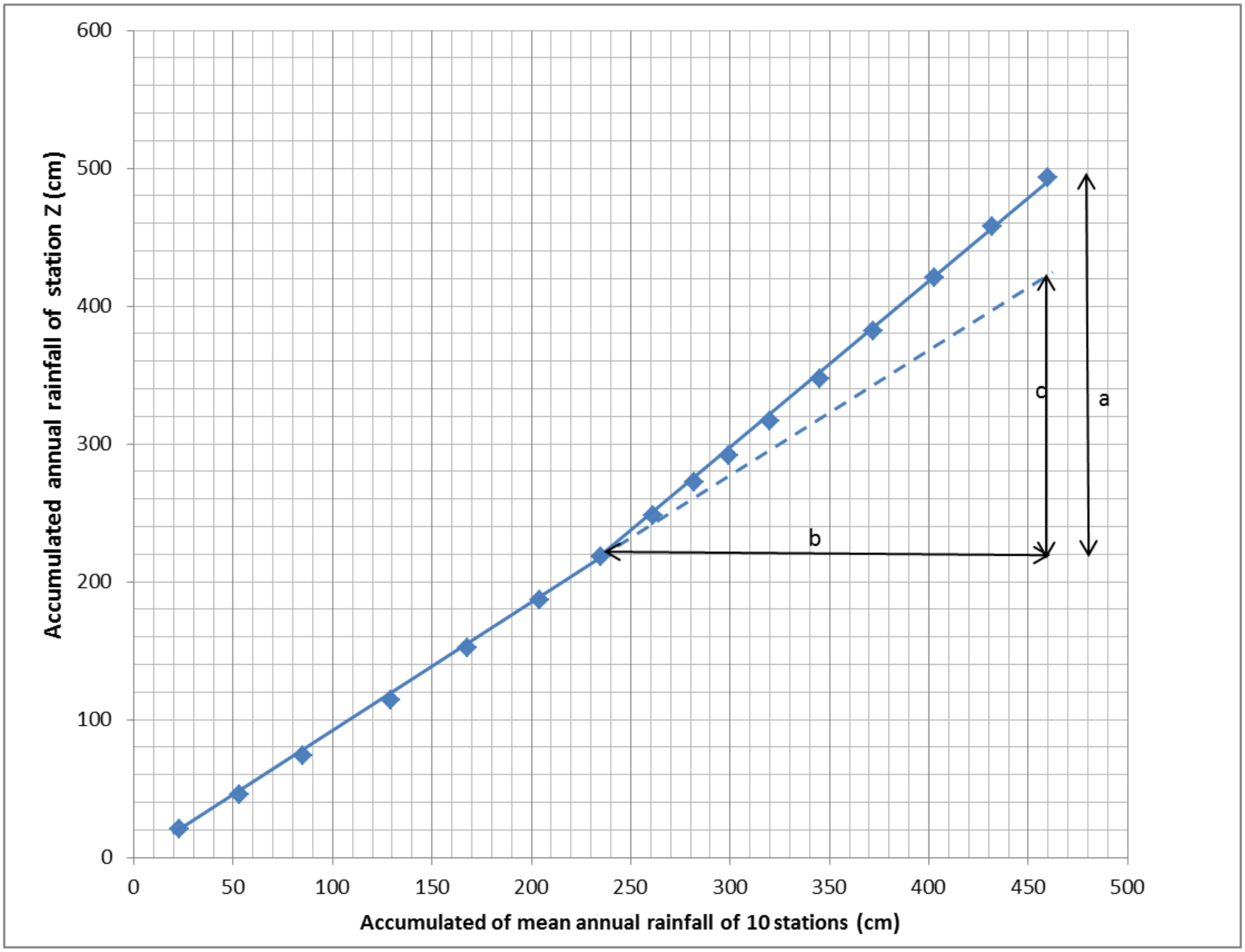
Difference in slope $=\frac{0.91-1.22}{0.91} x100$= -34%

As the difference in slope is more than 10%, correction needs to be applied.

Correction ratio = Mc/Ma = c/a= 0.74

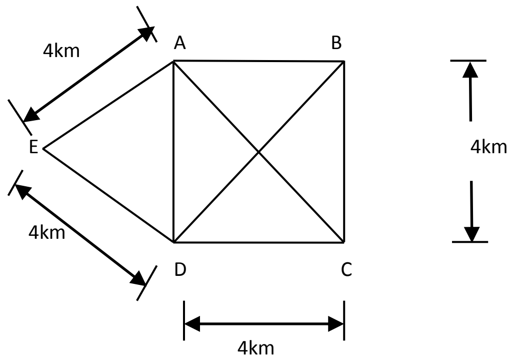
Corrected precipitation = (c/a)x observed precipitation

Corrected precipitation before 1980 is shown in above Table.

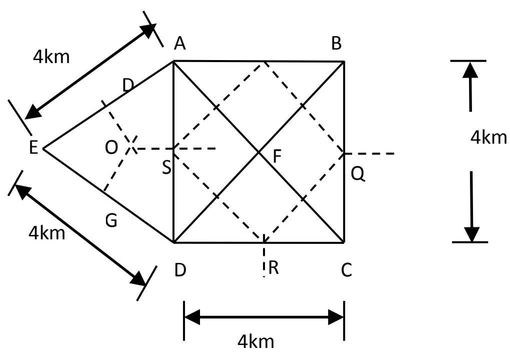


Double mass curve

5. The area of a catchment is composed of a square plus an equilateral triangular plot of side 4km as shown in the figure. The rainfall readings at A, B, C, D, E and F are 5cm, 11cm, 8cm, 7cm, 4cm and 3cm respectively. Find the mean precipitation by Thiessen method. (5.36cm)



Solution:



Precipitations at A, B, C, D, and E:
P_A = 5cm, P_B = 11cm, P_C = 8cm, P_D = 7cm, P_E = 4cm and P_F = 3cm

AB=BC=CD = DA=4km
Area of triangle PAS = $\frac{1}{2} As.AP = \frac{1}{2} x2x2$ = 2 sq.km.
Similarly, the area of triangles BPQ, CQR and DRS is also 2 sq.km.
Sides of square PQRS = $\sqrt{2^2+2^2} = 2\sqrt{2}$ km
Area of square PQRS= $2\sqrt{2}x2\sqrt{2}$ = 8 sq.km

Height of equilateral triangle = 2tan60 = 3.464km
Area of equilateral triangle = $\frac{1}{2} x4x3.464$ = 6.928 km^2
The equilateral triangle is equally divided into three areas.
Area of DOSA = Area of DOGE = Area of SOGD = $\frac{1}{3} x6.928$ = 2.309 km^2
Using above calculation, the areas of each polygon for each station are as follows:
Area of polygon for A = Area of (triangle PAS+ area DOSA) = 2 +2.309 = 4.309 sq.km.

Area of polygon for B = Area of triangle BPQ =2 sq.km.
Area of polygon for C = Area of triangle CQR =2 sq.km.
Area of polygon for D = Area of (triangles DRS+ Area SOGD) = 2 +2.309 = 4.309 sq.km.
Area of polygon for E = Area of DOGE = 2.309 sq.km.
Area of polygon for F = Area of square PQRS = 8 sq.km.
Precipitations at A, B, C, D and E, F: 10cm, 15cm, 18cm, 6cm, 9cm, 12cm
P_{avg} = ?

Using Thiessen method

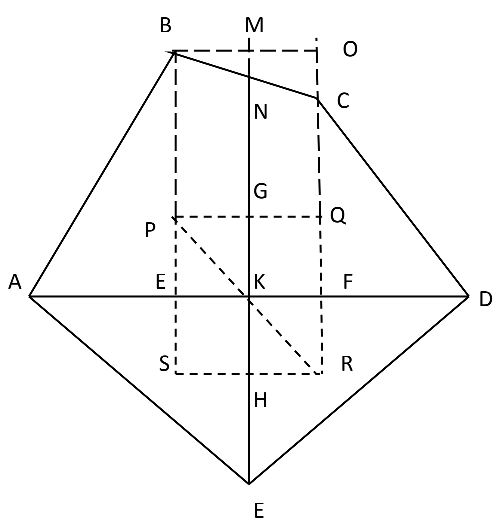
$$P_{avg} = \frac{\sum P_i A_i}{A}$$

$$= \frac{5x4.309 + 11x2 + 8x2 + 7x4.309 + 4x2.309 + 3x8}{4.309 + 2 + 2 + 4.309 + 2.309 + 8}$$

= 5.36cm

8. The shape of a catchment is in the form of a pentagon ABCDE. There are 4 raingauge stations P, Q, R and S inside the catchment. The position co-ordinates in km are: A(0, 0), B(50, 75), C(100, 70), D(150, 0), E(75, -50), P(50, 25), Q(100, 25), R(100, -25) and S(50, -25). If rainfalls recorded at P, Q, R and S are 88, 102, 112 and 116 mm respectively, determine the mean rainfall by Thiessen Polygon method.

Solution:



CO = 75-70 = 5km

$$\frac{BO}{CO} = \frac{BM}{MN}$$

$$\frac{100}{5} = \frac{50}{MN}$$

MN = 2.5km

Computing area of polygon bounding each station

$A_P = Area\ of\ \Delta ABE + Area\ BNKE$

$A_P = \frac{1}{2}x50x75 + \left(25x75 - \frac{1}{2}x2.5x25\right)$ = 3718.75 km^2

$A_Q = Area\ of\ \Delta CDF + Area\ NCFK$

$A_Q = \frac{1}{2}x50x70 + \left(25x75 - \frac{1}{2}x2.5x25 - 2.5x25\right)$ = 3531.25 km^2

$A_R = Area\ of\ \Delta DKE = \frac{1}{2}x75x50$ = 1875 km^2

$A_S = Area\ of\ \Delta AKE = \frac{1}{2}x75x50$ = 1875 km^2

Total area (A) = 11000 km^2

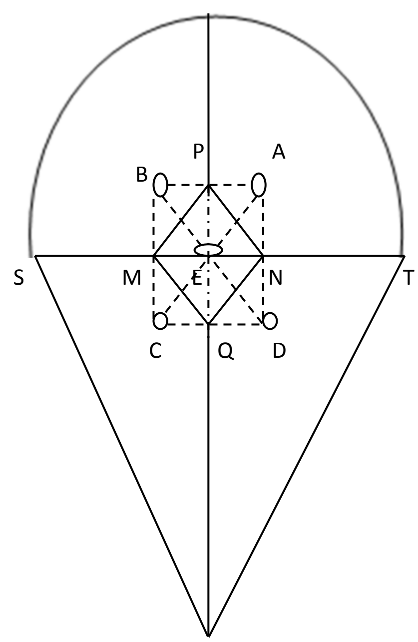
$P_{avg} = \frac{\sum P_i A_i}{A}$

$= \frac{88x3718.75+102x3531.25+112x1875+116x1875}{11000}$

= 101.36mm

9. A semicircle of diameter of 40km with an equilateral triangle of side of 40km below its diameter is a close approximation to a river basin. The position co-ordinates of 5 rain gauge stations A, B, C, D and E located within the basin with respect to the co-ordinate axes system whose X axis and origin coincident with diameter and center of the circle are (10, 10), (-10, 10), (-10, -10), (10, -10) and (0, 0) km respectively. If the rainfall recorded at these rain gauges are 80, 95, 76, 82, 107mm respectively, determine the average depth of rainfall using Thiessen polygon method.

Solution:



Diameter of semicircle = 40km

Co-ordinates of stations with E as origin

A (10, 10), B(-10, 10), C(-10, -10), D(10,-10), E (0, 0)

Precipitation:

P_A = 80mm, P_B = 95mm, P_C = 76mm, P_D = 82mm, P_E = 107mm

Sides of equilateral triangle = 40km

The Thiessen polygon which bounds each station is shown by solid lines.

PN=NQ=PM=PM = $\sqrt{10^2 + 10^2}$ = $10\sqrt{2}$ m

ER = $\sqrt{40^2 - 20^2}$ = 34.64m

Area of Thiessen polygon for station A (A_A) = Area of quarter circle-Area of triangle PEN

$= \frac{1}{4}\pi x 20^2 - \frac{1}{2} x 10 x 10$ = 264.16 m^2

Area of Thiessen polygon for station B (A_B) = A_A =264.16 m^2

Area of Thiessen polygon for station C (A_C) = Area of triangle SER-Area of triangle MEQ

$= \frac{1}{2} x 20 x 34.64 - \frac{1}{2} x 10 x 10$ = 296.4 m^2

Area of Thiessen polygon for station D (A_D) = A_C =296.4 m^2

Area of Thiessen polygon for station E (AE) = $10\sqrt{2}\, x 10\sqrt{2}$ = 200 m^2

Total area of basin (A) = 264.16+264.16+296.4+296.4+200 = 1321.12 m^2

Average rainfall (P_{av}) = ?

$$P_{av} = \frac{\sum P_i A_i}{A}$$

$$P_{av} = \frac{P_A A_A + P_B A_B + P_C A_C + P_D A_D + P_E A_E}{A}$$

$$P_{av} = \frac{80x264.16 + 95x264.16 + 76x296.4 + 82x296.4 + 107x200}{1321.12}$$

= 86.64 mm

Evaporation

2. An evaporation pan 1.2m diameter was used to find out the evaporation loss from the reservoir. The pan was initially filled up with water up to a depth of 8cm. During the period of observation, a rainfall of 4cm was recorded. At the end of the observation, the depth of water in the pan was found to be 8.5cm.Taking pan coefficient = 0.7, determine the volume of water evaporated from the reservoir. Take water spread of reservoir = 20km^2.

Solution:
Diameter of pan (d) = 1.2m
Area of pan (Ap) = $\frac{\pi}{4} x 1.2^2$ =1.767 m^2
Initial water level (I) = 8cm
Precipitation (P) = 4cm
Final water level (O) = 8.5cm
Evaporation (E) = I+P-O = 8+4-8.5 = 3.5cm = 0.035m
Pan coefficient (K) = 0.7
Water spread area (A) = 20km^2 = 20x10^6 m^2
Time duration = t h
Evaporation from reservoir = ?
Rate of evaporation (Er) = $\frac{E}{Apxt} = \frac{0.035}{1.767t} x1000$ = 0.0198/t in m/m^2/h

Evaporation from reservoir = K Er A t = 0.7x0.0198/tx20x10^{6}xt =13917m^3

4. Calculate the potential evapotranspiration for an area over Kathmandu in the month of January by Penman method using following data:
Mean monthly temperature = 11.5^{0}C, Mean RH = 75%, Mean sunshine hours = 9h, Potential sunshine hours = 11.6 h, Wind velocity at 2m height = 100 km/day, Albedo = 0.15, Upper terrestrial solar radiation = 8mm of Hg/day
Other values
Latitude = 26.5^0, Longitude = 84.5^0
Saturated vapor pressure at 11.5^{0}C = 10.4 mm of Hg, Slope of saturated vapor pressure curve = 1.24mm/^{0}C, Psychrometric constant = 0.49mm/^{0}C, Boltzman constant = 2.01x10^{-9} mm/day
Solution:

Mean monthly temperature (Ta) = 11.5^{0}C = 11.5+273 = 284.5K
Mean RH (RH) = 75%
Mean sunshine hours (n) = 9h
Potential sunshine hours (N) = 11.6 h

Wind velocity at 2m height (u2) = 100 km/day
Albedo (r) = 0.15
Upper terrestrial solar radiation (H_a) = 8mm/day
Latitude (ϕ) = 26.5
Saturated vapor pressure at 11.5^{0}C (e_w) = 10.4 mm of Hg
Slope of saturated vapor pressure curve (A) = 1.24mm/^{0}C
Psychrometric constant (γ) = 0.49mm/^{0}C
Boltzman constant (σ) = 2.01x10^{-9} mm/day
potential evapotranspiration (PET) = ?

