



Civinnovate

Discover, Learn, and Innovate in Civil Engineering

UNIT - 1

STRESS, STRAIN AND DEFORMATION OF SOLIDS

PART A

1. Define tensile stress and tensile strain.

The stress induced in a body, when subjected to two equal and opposite pulls, as a result of which there is an increase in length, is known as tensile stress. The ratio of increase in length to the original length is known as tensile strain.

2. Define compressive stress and compressive strain.

The stress induced in a body, when subjected to two equal and opposite pushes, as a result of which there is a decrease in length, is known as compressive stress. The ratio of increase in length to the original length is known as compressive strain.

3. Define shear stress and shear strain.

The stress induced in a body, when subjected to two equal and opposite forces, which are acting tangentially across the resisting section as a result of which the body tends to shear off across the section is known as shear stress and corresponding strain is known as shear strain.

4. Define Poisson's ratio (May 2009)

The ratio of lateral strain to the linear strain is a constant for a given material, when the material is stressed within the elastic limit. This ratio is Poisson's ratio and it is generally Poisson's ratio

5. Write the relationship between modulus of elasticity, modulus of rigidity and Poisson's ratio, Bulk modulus (May 2010,12)

The relationship between modulus of elasticity, modulus of rigidity and Poisson's ratio is given by

$$E=2C(1+\frac{1}{m})$$

$$E=3K(1-\frac{2}{m})$$

E=Modulus of elasticity, C=Modulus of rigidity, K=Bulk modulus, 1/m=poisson's ratio

6. State Hooke's law. (May 2010,13)

Hooke's law is stated as when a material is loaded within elastic limit, the stress is proportional to the strain produced by stress, or Stress/strain=constant. This constant is termed as modulus of elasticity.

7. Define stress and strain.

Stress: The force of resistance per unit area, offered by a body against deformation is known as stress.

Strain: The ratio of change in dimension to the original dimension when subjected to an external load is termed as strain and is denoted by e . It has no unit.

8. Define factor of safety

It is defined as the ratio of ultimate stress to the working stress or permissible stress.

9. Define modulus of rigidity

The ratio of shear stress to the corresponding shear strain when the stress is within the elastic limit is known as modulus of rigidity or shear modulus and is denoted by C or G or N

10. Define modulus of elasticity. (May 2012)

The ratio of tensile stress or compressive stress to the corresponding strain is known as modulus of elasticity or young's modulus and is denoted by E .

11. What is the radius of mohr's circle?

Radius of mohr's circle is equal to the maximum shear stress.

12. Define principal stresses and principal plane.

Principal stresses:

The magnitudes of normal stress, acting on the principal planes are known as principal stresses.

Principal plane:

The planes which have no shear stress are known as principal plane.

UNIT-I

STRESS, STRAIN + DEFORMATION OF SOLIDS

PART B

1. Two vertical rods one of steel and other of copper are each rigidly fixed at the top and 600mm apart. Diameters and length of the rods are 25mm and 5m respectively. A cross bar fixed to the rods at the lower end carries a load of 7kN such that the cross bar remains horizontal even after loading. Find the steps in each rod and the position of the load on the cross bar. Assume the modulus of elasticity for steel and copper as 200 kN/mm^2 and 100 kN/mm^2 respectively.

Given data

Steel

$$D_s = 25 \text{ mm}$$

$$L_s = 5 \text{ m} = 5 \times 10^3 \text{ mm}$$

$$E_s = 200 \text{ kN/mm}^2$$

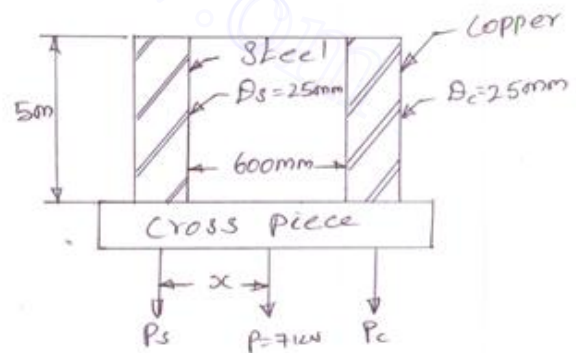
$$= 200 \times 10^3 \text{ N/mm}^2$$

Copper

$$D_c = 25 \text{ mm}$$

$$L_c = 5 \text{ m} = 5 \times 10^3 \text{ mm}$$

$$E_c = 100 \text{ kN/mm}^2 = 100 \times 10^3 \text{ N/mm}^2$$



To find

Load (P)

Solution

The load is placed in such a manner that the cross piece remains horizontal

$$e_s = e_c$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \frac{E_s}{E_c} \times \sigma_c$$

$$= \frac{200 \times 10^3}{100 \times 10^3} \times \sigma_c$$

$$\sigma_s = 2\sigma_c$$

P_s & P_c be the loads shared by the steel and copper respectively.

$$\therefore P = P_s + P_c$$

$$[P = \sigma \times A]$$

$$= \sigma_s A_s + \sigma_c A_c$$

$$= \sigma_s \times \pi/4 \times (D_s)^2 + \sigma_c \times \pi/4 \times (D_c)^2$$

$$= 2\sigma_c \times \pi/4 \times 25^2 + \sigma_c \times \pi/4 \times 25^2$$

$$7 \times 10^3 = 2\sigma_c \times 490.625 + \sigma_c \times 490.625$$

$$= 981.25\sigma_c + 490.625\sigma_c$$

$$= 1471.875\sigma_c$$

$$\sigma_c = 4.755 \text{ N/mm}^2$$

$$\sigma_s = 2 \times \sigma_c \quad ; \quad = 2 \times 4.755$$

$$\sigma_s = 9.51 \text{ N/mm}^2$$

$$P_c = \sigma_c \times A_c$$

$$= 4.755 \times \pi/4 \times 25^2$$

$$P_c = 2332.92 \text{ N}$$

Let 'x' be the distance from the steel rod where the load 'P' should be placed so that the cross piece remains horizontal after being loaded.

Take moment about steel rod,

$$P_c \times 600 = P \times x$$

$$2332.92 \times 600 = 7 \times 10^3 \times x$$

$$x = 199.96 \text{ mm.}$$

RESULT

$$\sigma_c = 4.755 \text{ N/mm}^2$$

$$\sigma_s = 9.51 \text{ N/mm}^2$$

$$P_c = 2332.92 \text{ N}$$

$$x = 199.96 \text{ mm.}$$

2. A steel tube of 30mm external diameter and 20mm internal diameter encloses a copper rod of 15mm diameter to which its rigidly joined at each end. If at a temperature of 60°C there is no longitudinal stresses, calculate the stresses in the rod and tube when the temperature is raised to 200°C. Take E for steel and copper as $2.1 \times 10^5 \text{ N/mm}^2$ and $1 \times 10^5 \text{ N/mm}^2$ respectively. The value of co-efficient

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If linear expansion for Steel and Copper is given as $11 \times 10^{-6} / ^\circ\text{C}$ and $18 \times 10^{-6} / ^\circ\text{C}$ respectively.

Given data

Copper rod

$$D_c = 15 \text{ mm}$$

$$A_c = \frac{\pi}{4} \times D_c^2 \\ = \frac{\pi}{4} \times 15^2 = 56.25\pi \text{ mm}^2$$

$$E_c = 1 \times 10^5 \text{ N/mm}^2$$

$$\alpha_c = 18 \times 10^{-6} / ^\circ\text{C}$$

Steel Tube

$$D_s = 30 \text{ mm}$$

$$d_s = 20 \text{ mm}$$

$$A_s = \frac{\pi}{4} (30^2 - 20^2) = 125\pi \text{ mm}^2$$

$$E_s = 2.1 \times 10^5 \text{ N/mm}^2$$

$$\alpha_s = 11 \times 10^{-6} / ^\circ\text{C}$$

Rise of Temperature $T = (200 - 10) = 190 ^\circ\text{C}$
To find

Stress

Solution

For equilibrium of the system

$$\sigma_c \cdot A_c = \sigma_s \cdot A_s$$

$$\sigma_c = \sigma_s \times \frac{A_s}{A_c} = \sigma_s \times \frac{125\pi}{56.25\pi} = 2.22 \times \sigma_s$$

$$\sigma_c = 2.22 \sigma_s$$

Actual expansion of Steel = free expansion of steel
+
Expansion due to tensile stress.

Expansion of Steel = Actual expansion of Copper⁵

$$\alpha_s \times T \times L + \frac{\sigma_s}{E_s} \times L = \alpha_c \times T \times L - \frac{\sigma_c}{E_c} \times L$$

$$\alpha_s \times T + \frac{\sigma_s}{E_s} = \alpha_c \times T - \frac{\sigma_c}{E_c}$$

$$11 \times 10^{-6} \times 190 + \frac{\sigma_s}{2.1 \times 10^5} = 18 \times 10^{-6} \times 190 - \frac{2.22 \sigma_s}{1 \times 10^5}$$

$$\frac{\sigma_s}{2.1 \times 10^5} + \frac{2.22 \sigma_s}{1 \times 10^5} = 18 \times 10^{-6} \times 190 - 11 \times 10^{-6} \times 190$$

$$\frac{\sigma_s + 2.1 \times 2.22 \sigma_s}{2.1 \times 10^5} = 5 \times 10^{-6} \times 190$$

$$5.662 \sigma_s = 199.5$$

$$\sigma_s = \frac{199.5}{5.662} = 35.235 \text{ N/mm}^2$$

$$\sigma_c = 2.22 \times 35.235 = 78.22 \text{ N/mm}^2$$

Result

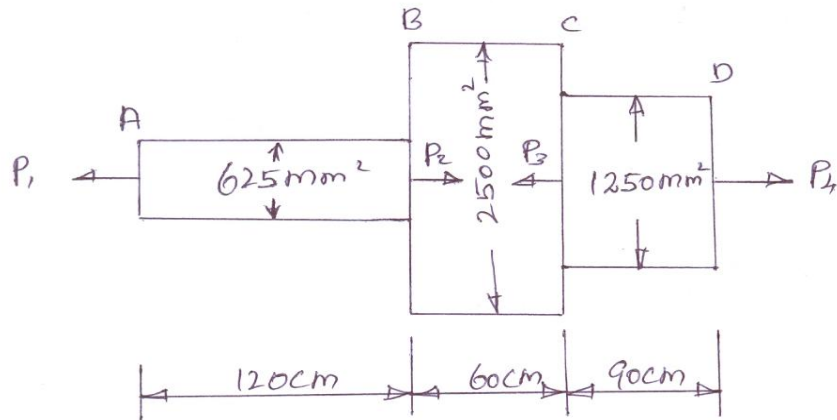
$$\sigma_s = 35.235 \text{ N/mm}^2$$

$$\sigma_c = 78.22 \text{ N/mm}^2$$

3. A member ABCD is subjected to point loads P_1, P_2, P_3 + P_4 as shown in fig. Calculate the force P_2 necessary for equilibrium, if $P_1 = 45 \text{ kN}$, $P_3 = 450 \text{ kN}$ and $P_4 = 130 \text{ kN}$. Determine the total

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elongation of the member, Assume the modulus of elasticity to be $2.1 \times 10^5 \text{ N/mm}^2$.



Given data

Part AB $A_1 = 625 \text{ mm}^2$

$L_1 = 120 \text{ cm} = 1200 \text{ mm}$

Part BC $A_2 = 2500 \text{ mm}^2$

$L_2 = 60 \text{ cm} = 600 \text{ mm}$

Part CD $A_3 = 1250 \text{ mm}^2$

$L_3 = 90 \text{ cm} = 900 \text{ mm}$

$E = 2.1 \times 10^5 \text{ N/mm}^2$

To find

force P_2

Total elongation

Solution.

Value of P_2 necessary for equilibrium

$$P_1 + P_3 = P_2 + P_4$$

Assume \rightarrow +ve

$$45 + 450 = P_2 + 130$$

$$P_2 = 495 - 130$$

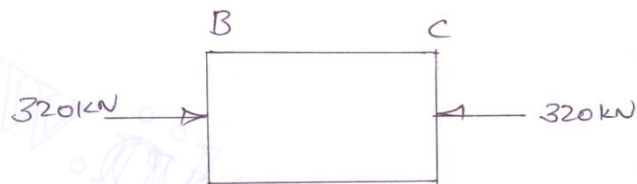
$$= 365 \text{ kN}$$



$$LHF = 45 \text{ kN}$$

$$RHF = 365 - 450 + 130$$

$$= 45 \text{ kN}$$

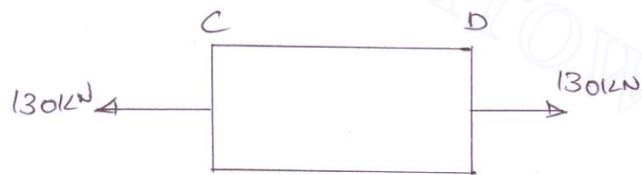


$$LHF = 45 - 365$$

$$= -320 \text{ kN}$$

$$RHF = -450 + 130$$

$$= -320 \text{ kN}$$



$$LHF = 45 - 365 + 450$$

$$= 130 \text{ kN}$$

$$RHF = 130 \text{ kN}$$

$$\text{Change in length } AB = \frac{P}{A_1 E} \times L_1$$

$$= \frac{45 \times 10^3}{625 \times 2.1 \times 10^5} \times 1200$$

$$= 0.4114 \text{ mm. (Tensile)}$$

$$\text{Change in length } BC = \frac{P}{A_2 E} \times L_2 = \frac{-320 \times 10^3}{2500 \times 2.1 \times 10^5} \times 600$$

$$= 0.3657 \text{ (compressive)}$$

$$\text{Change in length } \Delta P = \frac{P}{A_3 E} \times L_3 = \frac{130 \times 10^3}{1250 \times 2 \times 10^5} \times 900$$

$$= 0.4457 \text{ mm}$$

$$\text{Total change in length} = 0.4114 - 0.3657 + 0.4457$$

$$= 0.4914 \text{ mm}$$

RESULT

$$P_2 = 365 \text{ kN}$$

$$\text{Total elongation} = 0.4914 \text{ mm}$$

4. A cast iron flat 300mm long and 30mm (thickness) \times 60mm (width) uniform cross section, is acted upon by the following forces:

30kN Tensile in the direction of the length

360 kN Compression in the direction of the width

240 kN Tensile in the direction of the thickness

Calculate the direct strain, net strain in each direction and change in volume of the flat. Assume the modulus of elasticity and poisson's ratio for cast iron as 110 kN/mm^2 and 0.25 respectively.