$a = 0.29Cos\varphi$ = $0.29Cos26.5$ = 0.26
b = 0.52
Actual vapor pressure (e_a) = RH x e_w = 0.75x10.4 = 7.8 mm of Hg
Net radiation (H_n) is computed by

$$H_n = H_a(1-r)\left(a + b\frac{n}{N}\right) - \sigma T_a^4\left(0.56 - 0.092\sqrt{e_a}\right)\left(0.1 + 0.9\frac{n}{N}\right)$$

$$= 8(1-0.15)\left(0.26 + 0.52\frac{9}{11.6}\right) - 2.01x10^{-9}x284.5^4\left(0.56 - 0.092\sqrt{7.8}\right)\left(0.1 + 0.9\frac{9}{11.6}\right)$$

= 1.326 mm/day

Ea is computed by

$$E_a = 0.35\left(1 + \frac{u2}{160}\right)(ew - ea)$$

$$= 0.35\left(1 + \frac{100}{160}\right)(10.4 - 7.8)$$

=1.479 mm/day

According to Penman equation,

$$PET = \frac{AH_n + E_a\gamma}{A + \gamma}$$

$$PET = \frac{1.24x1.326 + 1.479x0.49}{1.24 + 0.49}$$

=1.369 mm/day

5. Estimate daily evaporation from a lake at 30^0 N for April by Penman method with following mean monthly data.

T_a, Kelvin	RH, %	n, hrs	u2, m/s	H_a, mm/day	N, hrs
293	65	10	1.2	14.8	12.9

Solution:
Mean monthly temperature (T_a) = 293K, Mean RH (RH) = 65%

Mean sunshine hours (n) = 10h
Potential sunshine hours (N) = 12.9 h
Wind velocity at 2m height (u2) = 1.2 m/s = (1.2x3600x24/1000) = 103.68km/day
Upper terrestrial solar radiation (H_a) = 14.8mm/day
Latitude (ϕ) = 30^0

Take Albedo (r) = 0.05 for water bodies, Psychrometric constant (γ) = 0.49mm/^{0}C and Boltzman constant (σ) = $2.01\text{x}10^{-9}$ mm/day
Potential evapotranspiration (PET) = ?
$a = 0.29Cos\varphi$ = $0.29Cos30$ = 0.25
b = 0.52
Saturated vapor pressure
$e_s = 4.584\ exp\left(\frac{17.27T}{237.3+T}\right)$ where T is in deg C
T = 293-273 =20 ^{0}c
$e_s = 4.584\ exp\left(\frac{17.27x20}{237.3+20}\right)$ = 17.5 mm of Hg
Actual vapor pressure (e_a) = RH x e_s = 0.65x17.5 = 11.375 mm of Hg
Slope of saturated vapor pressure curve (A) $= \frac{4098e_s}{(237.3+T)^2} = \frac{4098x17.5}{(237.3+20)^2}$ = 1.08
Net radiation (H_n) is computed by

$$H_n = H_a(1-r)\left(a + b\frac{n}{N}\right) - \sigma T_a^4\left(0.56 - 0.092\sqrt{e_a}\right)\left(0.1 + 0.9\frac{n}{N}\right)$$

$$= 14.8(1-0.05)\left(0.25 + 0.52\frac{10}{12.9}\right) - 2.01x10^{-9}x293^4\left(0.56 - 0.092\sqrt{11.375}\right)\left(0.1 + 0.9\frac{10}{12.9}\right)$$

= 6.23 mm/day

E_a is computed by

$$E_a = 0.35\left(1 + \frac{u2}{160}\right)(e_s - e_a)$$

$= 0.35\left(1 + \frac{103.68}{160}\right)(17.5 - 11.375)$ =3.53mm/day

According to Penman equation,

$$PET = \frac{AH_n + \gamma E_a}{A+\gamma}$$

$PET = \frac{1.08x6.23 + 0.49x3.53}{1.08+0.49}$ = 5.4 mm/day

4. An infiltration capacity curve prepared for a catchment indicated an initial infiltration 2.5 cm/hr and attains a constant value of 0.5 cm/hr after 10 hours. The total infiltration volume is 5.5cm. Calculate the Horton Constant (K).

Solution:
Initial infiltration (f_0) = 2.5 cm/hr
Constant infiltration (f_c) = 0.5 cm/hr
Total infiltration volume (F) = 5.5cm
Time (t) = 10 hrs
Horton constant (K) =?

Horton's Formula for infiltration is given by
$f = f_c + (f_0 - f_c)e^{-Kt}$
Cumulative infiltration is given by

$$F(t) = \int_o^t f(t)dt$$

$$= \int_o^t [f_c + (f_0 - f_c)e^{-kt}]dt$$

$$= f_c t + (f_0 - f_c)\left|\frac{e^{-kt}}{-k}\right|_0^t$$

$$F(t) = f_c t + \frac{f_0 - f_c}{k}\left(1 - e^{-kt}\right)$$

For large t, the value of e^{-kt} becomes negligible. Hence above equation reduces to

$$F(t) = f_c t + \frac{f_0 - f_c}{k}$$

$$k = \frac{f_0 - f_c}{F(t) - f_c t}$$

$K = \frac{2.5-0.5}{5.5-0.5x10}$ = 4/hr

5. The ordinates of a rainfall mass curve of a storm over a basin of area 950 km^2 measured in mm at 1hr interval are:
0, 10, 21, 30, 38, 46, 51, 56, 60, 65, 70

If the infiltration during this storm can be represented by Horton's equation with f_0 = 6.5 mm/h, f_c = 1.5 mm/h and K = 0.15/h, compute infiltration at each time interval and estimate the resulting runoff volume. ($24.82x10^6 m^3$)

Solution:

time (t)	cumulative rainfall (mm)	Incremental rainfall, R (mm)	Infiltration, F (mm)	Runoff, Q (mm)
0	0	0	0	0
1	10	10	6.1	3.9
2	21	11	5.5	5.5
3	30	9	4.9	4.1
4	38	8	4.5	3.5
5	46	8	4.0	4.0
6	51	5	3.7	1.3
7	56	5	3.4	1.6
8	60	4	3.1	0.9
9	65	5	2.9	2.1
10	70	5	2.7	2.3
			Total	29.2

$f(t) = f_c + (f_0 - f_c)e^{-Kt} = 1.5 + (6.5 - 1.5)e^{-0.15t} = 1.5 + 5e^{-0.15t}$

Column 4: Total infiltration between time t1 to t2 is computed as

$F = \int_{t1}^{t2} f(t)dt = \int_{t1}^{t2}(1.5 + 5e^{-0.15t})dt = 1.5(t2 - t1) + \frac{5}{-0.15}(e^{-0.15t2} - e^{-0.15t1})$

Column 5: Q = R-F

Total Runoff = 29.2 mm

Runoff volume = $(29.2/1000)x(850x10^6)\ m^3 = 24.82x10^6 m^3$

6. A 24hr storm occurred over a catchment of $1.8km^2$ and the total rainfall observed is 10cm. An infiltration capacity of 1cm/hr initially and finally 0.3cm/hr is obtained from Horton's curve with K = 0.5 hr^{-1}. An evaporation pan installed in the catchment indicated a decrease of 0.6cm in the water level (after allowing for rainfall) during 24 hours of its operation. Determine the runoff from the catchment. Take pan coefficient = 0.7.

Solution:

Catchment area (A) = $1.8km^2 = 1.8x10^6\ m^2$

Rainfall (R) = 10cm

Initial rate of infiltration (f_0) = 1cm/hr

Finial rate of infiltration (f_c) = 0.3cm/hr

Horton coeff. (K) = 0.5 hr^{-1}

Pan Evaporation (E_p) = 0.6cm

Pan coefficient (K) = 0.7
Runoff (Q) = ?

Total infiltration (F) during t = 24 hr is

$$F = f_c t + \frac{f_0 - f_c}{K}(1 - e^{-Kt})$$
$$= 0.3x24 + \frac{1 - 0.3}{0.5}(1 - e^{-0.5x24})$$
$$= 8.6cm$$

Evaporation (E) = K E_p = 0.7x0.6 = 0.42cm
Runoff (Q) = R-F-E = 10-8.6-0.42 = 0.98cm
Runoff volume = Qx A =(0.98/100)x1.8x10^6 = 17640m^3

7. The infiltration of a catchment can be represented by the equation f= 15+50$e^{-0.9t}$. If the rainfall intensity of 45mm/hr occurs continuously for 10 hr from a catchment of area 12km2, calculate
I. total runoff volume generated from the catchment.
II. total infiltration volume at the period
III. calculate time from the start of rainfall from which runoff started.
IV. Show your all (above three) results in infiltration curve.

Solution:
Rainfall intensity = 45mm/hr
f= 15+50$e^{-0.9t}$
$f(t) = f_c + (f_0 - f_c)e^{-Kt}$
f_c = 15, f_0-f_c = 50
f_0=6 5
K = -0.9
As rainfall intensity <f_0, the runoff does not start at t=0.
When infiltration rate (f) becomes equal to or less than rainfall intensity (i), runoff occurs.
At f = 45mm/hr
f= 15+50$e^{-0.9t}$
45= 15+50$e^{-0.9t}$
t = 0.57hr = 34.2 minute

Total Rainfall = 45x10 = 450mm
Until 0.57 hr, all the rainfall is infiltrated. Total infiltration up to 0.57hr = 45x0.57=25.7mm
Total Infiltration from 0.57 hr to 10 hr= $\int_{0.57}^{10} f(t)dt = \int_{0.57}^{10}(15 + 50e^{-0.9t})dt$
$= \left| 15t + \frac{1}{-0.9}50e^{-0.9t} \right|_{0.57}^{10}$ = 174.7mm

Total infiltration = 25.7+174.7=200.4

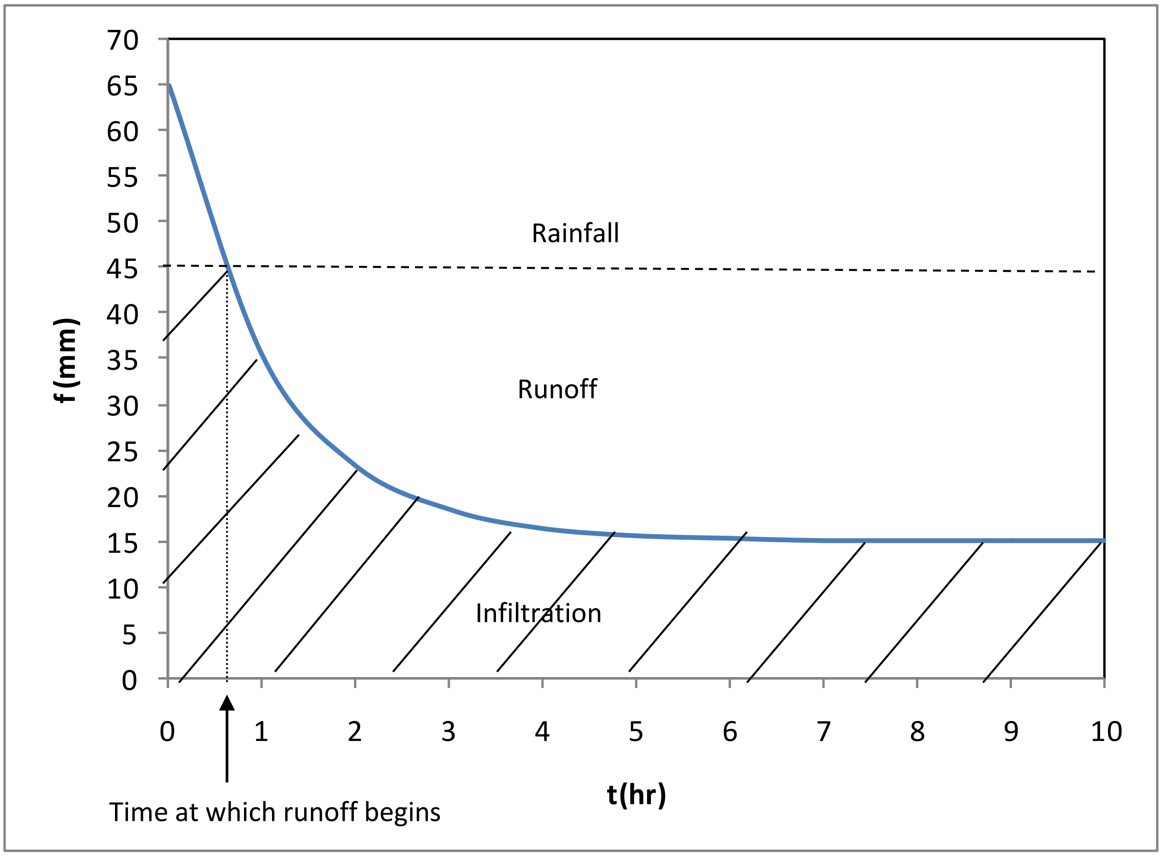
Total runoff = 450-200.4 = 249.6mm

a. Runoff volume = $249.6x10^{-3}x12x10^{6}$ = $2.995x10^{6}m^{3}$

b. Total infiltration volume = $200.4x10^{-3}x12x10^{6}$ = $2.404x10^{6}m^{3}$

c. Time from the start of rainfall for which runoff occurs = 0.57hr =34 minute

d.



8. Determine the best values of the parameters of Horton's infiltration equation for the following data pertaining to infiltration test on a soil using double ring infiltrometer.

Time since start (min)	5	10	15	20	30	40	60	80	100
cumulative infiltration (mm)	21.5	37.7	52.2	65.8	78.4	89.5	101.8	112.6	123.3

Solution:

Horton's infiltration equation is given by

$f = f_c + (f_0-f_c)\, e^{-Kt}$

$f - f_c = (f_0-f_c)\, e^{-Kt}$

Taking lnon both sides

$\ln (f - f_c) = \ln (f_0 - f_c) - Kt$

Let $y = \ln (f - f_c)$, $c = \ln (f_0 - f_c)$. Then above equation reduces to y = -Kt + c : linear equation

Determination of constants by least square method

time (min)	t(hr) = X	cumulative infiltration (mm)	Incremental infiltration (mm)	f (mm/h)	y=ln(f-fc)	xy	x^2
5	0.08	21.5	21.5	258	5.420092	0.451674	0.00694
10	0.17	37.7	16.2	194.4	5.089446	0.848241	0.02778
15	0.25	52.2	14.5	174	4.955123	1.238781	0.06250
20	0.33	65.8	13.6	163.2	4.87596	1.62532	0.11111
30	0.50	78.4	12.6	75.6	3.772761	1.88638	0.25000
40	0.67	89.5	11.1	66.6	3.540959	2.36064	0.44444
60	1.00	101.8	12.3	36.9	1.568616	1.568616	1.00000
80	1.33	112.6	10.8	32.4	-1.20397	-1.6053	1.77778
100	1.67	123.3	10.7	32.1			
Sum	4.33				28.02	8.37	3.68

(f-fc = 0 for last data. so it is neglected in the calculation. Hence n = 8)

Take f_c = 32.1mm/h

$K = -\frac{N\sum xy - \sum x \sum y}{N\sum x^2 - (\sum x)^2} = -\frac{8x8.37 - 4.33x28.02}{8x3.68 - (4.33)^2}$ = 5.1/hr

$c = \frac{\sum y - (-K)\sum x}{N} = \frac{28.02 + 5.1x4.33}{8}$ = 6.26

c = ln (f_0 -f_c)

f_0 -f_c = exp(c) = exp(6.26)= 523.2

f_0-32.1 = 523.2

f_0 = 555.3mm/h

9. Derive Horton equation from following infiltration data and compute the infiltration from 1.0 to 2.5 hour.

Time in hr	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Infiltration rate (cm/hr)	9.9	6.6	4.5	2.8	1.7	1.1	0.7	0.5	0.5

At t = 0, infiltration = 9.9cm/h. So, f_0 = 9.9cm/h

As the infiltration becomes constant at 0.5cm/hr (from the data), f_c = 0.5cm/h

Horton's infiltration equation is given by

$f = f_c + (f_0 - f_c)\ e^{-Kt}$

$f = 0.5 + (9.9 - 0.5)\ e^{-Kt}$

$f = 0.5+9.4e^{-Kt}$

Statistical method to determine K

$f = f_c + (f_0-f_c)\ e^{-Kt}$

$f - f_c = (f_0-f_c)\ e^{-Kt}$

Taking log on both sides

$\ln(f - f_c) = \ln(f_0 - f_c) - Kt$

Let $y = \ln(f - f_c)$, $c = \ln(f_0 - f_c)$. Then above equation reduces to

$y = -Kt + c$: linear equation

(As f-fc = 0 for second last and last data, neglect these data in the calculation. so n = 7)

	t =x	f	ln(f-fc) = y	xy	x^2
	0	9.9	2.2407097	0	0
	0.5	6.6	1.8082888	0.904144	0.25
	1	4.5	1.3862944	1.386294	1
	1.5	2.8	0.8329091	1.249364	2.25
	2	1.7	0.1823216	0.364643	4
	2.5	1.1	-0.510826	-1.27706	6.25
	3	0.7	-1.609438	-4.82831	9
Sum	10.5		4.33026	-2.20093	22.75

$K = -\frac{N\sum xy - \sum x \sum y}{N\sum x^2 - (\sum x)^2} = -\frac{7x-2.20093-10.5x4.33026}{7x22.75-(10.5)^2}$ = 1.24

Horton equation

$$f = 0.5 + 9.4e^{-1.24t}$$

Infiltration from 1 to 2.5 hr

$$F(t) = \int_1^{2.5} f(t)dt$$

$$= \int_1^{2.5} [0.5 + 9.4e^{-1.24t}]dt$$

$= [0.5t - 7.58e^{-1.24t}]_1^{2.5}$ = 2.6cm

11. The mass curve of a rainfall of duration 100 min. is given below

Time from start of rainfall (min)	0	20	40	60	80	100
Cumulative rainfall (cm)	0	0.5	1.2	2.6	3.3	3.5

If the catchment had an initial loss of 0.6 cm and a ϕ-index of 0.6 cm/hr, calculate the total surface runoff from the catchment.