Given data

$$\text{Load in x direction} = 30 \text{ kN}$$

$$= 30 \times 10^3 \text{ N [Tensile]}$$

$$\text{Load in y direction} = -360 \text{ kN}$$

$$= 360 \times 10^3 \text{ N [Compression]}$$

Load in z direction = 240 kN

$$E = 140 \text{ kN/mm}^2$$

$$= 140 \times 10^3 \text{ N/mm}^2$$

$$\text{Poisson's ratio } (\mu) = 0.25$$

To find

Direct strain

Net strain

Change in volume

Soln

$$\begin{aligned}\sigma_x &= \frac{\text{Load in x direction}}{y \times z} \\ &= \frac{30 \times 10^3}{60 \times 30} = 16.67 \text{ N/mm}^2\end{aligned}$$

$$\begin{aligned}\sigma_y &= \frac{\text{load in y direction}}{z \times x} \\ &= \frac{-360 \times 10^3}{30 \times 300} = -40 \text{ N/mm}^2\end{aligned}$$

$$\begin{aligned}\sigma_z &= \frac{\text{Load in z direction}}{x \times y} \\ &= \frac{240 \times 10^3}{300 \times 60} = 13.33 \text{ N/mm}^2\end{aligned}$$

The strain along x direction

$$e_x = \frac{1}{E} \left[\sigma_x - \mu (\sigma_y + \sigma_z) \right]$$

$$= \frac{1}{140 \times 10^3} [16.67 - 0.25(-40 + 13.3)]$$

$$e_x = 1.667 \times 10^{-4}$$

$$e_y = \frac{1}{E} [\sigma_y - \frac{1}{m} (\sigma_z + \sigma_x)]$$

$$= \frac{1}{140 \times 10^3} [-40 - (0.25)(13.33 + 16.67)]$$

$$\sigma_y = -3.39 \times 10^{-4}$$

$$e_z = \frac{1}{E} [\sigma_z - \frac{1}{m} (\sigma_x + \sigma_y)]$$

$$= \frac{1}{140 \times 10^3} [13.33 - 0.25(16.67 - 40)]$$

$$= 1.368 \times 10^{-4}$$

Volumetric strain

$$e_v = e_x + e_y + e_z$$

$$= 1.667 \times 10^{-4} - 3.39 \times 10^{-4} + 1.368 \times 10^{-4}$$

$$e_v = 0.355 \times 10^{-4} \text{ [compressive]}$$

Change in volume

$$\delta v = e_v \times v$$

$$= -0.355 \times 10^{-4} \times x \times y \times z$$

$$= -0.355 \times 10^{-4} \times 300 \times 60 \times 30$$

$$\delta v = -19.17 \text{ mm}^3$$

RESULT

$$e_x = 1.667 \times 10^{-4}$$

$$e_y = -3.39 \times 10^{-4}$$

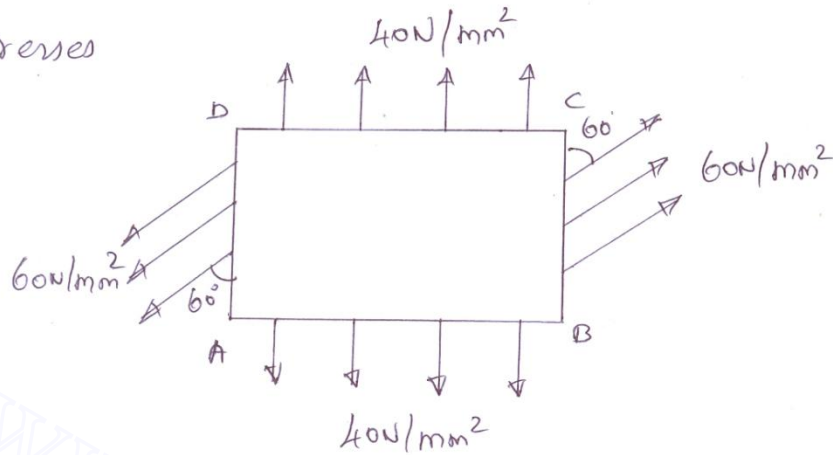
$$e_z = 1.368 \times 10^{-4}$$

$$e_v = -0.355 \times 10^{-4}$$

$$\delta v = -19.17 \text{ mm}^3$$

5.

A point in a strained material is subjected to the stresses as shown in fig. Locate the principal planes, and evaluate the principal stresses



Given data

The stresses BC (or) AD normal stress inclined to 60°
 Stresses along the BC + AD shear stress

To find

Locate principal plane
 Principal stresses

Soln

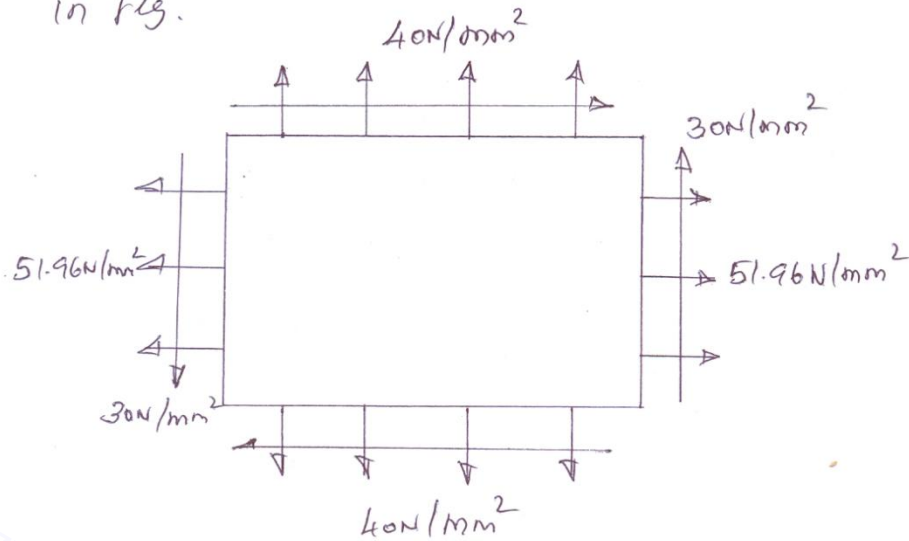
Stress normal to the face BC or AD

$$= 60 \times \sin 60^\circ = 60 \times 0.866 = 51.96 \text{ N/mm}^2$$

Stresses along the face BC or AD

$$= 60 \times \cos 60^\circ = 60 \times 0.5 = 30 \text{ N/mm}^2$$

The stresses acting on the material are shown in fig. 12



major tensile stress $\sigma_1 = 51.96 \text{ N/mm}^2$

minor tensile stress $\sigma_2 = 40 \text{ N/mm}^2$

shear stress $\tau = 30 \text{ N/mm}^2$

Location of principal planes

$\theta =$ Angle, which one of the principal planes make with the stress 40 N/mm^2 .

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} = \frac{2 \times 30}{51.96 - 40} = 4.999$$

$$\theta = 39^\circ 21'$$

The major principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$\begin{aligned}
 &= \frac{51.96 + 40}{2} + \sqrt{\left(\frac{51.96 - 40}{2}\right)^2 + 30^2} \\
 &= 45.98 + 30.6 \\
 &= 76.58 \text{ N/mm}^2
 \end{aligned}$$

Minor principal stresses

$$\begin{aligned}
 &= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2} \\
 &= \frac{51.96 + 40}{2} - \sqrt{\left(\frac{51.96 - 40}{2}\right)^2 + 30^2} \\
 &= 45.98 - 30.6 \\
 &= 15.38 \text{ N/mm}^2
 \end{aligned}$$

RESULT

Major principal stress = 76.58 N/mm^2

Minor principal stress = 15.38 N/mm^2

PART C

6. A bar of 30mm diameter is subjected to a pull of 60kN. The measured extension on gauge length of 200mm is 0.09mm and the change in diameter is 0.0039. Calculate the Poisson's ratio + the volume of the three moduli.

Given data

$$d = 30 \text{ mm}$$

$$P = 60 \text{ kN} = 60 \times 10^3 \text{ N}$$

$$L = 200 \text{ mm}$$

$$\delta L = 0.09 \text{ mm}$$

$$\delta d = 0.0039 \text{ mm}$$

To find

1. Poisson's ratio μ_m
2. Young's modulus E
3. Bulk modulus K
4. modulus of rigidity G

Solution

$$1. \text{ Poisson's ratio } \mu_m = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\mu_m = \frac{e_t}{e_l}$$

$$\begin{aligned} \text{Lateral strain } e_t &= \frac{\delta b}{b} \text{ (or) } \frac{\delta d}{d} \text{ (or) } \frac{\delta L}{L} \\ &= \frac{\delta d}{d} = \frac{0.0039}{30} \\ &= 1.3 \times 10^{-4} \end{aligned}$$

$$\begin{aligned} \text{Longitudinal strain } e_l &= \frac{\delta L}{L} = \frac{0.09}{200} \\ &= 4.5 \times 10^{-4} \end{aligned}$$

$$\text{Poisson ratio } \mu_m = \frac{1.3 \times 10^{-4}}{4.5 \times 10^{-4}} = 0.28$$

$$\mu_m = 0.28$$

$$2. \text{ Young's modulus } E = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{\sigma}{e_l}$$

$$= \frac{P}{A e_l} \quad \left\{ \sigma = \frac{P}{A} \right.$$

$$= \frac{60 \times 10^3}{\pi/4 \times d^2 \times 4.5 \times 10^{-4}}$$

$$= \frac{60 \times 10^3}{\pi/4 (30)^2 \times 4.5 \times 10^{-4}}$$

$$E = 1.8 \times 10^5 \text{ N/mm}^2$$

$$3. \quad E = 2G (1 + \nu/m)$$

$$1.8 \times 10^5 = 2 \times G (1 + 0.28)$$

$$G = 7.0 \times 10^4 \text{ N/mm}^2$$

$$4. \quad E = 3K (1 - 2/m)$$

$$1.8 \times 10^5 = 3K [1 - 2(0.28)]$$

$$K = 1.36 \times 10^5 \text{ N/mm}^2$$

RESULT

1. Poisson's ratio (ν/m) = 0.28
2. Young's modulus (E) = $1.8 \times 10^5 \text{ N/mm}^2$
3. Modulus of Rigidity (G) = $7 \times 10^4 \text{ N/mm}^2$
4. Bulk modulus (K) = $1.36 \times 10^5 \text{ N/mm}^2$

7. The normal stress at a point on two mutually perpendicular planes are 100mpa (tensile) and 50mpa (compressive). Determine the shear stresses on these planes if the maximum principal stress is limited to 150mpa (tensile). Determine the following.

- i) minimum principal stress
- ii) maximum shear stress and its plane
- iii) Normal, shear and resultant stresses on a plane which is inclined at 30° anticlockwise to x plane.

Given data

$$\sigma_1 = 140 \text{ mpa} = 140 \text{ N/mm}^2 \text{ [Tensile]}$$

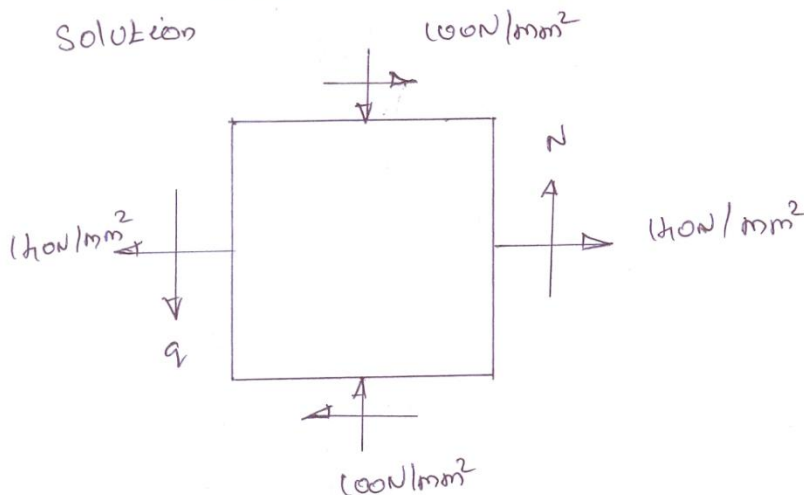
$$\sigma_2 = -100 \text{ mpa} = 100 \text{ N/mm}^2 \text{ [Compressive]}$$

$$\text{Major principal stress } \sigma_{n1} = 150 \text{ mpa} = 150 \text{ N/mm}^2$$

To find

- i) Shear stress q
- ii) minimum principal stress σ_{n2} + its position
- iii) Normal stresses (σ_n), Shear stress (σ_s) + σ_s which is inclined at 30° anticlockwise with x plane.
- iv) maximum shear stress + its plane.

Solution



$$\sigma_{n1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + 4q^2}$$

$$150 = \frac{140 - 100}{2} + \frac{1}{2} \sqrt{[140 - (-100)]^2 + 4q^2}$$

$$150 = \frac{40}{2} + \frac{1}{2} \sqrt{240^2 + 4q^2}$$

$$300 = 40 + \sqrt{240^2 + 4q^2}$$

$$260 = \sqrt{240^2 + 4q^2}$$

$$260^2 = 240^2 + 4q^2$$

$$4q^2 = 260^2 - 240^2$$

$$= 10,000$$

$$q^2 = 2500$$

$$q = 50 \text{ N/mm}^2$$

$$\sigma_{n2} = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + 4q^2}$$

$$= \frac{140 - 100}{2} - \frac{1}{2} \sqrt{(140 + 100)^2 + 4(50^2)}$$

$$= 20 - \frac{1}{2} \sqrt{240^2 + 10000}$$

$$\sigma_{n2} = -110 \text{ N/mm}^2$$

maximum shear stress (σ_{\pm})

$$\sigma_{\pm \text{ max}} = \frac{1}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + 4q^2}$$

$$= \frac{1}{2} \sqrt{(140 + 100)^2 + 4 \times 50^2}$$

$$\sigma_{\pm \text{ max}} = 130 \text{ N/mm}^2$$

Location of principal plane

$$\begin{aligned}\tan 2\theta &= \frac{2q}{\sigma_1 - \sigma_2} \\ &= \frac{2 \times 50}{140 - 100} = 0.416\end{aligned}$$

$$2\theta = \tan^{-1}(0.416) = 22.61^\circ$$

$$\theta = 11.30^\circ$$

maximum shear stress } = location of principal plane $+45^\circ$
plane

$$\begin{aligned}\theta &= 11.30^\circ + 45^\circ \\ &= 56.30^\circ\end{aligned}$$

To find σ_n, σ_\perp & σ_R at 30° inclined σ_n

$$\begin{aligned}\sigma_n &= \frac{\sigma_1 + \sigma_2}{2} + \left(\frac{\sigma_1 - \sigma_2}{2} \right) \cos 2\theta + q \sin 2\theta \\ &= \left(\frac{140 - 100}{2} \right) + \left(\frac{140 + 100}{2} \right) \cos 2(30) + 50 \sin 2(30) \\ &= 20 + 120 \times \cos 60 + 50 \times \sin 60\end{aligned}$$

$$(\sigma_n)_{30} = 123.30 \text{ N/mm}^2$$

$$\sigma_\perp = \left(\frac{\sigma_1 - \sigma_2}{2} \right) \sin 2\theta - q \cos 2\theta$$

$$\sigma_\perp = \left(\frac{140 - 100}{2} \right) \sin 60 - 50 \cos 60$$

$$(\sigma_\perp)_{30} = 78.92 \text{ N/mm}^2$$

$$\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$$

$$= \sqrt{123.30^2 + 78.92^2}$$

$$(\sigma_R)_{30^\circ} = 146.39 \text{ N/mm}^2$$

RESULT

$$q = 50 \text{ N/mm}^2$$

$$\sigma_{n2} = -110 \text{ N/mm}^2$$

$$(\sigma_t)_{\text{max}} = 130 \text{ N/mm}^2$$

$$(\sigma_n)_{30^\circ} = 123.30 \text{ N/mm}^2$$

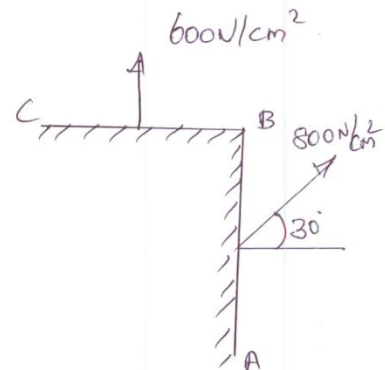
$$(\sigma_t)_{30^\circ} = 78.92 \text{ N/mm}^2$$

$$(\sigma_R)_{30^\circ} = 146.39 \text{ N/mm}^2$$

8. The intensity of resultant stress on a plane AB at a point in a material under stress is 800 N/cm^2 and it is inclined at 30° to the normal to the plane. The normal component of stress on another plane BC at right angle to plane AB is 600 N/cm^2 .