Solution:

Time from start of rainfall (min)	0	20	40	60	80	100
Cumulative rainfall (cm)	0	0.5	1.2	2.6	3.3	3.5
Incremental rainfall (cm)		0.5	0.7	1.4	0.7	0.2

Initial abstraction is 0.6cm. So there is no runoff in the first 20 minutes from rainfall of 0.5cm, and for the second 20 minutes, the abstraction of 0.1cm is subtracted to compute runoff.

ϕ-index = 0.6 cm/hr

Infiltration for 20 minute = ϕ Δt = 0.6x20/60 = 0.2 cm

$$Total\ runoff = \sum (Rainfall_i - Infiltration_i)$$

= 0 + (0.6-0.2) + (1.4-0.2)+ (0.7-0.2)

= 2.1cm

14. A mass curve of an isolated storm over a watershed is given below:

t (hr)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Cumulative rainfall (cm)	0	0.25	0.5	1.1	1.6	2.6	3.5	5.7	6.5	6.7	7.7

If the storm produces a direct runoff of 3.5cm at the outlet of the watershed, estimate the ϕ-index of the storm and duration of excess rainfall.

Solution:

t (hr)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Cumulative rainfall (cm)	0	0.25	0.5	1.1	1.6	2.6	3.5	5.7	6.5	6.7	7.7
Incremental rainfall (cm)		0.25	0.25	0.6	0.5	1	0.9	2.2	0.8	0.2	1

Total rainfall = 7.7cm

Runoff = 3.5cm

Infiltration = 7.7-3.5 = 4.2cm

First trial

Take effective duration of rainfall (te) = 5 hours

ϕ-index = 4.2/5 = 0.84 cm/h

Infiltration for 0.5hr = 0.84x0.5 = 0.42cm

With this ϕ-index, rainfall excess is

Rainfall excess = 0+ 0+(0.6-0.42)+(0.5-0.42) +(1.0-0.42)+(0.9-0.42)+(2.2-0.42)+(0.8-0.42)+0+(1-0.42)=4.06cm which is higher than net runoff of 3.5cm.

Second trial

The value of ϕ-index makes the rainfall of first 1 hour and second last half hour ineffective. So take te = 5-1.5 = 3.5 hours

ϕ-index = (7.7-3.5-(0.25+0.25+0.2))/3.5 = 1 cm/h

Take ϕ-index = 1.0 cm/h

Infiltration for 0.5hr = 1x0.5 = 0.5cm

With this ϕ-index, rainfall excess is

Rainfall excess = 0+ 0+(0.6-0.5)+0 +(1.0-0.5)+(0.9-0.5)+(2.2-0.5)+(0.8-0.5)+0+(1-0.5)=3.5cm which is equal to net runoff

ϕ-index = 1.0 cm/h. With this ϕ-index, four rainfall pulses become ineffective. That means ineffective duration = 2hr.

Effective duration of rainfall (te) = 5-2=3 hours

16. An isolated 3hr storm occurred over an area of 120ha as below:

Partial area of catchment (ha)	ϕ-index (cm/hr)	Rainfall (cm)		
		1st hr	2nd hr	3rd hr
36	0.9	0.6	2.4	1.3
18	1.1	0.9	2.1	1.5
66	0.5	1	2	0.9

Compute the hourly distribution of effective rainfall and the total rainfall on the catchment in this storm. Also estimate the runoff from the catchment. If the ϕ-index were to remain at the same value, what runoff would be produced by a uniform rainfall of 3.3 cm in 3 hours uniformly spread all over the catchment?

Solution:

Partial area of catchment (ha)	Weight, w_i	ϕ-index (cm/hr)	Rainfall (cm)			Subbasin Weighted rainfall cm	Subbasin Weighted Runoff cm
			1st hr	2nd hr	3rd hr		
36	0.3	0.9	0.6	2.4	1.3	1.29	0.3x[0+(2.4-0.9)+(1.3-0.9)]=0.57
18	0.15	1.1	0.9	2.1	1.5	0.675	0.15x[0+(2.1-1.1)+(1.5-1.1)]= 0.21
66	0.55	0.5	1	2	0.9	2.145	0.55x[(1-0.5)+(2-0.5)+(0.9-0.5)]= 1.32
					Total	4.11	2.1

i = subcatchment, $w_i = A_i/A$ where Ai = area of subcatchment, A = total area of catchment

Weighted rainfall = w_i * sum of rainfall of i of time t
Hourly Effective rainfall
1st hr=0.3*(0)+0.15*(0)+0.55*(1-0.5)=0.275cm
2nd hr = 0.3*(2.4-0.9)+0.15*(2.1-1.1)+0.55*(2-0.5)=1.425cm
3rd hr =0.3*(1.3-0.9)+0.15*(1.5-1.1)+0.55*(0.9-0.5)=0.4cm

Total rainfall = sum of weighted rainfall = 4.11cm
Runoff at time t = $w_i(Rainfall_t - \phi\Delta t)$ for $Rainfall_t > \ loss$, 0 0therwise
Weighted Runoff from subcatchment = sum of runoff at different time t
Total runoff = 2.1cm

Partial area of catchment (ha)	Weight, w_i	ϕ-index (cm/hr)	Rainfall in 3hr	Loss in 3 hr	Weighted Runoff
36	0.3	0.9	3.3	2.7	0.18
18	0.15	1.1	3.3	3.3	0
66	0.55	0.5	3.3	1.5	0.99
				Total	1.17

Weighted runoff = $w_i(Rainfall\ in\ 3\ hr - Loss\ in\ 3hr)$
Total runoff = 1.17cm

17. The following is the set of observed data for successive 15 minute period of 105 minutes storm in a catchment of 100 sq.km.
2, 2, 8, 7, 1.25, 1.25, 4 (cm/hr)
If the value of ϕ-index is 3 cm/hr, estimate the net runoff, the total rainfall, and the value of W-index.

Solution:
Runoff = Rainfall-infiltration for rainfall>infiltration
= 0 otherwise
Total runoff (Q) = $0 + 0 + (8-3)\frac{15}{60} + (7-3)\frac{15}{60} + 0 + 0 + (4-3)\frac{15}{60}$
= 2.5 cm

$Total\ rainfall\ (P) = 2x\frac{15}{60} + 2x\frac{15}{60} + 8x\frac{15}{60} + 7x\frac{15}{60} + 1.25x\frac{15}{60} + 1.25x\frac{15}{60} + 4x\frac{15}{60}$ = 6.375 cm
Time duration (tr) = 105/60 = 1.75 hr

$W - index = \frac{P-Q}{tr} = \frac{6.375-2.5}{1.75}$ = 2.2 cm/hr

19. A storm during a dry weather has rainfall intensities of 8, 12, 40, 38, 30, 26, 28, 5, 16, 32, 36, 24, 14 and 4 mm/h at half an hour intervals. What is the runoff volume from a basin area of 600 km^2 if initial abstractions are 10mm and the ϕ-index for the basin is 10 mm/h? What is the percent error in runoff estimate if the initial abstractions are neglected?

Solution:
Rainfall values of 30 min: 4, 6, 20, 19, 15, 13, 14, 2.5, 8, 16, 18, 12, 7, 2 mm
Initial loss = 10mm
The sum of the first and second rainfall value is 10mm, which is equal to initial loss.
ϕ-index = 10 mm/h
Infiltration for 30 minute = ϕ Δt = 10x30/60 = 5 mm
Runoff = Rainfall-infiltration for rainfall>infiltration
= 0 otherwise
Total runoff = 0+ 0 +(20-5) +(19-5) +(15-5) +(13-5) +(14-5) +0 +(8-5) +(16-5) +(18-5) +(12-5) +(7-5)+0
= 92mm
$= \frac{92}{1000} x 600 x 10^6 = 552 \times 10^5 \ m^3$

If initial loss is neglected,
Total runoff = 0+ (6-5) +(20-5) +(19-5) +(15-5) +(13-5) +(14-5) +0 +(8-5) +(16-5) +(18-5) +(12-5) +(7-5)+0
= 93mm

Percent error in runoff estimate = $\frac{93-92}{92} x 100$ = +1.09%

Surface runoff

3. Given the following data for a stream gauging operation in a river, compute discharge.

Distance from left bank (m)	Depth (m)	Velocity (m/s)	
		At 0.6d	At 0.8d
0	-	-	-
1.5	1.3	0.6	0.4
3.0	2.5	0.9	0.6
4.5	1.7	0.7	0.5
6.0	1.0	0.6	0.4
7.5	0.4	0.4	0.3
9.0	-	-	-

Solution:

Formulae

Average width for 1st section $(Wav_1) = \frac{\left(W_1 + \frac{W_2}{2}\right)^2}{2W_1}$, Average width for last section $(Wav_n) = \frac{\left(W_n + \frac{W_{n-1}}{2}\right)^2}{2W_n}$

Average width for other sections $(Wav_i) = \frac{W_i + W_{i+1}}{2}$

Depth = D_i

Cross-sectional area (A_i) = Wav_i D_i

Average velocity $(Vav_i) = \frac{V_{0.2d} + V_{0.8d}}{2}$

Segmental discharge (Q_i) = A_i Vav_i

Total discharge = $\sum Q_i$

Computation of discharge

Distance from left bank (m)	Width of section (m)	Average width, Wav_i (m)	Depth, D_i (m)	Cross-sectional area, A_i (m^2)	Average Velocity, Vav_i (m/s)	Segmental discharge, Q_i (m^3/s)
0						
1.5	1.5	1.6875	1.3	2.19375	0.5	1.097
3.0	1.5	1.5	2.5	3.75	0.75	2.813
4.5	1.5	1.5	1.7	2.55	0.6	1.530
6.0	1.5	1.5	1.0	1.5	0.5	0.75
7.5	1.5	1.6875	0.4	0.675	0.35	0.236
9.0	1.5				Sum	6.426

Discharge = 6.426 m^3/s

4. The data pertaining to a stream-gauging operation at a gauging site are given below:

Distance from right bank (m)	0	1	3	5	7	9	11	12

Depth (m)	0	1.1	2	2.5	2	1.7	1	0
No. of revolutions	0	39	58	112	90	45	30	0
Time (S)	0	100	100	150	100	100	100	0

The rating equation of the current meter is V: 0.53N+0.05 m/s, where N = revolutions per second was used to measure the velocity at 0.6 depth. Calculate the discharge in the stream.

Solution:

Formulae

Average width for 1st section $(Wav_1) = \frac{\left(W_1+\frac{W_2}{2}\right)^2}{2W_1}$, Average width for last section $(Wav_n) = \frac{\left(W_n+\frac{W_{n-1}}{2}\right)^2}{2W_n}$

Average width for other sections $(Wav_i) = \frac{W_i+W_{i+1}}{2}$

Depth = D_i

Cross-sectional area (A_i) = Wav_i D_i

Velocity (V) = 0.53 N + 0.05 where V is velocity at 0.6d and N_s is no. of revolutions per second (Revolution/time).

Average velocity (Vav_i) = $V_{0.6d}$

Segmental discharge (Q_i) = A_i Vav_i

Total discharge = $\sum Q_i$

Computation of discharge

Distance from right bank (m)	Width of section (m)	Average width, Wav_i (m)	Depth, D_i (m)	Cross-sectional area, A_i (m^2)	N(No. of revolution/time)	Average Velocity, Vav_i (m/s)	Segmental discharge, Q_i (m^3/s)
0							
1	1	2	1.1	2.2	0.39	0.2567	0.565
3	2	2	2	4	0.58	0.3574	1.430
5	2	2	2.5	5	0.746	0.44538	2.227
7	2	2	2	4	0.9	0.527	2.108
9	2	2	1.7	3.4	0.45	0.2885	0.981
11	2	2	1	2	0.3	0.209	0.418
12	1					Sum	7.728

Q = 7.728 m^3/s

6. A small stream has rectangular section having 10m width in a reach of 5km and Manning's roughness = 0.03. During a flood the high water records at the end of the reach is given below. Estimate the flood discharge of the river.

Section	Elevation of bed (m)	Water surface elevation (m)
Upstream	100.5	103.4
Downstream	97.2	100.3

Solution:

Width (b) = 10m, Length (L) = 5km = 5000m, Manning's roughness (n) = 0.03
Section 1
Water depth (y1) = 103.4-100.5 = 2.9 m
Cross-sectional area (A1) = by1 = 10x2.9 =29 m2
Wetted perimeter (P1) = b+2y1 = 10+2*2.9 = 15.8m
Hydraulic radius (R1) = A1/P1 = 29/15.8 = 1.8354 m
Conveyance $(K1) = \frac{1}{n} A1\ R1^{2/3} = \frac{1}{0.03} x29x\ 1.8354^{2/3}$ = 1449.1
Section 2
Water depth (y2) = 100.3-97.2 = 3.1 m
Cross-sectional area (A2) = by2 = 10x3.1 =31 m2
Wetted perimeter (P2) = b+2y1 = 10+2*3.1 = 16.2m
Hydraulic radius (R2) = A2/P2 = 31/16.2 = 1.9135 m
Conveyance $(K2) = \frac{1}{n} A2\ R2^{2/3} = \frac{1}{0.03} x31x\ 1.9135^{2/3}$ = 1592.67

Average K is
$K = \sqrt{K1\ K2} = \sqrt{1449.1x1592.67}$ = 1519.18

Formula to compute head loss (h_f)

$$h_f = (h1 - h2) + \left(\frac{V1^2}{2g} - \frac{V2^2}{2g}\right)$$

Here, Fall in water head (h1-h2) = 103.4-100.3 = 3.1 m

$$S_f = \frac{h_f}{L}, Q = K\sqrt{S_f}, V1 = Q/A1, V2 = Q/A2$$

Refined value of h_f is computed by

$$h_f = 3.1 + \left(\frac{V1^2}{2g} - \frac{V2^2}{2g}\right)$$

Take h_f = 3.1m for the first trial
Iterations

Trial	h_f (trial)	S_f	Q	V1	V2	h_f (refined)
1	3.1	0.00062	37.83	1.304396	1.220242	3.1108
2	3.1108	0.000622	37.89	1.306672	1.222371	3.1109
3	3.1109	0.000622	37.89	1.30668	1.222378	3.1109

The difference in h_f for trial 2 and 3 is negligible. So the iteration is stopped after 3rd trial. Discharge of the final trial is the peak discharge.
Peak discharge = 37.89 m^3/s

8. Three points on rating curve of a stream gauging station obtained from observed data have the following co-ordinates: (2 m^3/s, 10.65m), (4 m^3/s, 10.85m) and (8 m^3/s, 11.25m). Determine the equation of rating curve and compute the discharge in the stream corresponding to a stage of 11.5m.

Solution:

Q (m^3/s)	G (m)
2	10.65
4	10.85
8	11.25

Form of rating equation: $Q = C_r(G - a)^{\beta}$

Here, $Q_2 = \sqrt{Q_1 Q_3}$

$a = \frac{G_1 G_3 - G_2^2}{G_1 + G_3 - 2G_2} = \frac{10.65x11.25 - 10.85^2}{10.65 + 11.25 - 2x10.85}$ = 10.45

Finding β, C_r algebraically,

$$\frac{Q_2}{Q_1} = \frac{C_r(G_2 - a)^{\beta}}{C_r(G_1 - a)^{\beta}}$$

$$\frac{4}{2} = \frac{C_r(10.85 - 10.45)^{\beta}}{C_r(10.65 - 10.45)^{\beta}}$$

$$2 = (2)^{\beta}$$

$$\beta = 1$$

$$Q_1 = C_r(G_1 - a)^{\beta}$$

$$2 = C_r(10.65 - 10.45)^1$$

$$C_r = 10$$

The equation of rating curve is

$$Q = 10(G - 10.45)^1$$

For G = 11.5m

$Q = 10(11.5 - 10.45)^1$ = 10.5 m^3/s

9. The stage and discharge data of a river are given below, Derive stage-discharge (rating curve) relationship to predict the discharge for a given stage. Assume the value of stage for zero discharge as 149.10m. Compute the correlation coefficient of the established relationship. Determine the discharge corresponding to a stage of 152.45m.

Stage (m)	Discharge (m^3/s)
151.35	20
151.66	60

151.92	110
152.78	300
153.41	505
153.83	650
154.51	1050
155.42	1390
155.76	1600

Solution:
The form of rating curve equation is

$Q = C_r (G-a)^{\beta}$

$\text{Log}Q = \beta \text{ Log } (G-a) + \text{Log } C_r$

$Y = \beta X + m$

Stage for zero flow (a) = 149.10m

Stage, G (m)	Discharge, Q (m^3/s)	G-a	X = log(G-a)	Y = logQ	X^2	Y^2	XY
151.35	20	2.25	0.3522	1.301	0.124	1.693	0.458
151.66	60	2.56	0.4082	1.778	0.167	3.162	0.726
151.92	110	2.82	0.4502	2.041	0.203	4.167	0.919
152.78	300	3.68	0.5658	2.477	0.320	6.136	1.402
153.41	505	4.31	0.6345	2.703	0.403	7.308	1.715
153.83	650	4.73	0.6749	2.813	0.455	7.912	1.898
154.51	1050	5.41	0.7332	3.021	0.538	9.128	2.215
155.42	1390	6.32	0.8007	3.143	0.641	9.879	2.517
155.76	1600	6.66	0.8235	3.204	0.678	10.266	2.639
		sum	4.6198	19.2781	2.8503	49.3843	11.8502

$\beta = \frac{N(\sum XY)-(\sum X)(\sum Y)}{N(\sum X^2)-(\sum X)^2} = \frac{9x11.8502-4.6198x19.2781}{9x2.8503-(4.6198)^2} = 4.08$

$m = \frac{\sum Y-\beta\sum X}{N} = \frac{19.2781-4.08x4.6198}{9} = 0.048$

$\text{Log}C_r = m$

$C_r = 10^m = 10^{0.048} = 1.1$

Rating curve equation is

$Q = 1.1(G-149.10)^{4.08}$

Coefficient of correlation (r)

$$r = \frac{N(\sum XY) - (\sum X)(\sum Y)}{\sqrt{N(\sum X^2) - (\sum X)^2}\sqrt{N(\sum Y^2) - (\sum Y)^2}}$$

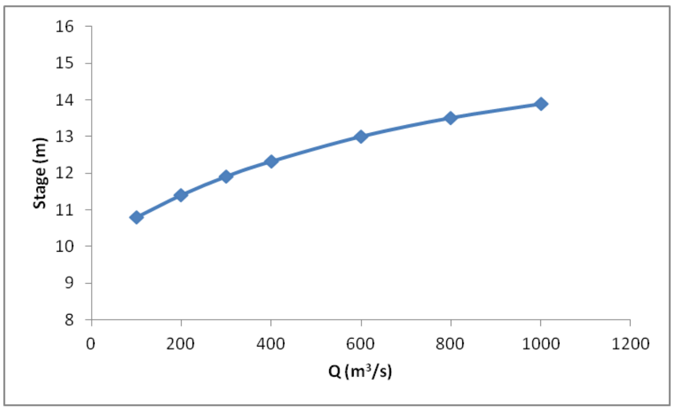
$$= \frac{9x11.8502-4.6198x19.2781}{\sqrt{9x2.8503-(4.6198)^2}\sqrt{9x49.3843-(19.2781)^2}} = 0.99$$

For G = 152.45

Q = 1.1(152.45-149.1)$^{4.08}$ = 153 m^3/s

12. Determine the stage corresponding to zero discharge for following data of a smooth rating curve.

Stage (m)	10.8	11.41	11.92	12.33	13.01	13.52	13.90
Discharge (m^3/s)	100	200	300	400	600	800	1000



To find stage for zero flow (a),

Take three values of Q such that $Q_2 = \sqrt{Q_1 Q_3}$ or $\frac{Q_2}{Q_1} = \frac{Q_3}{Q_2}$

Q_1 = 100 m^3/s, G_1 = 10.8m

Q_2 = 200 m^3/s, G_2 = 11.41m

Q_3 = 400 m^3/s, G_3 = 12.33m

$$a = \frac{G_1 G_3 - G_2^2}{G_1 + G_3 - 2G_2} = \frac{10.8x12.33 - 11.41^2}{10.8 + 12.33 - 2x11.41} = 9.6\text{m}$$

Unit Hydrograph

3. Given below are the observed flows from a storm of 4hr duration on a stream with a catchment area of 613 km^2. Derive 4hr unit hydrograph. Make suitable assumptions regarding base flow.

Time (hr)	0	4	8	12	16	20	24	28	32	36	40	44	48
Observed flow (m3/s)	10	110	225	180	130	100	70	60	50	35	25	15	10

Solution:

Catchment area (A) = 613 km^2

Assume base flow (BF) = 10 m^3/s

Direct runoff (Q_{dr}) = Q-BF

Volume of runoff (V) = $\sum Q_{dr}\, \Delta t$

Runoff depth (r_d) = V/A

Divide Q_{dr} by r_d to get UH ordinate.

Δt is same for each runoff ordinate.

Δt = 4 hour = 4x3600 s

V = $\sum Q_{dr}\, \Delta t = \Delta t \sum Q_{dr}$ = 890x4x3600

$r_d = \frac{V}{A} = \frac{890\times4\times3600}{613\times10^6}$ = 0.02m =2cm

Computation of UH

Time (hr)	0	4	8	12	16	20	24	28	32	36	40	44	48
Q (m^3/s)	10	110	225	180	130	100	70	60	50	35	25	15	10
BF (m^3/s)	10	10	10	10	10	10	10	10	10	10	10	10	10
Qdr (m^3/s)	0	100	215	170	120	90	60	50	40	25	15	5	0
UH (m^3/s)	0	50	108	85	60	45	30	25	20	13	7.5	2.5	0

6. The ordinate of a 4-h UH of a catchment of area 1000km^2 are given below. Calculate flood hydrograph resulting from two successive 4-h storms having rainfall of 1.5cm and 1cm. Assume uniform base flow of 10 m^3/s and ϕ-index equal to 0.10 cm/hr.

t(hr)	0	4	8	12	16	20	24	28	32	36	40	44
4hr UH (m^3/s)	0	20	60	150	120	90	66	50	32	20	10	0

Solution:

ϕ-index (infiltration loss) = 0.1 cm/hr

For 4 hour, loss (L) = 4x0.1 =0.4 cm

Rainfall values, R1 = 1.5 cm and R2 = 1.5 cm

Rainfall excess (R_{e1}) = R1-L = 1.5-0.4 =1.1cm

Rainfall excess (R_{e2}) = R2-L = 1.0-0.4 =0.6cm

DHR1 = UHxR_{e1}

DRH2 =UHxR_{e2} (lagged by 4 hour)

DRH = DRH1+DRH2

Q = DRH + BF

Computation of flood hydrograph

t(h)	4 hr UH (m^3/s)	DRH1(m^3/s)	DRH2(m^3/s)	DRH(m^3/s)	BF(m^3/s)	Q (m^3/s)
0	0	0		0	10	10
4	20	22	0	22	10	32
8	60	66	12	78	10	88
12	150	165	36	201	10	211
16	120	132	90	222	10	232
20	90	99	72	171	10	181
24	66	72.6	54	126.6	10	136.6
28	50	55	39.6	94.6	10	104.6
32	32	35.2	30	65.2	10	75.2
36	20	22	19.2	41.2	10	51.2
40	10	11	12	23	10	33
44	0	0	6	6	10	16
(48)			0	0	10	10

9. Given the following data about a catchment of area 1000 km^2, determine the peak discharge corresponding to a storm of 5cm in 1 hr.

Time (h)	0	1	2	3	4	5
Rainfall (cm)	0	2.5	0	0	0	0
Runoff (m^3/s)	300	300	1200	450	300	300

Solution:

Δt = 1 hour = 1x3600 s

Given runoff is direct runoff (Q_{dr}).

Volume of runoff (V) = $\sum Q_{dr}\,\Delta t = \Delta t \sum Q_{dr}$ = 4x3600x2850

$r_d = \frac{V}{A} = \frac{4\times 3600x2850}{1000\times 10^6}$= 0.0103m =1.03cm

Divide Q_{dr} by rd to get UH.

Time (h)	0	1	2	3	4	5
Runoff (m^3/s)	300	300	1200	450	300	300
UH(m^3/s)	292.4	292.4	1169.6	438.6	292.4	292.4

Peak of UH = 1169.6 m^3/s
For 5cm rainfall, peak of DRH = 1169.6x5 = 5848 m^3/s
Peak discharge = 5848 m^3/s+ Base flow

11. Ordinates of an 1hr unit hydrograph at 1hr interval are 5, 8, 5, 3 and 1 m3/s. Calculate
a) watershed area represented by this unit hydrograph
b) S-curve hydrograph derived from this unit hydrograph
c) 2hr unit hydrograph for this catchment.

Solution:
a)

time(hr)	0	1	2	3	4	5
Q (m^3/s)	0	5	8	5	3	1

Δt = 1hr = 1x3600S
Volume of runoff (V) = $\sum Q\Delta t = \Delta t \sum Q$ = 1x3600x22 = 79200m^3
For UH, runoff depth (r_d) = 1cm
Catchment area (A) = V/r_d = 79200/1x10^{-2} = 7920000m^2 = 7.92 km^2
b) and c)
Duration of given UH (D) = 1hr
S-curve addition = Ordinate of S-curve at (t-D)
Ordinate of S curve (S1) = ordinate of UH+ S-curve addition
Required duration of UH (D') = 2hr
S 2 = S1 lagged by 2 hr hour
2hr-UH = (S1-S2)/(D'/D) = (S1-S2)/(2/1)

time(hr)	UH (m^3/s)	S-curve addition	S curve (S1)	S2	2hr UH(m^3/s)
0	0		0		0
1	5	0	5		2.5
2	8	5	13	0	6.5
3	5	13	18	5	6.5
4	3	18	21	13	4
5	1	21	22	18	2
6			22	21	0.5
7			22	22	0

12. The following table gives the ordinates of a DRH resulting from two successive 3-hour durations of rainfall excess value of 2 cm and 4 cm respectively.

t (hr)	0	3	6	9	12	15	18	21	24	27	30
DRH (m^3/s)	0	120	480	660	460	260	160	100	50	20	0

a. Derive the ordinates of 3-hr UH.
b. Using the 3-hr UH derived in a, calculate the flood hydrograph for 2 successive storms of 6.5 cm and 10.5 cm of 3 hours duration and ϕ-index of 0.2cm/hr. Assume base flow = 20 m^3/s

Solution:
a. Effective rainfall, R_1 = 2 cm and R_2 = 4cm
It is a case of multiple storms. We have to use discrete time convolution equation to compute UH ordinate. The equation is

$$Q_n = \sum_{m=1}^{n \leq m} R_m \, U_{n-m+1}$$

Q = Direct runoff, R = Excess rainfall, U = UH ordinate
Here, total no. of runoff ordinates (n) = 9
Total number of rainfall excess values (m) =2

For n =1, m =1
$Q_1 = R_1U_1$
$U_1 = Q_1/R_1$ =120/2 = 60
For n = 2, m=1, 2
$Q_2 = R_1U_2+R_2U_1$
$U_2 = (Q_2-R_2U_1)/R_1$ = (480-4x60)/2 = 120
For n = 3 onwards, m= 1, 2. So, we can use the similar expression as that of U_2 for n = 3 onwards.
$U_n = (Q_n-R_2U_{n-1})/R_1$

$U_3 = (Q_3-R_2U_2)/R_1$ = (660-4x120)/2 = 90
$U_4 = (Q_4-R_2U_3)/R_1$ = (460-4x90)/2 = 50
$U_5 = (Q_5-R_2U_4)/R_1$ = (260-4x50)/2 = 30
$U_6 = (Q_6-R_2U_5)/R_1$ = (160-4x30)/2 = 20
$U_7 = (Q_7-R_2U_6)/R_1$ = (100-4x20)/2 = 10
$U_8 = (Q_8-R_2U_7)/R_1$ = (50-4x10)/2 = 5
$U_9= (Q_9-R_2U_8)/R_1$ = (20-4x5)/2 = 0

UH

t (hr)	0	3	6	9	12	15	18	21	24	27	30
UH(m^3/s)	0	60	120	90	50	30	20	10	5	0	0

b. ϕ-index (infiltration loss) = 0.2 cm/hr
For 3 hour, loss (L) = 3x0.2 =0.6cm
Rainfall values, R1 = 6.5 cm and R2 = 10.5 cm
Rainfall excess (Re1) = R1-L = 6.5-0.6=5.9cm
Rainfall excess (Re2) = R2-L = 10.5-0.6 =9.9cm