Determine the following

- (i) The resultant stress on the plane BC
- (ii) The principal stresses & their direction
- (iii) The maximum shear stresses and their planes.



Given data

Resultant stress on plane AB = 800 N/cm^2

Angle of inclination of the above } = 30°
 stress }

Normal stress on plane BC = 600 N/cm^2

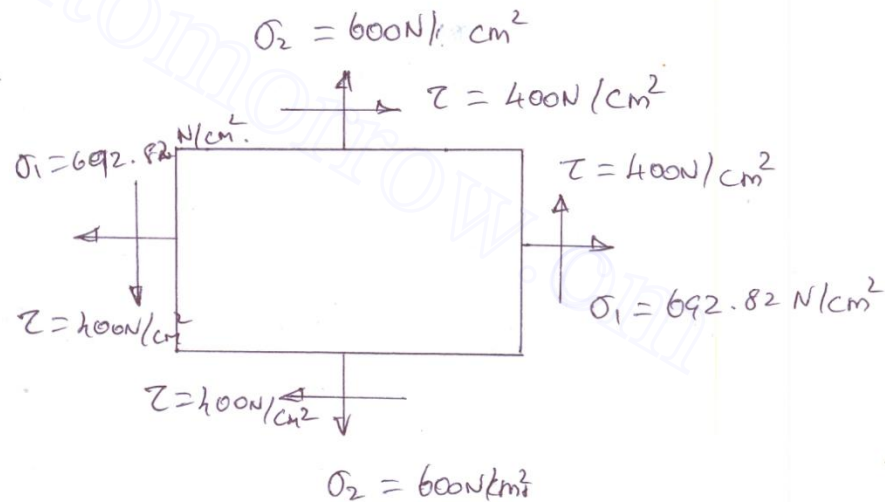
To find

The resultant stress (BC)

The principal stress

maximum shear stress.

Solution.



The normal stress on plane AB

$$= 800 \times \cos 30 = 692.82 \text{ N/cm}^2$$

The tangential stress on plane AB

$$= 800 \times \sin 30 = 400 \text{ N/cm}^2$$

(i) Resultant stress on plane BC

$$\sigma_2 = 600 \text{ N/cm}^2$$

$$\tau = 400 \text{ N/cm}^2$$

$$= \sqrt{\sigma_2^2 + \tau^2}$$

$$= \sqrt{600^2 + 400^2} = 721 \text{ N/cm}^2$$

$$\tan \theta = \frac{\sigma_2}{\tau} = \frac{600}{400} = 1.5$$

$$\theta = \tan^{-1}(1.5) = 56.3^\circ$$

(ii) major principle stress + their direction

$$\text{major stress} = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{692.82 + 600}{2} + \sqrt{\left(\frac{692.82 - 600}{2}\right)^2 + 400^2}$$

$$= 646.41 + 402.68$$

$$= 1049.09 \text{ N/cm}^2$$

minor principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{692.82 + 600}{2} - \sqrt{\left(\frac{692.82 - 600}{2}\right)^2 + 400^2}$$

$$= 646.41 - 402.68$$

$$= 243.73 \text{ N/cm}^2$$

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} = \frac{2 \times 400}{692.82 - 600} = \frac{800}{92.82} = 8.61$$

$$2\theta = \tan^{-1}(8.61) = 83.38^\circ$$

$$\theta = 41.69^\circ$$

(ii) The maximum shear stress \rightarrow these planes

$$\begin{aligned} (\sigma_{\pm})_{\max} &= \frac{1}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2} = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2} \\ &= \sqrt{\left(\frac{692.82 - 600}{2}\right)^2 + 400^2} \\ &= 402.68 \text{ N/cm}^2 \end{aligned}$$

UNIT-II

TRANSVERSE LOADING ON BEAMS AND STRESSES IN BEAM

PART A

1. What are the different types of beams? (Dec 2010,11, May 2012)

- i. **Cantilever beam:** A beam which is fixed at one end and at the other end is known as cantilever beam.
- ii. **Simply supported beam:** A beam supported or resting freely on the supports at its both end is known as simply supported beam
- iii. **Fixed beam:** A beam whose both end are fixed or built-in walls is known as fixed beam.
- iv. **Overhanging beam:** if the end portion of a beam is extended beyond the support is known as overhanging beam.
- v. **Continuous beam:** A beam which is having more than two supports

is known as continuous beam

2. Name the various types of load.

- concentrated load or point load
- Uniformly load
- Uniformly distributed load

3. Define shear force & bending moment at a section of a beam.

The algebraic sum of the vertical force at any section of a beam to the right or left of the section is known as shear force.

The algebraic sum of the moments of all the force acting to the right or left of the section is known as bending of the beam.

4. What is meant by point of contra flexure? (May 2008, 10, 13)

It is the point where the bending moment is zero where it change sign from positive to negative or vice –versa.

5. Mention the different types of supports? (May 2011)

- i. Fixed support
- ii. Hinged support
- iii. Roller support

6. What is section modulus?

The ratio of Moment of Inertia of a section about the neutral axis to the distance of the outer most layer from the neutral axis is known as Section Modulus. It is denoted by Z.

7. Write the bending equation?

$$E/R = M/I = f/Y$$

M = bending moment or f = bending stress

I = moment of inertia about N.A.

Y = distance of the fibre from N.A.

R = radius of curvature

E = young's modulus of beam

8. What are the assumptions made in the theory of simple bending? (May 2014)

The material of the beam is perfectly homogeneous and isotropic.

i. The beam material is stressed, within its elastic limit and thus obeys Hooke's law.

ii. The transverse sections, which were plane before bending, remain plane after bending also.

iii. Each layer of the beam is free to expand or contract, independently, of the layer, above or below it.

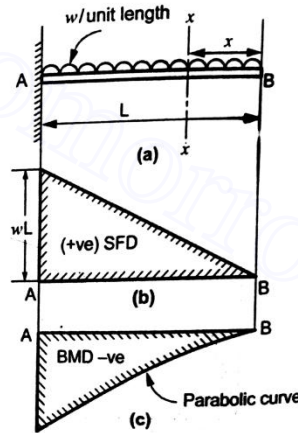
9. Define neutral axis of a cross section

The line of intersection of the neutral surface on a cross-section is called the neutral axis of a cross-section. There is no stress at the axis.

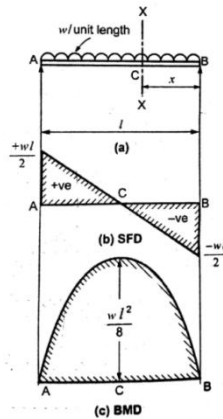
10. What is meant by negative or hogging BM?

BM is said to be negative if the moment on the left side of the beam is counter-clockwise or the right side of the beam is clockwise.

11. Draw the rough sketch of SF and BM for the beam given below.



12. Draw the SF and BM diagrams for the simply supported beam of length L subjected to UDL of w/m length throughout its length.

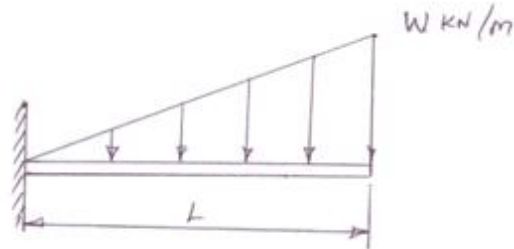


UNIT - II

TRANSVERSE LOADING ON BEAMS AND STRESSES IN BEAMS.

PART B

1. Draw Shear force and bending moment diagram for the cantilever beam as shown in fig.



Solution

$$\sum F_y = 0$$

$$\sum F_y = R_A - \frac{1}{2} WL$$

$$0 = R_A - \frac{1}{2} WL$$

$$R_A = \frac{WL}{2}$$

$$\sum M_A = 0$$

$$\sum M_A = M_A - (WL/2) \left(\frac{2L}{3} \right)$$

$$0 = M_A - \frac{WL^2}{3}$$

$$M_A = \frac{WL^2}{3}$$

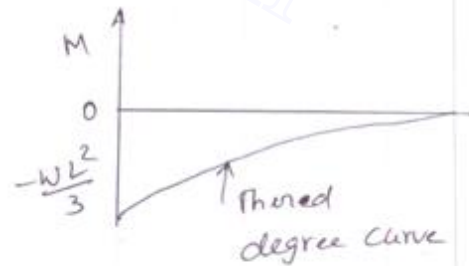
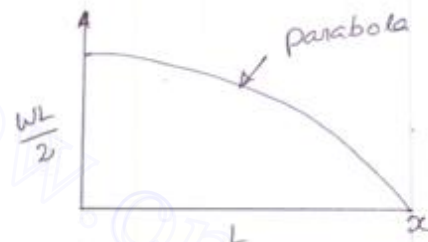
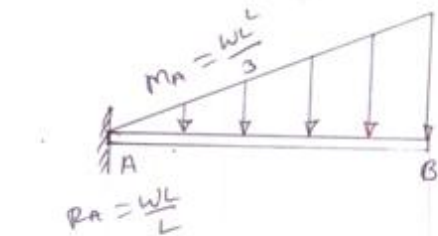
Shear force Diagram

$$V_A = 0$$

$$V_B = 0 + \frac{WL}{2} = \frac{WL}{2}$$

$$V_B = \frac{WL}{2} - \frac{1}{2} WL = 0.$$

The rate of downward loading increases from left to right, the SFD will be



Parabola with tangent rotating clockwise.

Bending moment

Take section at distance x , By similarity triangles, the height of triangle will be $\frac{wx}{L}$

$$m = -\frac{wL^2}{3} + \frac{wL}{2}x - \frac{1}{2}x\left(\frac{wx}{L}\right)\left(\frac{x}{3}\right)$$

$$= -\frac{wL^2}{3} + \frac{wL}{2}x - \frac{w}{6L}x^3$$

This is a third degree curve. Hence BMD will be a third degree curve.

$$x=0.$$

$$M_A = -\frac{wL^2}{3}$$

$$\text{At B, } x=L$$

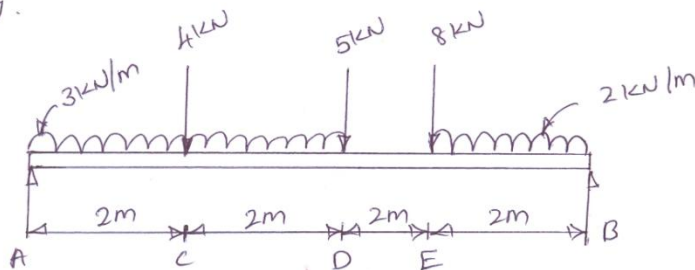
$$M_B = -\frac{wL^2}{3} + \frac{wL(L)}{2} - \frac{w(L^3)}{6L}$$

$$= -\frac{wL^2}{3} + \frac{wL^2}{2} - \frac{wL^2}{6}$$

$$= 0.$$

Shear force decrease from at OB, the BMD will be a third degree curve with tangent rotating clockwise from A to B.

2. Draw the shear force and bending moment diagram for the simply supported beam shown in fig.



Solution

$$\sum F_y = 0$$

$$R_A - 3 \times 4 - 4 - 5 - 8 - 2 \times 2 + R_B = 0.$$

$$R_A = 17 \text{ kN}$$

$$\sum M_A = 0.$$

$$-(3 \times 4) \times 2 - 4 \times 2 - 5 \times 4 - 8 \times 6 - 2 \times 2 \times 7 + R_B \times 8 = 0.$$

$$R_B = 16 \text{ kN}.$$

Shear force

$$V_{AL} = 0$$

$$V_{AR} = 0 + 17 = 17 \text{ kN}$$

$$V_{CL} = 17 - 3 \times 2 = 11 \text{ kN}$$

$$V_{CR} = 11 - 4 = 7 \text{ kN}$$

$$V_{DL} = 7 - 3 \times 2 = 1 \text{ kN}$$

$$V_{DR} = 1 - 5 = -4 \text{ kN} = V_{EL}$$

$$V_{ER} = -4 - 8 = -12 \text{ kN}$$

$$V_{BL} = -12 - 2 \times 2 = -16 \text{ kN}$$

Bending moment

$$M_A = 0$$

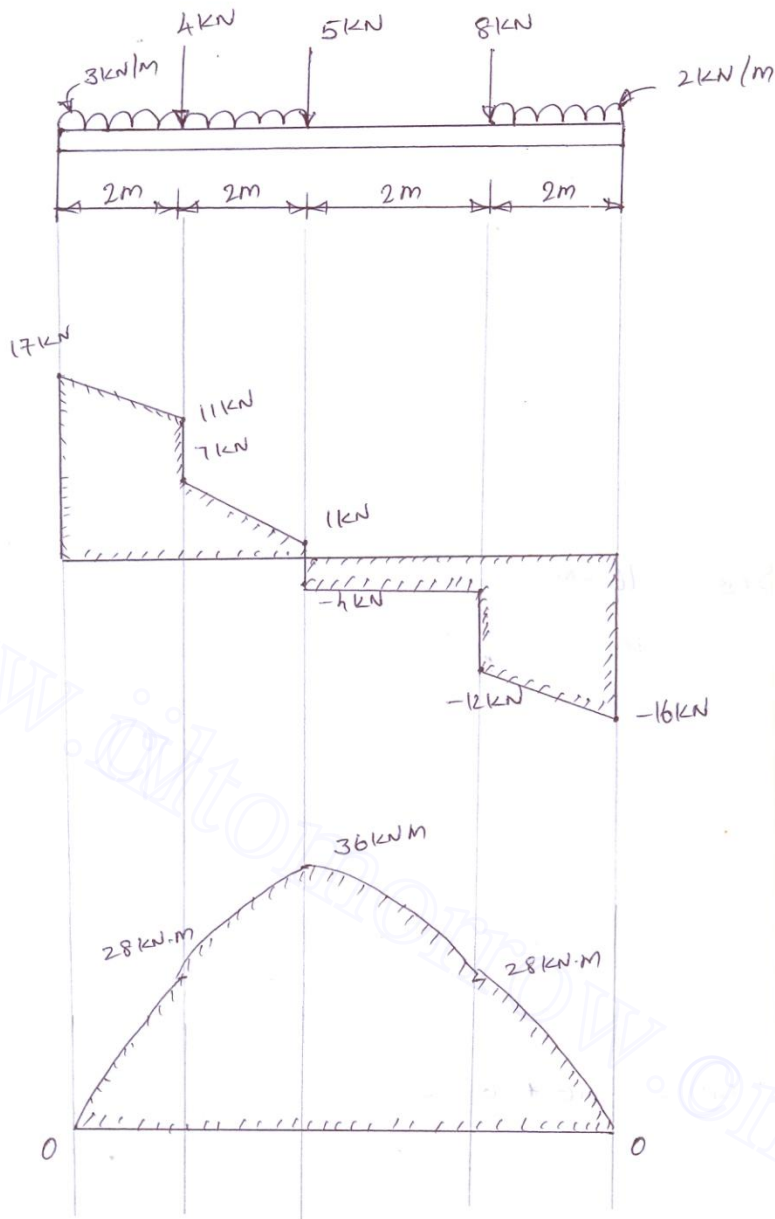
$$M_C = 0 + 17 \times 2 + \frac{1}{2} \times 2 \times 6 = 28 \text{ kN}\cdot\text{m}$$

$$M_D = 28 + 1 \times 2 + \frac{1}{2} \times 2 \times 6 = 36 \text{ kN}\cdot\text{m}$$

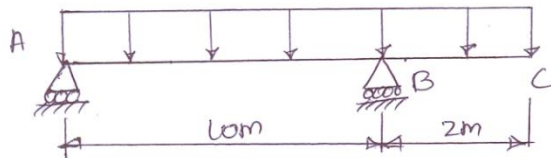
$$M_E = 36 - 4 \times 2 = 28 \text{ kN}\cdot\text{m}$$

$$M_B = 28 - 12 \times 2 - \frac{1}{2} \times 2 \times 4$$

$$= 0.$$



3. Draw the Shear force and Bending moment diagram for the beam shown in fig. Also determine the maximum bending moment and location of point of contra flexure.