DHR1 = UHxRe1
DRH2 =UHxRe2 (lagged by 6 hour)
DRH = DRH1+DRH2
Q = DRH + BF

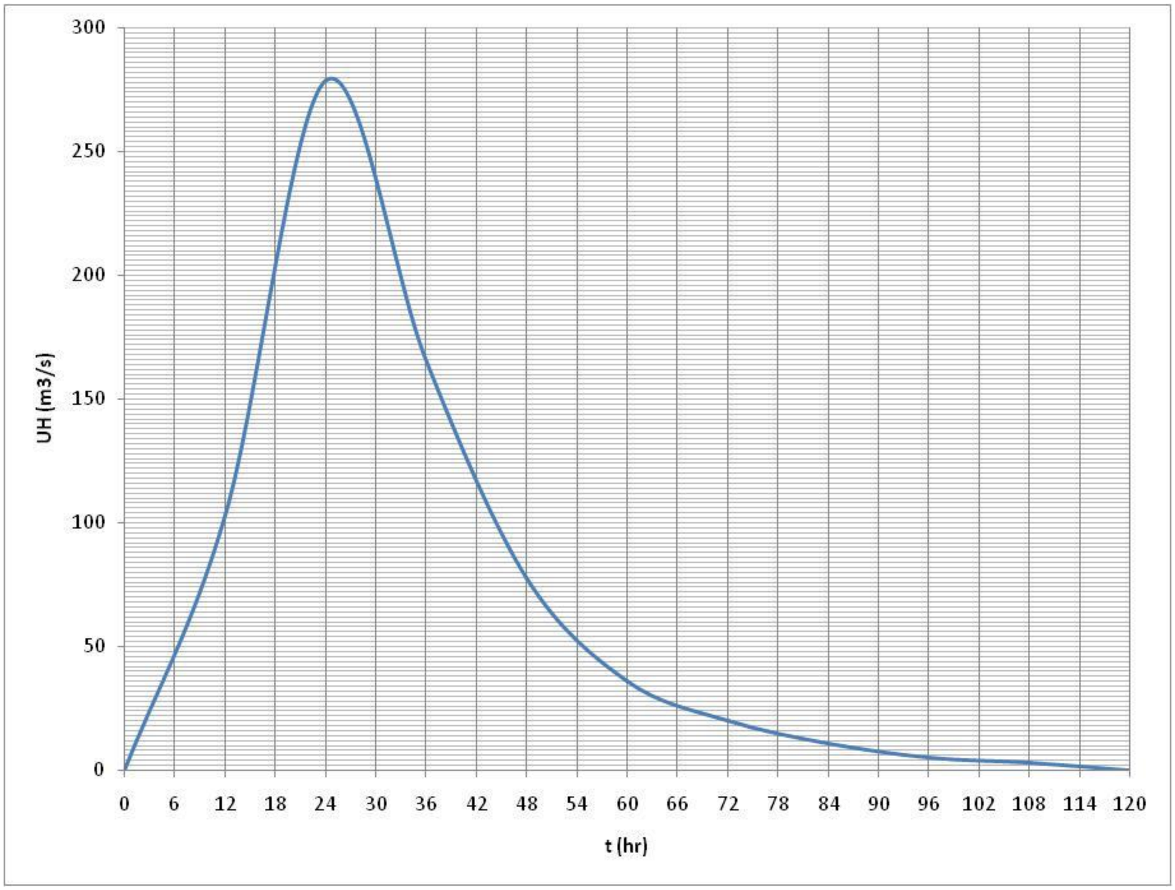
Computation of flood hydrograph

t (hr)	UH	DRH1	DRH2	DRH	BF	Q (m^3/s)
0	0	0		0	20	20
3	60	354	0	354	20	374
6	120	708	594	1302	20	1322
9	90	531	1188	1719	20	1739
12	50	295	891	1186	20	1206
15	30	177	495	672	20	692
18	20	118	297	415	20	435
21	10	59	198	257	20	277
24	5	29.5	99	128.5	20	148.5
27	0	0	49.5	49.5	20	69.5
30	0	0	0	0	20	20
33			0	0	20	20

14. Given below is a 12-hr UH. Derive 6-hr UH.

t (hr)	0	12	24	36	48	60	72	84	96	108	120
UH(m^3/s)	0	103	279	165	78	36	20	11	5	3	0

Solution:
Required duration of UH (D') =6 hr
Given duration (D) =12 hr
n = D'/D = 0.5 (real)
Here, D'<D. To derive UH of 6 hr, the interval of ordinates of given UH should be at least 6hour.
Plot given UH versus t on a graph paper and get the values of UH at 6 hour interval.



S-curve addition = Ordinate of S-curve at (t-D)

Ordinate of S curve (S1) = ordinate of UH+ S-curve addition

S2 = S1 lagged by 6 hour

6-hr UH = (S1-S2)/(D'/D) = (S1-S2)/0.5

Computation of 6-hr UH

t (hr)	UH(m3/s)	S curve addition	S1	S2	6-hr UH	6-hr UH (corrected)
0	0		0		0	0
6	48		48	0	96	96
12	103	0	103	48	110	110
18	191	48	239	103	272	272
24	279	103	382	239	286	286
30	238	239	477	382	190	190
36	165	382	547	477	140	140
42	117	477	594	547	94	94
48	78	547	625	594	62	62
54	53	594	647	625	44	44
60	36	625	661	647	28	28

66	27	647	674	661	26	26
72	20	661	681	674	14	14
78	15	674	689	681	16	10
84	11	681	692	689	6	6
90	8	689	697	692	10	4
96	5	692	697	697	0	0
102	4	697	701	697	8	0
108	3	697	700	701	-2	0
114	2	701	703	700	6	0
120	0	700	700	703	-6	0
126		703	700	700	0	0

The UH of 6 hour should be corrected manually from 90 hour onwards to make it smooth.

15. The ordinates of 4hr unit hydrograph are given below:

Time (hr)	0	2	4	6	8	10	12	14	16	18	20	22	24	26
UH ordinate (m^3/s)	0	50	150	350	600	900	850	500	350	250	150	60	10	0

A storm had 3 successive 2hr intervals of rainfall magnitude of 2, 4 and 3cm respectively. Assuming ϕ index of 0.2cm/hr and a base flow of 15 m^3/s, determine the resulting hydrograph of flow.

Solution:

Part I

First convert 4hr UH to 2hr UH by S-curve method

Required duration of UH (D') =2 hr

Given duration (D) =4 hr

n = D'/D = 0.5 (real)

S-curve addition = Ordinate of S-curve at (t-D)

Ordinate of S curve (S_1) = ordinate of UH+ S-curve addition

S_2 = S_1 lagged by 2 hour

2-hr UH = (S_1-S_2)/(D'/D) = (S1-S2)/0.5

t(hr)	6hr UH	S curve addition	S_1	S_2	2hr UH
0	0		0		0
2	50		50	0	100
4	150	0	150	50	200
6	350	50	400	150	500
8	600	150	750	400	700
10	900	400	1300	750	1100

12	850	750	1600	1300	600
14	500	1300	1800	1600	400
16	350	1600	1950	1800	300
18	250	1800	2050	1950	200
20	150	1950	2100	2050	100
22	60	2050	2110	2100	20
24	10	2100	2110	2110	0
26	0	2110	2110	2110	0

PartII

Computation of flood hydrograph

ϕ-index (infiltration loss) = 0.2 cm/hr

For 2 hour, loss (L) = 2x0.2 =0.4 cm

Rainfall values, R1 = 2 cm and R2 = 4 cm and R3 = 3cm

Rainfall excess (R_{e1}) = R1-L = 2-0.4 =1.6cm

Rainfall excess (R_{e2}) = R2-L = 4-0.4 =3.6cm

Rainfall excess (R_{e3}) = R3-L = 3-0.4 =2.6cm

DHR1 = UHxR_{e1}

DRH2 =UHxR_{e2} (lagged by 2 hour)

DRH2 =UHxR_{e3} (lagged by 4 hour)

DRH = DRH1+DRH2+DRH3

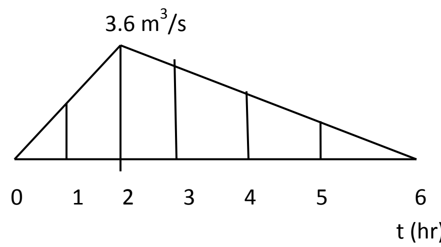
Q = DRH + BF

Time (hr)	UH	DRH1	DRH2	DRH3	DRH	BF	Q
	(m^3/s)	(m^3/s)	(m^3/s)	(m^3/s)	(m^3/s)	(m^3/s)	(m^3/s)
0	0	0			0	15	15
2	100	160	0		160	15	175
4	200	320	360	0	680	15	695
6	500	800	720	260	1780	15	1795
8	700	1120	1800	520	3440	15	3455
10	1100	1760	2520	1300	5580	15	5595
12	600	960	3960	1820	6740	15	6755
14	400	640	2160	2860	5660	15	5675
16	300	480	1440	1560	3480	15	3495
18	200	320	1080	1040	2440	15	2455
20	100	160	720	780	1660	15	1675
22	20	32	360	520	912	15	927

24	0	0	72	260	332	15	347
26	0	0	0	52	52	15	67
				0	0	15	15

16. A 1hr unit hydrograph of a small rural catchment is triangular with a peak value of $3.6m^3/s$ occurring at 2hr from the start and a base time of 6hours. Following urbanization over a period of two decades, the infiltration index ϕ has decreased from 0.7cm/h to 0.4cm/h. Also the 1hr unit hydrograph has now a peak of $6m^3/s$ at 1hr from start and a time base of 4 hours. If a design storm has intensities of 4cm/h and 3cm/h for two consecutive 1hr intervals, estimate the percentage increase in peak storm runoff and in the volume of flood runoff, due to urbanization.

Solution:
Before urbanization
Peak flow = $3.6m^3/s$ at 2hr from start
ϕ index = 0.7cm/h
Rainfall, R1 = 4cm and R2 = 3cm
R_{e1} = 4-0.7 = 3.3cm, R_{e2} = 3-0.7 = 2.3cm

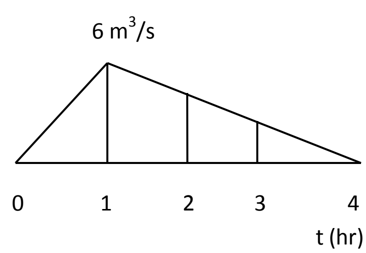


Obtain UH with time interval of 1hr .
(by similar triangles).
DRH_1 = UHxRe1
DRH_2 = UHxRe2 (lagged by 1hr)
DRH = DRH_1+DRH_2

Time (h)		UH (m^3/s)	DRH_1	DRH_2	DRH (Q)
0	(0/2)3.6	0	0		0
1	(1/2)3.6	1.8	5.94	0	5.94
2		3.6	11.88	4.14	16.02
3	(3/4)3.6	2.7	8.91	8.28	17.19
4	(2/4)3.6	1.8	5.94	6.21	12.15
5	(1/4)3.6	0.9	2.97	4.14	7.11
6		0	0	2.07	2.07
				0	0

Peak runoff = $17.19m^3/s$
Volume of runoff (V) = $\sum Q\Delta t = \Delta t \sum Q$ = 1x3600x60.48 = $217728m^3$

After urbanization
Peak flow = $6m^3/s$ at 1.0hr from start
ϕ index = 0.4cm/h
Rainfall, R1 = 4cm and R2 = 3cm
R_{e1} = 4-0.4 = 3.6cm, R_{e2} = 3-0.4 = 2.6cm



Obtain UH with time interval of 1hr
(by similar triangles).
$DRH_1 = UHxR_{e1}$
$DRH_2 = UHxR_{e2}$ (lagged by 1hr)
DRH = DRH1+DRH2

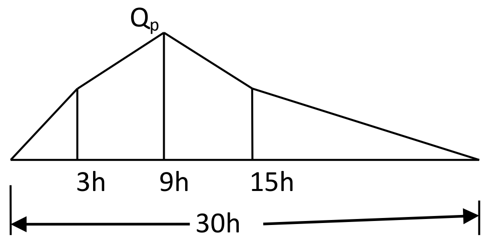
Time (h)		UH (m^3/s)	DRH_1	DRH_2	DRH (Q)
0		0	0		0
1		6	21.6	0	21.6
2	(2/3)6	4	14.4	15.6	30
3	(1/3)6	2	7.2	10.4	17.6
4		0	0	5.2	5.2
				0	0

Peak runoff = $30m^3/s$
Volume of runoff (V) = $\sum Q\Delta t = \Delta t \sum Q$ = 1x3600x74.4 = $267840m^3$
a. Percentage increase in peak runoff $= \frac{30-17.19}{17.19} x100$ = 74.5%
b. Volume of flood runoff due to urbanization = 267840-217728=$50112m^3$

17. Unit hydrograph of 3h duration from a catchment area of 524.88ha has peak discharge value after 9 hours from the start of the storm. It has half of the peak discharge value at 3hours and 15 hours after the start of the storm respectively. If time base (t_b) is 30 hours, constitute the unit hydrograph and calculate the flood hydrograph from the following rainfall data. Assume ϕ index as 0.333 cm/h and base flow as 20 cumecs.

Time (hr)	0	6	9	12
Cumulative rainfall (cm)	0	12	12	23

Solution:
Catchment area (A) = 524.88ha = $524.88x10^4$ m^2
Time base (t_b) = 30hr



Q_p = peak discharge

Area of UH = Volume of 1cm depth of runoff over the catchment

$$\frac{1}{2}x3x3600x\frac{Q_p}{2} + \frac{1}{2}\left(Q_p + \frac{Q_p}{2}\right)x6x3600 + \frac{1}{2}\left(Q_p + \frac{Q_p}{2}\right)x6x3600 + \frac{1}{2}x15x3600x\frac{Q_p}{2} = Ax1cm$$

$$13.5x3600xQ_p = 524.88x10^4x1x10^{-2}$$

Q_p = 1.08 m^3/s

Peak of UH = 1.08 m^3/s at 9hr
At 3 hr and 15 hr from the start of storm, ordinate of UH = 1.08/2 = 0.54 m^3/s
Interpolating for other durations
At 6hr, UH ordinate = $0.54 + \frac{1.08-0.54}{9-3} x3$ = 0.81 m^3/s
At 12hr, UH ordinate = $1.08 - \frac{1.08-0.54}{6} x3$ = 0.81 m^3/s
At 18 hr, UH ordinate = $0.54 - \frac{0.54-0}{15} x3$ = 0.432 m^3/s
At 21 hr, UH ordinate = $0.54 - \frac{0.54-0}{15} x6$ = 0.324 m^3/s
At 24 hr, UH ordinate = $0.54 - \frac{0.54-0}{15} x9$= 0.216 m^3/s
At 27 hr, UH ordinate = $0.54 - \frac{0.54-0}{15} x12$= 0.108 m^3/s
At 30 hr, UH ordinate = 0

Resulting UH

Time (hr)	0	3	6	9	12	15	18	21	24	27	30
UH (m^3/s)	0	0.54	0.81	1.08	0.81	0.54	0.432	0.324	0.216	0.108	0

ϕ index = 0.333 cm/h
Incremental rainfall: 12-0 =12 cm in 6 hr, 12-12= 0cm in 9hr and 23-12= 11cm in 12hr
Effective rainfall
R_{e1} = 12-6x0.333 = 10.002cm
R_{e2} = 0
R_{e3} = 11-3x0.333 = 10.001cm
DHR1 = UHxR_{e1}
DRH2 =UHxR_{e2} (lagged by 3 hour)
DRH2 =UHxR_{e3} (lagged by 6 hour)

DRH = DRH1+DRH2+DRH3
Q = DRH + BF where BF = Base flow = 20 m^3/s

Computation of flood hydrograph

Time (hr)	UH (m^3/s)	DRH1	DRH2	DRH3	DRH	Q (m^3/s)
0	0	0			0	20
3	0.54	5.401	0		5.401	25.401
6	0.81	8.102	0	0	8.102	28.102
9	1.08	10.802	0	5.401	16.203	36.203

12	0.81	8.102	0	8.101	16.202	36.202
15	0.54	5.401	0	10.801	16.202	36.202
18	0.432	4.321	0	8.101	12.422	32.422
21	0.324	3.241	0	5.401	8.641	28.641
24	0.216	2.160	0	4.320	6.481	26.481
27	0.108	1.080	0	3.240	4.321	24.321
30	0	0	0	2.160	2.160	22.160
33			0	1.080	1.080	21.080
36				0	0	20

Statistical Hydrology

3. Flood frequency records on a river have been collected for 17 years starting from 1951 to 1967 and the peak values of the flood observed during these 17 years are given below:

Year	1951	1952	1953	1954	1955	1956	1957	1958	1959
Peak discharge (m^3/s)	3000	4400	6000	3500	2900	4800	3900	3300	6700
Year	1960	1961	1962	1963	1964	1965	1966	1967	
Peak discharge (m^3/s)	5400	4300	3700	4200	9000	4000	3600	5100	

a. Prepare a Gumbel's extreme value probability paper.
b. Estimate graphically 100 year and 500 year flood. Also check the result by analytical method. Take mean and standard deviation of reduced variate as 0.518 and 1.041 respectively.