Solution

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$$\sum M_A = 0.$$

$$R_B \times 10 = 100 \times 12 \times 12/2$$

$$R_B = 720 \text{ N}$$

$$\sum y = 0$$

$$R_A + R_B = 100 \times 12$$

$$= 1200$$

$$R_A = 1200 - 720$$

$$R_A = 480 \text{ N}$$

Shear force

$$SFA = R_A = 480 \text{ N}$$

$$SFB = 480 - 100 \times 10 = -520 \text{ N}$$

$$SFC = 480 - (100 \times 12) - 720 = 0.$$

The maximum bending moment is situated at a distance of x from the point A, where the SF changes its sign. SF equation at that point is

$$R_A - 100x = 0$$

$$480 - 100x = 0$$

$$x = 4.8 \text{ m.}$$

Bending moment

$$BMA = 0.$$

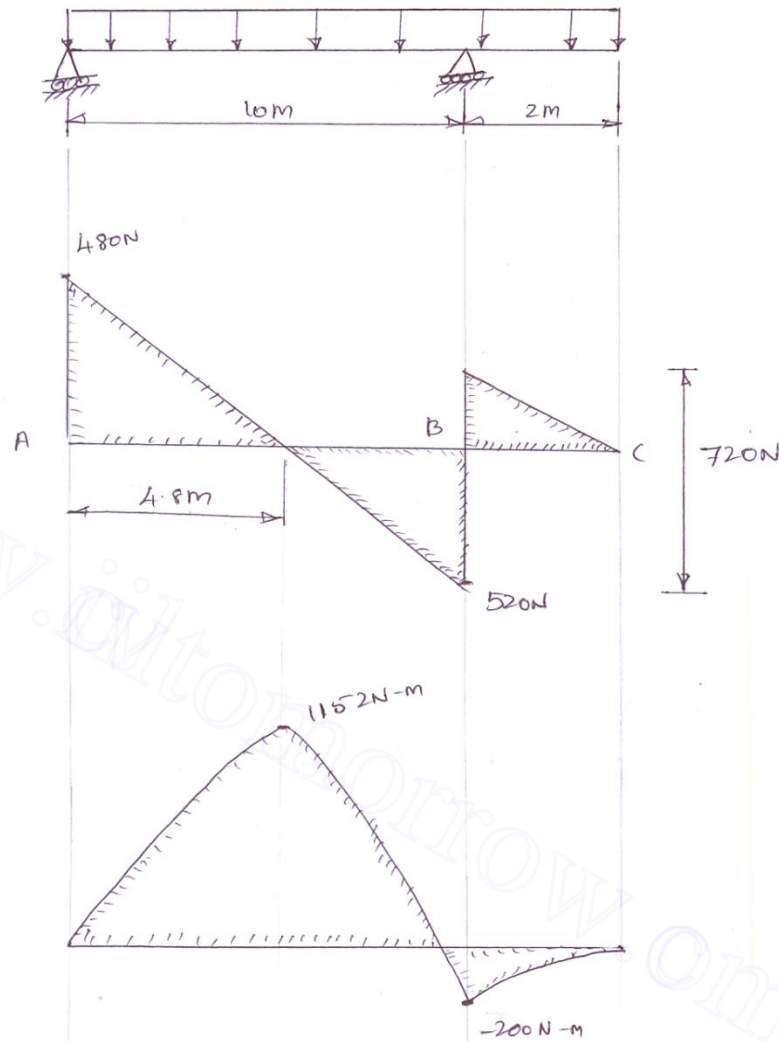
$$BMB = 480 \times 10 - \frac{100}{2} (10)^2 = -200 \text{ N-m}$$

$$BMC = 480 \times 12 - \frac{100}{2} 12^2 + 720 \times 2$$

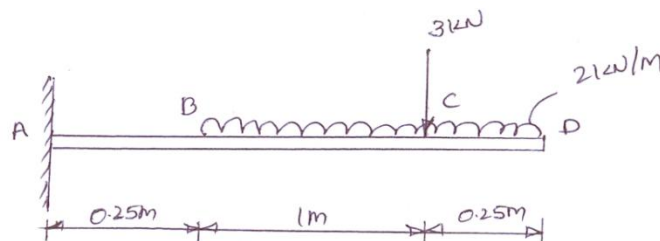
$$= 0.$$

$$Bm_x = 4.8 \text{ m} \times 480 - \frac{100}{2} (4.8)^2$$

$$= 115.2 \text{ N-m.}$$



4. A cantilever 1.5m long is loaded with a uniformly distributed load of 2 kN/m run over a length of 1.25 m from the free end. It also carries a point load of 3 kN at a distance of 0.25 m from the free end. Draw the shear force and bending moment diagram of the cantilever.



Shear force

$$S_{FD} = 0$$

$$S_{FC} = 2 \times 0.25 = 0.5 \text{ kN}$$

$$S_{FC} = (2 \times 0.25) + 3 = 3.5 \text{ kN}$$

$$S_{FB} = 2 \times 1.25 + 3 \text{ kN} = 5.5 \text{ kN}$$

$$S_{FA} = 5.5 \text{ kN}$$

Bending moment

$$BM_A = 0$$

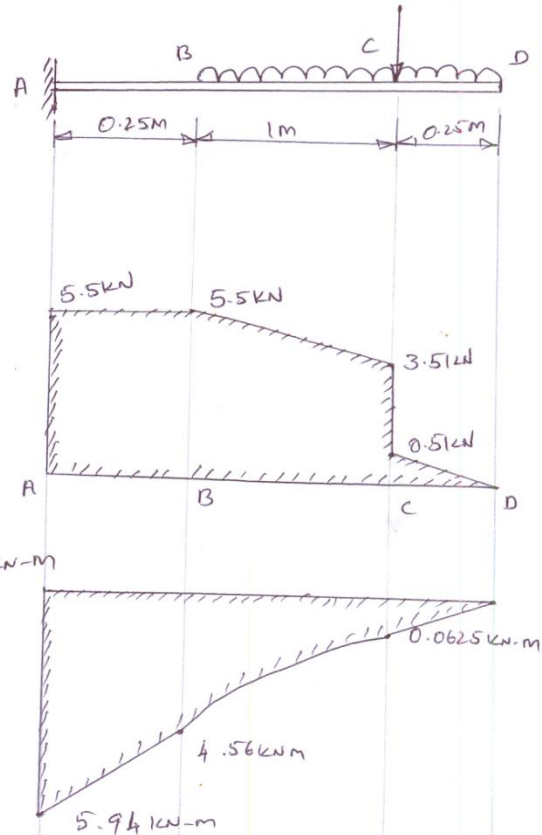
$$BM_C = -2 \times 0.25 \times \frac{0.25}{2} = -0.0625 \text{ kN-m}$$

$$BM_B = -\left(2 \times 1.25 \times \frac{1.25}{2}\right) - 3 \times 1$$

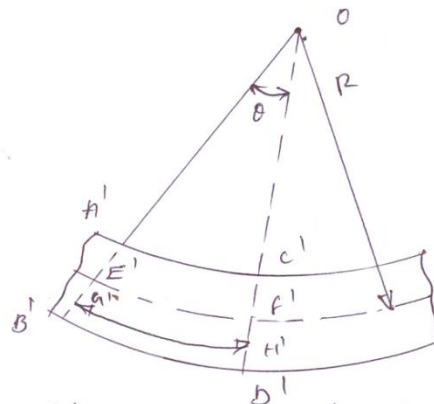
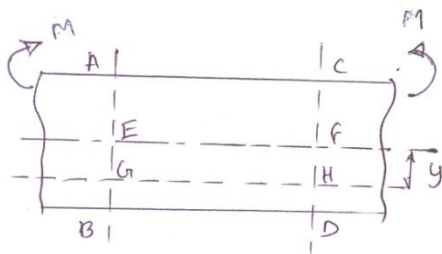
$$= -4.563 \text{ kN-m}$$

$$BM_A = -\left[2 \times 1.25 \times \left(\frac{1.25}{2} + 0.25\right)\right]$$

$$= -5.94 \text{ kN-m}$$



B. Bending theory



The BM induced by the moment tends to bend the beam in a concave manner, so the top surfaces (AC) are subjected to compressive stresses and contract

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while the bottom surfaces (BD) are subjected to tensile stresses and extend. However, there is a layer EF in between the top & bottom, which will retain its original length after bending $E'F'$. This layer EF which is neither compressed or stretched is known as the neutral layer or neutral plane. In fig. GH represents a typical layer of material at a distance 'y' from neutral plane & R is the radius of curvature of the portion of the neutral layer in the bend beam.

The following steps are involved in the development of bending theory

- (i) Determination of strain layer ϵ'
- (ii) Evaluation of stress in this layer by means of young's modulus.

Assumptions of simple bending

- * The material is perfectly homogenous and isotropic obeys hooke's law.
- * The value of young's modulus is same in tension as well as in compression.
- * Transverse sections which are plane before bending remains plane after bending.
- * The radius of curvature of the beam is very large compared to the cross sectional dimension of the beam.

* Each layer of the beam is free to expand
Contract, independently of the layer, above
below etc.

* The resultant force on a transverse section of
the beam is zero.

The Derivation of the Bending equation following
Determination of load carried by the strip of
cross section at a distance y from neutral plane.

Calculating the moment produced by this load
about neutral plane and summation of the total
moment of all such strip loads.

Determination of strain in layer $G'H'$ change in

length of layer $G'H$ after bending = $G'H' - G'H$

$$\begin{aligned}\text{Strain in layer } G'H &= \frac{\text{change in length}}{\text{Original length}} \\ &= \frac{G'H' - G'H}{G'H}\end{aligned}$$

But, $G'H = EF$ and $E'F' = E'F'$ [because of neutral
plane]

$$\text{Strain in layer } G'H = \frac{G'H' - E'F'}{E'F'}$$

Expressing the above equation in terms of

$R + \theta$

The arc length $G'H' = (R+y)\theta$

The arc length $E'F' = R\theta$

$$\begin{aligned} \text{Strain in layer } (r+y) &= \frac{(R+y)\theta - R\theta}{R\theta} \\ &= \frac{R\theta + y\theta - R\theta}{R\theta} = y/R \end{aligned}$$

Stress σ_b in layer $(r+y)$.

We know that young's modulus $E = \frac{\text{Stress } (\sigma_b)}{\text{Strain}}$

$$\sigma_b = E \times \text{Strain}$$

$$\sigma_b = E \times y/R$$

$$\frac{E}{R} = \frac{\sigma_b}{y} \quad \text{--- (1)}$$

Load carried by $(r+y)$

Let $a \Rightarrow$ area of cross section

of strip at $(r+y)$

We know that

$$\text{Stress} = \frac{\text{load}}{\text{Area}}$$

$$\text{load} = \text{Stress} \times \text{Area}$$

$$= \frac{E y}{R} \times a = \frac{E}{R} \times a y$$

$$\text{load} = \frac{E}{R} a y$$

Moment of layer at $(r+y)$

Moment (m) of the load on

this strip about neutral layer

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The weight of water and pipe together over one metre is the intensity of uniformly distributed load.

Weight of pipe over 1m length

$$W_p = 14.844 \times 10^{-3} \times 72 = 1.069 \text{ kN}$$

Weight of water over 1m length

$$W_w = 70.686 \times 10^{-3} \times 10$$

$$W_w = 0.707 \text{ kN}$$

Intensity of uniformly distributed load

$$W = W_p + W_w \\ = 1.069 + 0.707$$

$$W = 1.776 \text{ kN/m}$$

$$y_{\max} = \frac{300 + 2 \times 15}{2} = 165 \text{ mm}$$

$$M_{\max} = \frac{WL^2}{8} = \frac{1.776 \times 6^2}{8} = 7.992 \text{ kN.m}$$

$$I = \frac{\pi}{64} (330^4 - 300^4)$$

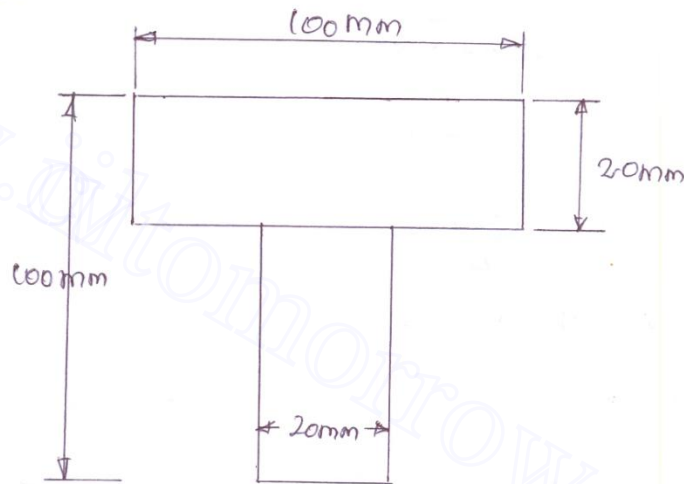
$$= 184.53 \times 10^6 \text{ mm}^4$$

$$\sigma_{\max} = \frac{M_{\max}}{I} \cdot y_{\max}$$

$$= \frac{7.992 \times 10^6}{184.53 \times 10^6} \times 165$$

$$= 7.146 \text{ MPa}$$

- 7 A T-Section of a simply supported beam has the width of flange 100mm, overall depth 100mm, thickness of flange and stem = 20mm. Determine the maximum stress in beam when a bending moment of 12 kN-m is acting on the beam. Also calculate the shear stress at the neutral axis and at the junction of web and flange. When shear force of 50 kN is acting on beam.