Solution:
a) The relationship between discharge (x) and the reduced variate (y_T) is linear. Therefore, to prepare Gumbel's extreme value probability paper, discharge is plotted on the Y-axis and the reduced variate is plotted on the X-axis in linear scale. To show the return period on the X-axis, for different values of return periods Y_T, return period T is computed as shown in table below.

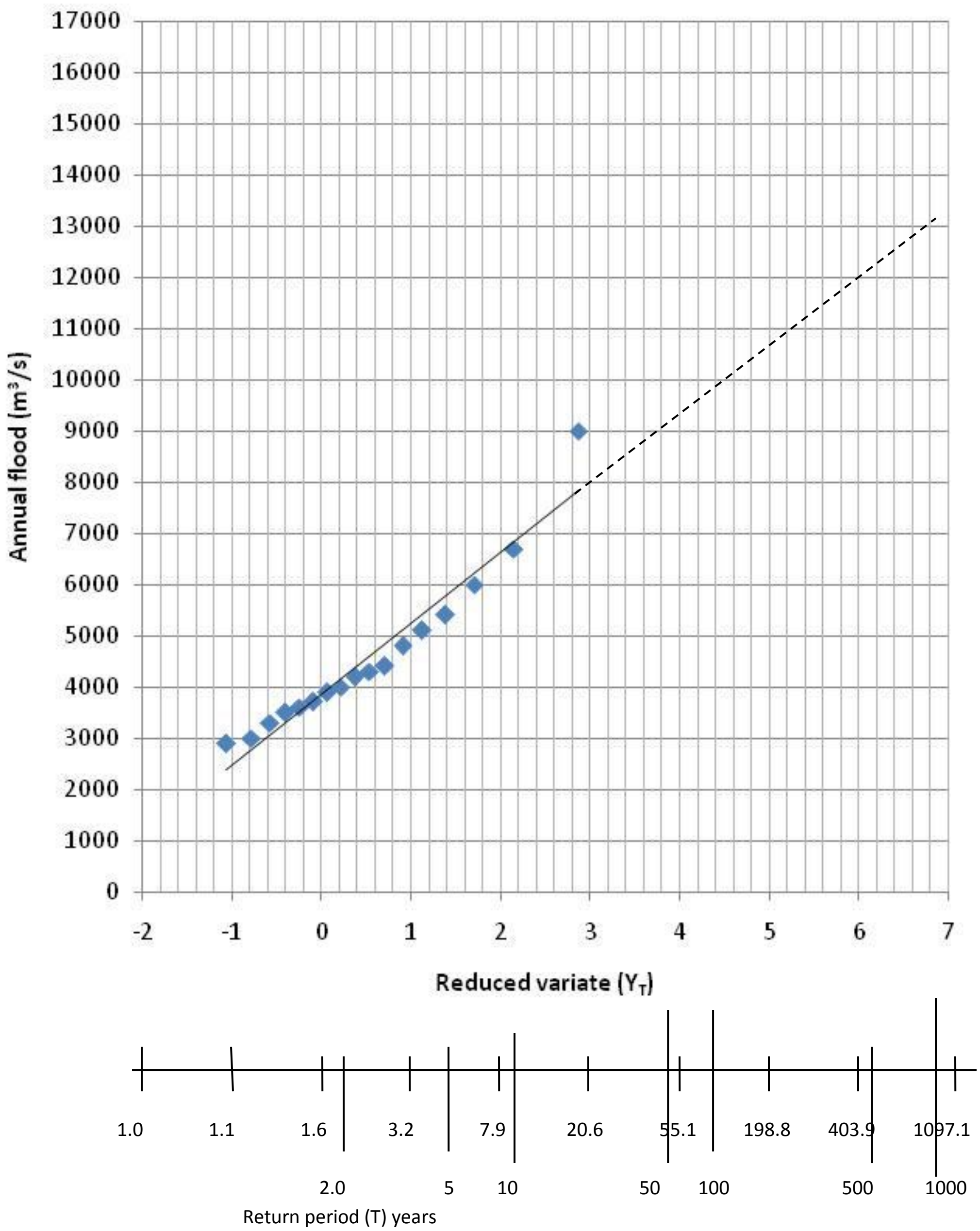
Y_T	T
-2	1.0
-1	1.1
0	1.6
1	3.2
2	7.9
3	20.6
4	55.1
5	148.9
6	403.9
7	1097.1

For corresponding value of Y_T, return period (T) is marked on the X-axis.

Table for plotting position

Discharge	Discharge in descending order (Q_i)	Rank (m)	Return period (T_i = n+1/m)	Reduced variate (YT = -ln(ln(T/T-1)
3000	9000	1	18.0	2.86
4400	6700	2	9.0	2.14

6000	6000	3	6.0	1.70
3500	5400	4	4.5	1.38
2900	5100	5	3.6	1.12
4800	4800	6	3.0	0.90
3900	4400	7	2.6	0.71
3300	4300	8	2.3	0.53
6700	4200	9	2.0	0.37
5400	4000	10	1.8	0.21
4300	3900	11	1.6	0.06
3700	3700	12	1.5	-0.09
4200	3600	13	1.4	-0.25
9000	3500	14	1.3	-0.41
4000	3300	15	1.2	-0.58
3600	3000	16	1.1	-0.79
5100	2900	17	1.1	-1.06



b) From the graph

X_{100} = 9900 m^3/s

X_{500} = 12100 m^3/s

Analytical method

$\overline{y_n}$ = 0.518 and S_n = 1.041

Discharge (X_i)	$(X_i - \bar{X})^2$
3000	2485259.5

	4400	31141.9
	6000	2026436.0
	3500	1158788.9
	2900	2810553.6
	4800	49965.4
	3900	457612.5
	3300	1629377.2
	6700	4509377.2
	5400	678200.7
	4300	76436.0
	3700	768200.7
	4200	141730.1
	9000	19567612.5
	4000	332318.3
	3600	953494.8
	5100	274083.0
Sum	77800	37950588.24

Mean $(\bar{x}) = \frac{\sum X_i}{n} = \frac{77800}{17} = 4576.47$

Standard deviation $(\sigma) = \sqrt{\frac{\sum(X_i - \bar{x})^2}{n-1}} = \sqrt{\frac{37950588.24}{16}} = 1540.1$

$y_{100} = -ln\left(ln\frac{T}{T-1}\right) = -ln\left(ln\frac{100}{100-1}\right)$=4.6

$y_{500} = -ln\left(ln\frac{T}{T-1}\right) = -ln\left(ln\frac{500}{500-1}\right)$=6.213

$$K_T = \frac{y_T - \overline{y_n}}{S_n}$$

$$K_{100} = \frac{y_{100} - \overline{y_n}}{S_n} = \frac{4.6 - 0.518}{1.1041} = 3.921$$

$$K_{500} = \frac{y_{500} - \overline{y_n}}{S_n} = \frac{6.213 - 0.518}{1.1041} = 5.47$$

$$X_T = \bar{x} + K_T\sigma$$

$X_{100} = \bar{x} + K_{100}\sigma = 4576.47 + 3.921x1540.1$= 10615 m^3/s

$X_{500} = \bar{x} + K_{500}\sigma = 4576.47 + 5.47x1540.1$=13000 m^3/s

4. a) Find out the frequency of a flood of magnitude 2000 m^3/s, given the following record of maximum yearly peak floods for 10 years.

Year	1	2	3	4	5	6	7	8	9	10
Q (m^3/s)	300	700	200	400	1000	900	800	500	100	600

Take values of reduced mean and reduced standard deviation in Gumbel's Extreme value distribution as 0.4952 and 0.9496 respectively.

b) What is the probability of exceedence of flood of magnitude 2000 m^3/s? What is the probability that this flood may occur in the next 100 years?

c) What are the 80% confidence limits of above floods if f(c) = 1.282?

Solution:

Computation of mean and standard deviation

	Discharge (X_i)	$(X_i - \bar{X})^2$
	300	62500
	700	22500
	200	122500
	400	22500
	1000	202500
	900	122500
	800	62500
	500	2500
	100	202500
	600	2500
Sum	5500	825000

Mean $(\bar{x}) = \frac{\sum X_i}{n} = \frac{5500}{10} = 550$

Standard deviation $(\sigma) = \sqrt{\frac{\sum(X_i - \bar{x})^2}{n-1}} = \sqrt{\frac{825000}{9}} = 302.765$

a) Frequency of flood (T) of magnitude 2000 m^3/s = ?

$\overline{y_n}$ = 0.4952 and S_n = 0.9496

X = 2000

$$X_T = \bar{x} + K_T\sigma$$

2000 = 550 + K x 302.765

K = 4.789

$$K_T = \frac{y_T - \overline{y_n}}{S_n}$$

$$4.789 = \frac{y_T - 0.4952}{0.9496}$$

y_T = 5.043

$$y_T = -ln\left(ln\frac{T}{T-1}\right)$$

$$5.043 = -ln\left(ln\frac{T}{T-1}\right)$$

$$ln\frac{T}{T-1} = \exp(-5.043) = 0.006454$$

$$\frac{T}{T-1} = \exp(0.006454) = 1.006475$$

T = 155.4 years

b) Exceedence probability of flood of 2000 m^3/s (P) = 1/T = 1/155.4 = 0.006435

Probability of occurrence of flood of 2000 m^3/s in next 100 years (Pr) = ?

n=100

Pr = $1-(1-P)^n = 1-(1-0.006435)^{100}$ = 0.475 = 47.5%

c) 80% confidence limit for this flood =?

f(c) =1.282 for 80% confidence limit

K = 4.789 (computed in a)

$b = \sqrt{1 + 1.13K + 1.1K^2} = \sqrt{1 + 1.13x4.789 + 1.1x4.789^2}$ =5.6968

$$S_e = b\frac{\sigma}{\sqrt{N}} = 5.6968\frac{302.756}{\sqrt{10}} = 545.43$$

$$X_{UL} = X \pm f(c)S_e$$

80% confidence limit

$X_{UL} = 2000 \pm 1.282x545.43$ =2699.24 m^3/s, 1300.76 m^3/s

7. A hydraulic structure on a stream has been designed for a discharge of 350m^3/s. If the available flood data on the stream is for 20 years and the mean and standard deviation for annual flood series are 121 and 60 m3/s respectively, calculate the return period for the design flood using Gumbel's method. Take $\overline{y_n}$ and S_n from Gumbel table.

Solution:

Discharge (Q) = 350m^3/s

Mean of flood ($\bar{x}$) =121m^3/s

Standard deviation of flood (σ) = 60 m^3/s
Return period (T) = ?
$\overline{y_n}$ = 0.5236 and S_n = 1.0628 for 20 years

$$Q = \bar{x} + K\sigma$$
$$350 = 121 + Kx60$$
$$K = 3.816$$

$$K = \frac{y_T - \overline{y_n}}{S_n}$$
$$3.816 = \frac{y_T - 0.5236}{1.0628}$$
$$y_T = 4.579$$

$$y_T = -ln\left(ln\frac{T}{T-1}\right)$$
$$4.5792 = -ln\left(ln\frac{T}{T-1}\right)$$
$$\frac{T}{T-1} = \exp(\exp(-4.579)) = 1.01$$
T = 101 years

11. For a data of maximum recorded annual floods of a river, the mean and standard deviation are 4200 m^3/s and 1705 m^3/s respectively. Using Gumbel's extreme value distribution, estimate the return period of a design flood of 9500 m^3/s. Assume an infinite sample size.

Solution:
Mean of flood ($\bar{Q}$) = 4200m^3/s
Standard deviation of flood (σ) = 1705m^3/s
Flood discharge (Q) = 9500 m^3/s
Return period (T) of 9500 m^3/s flood = ?

$$Q = \bar{Q} + K_T\sigma$$
$$9500 = 4200 + K_T x1705$$
$$K_T = 3.108$$

For large sample size

$$K_T = -\left[0.45 + 0.78ln\left(ln\frac{T}{T-1}\right)\right]$$
$$3.108 = -\left[0.45 + 0.78ln\left(ln\frac{T}{T-1}\right)\right]$$
$$\frac{T}{T-1} = 1.0$$

T = 100 years

Alternative formula for K_T

$\overline{y_n}$ = 0.577 and S_n = 1.2825 (for large sample size)

$$K_T = \frac{y_T - \overline{y_n}}{S_n}$$

$$3.108 = \frac{y_T - 0.577}{1.2825}$$

y_T = 4.56

$$y_T = -ln\left(ln\frac{T}{T-1}\right)$$

$$4.56 = -ln\left(ln\frac{T}{T-1}\right)$$

T = 100 years

12. A river has 40 years of annual flood flow record. The discharge values are in m^3/s. The logarithms to the base 10 of these discharge values show a mean value of 3.2736, standard deviation of 0.3037 and a coefficient of skewness of 0.07. Calculate the 50 year return period annual flood discharge by Log-Pearson type III method. Use Table to compute the value of K.

Solution:

Mean ($\bar{y}$)= 3.2736

Standard deviation (σ) = 0.3037

Coefficient of skewness(C_s)= 0.07

Return period (T) = 50 year

Computing K for C_s = 0.07 and T = 50 years using table

C_s = 0.0, K = 2.054

C_s = 0.1, K = 2.107

Interpolating

C_s = 0.07, $K = 2.054 + \frac{2.107-2.054}{0.1-0.0}(0.07 - 0.0)$ = 2.0911

$y_T = \bar{y} + K_T\sigma$ = 3.2736+2.0911x0.3037 = 3.9087

Flood of 50 year return period (X_{50}) = $10^{3.9087}$= 8104 m^3/s

13. The following are the annual peak flow data (m^3/s) of Bagmati River at Pandheradobhan station from 1987 to 2006:

2890, 2670, 4410, 5620, 3090, 2100, 16000, 3700, 3500, 3060, 3300, 4660, 5080, 6200, 3140, 8000, 3380, 6850, 5580, 3500

Compute flood magnitude with 25year return period using Lognormal distribution and Log-Pearson type III distribution. (Obtain frequency factor value by using Table).