Solution

$$\bar{y} = \frac{(20 \times 80) \times 40 + (100 \times 20) \times 90}{20 \times 80 + 100 \times 20}$$

$$\bar{y} = 67.78 \text{ mm}$$

$$I = \left[\frac{20 \times 80^3}{12} + 20 \times 80 \times (40 - 67.78)^2 \right] + \left[\frac{100 \times 20^3}{12} + 100 \times 20 \times (90 - 67.78)^2 \right]$$

$$I = 3.142 \times 10^6 \text{ mm}^4$$

By flexure formula

$$\frac{\sigma_{\max}}{y_{\max}} = \frac{M}{I}$$

$$M = 12 \text{ kN} = 12 \times 10^6 \text{ N mm}$$

$$y_{\max} = \bar{y} = 67.78 \text{ mm}$$

$$\frac{\sigma_{\max}}{67.78} = \frac{12 \times 10^6}{3.142 \times 10^6}$$

$$\sigma_{\max} = 258.87 \text{ MPa}$$

$$I = \frac{V A \bar{y}}{b I}$$

$$V = 50 \text{ kN} = 50 \times 10^3 \text{ N}$$

Neutral axis

$$A \bar{y} = (20 \times 67.78) \times \frac{67.78}{2} = 45.94 \times 10^3 \text{ mm}^3$$

$$b = 20 \text{ mm}$$

$$I_{NA} = \frac{50 \times 10^3 \times 45.94 \times 10^3}{20 \times 3.142 \times 10^6}$$

$$I_{NA} = 36.55 \text{ MPa}$$

At Junction of web and flange

$$A \bar{y} = 100 \times 20 \times (90 - 67.78) = 4440 \text{ mm}^3$$

Just above the junction at web + flange

$$b = 100 \text{ mm}$$

$$I_1 = \frac{50 \times 10^3 \times 44440}{100 \times 3.142 \times 10^6}$$

$$I_1 = 7.072 \text{ mpa}$$

Just below the junction of web + flange

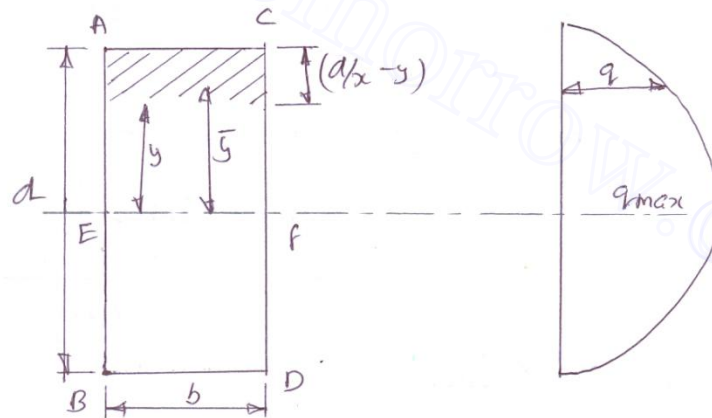
$$b = 20 \text{ mm}$$

$$I_2 = \frac{50 \times 10^3 \times 44440}{20 \times 3.142 \times 10^6}$$

$$I_2 = 35.36 \text{ mpa.}$$

8. Shear stress distribution over a rectangular section

Consider a rectangular beam of width b + depth d as shown in fig.



Shear stress at a distance y from neutral layer

$$q = F \cdot \frac{A\bar{y}}{Ib}$$

Where A - Area of the section above y

$$A = b \times \left(\frac{d}{2} - y\right)$$

\bar{y} - Distance of CG of area A from neutral axis.

$$\bar{y} = y + \frac{(d/2 - y)}{2} = (y + d/4 - y/2) = 1/2 (y + d/2)$$

I - moment of inertia

$$I = \frac{bd^3}{12}$$

$$q = f \times \frac{b(d/2 - y) \times 1/2 (y + d/2)}{Ib}$$

$$= \frac{f}{2I} \left(\frac{d^2}{4} - y^2 \right)$$

At Neutral Axis $y=0$.

$$q_{\max} = \frac{f}{2I} \left(\frac{d^2}{4} - 0^2 \right) = \frac{f}{2I} \frac{d^2}{4}$$

$$= \frac{fd^2}{8I} = \frac{fd^2}{8 \times \frac{bd^3}{12}} = \frac{12}{8} \frac{f}{bd}$$

$$= \frac{3}{2} \frac{f}{bd} = \frac{3}{2} \times \text{Average Stress}$$

Average Stress

$$q_{\text{ave}} = \frac{\text{Shear force}}{\text{Area}} = \frac{f}{b \times d}$$

$$q_{\max} = \frac{3}{2} \times q_{\text{ave}}$$

Bottom edge $y = d/2$

$$q = \frac{f}{2I} \left[\frac{d^2}{4} - \left(\frac{d}{2} \right)^2 \right] = 0$$

\therefore The top and bottom edge, the shear stress is zero.

UNIT – III

TORSION

PART A

1. Define torsion

(May 2014)

A shaft is said to be in torsion, when equal and opposite torques are applied at the two ends of the shaft. The torque is equal to the product of the force applied (tangentially to the ends of a shaft) and radius of the shaft.

2. What are the assumptions made in the theory of torsion?

(May 2010)

The material of the shaft is uniform throughout.

The twist along the shaft is uniform.

Normal cross sections of the shaft, which were plane and circular before

Twist; remain plane and circular after twist.

All diameters of the normal cross section which were straight before twist, remain straight with their magnitude unchanged, after twist.

3. Write the expression for power transmitted by a shaft in Watts

$$P = \frac{2\pi NT}{60}$$

Where

N--- Speed of the shaft in rpm

T—Mean torque transmitted in Nm

P---- Power

4. The torque transmitted by a hollow shaft is given by

$$T = \frac{\pi}{16} \times \tau (D^4 - d^4) / D$$

Where τ -maximum shear stress induced at the outer surface.

D- External diameter

d-internal diameter

5. Define polar modulus.

Polar modulus is defined as the ratio of the polar moment of inertia to the radius of the shaft. It is also called torsional section modulus and is denoted by Z_p .

6. Define torsional rigidity

(May 2012)

Let a twisting moment T produce a twist of radian in a length l then

$$T/J = C\theta/L$$

Where C—modulus of rigidity of the material.

7. Why hollow circular shafts are preferred when compared to solid circular shafts?

Comparison by strength;

The torque transmitted by the hollow shaft is greater than the solid shaft, thereby hollow shaft is stronger than the solid shaft.

Comparison by weight:

For the same material, length and given torque, weight of a hollow shaft will be less. So hollow shafts are economical when compared to solid shafts, when torque is acting.

8. What is mean by spring? Name the two important types of springs.

Spring is a device which is used to absorb energy by taking very large change in its form without permanent deformation and then release the same when its required.

TYPES

Torsion spring

Bending spring

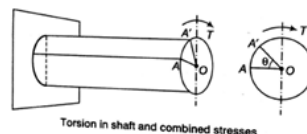
9. Distinguish between close and open helical coil springs.

If the angle of the helix of the coil is so small that the bending effects can be neglected, then the spring is called a closed –coiled spring. Close –coiled spring is a torsion spring The pitch between two adjacent turns is small. If the slope of the helix of the coil is quite appreciable then both the bending as well as torsional shear stresses are introduced in the spring, then the spring is called open coiled spring.

10. Define stiffness of a spring? In what unit it is measured?

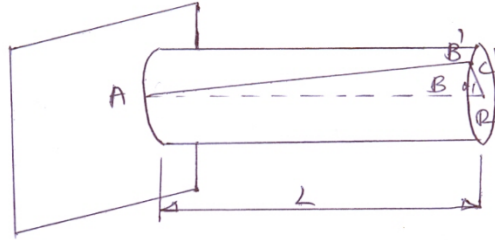
Stiffness of a spring is defined as load per unit deflection. It is denoted by K and unit is N/mm.

11. Draw shear stress distribution of a circular section due to torque.



UNIT-III
TORSION
PART B

1. Derive Torsion equation.



Consider a shaft of length \$L\$ and circular cross section of radius \$R\$ subjected to torque \$T\$ as shown in figure.

A line \$AB\$ on the surface of the shaft, which is straight in absence of the torque, becomes distorted.

The point \$B\$ moves to the \$B'\$ as shown

\$\theta = \angle BOB'\$ is known as the angle of twist.

\$\phi = \angle BAB'\$ is the shear strain.

For the extreme fibres (which are at distance \$R\$ from axis of the shaft) like \$AB\$,

$$BB' = L\theta = R\phi$$

$$\phi = \frac{R\theta}{L} \rightarrow \text{①}$$

for a fibre at any distance \$r\$ from axis of shaft (\$r < R\$),

$$cc' = L\phi' = r\theta$$

where \$\phi'\$ is the shear strain on fibre at \$c\$.

$$\phi = \frac{r\theta}{L}$$

$\frac{\text{Shear stress}}{\text{Shear strain}} = \text{Modulus of rigidity}$

$$\frac{\tau'}{\phi'} = G$$

where,

$\tau' =$ shear stress on the fibre at r .

$$\tau' = G\phi'$$

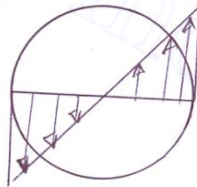
$$\tau' = \frac{G\theta r}{L} \rightarrow (2)$$

As G , θ and L are constants,

$$\tau' \propto r$$

for $r = 0$, $\tau' = 0$ and for $r = R$, $\tau' = \tau$, the maximum shear stress.

The variation of shear stress with radial distance from the axis of the shaft is shown in figure (2).



At $r = R$,

$$\tau = \frac{GR\theta}{L} \rightarrow (3)$$

from equations (1) & (2),

$$\frac{\tau'}{r} = \frac{\tau}{R} = \frac{G\theta}{L} \rightarrow (4)$$

Consider an elementary area da at a section of the shaft as shown in figure (3).

Let $df = \frac{\text{Force on area}}{\text{element } da}$.

Then, the torque on area element is

$$dT = df \cdot r$$

$$T = \int_0^R df \cdot r$$

$$dF = \tau' dA = \left(\frac{G\theta r}{L} \right) dA.$$

From equation ②,

$$\begin{aligned} T &= \int_0^R G\theta/L r^2 dA \\ &= G\theta/L \int_0^R r^2 dA \end{aligned}$$

But, $\int_0^R r^2 dA = J$, the polar moment of Inertia.

$$T = \frac{G\theta}{L} J$$

$$T/J = \frac{G\theta}{L} \rightarrow \text{③}$$

From eqn ④ & ③,

$$T/J = \tau/R = \frac{G\theta}{L} \text{ is known as Torsion equation.}$$

2. A hollow shaft with diameter ratio $3/5$ is required to transmit 450 kW at 120 rpm . The shearing stress in the shaft must not be exceed 60 N/mm^2 and twist in a length of 2.5 m is not to exceed 1° . Calculate the minimum external diameter of the shaft
 $C = 80 \text{ kN/mm}^2$.

Given:

$$d/D = 3/5$$

$$d = 0.6D$$

$$P = 450 \text{ kW}$$

$$N = 120 \text{ rpm}$$

$$\tau = 60 \text{ N/mm}^2$$

$$L = 2.5 \text{ m} = 2500 \text{ mm}$$

Twist in the shaft $= 1^\circ = \frac{1 \times \pi}{180} = 0.0174$ radians

$$C = 80 \text{ kN/mm}^2 = 80 \times 10^3 \text{ N/mm}^2.$$

To.

External diameter of the hollow shaft

Solution:

$$P = 2\pi NT / 60$$

$$450 = \frac{2\pi \times 120 \times T}{60}$$

$$T = 35.80 \text{ kN-m.}$$

CASE I:

Shear stress considering.

$$T = \frac{\pi}{16} \times I \times \left[\frac{D^4 - d^4}{D} \right]$$

$$35.80 \times 10^6 = \frac{\pi}{16} \times 60 \times \left[\frac{D^4 - (0.6D)^4}{D} \right]$$

$$35.80 \times 10^6 = \frac{\pi}{16} \times 60 \times \frac{D^4}{D} [1 - (0.6)^4]$$

$$35.80 \times 10^6 = 11.78 \times D^3 \times (0.870)$$

$$D^3 = 3493160.041$$

$$D = 151.73 \text{ mm}$$

CASE II:

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$J = \frac{\pi}{32} [D^4 - d^4]$$

$$= \frac{\pi}{32} [D^4 - (0.6D)^4]$$

$$\frac{35.80 \times 10^6}{\frac{\pi}{32} [D^4 - (0.6D)^4]} = \frac{80 \times 10^3 \times 0.0174}{2500}$$

$$\frac{35.80 \times 10^6}{\frac{\pi}{32} \times D^4 [1 - (0.6)^4]} = 0.5568$$

$$\frac{35.80 \times 10^6}{\pi/32 D^4 (0.870)} = 0.5568$$

$$\frac{35.80 \times 10^6}{\pi/32 \times 0.870 \times 0.5568} = D^4$$

$$7.527 \times 10^8 = D^4$$

$$D = 165.6 \text{ mm}$$

3. A close coiled helical spring is to have a stiffness of 1.5 N/mm of compression. Under a maximum load of 60 N. The maximum shearing stress produced in the wire of the spring is 125 N/mm². Solid length of the spring is 50 mm. Find the diameter of coil, diameter of wire and number of coils $C = 4.5 \times 10^4$ N/mm².

Given:

$$K = 1.5 \text{ N/mm}$$

$$W = 60 \text{ N}$$

$$I_{\text{max}} = 125 \text{ N/mm}^2$$

$$G = C = 4.5 \times 10^4 \text{ N/mm}^2$$

$$\text{Solid length (nd)} = 50 \text{ mm}$$

To:

Number of turns

Diameter of the wire.

Solution:

$$n = 50/d$$

$$K = \frac{Gd^4}{64R^3n}$$

$$1.5 = \frac{4.5 \times 10^4 d^4}{64 R^3 n}$$

$$1.5 = \frac{4.5 \times 10^4 d^4}{64 R^3 (50/d)}$$

$$\frac{d^5}{R^3} = 0.1067 \rightarrow \textcircled{1}$$

$$I_{\max} = \frac{16WR}{\pi d^3}$$

$$125 = \frac{16 \times 60R}{\pi d^3}$$

$$R = \frac{d^3}{2.445} \rightarrow \textcircled{2}$$

Substitute in eqn ①.

$$\frac{d^5}{\left(\frac{d^3}{2.445}\right)^3} = 0.1067$$

$$\frac{d^5 \times 2.445^3}{d^9} = 0.1067$$

$$d^4 = \frac{2.445^3}{0.1067}$$

$$d = 3.42 \text{ mm}$$

$$\text{Radius of coil (R)} = \frac{3.42^3}{2.445}$$

$$R = 16.36 \text{ mm}$$

$$\text{Diameter of coil (D)} = 2R$$

$$= 2 \times 16.36$$

$$D = 32.72 \text{ mm}$$

$$n = \frac{50}{d}$$

$$n = 50 / 3.42$$

$$n = 14.62$$

$$n = 15.$$

4. A steel shaft is required to transmit 75 kW power at 100 rpm and the maximum twisting moment is 30% greater than the mean. Find the diameter of the steel shaft if the maximum stress is 70 N/mm^2 . Also determine the angle of twist in a length of 3m of the shaft. Assume the modulus of rigidity for steel as 90 kN/mm^2 .