Solution:
Log transformed data ($y=\log_{10}(x)$)
3.461, 3.427, 3.644, 3.750, 3.490, 3.322, 4.204, 3.568, 3.544, 3.486, 3.519, 3.668, 3.706, 3.792, 3.497, 3.903, 3.529, 3.836, 3.747, 3.544
Mean $(\bar{y}) = \frac{\sum y}{n}$= 3.632
Standard deviation $(\sigma) = \sqrt{\left(\frac{\sum(y_i-\bar{y})^2}{n-1}\right)}$ = 0.201
Coefficient of skewness$(C_s) = \frac{1}{\sigma^3}\frac{n}{(n-1)(n-2)}\sum(y_i - \bar{y})^3$ = 1.2

Lognormal
For lognormal distribution, $C_s = 0$
For T= 25 years and $C_S = 0$, $K_T = 1.751$
$y_T = \bar{y} + K_T\sigma$ = 3.632+1.751x0.201 = 3.98
Flood of 25 year return period (X_{25}) = $10^{3.98}$ = 9550 m^3/s
Log Pearson III
For T= 25 years and $C_S = 1.2$, $K_T = 2.087$
$y_T = \bar{y} + K_T\sigma$ = 3.632+2.087x0.201 = 4.05
Flood of 25 year return period (X_{25}) = $10^{3.98}$ = 11220 m^3/s

14. For a river, the estimated flood peaks for two return periods by the use of Gumbel's method are as follows:

Return period (years)	Peak flood (m^3/s)
100	435
50	395

What flood discharge in the river will have a return period of 1000 years?
Solution:
For T = 100 years, $X_{100} = 435 m^3/s$
For T = 50 years, $X_{50} = 395\ m^3/s$

$$y_T = -ln\left(ln\frac{T}{T-1}\right)$$

$y_{50} = -ln\left(ln\frac{50}{50-1}\right)$ = 3.902
$y_{100} = -ln\left(ln\frac{100}{100-1}\right)$ = 4.6
$y_{100} = -ln\left(ln\frac{1000}{1000-1}\right)$ = 6.907

$$X_T = \bar{x} + K_T\sigma$$
$$X_{50} = \bar{x} + \frac{y_{50} - \overline{y_n}}{S_n}\sigma$$

$395 = \bar{x} + \frac{3.902-\overline{y_n}}{S_n}\sigma$ (I)

$$X_{100} = \bar{x} + \frac{y_{100} - \overline{y_n}}{S_n}\sigma$$

$435 = \bar{x} + \frac{4.6 - \overline{y_n}}{S_n}\sigma$ (II)

Subtracting I from II

$$40 = 0.698\frac{\sigma}{S_n}$$

$\frac{\sigma}{S_n} = 57.3$ (III)

$$X_{1000} = \bar{x} + \frac{y_{1000} - \overline{y_n}}{S_n}\sigma$$

$X_{1000} = \bar{x} + \frac{6.907 - \overline{y_n}}{S_n}\sigma$ (IV)

Subtracting II from IV

$$X_{1000} - 435 = 2.307\frac{\sigma}{S_n}$$

Substituting the value of $\frac{\sigma}{S_n}$ from III

$X_{1000} = 435 + 2.307x57.3$= 567 m^3/s

15. Frequency analysis of a flood data of a river by using log-pearson type III distribution yielded the following data:

Return period (T) years	Peak flood (m^3/s)
50	10000
200	15000

and variation of the frequency factor K with return period T for C_s = 0.4 is

Return period (T)	50	200	1000
Frequency (K)	2.26	2.95	3.67

Estimate the flood magnitude in the river with return period of 1000 years.

Solution:

T (yr)	Q(m3/s)	Y_T = logQ	K_T
50	10000	4	2.26
200	15000	4.17	2.95

$$Y_T = \bar{Y} + K_T\sigma$$

$4 = \bar{Y} + 2.26\sigma$ (a)

$4.17 = \bar{Y} + 2.95\sigma$ (b)

Solving a and b

$\bar{Y} = 3.44$, $\sigma = 0.25$

$$Y_{1000} = \bar{Y} + K_{1000}\sigma$$

= 3.44+0.25x3.67 = 4.36
$Q_{1000} = 10^{4.36} = 22910\ m^3/s$

16. A highway bridge has to be designed with an expected life of 50 years and an allowable flood risk of 4%. The flood data of bridge site were well fitted to Gumbel EV distribution and discharges for 50 and 300 years return period are found to be 150 and 650 m^3/s respectively. Estimate the frequency and magnitude of design flood for the bridge.

Solution:
Design life (n) =50 years
Risk (R) = 4% = 0.04
Return period (T) = ?

$$R = 1 - \left(1 - \frac{1}{T}\right)^n$$
$$0.04 = 1 - \left(1 - \frac{1}{T}\right)^{50}$$
$$\left(1 - \frac{1}{T}\right)^{50} = 0.96$$

T= 1225 years

For T = 50 years, $X_{50} = 150\ m^3/s$
For T = 300 years, $X_{300} = 650\ m^3/s$
For T = 1225 years, X_{1225}= ?

$$y_T = -ln\left(ln\frac{T}{T-1}\right)$$

$y_{50} = -ln\left(ln\frac{50}{50-1}\right)$ = 3.902
$y_{300} = -ln\left(ln\frac{300}{300-1}\right)$ = 5.702
$y_{1225} = -ln\left(ln\frac{1225}{1225-1}\right)$ = 7.11

$$X_T = \bar{x} + K_T\sigma$$
$$X_{50} = \bar{x} + \frac{y_{50} - \overline{y_n}}{S_n}\sigma$$

$150 = \bar{x} + \frac{3.902 - \overline{y_n}}{S_n}\sigma$ (I)

$$X_{300} = \bar{x} + \frac{y_{300} - \overline{y_n}}{S_n}\sigma$$

$650 = \bar{x} + \frac{5.702 - \overline{y_n}}{S_n}\sigma$ (II)
Subtracting I from II

$$500 = 1.8\frac{\sigma}{S_n}$$

$\frac{\sigma}{S_n} = 277.77$ (III)

$$X_{1225} = \bar{x} + \frac{y_{1225} - \overline{y_n}}{S_n}\sigma$$

$X_{1225} = \bar{x} + \frac{7.11 - \overline{y_n}}{S_n}\sigma$ (IV)

Subtracting II from IV

$$X_{1225} - 650 = 1.408\frac{\sigma}{S_n}$$

Substituting the value of $\frac{\sigma}{S_n}$ from III

$X_{1225} = 650 + 1.408x277.77$ = 1041 m^3/s

17. The analysis of annual flood series of a river data yielded $\bar{Q}$ = 1200m^3/s and standard deviation σ = 650m^3/s. For what discharge would you design the structure to provide 95% assurance that the structure would not fail in next 50 years. Use Gumbel's method and assume sample size to be infinite.

Solution:

Mean of flood ($\bar{Q}$) = 1200m^3/s

Standard deviation of flood (σ) = 650m^3/s

Expected life (n) = 50 years

Reliability (Re) = 95% = 0.95

Flood discharge (Q) = ?

Risk (R) = 1-Re = 1-0.95 = 0.05

Finding return period (T)

$$R = 1 - \left(1 - \frac{1}{T}\right)^n$$

$$0.05 = 1 - \left(1 - \frac{1}{T}\right)^{50}$$

$$\left(1 - \frac{1}{T}\right)^{50} = 0.95$$

$$1 - \frac{1}{T} = (0.95)^{1/50} = 0.998975$$

T = 975 years

$K = -\left[0.45 + 0.78ln\left(ln\frac{T}{T-1}\right)\right] = -\left[0.45 + 0.78ln\left(ln\frac{975}{975-1}\right)\right]$ = 4.92

$$Q = \bar{x} + K_T\sigma$$

$$= 1200 + 4.92x650 = 4398 m^3/s$$

Design discharge = 4398 m^3/s

[Alternative method to compute K

$\overline{y_n}$ = 0.577 and S_n = 1.2825 (for large sample size)

$y_T = -ln\left(ln\frac{T}{T-1}\right) == -ln\left(ln\frac{975}{975-1}\right)$ = 6.882

Frequency factor (K_T) is

$K_T = \frac{y_T-\overline{y_n}}{S_n} = \frac{6.882-0.577}{1.2825} = 4.92$]

18. Analysis of the annual flood peak of a river for 21 years yielded a mean of 8520 m^3/s and a standard deviation of 3900 m^3/s. A proposed water control project on this river is to have an expected life of 40 years. Policy decision of the project allows an acceptable reliability of 85%.

(a) Using Gumbel's method, recommend the flood discharge for this project. Take $\overline{y_n}$ = 0.5436 and S_n = 1.1413 for 40 years.

(b) If a safety factor for flood magnitude of 1.3 is desired, what discharge is to be adopted? What would be the corresponding safety margin?

Solution:

Mean of flood ($\bar{x}$) =8520 m^3/s

Standard deviation of flood (σ) = 3900 m^3/s

Expected life (n) = 40 years

Reliability (Re) = 85% = 0.85

(a) Flood discharge (Q) = ?

Risk (R) = 1-Re = 1-0.85 = 0.15

Finding return period (T)

$$R = 1 - \left(1 - \frac{1}{T}\right)^n$$

$$0.15 = 1 - \left(1 - \frac{1}{T}\right)^{40}$$

$$\left(1 - \frac{1}{T}\right)^{40} = 0.85$$

$$1 - \frac{1}{T} = (0.85)^{1/40} = 0.995945$$

T = 247 years

Reduced variate is

$y_T = -ln\left(ln\frac{T}{T-1}\right) == -ln\left(ln\frac{247}{247-1}\right)$ = 5.50736

$\overline{y_n}$ = 0.5436 and S_n = 1.1413 for 40 years
Frequency factor (K_T) is

$$K_T = \frac{y_T - \overline{y_n}}{S_n}$$

$$= \frac{5.50736 - 0.5436}{1.1413} = 4.3492$$

$$Q = \bar{x} + K_T\sigma$$

$$= 8520 + 4.3492x3900 = 25482 m^3/s$$

Design discharge = 25482m^3/s say 25500 m^3/s

b) For safety factor = 1.3, Actual discharge = ?, safety margin = ?

$$Saftey\ factor = \frac{Actual\ discharge}{Design\ discharge}$$

$$1.3 = \frac{Actual\ discharge}{25500}$$

Actual discharge = 33150 m^3/s
Safety margin = Actual discharge- Design discharge = 33150-25500 = 7650 m^3/s

19. The project life of a headworks is 50 years. The flood discharge at risk 63.5803% is 4200 cumecs. The average flood is 3500 cumecs, which is derived from long term historical data using Gumbel distribution. Calculate the discharge from 500 year return period and risk 39.49939%. Prepare a Gumbel graph using arithmetic graph paper. Plot these three discharges on Gumbel paper.

Solution:
Mean of flood ($\bar{Q}$) = 3500m^3/s
Project life (n) = 50 years
At Risk (R) = 63.5803%, flood = 4200 m^3/s

$R = 1 - \left(1 - \frac{1}{T}\right)^n$

$0.635803 = 1 - \left(1 - \frac{1}{T}\right)^{50}$

$\left(1 - \frac{1}{T}\right)^{50} = 0.364197$

T= 50 years
Q_{50} = 4200 m^3/s
For risk of 39.499939%

$R = 1 - \left(1 - \frac{1}{T}\right)^n$

$0.39499939 = 1 - \left(1 - \frac{1}{T}\right)^{50}$

$\left(1 - \frac{1}{T}\right)^{50} = 0.605006$

T= 100 years

For large sample size

$K_T = -\left[0.45 + 0.78 ln\left(ln\frac{T}{T-1}\right)\right]$

$K_{50} = -\left[0.45 + 0.78 ln\left(ln\frac{50}{50-1}\right)\right]$= 2.59

$K_{100} = -\left[0.45 + 0.78 ln\left(ln\frac{100}{100-1}\right)\right]$=3.138

$K_{500} = -\left[0.45 + 0.78 ln\left(ln\frac{500}{500-1}\right)\right]$=4.396

(Alternative formula for K_T

$\overline{y_n}$ = 0.577 and S_n = 1.2825 (for large sample size)

$y_T = -ln\left(ln\frac{T}{T-1}\right)$

$K_T = \frac{y_T - \overline{y_n}}{S_n}$. This also gives similar value as above for K50, K100 and K500.)

$Q_T = \bar{Q} + K_T\sigma$

$Q_{50} = \bar{Q} + K_{50}\sigma$

$4200 = 3500 + 2.59\sigma$

σ=270.27

$Q_{100} = \bar{Q} + K_{100}\sigma$ = 3500+3.138x270.27 = 4348 m^3/s

$Q_{500} = \bar{Q} + K_{500}\sigma$ = 3500+4.396x270.27 = 4688 m^3/s

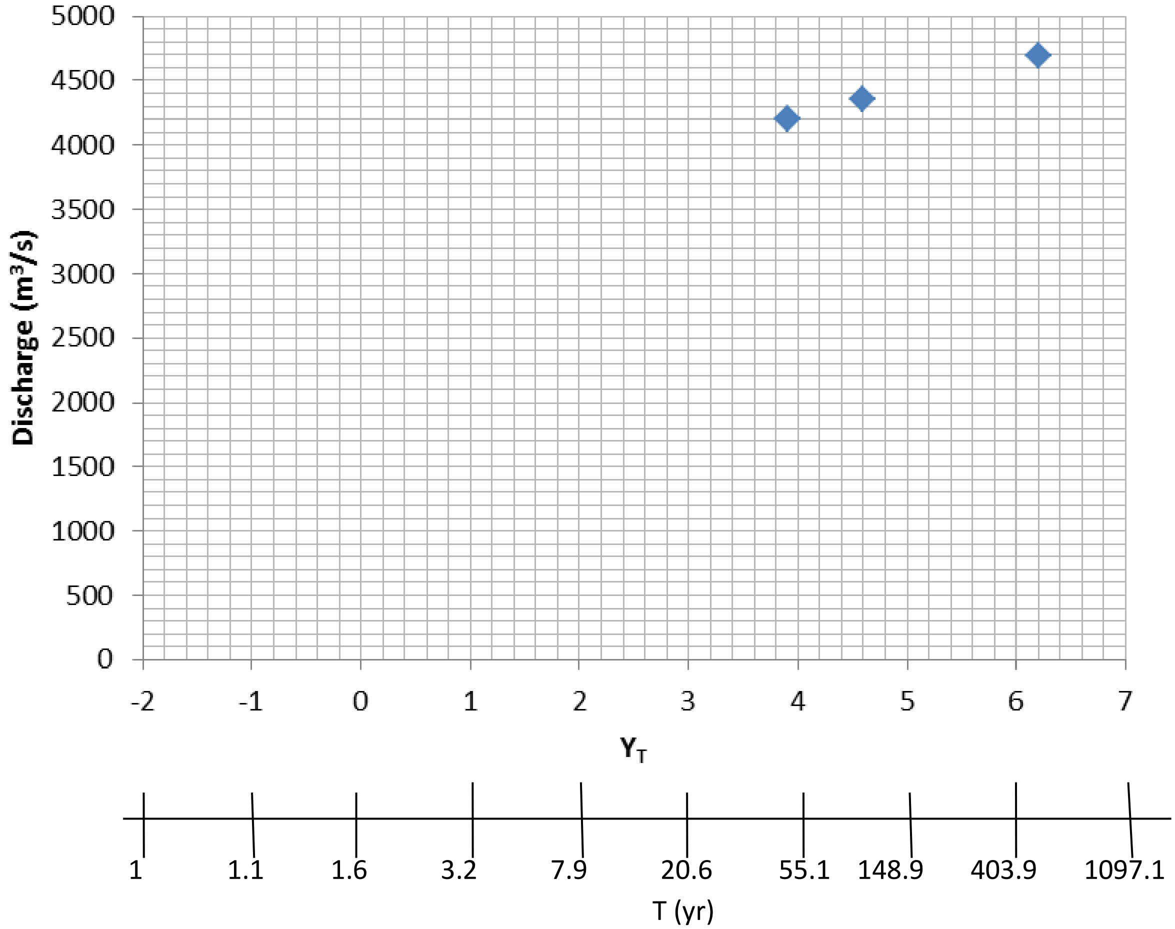
Plotting

The relationship between discharge (x) and the reduced variate (y_T) is linear. Therefore, to prepare Gumbel's extreme value probability paper, discharge is plotted on the Y-axis and the reduced variate is plotted on the X-axis in linear scale. To show the return period on the X-axis, for different values of Y_T, return period T is computed as shown in table below.

Y_T	-2	-1	0	1	2	3	4	5	6	7
T	1	1.1	1.6	3.2	7.9	20.6	55.1	148.9	403.9	1097.1

For corresponding value of Y_T, return period (T) is marked on the X-axis below Y_T.

T	YT
50	3.9
100	4.6
500	6.2



Gumbel plot

20. An analysis of an annual flood series covering the period 1900 to 2000 (100 years) on a certain river shows that the 100 year flood has a magnitude of 640,000 m^3/s and 2 year has a magnitude of 225,000 m^3/s. assume the annual floods fit with Gumbel distribution.

a) What is the probability of having a flood as great as or greater than 440,000 m^3/s?

b) What is the magnitude of the flood having recurrence interval of 50 years?

c) What is the probability of having 575,000 m^3/s flood or a greater flood in coming 25 years time?

d) Find the mean and standard deviation of the annual floods and occurrence interval of the mean flood.