Solution:

$$P = \frac{2\pi n T_{\text{mean}}}{60,000} \text{ kW}$$

$$75 = \frac{2\pi \times 100 T_{\text{mean}}}{60,000}$$

$$T_{\text{mean}} = 7.162 \times 10^3 \text{ Nm}$$

$$T_{\text{max}} = T_{\text{mean}} + \frac{30}{100} T_{\text{mean}} = 1.3 T_{\text{mean}}$$

$$T_{\text{max}} = 1.3 \times 7.162 \times 10^3$$

$$T_{\text{max}} = 9.3106 \times 10^3 \text{ Nm}$$

$$I_{\text{max}} = \frac{TR}{J} = \frac{T_{\text{max}} (D/2)}{(\pi/32) D^4}$$

$$T_{\text{max}} = \pi/16 D^3 I_{\text{max}}$$

$$I_{\text{max}} = 70 \text{ N/mm}^2$$

$$T_{\max} = \pi/16 D^3 \times I$$

$$9.3106 \times 10^6 = \pi/16 D^3 \times 70$$

$$D = 87.82 \text{ mm}$$

$$\theta = TL/GJ$$

$$J = \pi/32 D^4 = \pi/32 \times 87.82^4$$

$$J = 5.84 \times 10^6 \text{ mm}^4$$

$$L = 3 \text{ m} = 3 \times 10^3 \text{ mm}$$

$$G = 90 \text{ kN/mm}^2$$

$$= 90 \times 10^3 \text{ N/mm}^2$$

$$T = 9.3106 \times 10^6 \text{ Nmm}$$

$$\theta = \frac{9.3106 \times 10^6 \times 3 \times 10^3}{90 \times 10^3 \times 5.84 \times 10^6}$$

$$\theta = 0.053145 \text{ rad}$$

$$= 0.053143 \times 180/\pi \text{ degree}$$

$$\theta = 3.045^\circ$$

5. A solid circular shaft transmits 75 kW power at 200 rpm. Calculate the shaft diameter, if the twist in the shaft is not to exceed 1° in 2 m length of shaft, and shear stress is limited to 50 N/mm^2 . Take $C = 1 \times 10^5 \text{ N/mm}^2$.

Given data

$$P = 75 \text{ kW} = 75 \times 10^3 \text{ W}$$

$$N = 200 \text{ rpm}$$

$$\theta = 1^\circ$$

$$= \frac{\pi}{180} \times 1 = 0.01745 \text{ rad.}$$

$$L = 2 \text{ m} = 2 \times 10^3 \text{ mm}$$

$$\tau = 50 \text{ N/mm}^2$$

$$P = \frac{2\pi NT}{60} \Rightarrow 75 \times 10^3 = \frac{2\pi \times 200 \times T}{60}$$

$$T = \frac{75 \times 10^3 \times 60}{2\pi \times 200} = 3580.98 \text{ Nm}$$

To find

d.

Solution

(i) Diameter of the shaft when maximum shear stress is limited to 50 N/mm^2 .

$$T = \frac{\pi}{16} \tau D^3$$

$$3580980 = \frac{\pi}{16} \times 50 \times D^3$$

$$D = 71.3 \text{ mm.}$$

(ii) Diameter of the shaft when the twist in the shaft is not to exceed 1°

$$\frac{\theta}{L} = \frac{C\theta}{J}$$

$$\frac{3580980}{\pi/32 D^4} = \frac{10^5 \times 0.01745}{2000}$$

$$J = \pi/32 D^4$$

$$D = \left[\frac{32 \times 2000 \times 3580980}{\pi \times 10^5 \times 0.01745} \right]^{1/4} = 80.4 \text{ mm.}$$

PART C

6. Two solid shafts AB + BC of aluminium and steel respectively are rigidly fastened together at B and attached to two rigid supports at A and C. Shaft AB is 7.5 cm in diameter and 2 m in length. Shaft BC is 5.5 cm in diameter and 1 m length. A torque of 20000 N-m is applied at the junction B. Compute the maximum shearing stresses in each material. What is the angle of twist at the junction. Take

the modulus of rigidity of the materials

$$C_{AL} = 0.3 \times 10^5 \text{ N/mm}^2 + C_{SE} = 0.9 \times 10^5 \text{ N/mm}^2.$$

Given data

Aluminium

$$L_1 = 2 \text{ m} = 2000 \text{ mm}$$

$$d_1 = 7.5 \text{ cm} = 75 \text{ mm}$$

$$C_1 = 0.3 \times 10^5 \text{ N/mm}^2$$

Steel

$$L_2 = 1 \text{ m} = 1000 \text{ mm}$$

$$d_2 = 5.5 \text{ cm} = 55 \text{ mm}$$

$$C_2 = 0.9 \times 10^5 \text{ N/mm}^2.$$

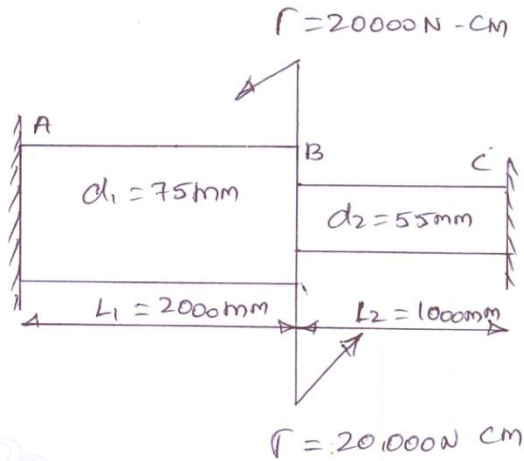
$$\begin{aligned} T &= 20,000 \text{ N cm} \\ &= 20000 \text{ N mm} \end{aligned}$$

To find

maximum Shear Stress

Solution

The Torque is applied at junction B, hence angle of twist in shaft AB + BC will be same ($\theta_1 = \theta_2 = \theta$)



$$T_1 + T_2 = 20,000 \text{ N mm}$$

$$\frac{T}{J} = \frac{C\theta}{L}$$

Shaft AB

$$\frac{T_1}{J_1} = \frac{C_1 \theta_1}{L_1} \quad \therefore \theta_1 = \frac{T_1 \times L_1}{J_1 \times C_1}$$

$$\begin{aligned} J_1 &= \frac{\pi}{32} d_1^4 \\ &= \frac{\pi}{32} \times 75^4 \end{aligned}$$

$$\theta_1 = \frac{T_1 \times 2000}{\frac{\pi}{32} \times 75^4 \times 0.3 \times 10^5} = \frac{T_1 \times 2000 \times 32}{\pi \times 75^4 \times 0.3 \times 10^5}$$

Shaft BC

$$\theta_2 = \frac{T_2 \times L_2}{J_2 \times C_2}$$

$$J_2 = \frac{\pi}{32} \times 55^4$$

$$= \frac{r_2 \times 1000}{\pi/32 \times 55^2 \times 0.9 \times 10^5}$$

$$= \frac{r_2 \times 1000 \times 32}{\pi \times 55^4 \times 0.9 \times 10^5}$$

$$\theta_1 = \theta_2$$

$$\frac{r_1 \times 2000 \times 32}{\pi \times 75^4 \times 0.3 \times 10^5} = \frac{r_2 \times 1000 \times 32}{\pi \times 55^4 \times 0.9 \times 10^5}$$

$$\frac{2r_1}{75^4 \times 0.3} = \frac{r_2}{55^4 \times 0.9}$$

$$r_1 = \frac{75^4 \times 0.3}{55^4 \times 0.9 \times 2} r_2$$

$$r_1 = 0.576 r_2$$

$$r_1 + r_2 = 20,000$$

$$0.576 r_2 + r_2 = 200000$$

$$1.576 r_2 = 20,0000$$

$$r_2 = \frac{200000}{1.576} = 126900 \text{ N mm}$$

$$r_1 + r_2 = 200000$$

$$r_1 = 200000 - r_2 = 200000 - 126900$$

$$= 73100 \text{ N mm}$$

$$\frac{r}{J} = \frac{\tau}{R}$$

$$\frac{r_1}{J_1} = \frac{\tau_1}{R_1}, \quad \tau_1 = \frac{r_1 \times R_1}{J_1} = \frac{73100 \times 37.5}{\pi/32 \times 75^4}$$

$$\tau_1 = 0.882 \text{ N/mm}^2$$

$$\frac{r_2}{\sqrt{2}} = \frac{z_2}{R_2}$$

13

$$\begin{aligned} z_2 &= \frac{r_2 \times R_2}{\sqrt{2}} = \frac{126900 \times 27.5}{\frac{\pi}{32} \times 55^4} \\ &= \frac{126900 \times 27.5 \times 32}{\pi \times 55^4} = 3.884 \text{ N/mm}^2 \end{aligned}$$

RESULT

$$z_1 = 0.882 \text{ N/mm}^2$$

$$z_2 = 3.884 \text{ N/mm}^2$$

7. A laminated spring 1m long is made up of plates each 5cm wide and 1cm thick. If the bending stress in the plate is limited to 100 N/mm², how many plates would be required to enable the spring to carry a central point load of 2kN. If $E = 2.1 \times 10^5 \text{ N/mm}^2$ what is the deflection under the load.

Given data

$$l = 1\text{m} = 1000\text{mm}$$

$$b = 5\text{cm} = 50\text{mm}$$

$$t = 1\text{cm} = 10\text{mm}$$

$$\sigma = 100 \text{ N/mm}^2$$

$$W = 2 \text{ kN} = 2000 \text{ N}$$

$$E = 2.1 \times 10^5 \text{ N/mm}^2$$

To find

n - no of plates

δ - Deflection

$$\sigma = \frac{3Wt}{2nb^2}$$

$$100 = \frac{3 \times 2000 \times 1000}{2 \times n \times 50 \times 10^2}$$

$$n = \frac{3 \times 2000 \times 1000}{100 \times 2 \times 50 \times 100} = 6$$

Deflection

$$\delta = \frac{\sigma \times l^3}{4E \times b} = \frac{100 \times 1000^2}{4 \times 2.1 \times 10^5 \times 10} = 11.9 \text{ mm.}$$

Result

$$n = 6$$

$$\delta = 11.9 \text{ mm.}$$

8. An open coil helical spring made of 5mm diameter wire has 16 coils 100mm inner diameter with helix angle of 16°. Calculate the deflection maximum direct + shear stresses induced due to an axial load of 300N. Take G = 90GPa + E = 200GPa.

Given data

$$d = 5 \text{ mm}$$

$$n = 16$$

$$D_i = 100 \text{ mm}$$

$$D = D_i + d = 100 + 5 = 105 \text{ mm}$$

$$R = 52.5 \text{ mm.}$$

$$\alpha = 16^\circ$$

$$W = 300\text{N}$$

$$E = 200\text{GPa} = 200 \times 10^3 \text{N/mm}^2$$

$$G = 90\text{GPa} = 90 \times 10^3 \text{N/mm}^2$$

To find

Deflection

Maximum direct + Shear Stresses.

Solution

$$\delta = \frac{64WR^3 n \sec \alpha}{d^4} \left[\frac{\cos^2 \alpha}{C} + \frac{2 \sin^2 \alpha}{E} \right]$$

$$= \frac{64 \times 300 \times 52.5^3 \times 16 \times \sec 16^\circ}{5^4}$$

$$= 815.36 \text{ mm.}$$

$$\text{Bending Stress } \sigma_b = \frac{32WR \sin \alpha}{\pi d^3}$$

$$= \frac{32 \times 300 \times 52.5 \times \sin 16}{\pi \times 5^3}$$

$$= 353.76 \text{ N/mm}^2$$

$$\text{Shear Stress } \tau = \frac{16WR \cos \alpha}{\pi d^3}$$

$$= \frac{16 \times 300 \times 52.5 \times \cos 16}{\pi \times 5^3}$$

$$\tau = 616.85 \text{ N/mm}^2.$$

Maximum shear stress

$$\begin{aligned}\tau_{\max} &= \frac{16WR}{\pi d^3} \\ &= \frac{16 \times 300 \times 52.5}{\pi \times 5^3} \\ &= 641.71 \text{ N/mm}^2\end{aligned}$$

Maximum principle stress

$$\begin{aligned}\sigma_{b1} &= \frac{16WR}{\pi d^3} (\sin \alpha + 1) \\ &= \frac{16 \times 300 \times 52.5}{\pi \times 5^3} (\sin 16^\circ + 1)\end{aligned}$$

$$\sigma_{b1} = 818.59 \text{ N/mm}^2$$

Result

$$f = 815.87 \text{ mm}$$

$$\tau_{\max} = 641.71 \text{ N/mm}^2$$

$$\sigma_{b1} = 818.59 \text{ N/mm}^2$$

Unit - IV
BEAM DEFLECTION
PART A

1. Write the maximum value of deflection for a cantilever beam of length L , constant EI and carrying concentrated load W at the end.

Maximum deflection at the end of a cantilever due to the load $=WL^3/3EI$

2. What are the different methods used for finding deflection and slope of beams?

Double integration method

Mecaulay's method

Strain energy method

Moment area method

Unit load method

3. State the two theorems in moment area method. (May 2014)

Mohr's Theorem-I: the angle between tangents at any two points A and B on the bent beam is equal to total area of the corresponding position of the bending moment diagram divided by EI .

Mohr's Theorem-II: The deviation of B from the tangent at A is equal to the statically moment of the B.M.D. area between A and B with respect to B divided by EI .

4. What is meant by elastic curve?

The deflected shape of a beam under load is called elastic curve of the beam, Within elastic limit.

5. When Macaulay's method is preferred?

This method is preferred for determining the deflections of a beam subjected to several concentrated loads or a discontinuous load.

6. What are the boundary conditions for a cantilever beam?

The boundary conditions for a cantilever beam are:

- (i) Deflection at the fixed end is zero.
- (ii) Slope is zero at the fixed end.

7. What is meant by Double-Integration method? (May 2013)

Double-integration method is a method of finding deflection and slope of a Bent beam. In this method the differential equation of curvature of bent beam, EI

$$d^2y/dx^2=M$$

M is integrated once to get slope and twice to get deflection. Here the constants of integration C1 and C2 are evaluated from known boundary condition.

8. What is Modulus of resilience? (May/June 2013)

It is the proof resilience of the material per unit volume.

Modulus of resilience= proof resilience / Volume of the body

9. What are the limitations of double integration method? (Dec 2014)

1. Double integration method can be used only for beams with uniform cross section
2. It is useful only in cases where there is no change in loading.

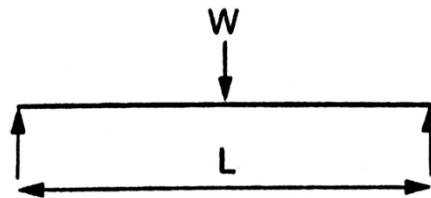
10. Define strain energy. (Dec 2014)

Strain energy is the energy absorbed or stored by a member when work is done on it to deform it.

11. State Maxwell's reciprocal theorem.

The work done by the first system of loads due to displacements caused by a second system of loads equal the work done by the second system of loads due to displacements caused by the first system of loads.

12. Write down the equation for the maximum deflection of a cantilever beam carrying a central point load W.



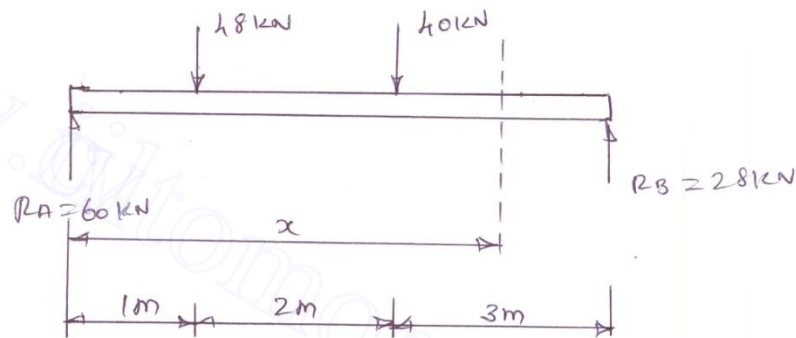
$$\text{DEFLECTION } y_c = \frac{W L^3}{48 E I}$$

UNIT-IV

DEFLECTION OF BEAMS

PART B

1. A beam of length 6m is simply supported at the ends and carries two point loads of 48 kN and 40 kN at a distance of 1m and 3m respectively from the left support. Compute the slope and deflection under each loads. Assume $EI = 17000 \text{ kN-m}^2$.