Solution:

100 year flood (X_{100}) = 640,000 m^3/s

2 year flood (X_2) = 225,000 m^3/s

Formula for Gumbel distribution

$\alpha = 0.78S$ and $u = \bar{x} - 0.45S$ where α and u are Parameters of Gumbel distribution, $\bar{x}$ and S are mean and standard deviation of flood series.

Reduced variate y is given by

$$y = \frac{X - u}{\alpha}$$

$y_T = \frac{X - \bar{x} + 0.45S}{0.78S}$ (a)

Reduced variate is also related to return period T as

$y = -ln\left(ln\frac{T}{T-1}\right)$ (b)

Using equation a and b for 100 year flood

$y_{100} = \frac{X_{100} - \bar{x} + 0.45S}{0.78S} = \frac{640000 - \bar{x} + 0.45S}{0.78S}$ (c)

$y_{100} = -ln\left(ln\frac{T}{T-1}\right) = -ln\left(ln\frac{100}{100-1}\right)$=4.6 (d)

Equating c and d

$$4.6 = \frac{640000 - \bar{x} + 0.45S}{0.78S}$$

$\bar{x} + 3.138S = 640000$ (e)

Similarly using equation a and b for 2 year flood

$y_2 = \frac{X_2 - \bar{x} + 0.45S}{0.78S} = \frac{225000 - \bar{x} + 0.45S}{0.78S}$ (f)

$y_2 = -ln\left(ln\frac{T}{T-1}\right) = -ln\left(ln\frac{2}{2-1}\right)$= 0.3665 (g)

Equating f and g

$$0.3665 = \frac{225000 - \bar{x} + 0.45S}{0.78S}$$

$\bar{x} - 0.164S = 225000$ (h)

Solving e and h

S = 125681 and $\bar{x}$ = 245613

a) $P(X \geq 440000)$= ?

For X = 440000, reduced variate y is

$$y = \frac{X-\bar{x}+0.45S}{0.78S} = \frac{44000-245613+0.45x125681}{0.78x125681} = 2.56$$

$$P(X \geq 440000) = 1 - exp\big(-exp(-y)\big) = 1 - exp\big(-exp(-2.56)\big) = 0.074$$

b) Magnitude of flood with 50 year return period (X_{50}) =?

Using equation a and b for 50 year flood

$$y_{50} = \frac{X_{50}-\bar{x}+0.45S}{0.78S} = \frac{X_{50}-245613+0.45x15681}{0.78x125681} \qquad \text{(i)}$$

$$y_{50} = -ln\left(ln\frac{T}{T-1}\right) = -ln\left(ln\frac{50}{50-1}\right) = 3.9 \qquad \text{(j)}$$

Equating I and j

$$3.9 = \frac{X_{50} - 245613 + 0.45x15681}{0.78x125681}$$

X_{50} = 571318 m^3/s

c) Probability of having 575,000 m^3/s flood or a greater flood in coming 25 years time =?

X = 575,000 m^3/s , n = 25 years

Using equation a

$$y = \frac{X-\bar{x}+0.45S}{0.78S} = \frac{575000-245613+0.45x15681}{0.78x125681} = 3.937$$

P = 1- exp(-exp(-y) = 1-exp(-exp(-3.937) = 0.019

Probability of having 575,000 m^3/s flood or a greater flood in coming 25 year's time $=1-(1-P)^n$

$= 1-(1-0.019)^{25} = 0.38$

d) As obtained above mean = 245613 and standard deviation = 125681

Occurrence interval of mean flood (T) =?

For mean flood

$$y = \frac{X-\bar{x}+0.45S}{0.78S} = \frac{\bar{x}-\bar{x}+0.45S}{0.78S} = 0.58$$

P = 1- exp(-exp(-y) = 1-exp(-exp(-0.58) = 0.428

Occurrence interval of mean flood (T) = 1/P = 1/0.428 = 2.33 years

Peak flow estimation

1. A catchment of area 120ha has a time of concentration of 30min and runoff coefficient of 0.3. If a storm of duration 45min results in 3cm of rain over the catchment, estimate the resulting peak flow rate.

Solution:

Catchment area (A) = 120ha = $\frac{120x10^4}{10^6}$ km^2 = 1.2 km^2

Runoff coefficient (C) = 0.3

Time of concentration (t_c) = 30 min

Rainfall in 45 min = 3cm = 30mm

Rainfall in 30min (R) = $\frac{30}{45}x30$ = 20mm

Rainfall intensity (i) at t_c = $\frac{20}{30}x60$ = 40mm/hr

Peak flow (Q_p) = ?

From rational formula,

$Q_p = \frac{CiA}{3.6}$

$= \frac{0.3x40x1.2}{3.6}$ = 4m^3/s

3. Information on the 50-year storm is given below.

Duration (min)	15	30	45	60	180
Rainfall (mm)	40	60	75	100	120

A culvert has to drain 200ha of land with a maximum length of travel of 1.25km. The general slope of the catchment is 0.001 and its runoff coefficient is 0.2. Estimate the peak flow by rational method for designing the culvert for a 50-year flood.

Solution:

Catchment area (A) = 200ha = $\frac{200x10^4}{10^6}$ km^2 = 2 km^2

Runoff coefficient (C) = 0.2

Maximum length of travel (L) = 1.25km = 1250m

Slope of catchment (S) = 0.001

Peak flow (Q_p) = ?

From Kirpich equation,

Time of concentration $(t_c) = 0.01947L^{0.77}S^{-0.385}$

$= 0.01947x(1250)^{0.77}(0.001)^{-0.385}$ = 67.45min

Finding rainfall at 67.45min from given data,

Rainfall = $100 + \frac{120-100}{180-60} x7.45$ = 101.24mm

Rainfall intensity (i) at t_c = $\frac{101.24}{67.45} x60$ = 90mm/hr

Peak flow (Q_p) = ?

From rational formula,

$Q_p = \frac{CiA}{3.6}$

$= \frac{0.2x90x2}{3.6}$ = 10m^3/s

6. A catchment area has a time of concentration of 20 minutes and an area of 20ha. Estimate the peak discharge corresponding to a return period of 25 years. Assume a runoff coefficient of 0.25. The intensity-duration-frequency for the storm in the area can be expressed by $i = \frac{KT^x}{(D+a)^n}$ where i = intensity in cm/h and D = Duration of storm in hours, with coefficients K = 6.93, x = 0.189, a = 0.5, n = 0.878.

Solution:

Catchment area (A) = 20ha = $\frac{20x10^4}{10^6}$ km^2 = 0.2 km^2

Time of concentration (t_c) = D = 20min

Return period (T)= 25 years

Runoff coefficient (C)= 0.25

Constants: K = 6.93, x = 0.189, a = 0.5, n = 0.878

Peak discharge (Q_p) = ?

Rainfall intensity is

$i = \frac{KT^x}{(D+a)^n} = \frac{6.93x25^{0.189}}{((20/60)+0.5)^{0.878}}$ =14.943 cm/h = 149.43mm/h

From rational formula,

$Q_p = \frac{CiA}{3.6}$

$= \frac{0.25x149.43x0.2}{3.6}$ = 2.08m^3/s

Flow routing

1. The storage, elevation and outflow data of a reservoir are given below:

Elevation (m)	299.5	300.2	300.7	301.2	301.7	302.2	302.7
Storage (Mm^3)	4.8	5.5	6	6.6	7.2	7.9	8.8
Outflow (m^3/s)	0	0	15	40	75	115	160

The following flood hydrograph is expected into the reservoir.

time (h)	0	3	6	9	12	15	18	21	24	27
Discharge (m^3/s)	10	20	52	60	53	43	32	22	16	10

If the reservoir surface is at elevation 300.2m at the commencement of the flow, route the flood to obtain outflow hydrograph. Also determine the attenuation in the peak flow and lag in the peak flow time. (0, 2, 13, 30, 39, 43, 40, 34, 30, 19 m^3/s, attenuation = 17m^3/s, lag = 6h)

Solution:

Using Modified Pul's method, the equation is

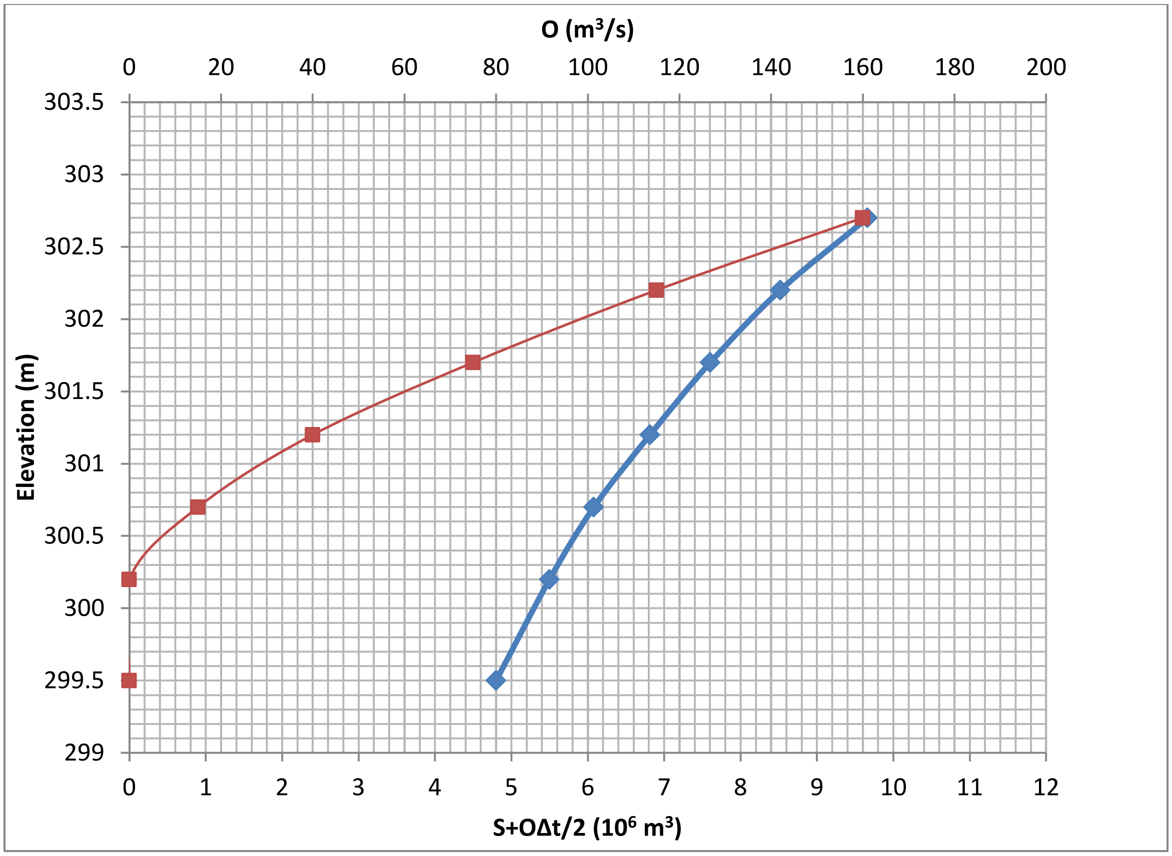
$$\left(\frac{I_1 + I_2}{2}\right)\Delta t + \left(S_1 - \frac{O_1 \Delta t}{2}\right) = \left(S_2 + \frac{O_2 \Delta t}{2}\right)$$

Δt = 3hr =3x60x60 = 10800s = 0.0108 Ms

Calculation of $S + \frac{Q\Delta t}{2}$

Elevation (m)	299.5	300.2	300.7	301.2	301.7	302.2	302.7
S(10^6xm^3)	4.8	5.5	6	6.6	7.2	7.9	8.8
Outflow, O(m^3/s)	0	0	15	40	75	115	160
$S + \frac{O\Delta t}{2}$(10^6xm^3)	4.8	5.5	6.081	6.816	7.605	8.521	9.664

Plot a graph between O vs. elevation and $S + \frac{O\Delta t}{2}$ vs. elevation.



In the above fig.

Blue: $S + \frac{O\Delta t}{2}$ vs. elevation curve

Red: O vs. elevation curve

The calculation in tabular form

time (h)	I (m^3/s)	$\left(\frac{I_1 + I_2}{2}\right)\Delta t$	$\left(S_1 - \frac{O_1\Delta t}{2}\right)$	$\left(S_2 + \frac{O\Delta t}{2}\right)$	Elevation (m)	Outflow, O_2 (m^3/s)
0	10				300.2	0
		0.162	5.5	5.662		
3	20				300.3	2
		0.3888	5.6404	6.0292		
6	52				300.65	13
		0.6048	5.8888	6.4936		
9	60				301	30
		0.6102	6.1696	6.7798		
12	53				301.15	39
		0.5184	6.3586	6.877		
15	43				301.25	43
		0.405	6.4126	6.8176		

18	32				301.2	40
		0.2916	6.3856	6.6772		
21	22				301.1	34
		0.2052	6.31	6.5152		
24	16				301	30
		0.1404	6.1912	6.3316		
27	10				300.85	19

Attenuation in peak = 60-43 = 17 m^3/s
Lag in peak = 15-9 = 6hr

Calculation procedure
At t = 0 hr, reservoir elevation = 300.2m, O = 0 m^3/s, S = 5.5 Mm^3
$\left(S_1 - \frac{O_1 \Delta t}{2}\right) = \left(5.5 - \frac{0x0.0108}{2}\right)$ = 5.5 Mm^3
$\left(\frac{I_1+I_2}{2}\right) \Delta t = \left(\frac{10+20}{2}\right) 0.0108$ = 0.162

$$\left(\frac{I_1 + I_2}{2}\right) \Delta t + \left(S_1 - \frac{O_1 \Delta t}{2}\right) = \left(S_2 + \frac{O_2 \Delta t}{2}\right)$$

$$0.162 + 5.5 = \left(S_2 + \frac{O_2 \Delta t}{2}\right)$$

$\left(S_2 + \frac{O_2 \Delta t}{2}\right)$ = 5.662Mm^3
From the curve
For $\left(S_2 + \frac{O_2 \Delta t}{2}\right)$ = 5.662Mm^3, elev = 300.3m
For 300.3m elv., O = 2 m^3/s
For the next step,
$\left(S_1 - \frac{O_1 \Delta t}{2}\right) = \left(S_2 + \frac{O_2 \Delta t}{2}\right) of\ previous\ step - O_2 \Delta t$ = 5.662-2x0.0108 = 5.6404 Mm^3
Repeat above procedure for other time steps.

4. A drainage basin has area = 110 km^2, Storage constant, K = 12h, and time of concentration = 7 h. The following isochrones area distribution data are available

Time (hr)	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18
Inter-isochrone area (km^2)	3	9	20	22	16	18	10	8	4

Determine the IUH of this catchment.

Storage constant (K) = 12h
Time interval (Δt) = 2h
Basin area = 110 km^2
Constants
$C_1 = \frac{0.5\Delta t}{K+0.5\Delta t} = \frac{0.5x2}{12+0.5x2}$ = 0.077

$C_2 = \frac{K-0.5\Delta t}{K+0.5\Delta t} = \frac{12-0.5x2}{12+0.5x2}$ = 0.846

Formulae

Inflow $(I) = 2.78\frac{A_r}{\Delta t}$ m^3/s where A_r = inter-isochrones area

$O_2 = 2C_1I_t + C_2O_{t-1} = 2x0.077I_t + 0.846O_{t-1} = 0.154I_t + 0.846O_{t-1}$

time (h)	A_r (km^2)	I (m^3/s)	O (m^3/s) = IUH
0	0	0	0
2	3	4.17	0.64
4	9	12.51	2.47
6	20	27.8	6.37
8	22	30.58	10.10
10	16	22.24	11.97
12	18	25.02	13.98
14	10	13.9	13.97
16	8	11.12	13.53
18	4	5.56	12.30
20			10.41
22			8.80
			so on

Civinnovate

Discover, Learn, and Innovate in Civil Engineering