Solution:

The Free body diagram is shown.

$$\sum M_A = 0$$

$$-48(1) - 40(3) + R_B(6) = 0$$

$$R_B = 28 \text{ kN.}$$

$$\sum f_y = 0$$

$$R_A - 48 - 40 + R_B = 0$$

$$R_A = 60 \text{ kN.}$$

Take section portion CD at distance x from A as shown in fig.

$$EI \frac{d^2y}{dx^2} = M$$

$$EI \frac{d^2y}{dx^2} = 60x \left\{ \begin{array}{l} | \\ | \\ | \end{array} \right. - 48(x-1) \left\{ \begin{array}{l} | \\ | \\ | \end{array} \right. - 40(x-3) \left\{ \begin{array}{l} | \\ | \\ | \end{array} \right. \text{ KNm.}$$

$$EI \frac{dy}{dx} = 30x^2 + C_1 \left\{ \begin{array}{l} | \\ | \\ | \end{array} \right. - 24(x-1)^2 \left\{ \begin{array}{l} | \\ | \\ | \end{array} \right. - 20(x-3)^2 \left\{ \begin{array}{l} | \\ | \\ | \end{array} \right. \text{ KNm}^2$$

$$EI y = 10x^3 + C_1 x + C_2 \left\{ \begin{array}{l} | \\ | \\ | \end{array} \right. - 8(x-1)^3 \left\{ \begin{array}{l} | \\ | \\ | \end{array} \right. - \frac{20(x-3)^3}{3} \left\{ \begin{array}{l} | \\ | \\ | \end{array} \right. \text{ KNm}^2$$

At $x=0, y=0,$

$$0 = C_2$$

At $x=6m, y=0,$

$$0 = 10 \times 6^3 + C_1 \times 6 - 8(6-1)^3 - \frac{20(6-3)^3}{3}$$

$$C_1 = -163.33$$

At $C, x=1m$

$$EI \left(\frac{dy}{dx} \right)_C = 30 \times 1^2 - 163.33$$

$$EI \left(\frac{dy}{dx} \right)_C = -133.33 \text{ KNm}^2$$

$$EI = 17000 \text{ KNm}^2.$$

$$17000 \left(\frac{dy}{dx} \right)_C = -133.33$$

$$\left(\frac{dy}{dx} \right)_C = -7.843 \times 10^{-3} \text{ radians.}$$

$$\left(\frac{dy}{dx} \right)_C = -0.45^\circ \Rightarrow \left(\frac{dy}{dx} \right)_C = 0.45^\circ \swarrow$$

At $C, x=3m$

$$17000 \left(\frac{dy}{dx} \right)_D = 30 \times 3^2 - 163.33 - 24(3-1)^2$$

$$\left(\frac{dy}{dx}\right)_D = -6.276 \times 10^{-4} \text{ rad}$$

$$\left(\frac{dy}{dx}\right)_D = -0.036^\circ$$

$$\left(\frac{dy}{dx}\right)_D = 0.036^\circ$$

At C,

$$17000 y_c = 10 \times 1^3 - 163.33 \times 1$$

$$y_c = -9.02 \times 10^{-3} \text{ m}$$

$$y_c = 9.02 \text{ mm} \downarrow$$

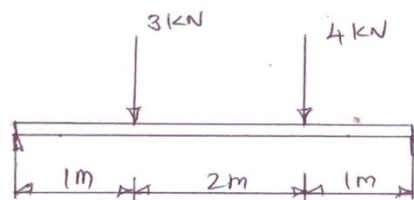
At D,

$$17000 y_D = 10 \times 3 - 163.33 \times 3 - 8$$

$$y_D = -0.0167 \text{ m}$$

$$y_D = 16.7 \text{ mm}$$

2. Using Conjugate beam method, determine the
- slope at each end and under each load.
 - Deflection under each load for the given beam shown in figure. Take $E = 2 \times 10^5 \text{ N/mm}^2$ and $I = 10^8 \text{ mm}^4$.



Solution:

$$\sum M_A = 0$$

$$-3(1) - 4(3) + R_D(4) = 0$$

$$R_D = 3.75 \text{ kN}$$

$$\sum f_y = 0$$

$$R_A - 3 - 4 + R_D = 0$$

$$R_A = 3.25 \text{ kN}$$

$$EI = 2 \times 10^5 \times 10^8$$

$$= 2 \times 10^{13} \text{ Nmm}^2$$

$$= 2 \times 10^{14} \text{ kN/m}^2$$

The beam, SFD, $\frac{M}{EI}$ diagram and Conjugate beam are shown in Fig (1).

$$\text{At A; } \frac{M}{EI} = 0.$$

$$\text{At B, } \frac{M}{EI} = \frac{0 + 3.25 \times 1}{2 \times 10^4} = 1.625 \times 10^{-4} \text{ m}^{-1}$$

$$\text{At C, } \frac{M}{EI} = \frac{3.25 + 0.25 \times 2}{2 \times 10^4} = 1.875 \times 10^{-4} \text{ m}^{-1}$$

$$\text{At D, } \frac{M}{EI} = 0.$$

For conjugate beam,

$$\sum M_A = 0$$

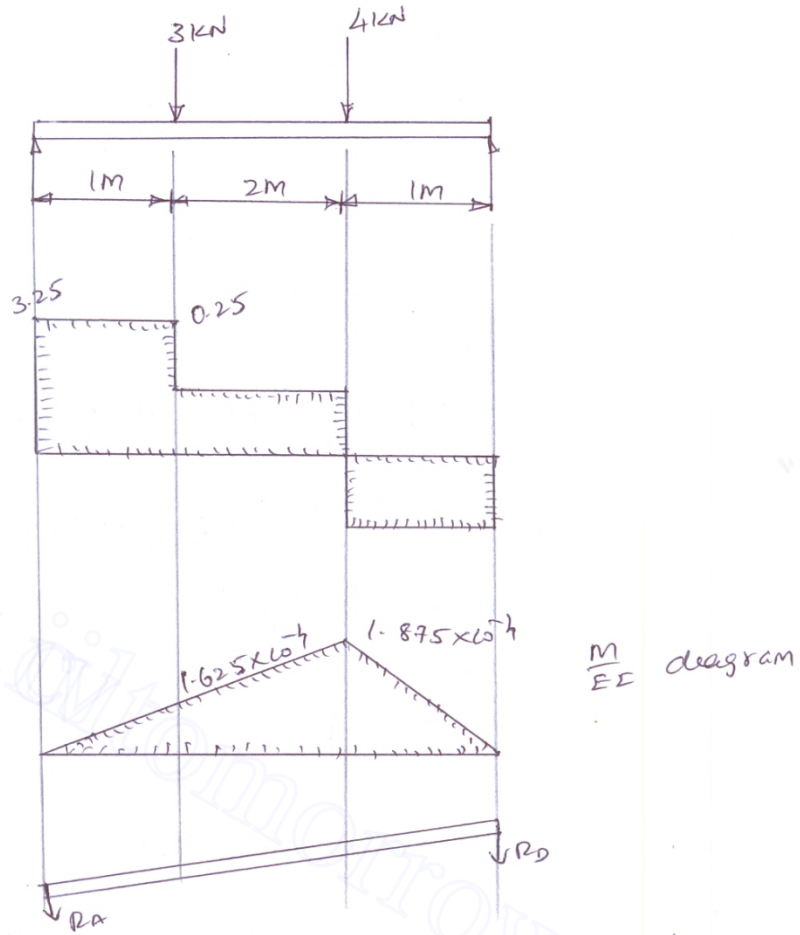
$$-R_D \times 4 + \left(\frac{1}{2} \times 1 \times 1.625 \times 10^{-4} \right) \times \left(\frac{2}{3} \right) + \left(1.625 \times 10^{-4} \times 2 \right) \times 2 \\ + \left(\frac{1}{2} \times 2 \times 0.25 \times 10^{-4} \right) \times \left(1 + \frac{4}{3} \right) + \left(\frac{1}{2} \times 1 \times 1.875 \times 10^{-4} \right) \times \left(3 + \frac{1}{3} \right) = 0$$

$$R_D = 2.6875 \times 10^{-4} \downarrow$$

$$\sum f_y = 0$$

$$-R_A - R_D + \frac{1}{2} \times 1 \times 1.625 \times 10^{-4} + 1.625 \times 10^{-4} \times 2 + \frac{1}{2} \\ \times 2 \times 0.25 \times 10^{-4} + \frac{1}{2} \times 1 \times 1.875 \times 10^{-4} = 0$$

$$R_A = 2.5625 \times 10^{-4}$$



Slope at A = Shear force at A on Conjugate beam,
 $= -2.5625 \times 10^{-4} \text{ rad}$
 $= -0.0147$
 $A = 0.0147^\circ \downarrow$

Slope at D = Shear force at D on Conjugate beam
 $= +2.6875 \times 10^{-4} \text{ rad}$
 $= 0.0154^\circ$
 $D = 0.0154^\circ \uparrow$

Slope at B = Shear force at B on Conjugate beam
 $= -2.5625 \times 10^{-4} \times \frac{1}{2} \times 1 \times 1.625 \times 10^{-4}$
 $= -1.75 \times 10^{-4} \text{ rad}$

$$= -0.01^\circ$$

$$B = 0.01^\circ \downarrow$$

Slope at C = shear force at C on Conjugate beam

$$= 2.6875 \times 10^{-4} - 1/2 \times 1 \times 1.875$$

$$= 1.75 \times 10^{-4} \text{ rad}$$

$$= 0.01^\circ$$

$$C = 0.01^\circ \uparrow$$

Deflection at B = Bending moment at B on Conjugate beam

$$y_B = -2.5625 \times 10^{-4} \times 1 + 1/2 \times 1 \times 1.625 \times 10^{-4} \times 1/3$$

$$= -2.2914 \times 10^{-4} \text{ m}$$

$$= -0.229 \text{ mm}$$

$$y_B = 0.229 \text{ mm} \downarrow$$

Deflection at C = Bending moment at C on conjugate beam

$$y_C = -2.6875 \times 10^{-4} \times 1 + (1/2 \times 1 \times 1.875 \times 10^{-4}) \times 1/3$$

$$= -2.375 \times 10^{-4} \text{ m}$$

$$y_C = 0.2375 \text{ mm} \downarrow$$

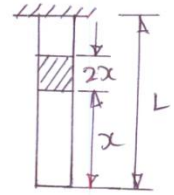
3. Determine the strain energy due to self weight of a bar of uniform cross section 'a' having length 'l' which is hanging vertically down.

Solution:

Consider an element at a distance 'x' from the lower end of the bar as shown in

fig.

Let 'dx' be the thickness of the element.
 The section 'x-x' will be acted upon by
 the weight of the bar of length 'x'.



Let $w_x =$ weight of the bar of length 'x'.

$$= \text{Volume of the bar of length } x \times \left(\begin{array}{l} \text{weight of} \\ \text{unit volume} \end{array} \right)$$

$$= [A \cdot x] \rho$$

$$w_x = P \cdot x.$$

As a result of this weight, the portion 'dx' will experience a small elongation 'ds' then

$$\text{Strain in portion } dx = \frac{\text{Elongation in } dx}{\text{Length of } x}$$

$$dx = \frac{ds}{dx}$$

$$\text{Stress in portion } dx = \frac{\text{weight acting on section } x-x}{\text{Area of section}}$$

$$= P \cdot x / A$$

$$= P \cdot x.$$

$$\text{Young's modulus, } E = \frac{\text{Stress}}{\text{Strain}} = \frac{P \cdot x}{\left(\frac{ds}{dx} \right)}$$

$$E = \frac{P \cdot x \cdot dx}{ds}$$

$$ds = \frac{P \cdot x \cdot dx}{E}$$

Now the strain energy stored in portion 'dx' is given by

$$dv = \text{Average weight} \times \text{Elongation}$$

$$= \left[\frac{1}{2} \times W_x \right] \times dx = \left[\frac{1}{2} \times P A x \right] \times \frac{P x dx}{E}$$

$$= \frac{1}{2} \times P^2 A x \frac{dx}{E}$$

Total strain energy stored within the bar due to its self weight 'w' is obtained by Integrating the above equation from 0 to L.

$$u = \int_0^L dv = \int_0^L \frac{1}{2} \times P^2 A x^2 dx / E$$

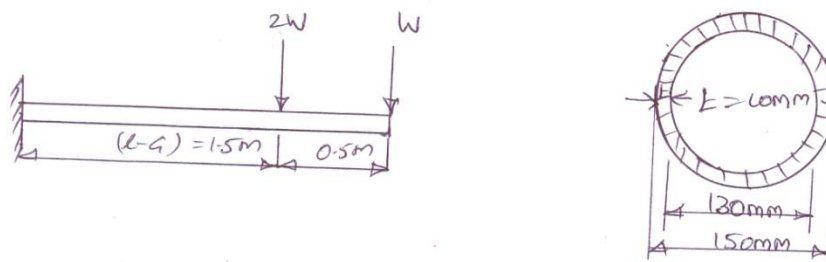
$$= \frac{1}{2} \frac{P^2 A}{E} \int_0^L x^2 dx = \frac{1}{2} \frac{P^2 A}{E} \left(\frac{x^3}{3} \right)_0^L$$

$$= \frac{1}{2} \frac{P^2 A}{E} \frac{L^3}{3} = \frac{AP^2 L^3}{6E}$$

$$U = \frac{AP^2 L^3}{6E}$$

Double Integration Method.

4. A 2 m long cantilever made up of steel tube section 150 mm external diameter and 10 mm thick is loaded as shown in Fig.



- Take $E = 200 \text{ GPa}$. Calculate
- The value of w , so that the max bending stress is 150 MPa .
 - The maximum deflection for the loading.

Solution:

Case (i).

$$M_A = 2W \times 1500 + W \times 2000$$

$$= 5000W$$

$$\frac{E}{R} = \frac{M}{I} = \frac{\sigma_b}{y}$$

$$I = \frac{\pi}{64} (D^4 - d^4)$$

$$= \frac{\pi}{64} (150^4 - 130^4)$$

$$I = 10.86 \times 10^6 \text{ mm}^4$$

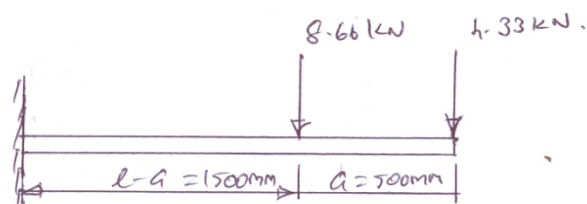
$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$y = R/d = 150/2 = 75 \text{ mm}.$$

Therefore,

$$\frac{5000W}{10.83 \times 10^6} = \frac{150}{75}$$

Case (ii).



Double Integration Method.

- Deflection at free end due to the load 4.33 kN alone.

$$= \frac{W \cdot L^3}{3EI}$$

(ii) Deflection at free end due to load 8.66 kN alone.

$$= \frac{WL(L-a)^3}{3EI} + \frac{WL(L-a)^2}{2EI} \cdot a$$

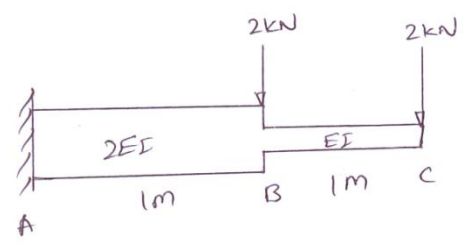
$$y_B = \frac{W_1 L^3}{3EI} + \frac{WL(L-a)^3}{3EI} + \frac{W_2(L-a)^2}{2EI} \cdot a$$

$$= \frac{4.33 \times 10^3 \times 2000^3}{3 \times 200 \times 10^3 \times 10.86 \times 10^6} + \frac{8.33 \times 10^3 \times 1500^3}{3 \times 200 \times 10^3 \times 10.86 \times 10^6} + \frac{8.33 \times 10^3 \times 1500^2}{2 \times 200 \times 10^3 \times 10.86 \times 10^6} \times 500$$

$$= 5.316 + 4.314 + 2.151$$

$$y_B = 11.78 \text{ mm.}$$

5. For the cantilever beam shown in fig. find the deflection and slope at the free end. $EI = 10000 \text{ kN/m}^2$.



$$A_1 = \frac{1}{2} \times 1 \times \frac{2}{EI} = \frac{1}{EI}$$

$$\bar{x}_1 = \frac{2}{3} \times 1 = \frac{2}{3} \text{ m}$$

$$A_2 = \frac{1}{EI} \times 1 = \frac{1}{EI}$$

$$\bar{x}_2 = 1.5 \text{ m}$$

$$A_3 = \frac{1}{2} \times 1 \times \frac{2}{EI} = \frac{1}{EI}$$

$$\bar{x}_3 = 1 + \frac{2}{3} \times 1 = \frac{5}{3} \text{ m}$$

$$\text{Slope } C = A_1 + A_2 + A_3$$

$$= \frac{1}{EI} + \frac{1}{EI} + \frac{1}{EI}$$

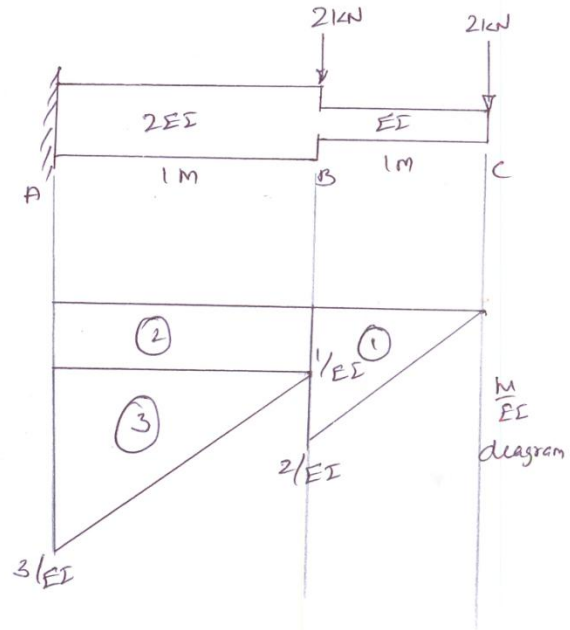
$$= \frac{3}{EI}$$

Deflection at 'C'

$$= A_1 \bar{x}_1 + A_2 \bar{x}_2 + A_3 \bar{x}_3$$

$$= \frac{1}{EI} \times \frac{2}{3} + \frac{1}{EI} \times 1.5 + \frac{1}{EI} \times \frac{5}{3}$$

$$= \frac{11.50}{3EI}$$



RESULT

$$\text{Slope} = \frac{3}{10,000} = 3 \times 10^{-4} \text{ rad}$$

$$\text{Deflection} = \frac{11.5}{30,000} = 3.83 \times 10^{-4} \text{ m}$$

PART C

6. A tension bar is made of two parts. The length of first part is 300cm and area is 20cm² while the second part of length 200cm and area 30cm². An axial load of 90kN is gradually applied. Find the total strain energy produced in the bar and compare this value with

that obtain in a Uniform bar of same length and having same volume under same load. Take $E = 2 \times 10^5 \text{ N/mm}^2$.

Given data

$$L_1 = 300 \text{ cm} = 3000 \text{ mm}$$

$$A_1 = 20 \text{ cm}^2 = 2000 \text{ mm}^2$$

$$V_1 = A_1 \times L_1 = 2000 \times 3000$$

$$= 6 \times 10^6 \text{ mm}^3$$

$$L_2 = 200 \text{ cm} = 2000 \text{ mm}$$

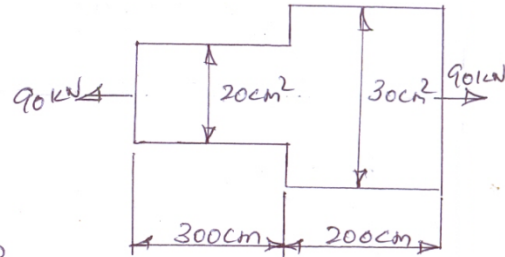
$$A_2 = 30 \text{ cm}^2 = 3000 \text{ mm}^2$$

$$V_2 = A_2 \times L_2 = 3000 \times 2000$$

$$= 6 \times 10^6 \text{ mm}^3$$

$$P = 90 \text{ kN} = 90 \times 10^3 \text{ N}$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$



To find

Total strain energy produced in the bar

To compare strain energy produced in this bar and uniform bar

Solution

$$\sigma_1 = \frac{P}{A} = \frac{90 \times 10^3}{2000} = 45 \text{ N/mm}^2$$

$$\text{Strain energy } U_1 = \frac{\sigma_1^2}{2E} \times V_1 = \frac{45^2}{2 \times 2 \times 10^5} \times 6 \times 10^6$$

$$= 30375 \text{ N-mm} = 30.3 \text{ Nm}$$

$$U_1 = 30.3 \text{ N-m}$$

$$\text{Part-II} \quad \sigma_2 = \frac{P}{A_2} = \frac{90000}{3000} \\ = 30 \text{ N/mm}^2$$

$$U_2 = \frac{\sigma_2^2}{2E} \times V_2 = \frac{30^2}{2 \times 2 \times 10^5} \times 6 \times 10^6 \\ = 13.5 \text{ Nm.}$$

Strain energy stored in a Uniform bar

$$V = V_1 + V_2 \\ = 6 \times 10^6 + 6 \times 10^6 = 12 \times 10^6 \text{ mm}^3$$

$$L = L_1 + L_2 \\ = 3000 + 2000 = 5000 \text{ mm.}$$

$$V = A \times L$$

$$12000000 = A \times 5000$$

$$A = 2400 \text{ mm}^2$$

$$\text{Stress in Uniform bar } \sigma = \frac{P}{A} = \frac{90000}{2400} \\ = 37.5 \text{ N/mm}^2$$

$$\text{Strain energy stored in Uniform bar } U = \frac{\sigma^2}{2E} \times V$$

$$= \frac{37.5^2}{2 \times 2 \times 10^5} \times 12000000$$

$$= 42,187 \text{ N-mm}$$

$$U = 42.187 \text{ N-m}$$

$$\frac{\text{Strain energy in given bar}}{\text{Strain energy in Uniform bar}} = \frac{43.8}{42.187} = 1.03$$

Result

$$U = 43.8 \text{ N-m}$$

$$\text{Ratio of Strain energy} = 1.03$$

THIN CYLINDER, SPHERES AND THICK CYLINDER

PATR A

1. Distinguish between thin walled cylinder and thick walled cylinder?

In thin walled cylinder, thickness of the wall of the cylindrical vessel is less than $1/15$ to $1/20$ of its internal diameter. Stress distribution is uniform over the thickness of the wall. If the ratio of thickness to its internal diameter is more than $1/20$, then cylindrical shell is known as thick cylinders. The stress distribution is not uniform over the thickness of the wall.

2. What are the two type of stress developed in thin cylinder subjected to internal pressure. (Dec 2011,May 2012)

1. Hoop stress
2. Longitudinal stress

3. Define hoop and longitudinal stress (May 2013,Dec 2014)

Hoop stress:

The stress acting along the circumference of the cylinder is called circumference or hoop stress

Longitudinal stress:

The stress acting along the length of the cylinder is known as longitudinal stress

4. For what purpose are the cylindrical and spherical shells used?

The cylindrical and spherical shells are used generally as containers for storage of liquids and gases under pressure.

5. What are assumptions made in the analysis of thin cylinders?

Radial stress is negligible.

Hoop stress is constant along the thickness of the shell.

Material obeys Hooke's law.

Material is homogeneous and isotropic.

6. Write the change in diameter and change in length of a thin cylindrical shell due to internal pressure, P.

$$\text{Change in diameter } \delta d = PD^2 / 2tE(1-1/2m)$$

Change in length $\delta l = PDl / 2tE(1/2 - 1/m)$

Where P=internal pressure of fluid D= diameter of the cylindrical shell

t = thickness of the cylindrical shell L= length of cylindrical

$1/m =$ Poisson ratio

7. What are the assumptions in lames theorem?

- i) The material is homogeneous and isotropic
- ii) The material is stressed within elastic limit

8. How many stresses are developed in thick cylinders? Name them.(May/Jun 2012)

Three types of stresses are developed in thick cylinders.

- i) Radial stress
- ii) Hoop stress
- iii) Longitudinal stress

9. Write lames equation to find out stress in thick cylinder(Dec 2014)

Radial stress $\sigma_r = b/r^2 - a$

Hoop stress $\sigma_c = b/r^2 + a$

10. In a thick cylinder will the radial stress vary over the thickness of wall?

Yes, in the thick cylinder radial stress is maximum at inner and minimum at the outer radius

11. Define radial pressure in thin cylinder.

The radial stress for a thick-walled cylinder is equal and opposite to the gauge pressure on the inside surface, and zero on the outside surface. The circumferential stress and longitudinal stresses are usually much larger for pressure vessels, and so for thin-walled instances, radial stress is usually neglected.

12. How does a thin cylinder fail due to internal fluid pressure?

Failure of materials under combined tensile and shear stresses is not simple to predict.

Maximum Principal Stress Theory

Component fails when one of the principal stresses exceeds the value that causes failure in simple tension

Maximum Shear Stress Theory

Component fails when maximum shear stress exceeds the shear stress that causes failure in simple tension

Maximum Strain Energy Theory

Component fails when strain energy per unit volume exceeds the value that causes failure in simple tension

UNIT-3

Thin Cylinders, spheres and Thick Cylinders

PART B

1. A Cylinder shell 800 mm in diameter, 3 m long is having 10 mm metal thickness. If the shell is subjected to an internal pressure of 2.5 N/mm^2 .

(i) The change in diameter.

(ii) The change in length

(iii) The change in volume.

Assume the modulus of elasticity and poisson's ratio of the material of the shell as 200 kN/mm^2 and 0.25 respectively.

Solution:

(i) Change in diameter is given by

$$\delta d = \frac{Pd^2}{4tE} (L - \nu)$$

$$\delta d = \frac{2.5 \times 800^2}{4 \times 10 \times 200 \times 10^3} (2 - 0.25)$$

$$\delta d = 0.35 \text{ mm.}$$

(ii) Change in length is given by

$$\delta L = \frac{PdL}{4tE} (1 - 2\nu)$$

$$\delta L = \frac{2.5 \times 800 \times 3 \times 10^3}{4 \times 10 \times 200 \times 10^3} (1 - 2 \times 0.25)$$

$$\bar{\Delta}L = 0.375 \text{ mm}$$

(iii) change in volume is given by

$$\bar{\Delta}v = \frac{Pd}{4tE} (5-4\nu)v$$

$$v = \pi/4 d^2 L$$

$$\bar{\Delta}v = \frac{2.5 \times 800}{4 \times 10 \times 200 \times 10^3} (5-4 \times 0.25) \times \frac{\pi}{4} \times \frac{800^2 \times 3 \times 10^3}{800^2 \times 3 \times 10^3}$$

$$\bar{\Delta}v = 1.508 \times 10^6 \text{ mm}^3.$$

2. A Spherical shell of 2m diameter is made up of 10 mm thick plates. Calculate the change in diameter and volume of the shell, when its subjected to an internal pressure of 1.6 MPa. Take $E = 200 \text{ GPa}$ and $\nu = 0.3$.

Solution:

$$\bar{\Delta}d = \frac{Pd^2}{4tE} (1-\nu)$$

$$P = 1.6 \text{ MPa}$$

$$d = 2 \text{ m} = 2 \times 10^3 \text{ mm}$$

$$t = 10 \text{ mm}$$

$$E = 200 \text{ GPa} = 200 \times 10^3 \text{ MPa}$$

$$\nu = 1/3 = 0.3$$

$$\bar{\Delta}d = \frac{1.6 \times (2 \times 10^3)^2 (1-0.3)}{4 \times 10 \times 200 \times 10^3}$$

$$\bar{\Delta}d = 0.56 \text{ mm}$$

$$\Delta V = \frac{\pi P d^4}{8 t E} (1 - \nu)$$

$$= \frac{\pi \times 1.6 \times (2 \times 10^3)^4 (1 - 0.3)}{8 \times 10 \times 200 \times 10^3}$$

$$\Delta V = 3.52 \times 10^6 \text{ mm}^4.$$

Thick Cylinder.

3. Find the thickness of metal necessary for a thick cylindrical shell of internal diameter 160 mm to withstand an internal pressure of 8 N/mm². The maximum hoop stress in the section is not exceed 35 N/mm².

Solution:

$$r = 80 \text{ mm}$$

$$P_r = 8 \text{ N/mm}^2$$

$$\sigma_r = 35 \text{ N/mm}^2$$

By Lame's equation,

$$P_r = \frac{B}{r^2} - A$$

$$8 = \frac{B}{80^2} - A \rightarrow \textcircled{1}$$

$$\sigma_r = \frac{B}{r^2} + A$$

$$35 = \frac{B}{80^2} + A \rightarrow \textcircled{2}$$

From equations $\textcircled{1}$ and $\textcircled{2}$

$$A = 13.5$$

$$B = 137600$$

$$P_x = \frac{137600}{x^2} - 13.5$$

At the outer surface, $P_x = 0$, $x = 80 + t$

$$0 = \frac{137600}{(80+t)^2} - 13.5$$

$$(80+t)^2 = \frac{137600}{13.5}$$

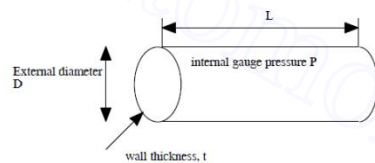
$$t = 20.96 \text{ mm}$$

RESULT:

$$\sigma_x = 35 \text{ N/mm}^2$$

$$t = 20.96 \text{ mm}$$

4. Derivation - Deformation in Thin Cylinder.



When a thin cylindrical shell is subjected to an internal pressure, there will be an increase in diameter as well as length of the shell there by subjected to lateral and linear strains,

Consider a thin cylinder of radius (r) and thickness (t).

Let,

L = Length of the cylindrical shell

r = Radius of the cylindrical shell.

t = Thickness of the cylindrical shell

P = Intensity of pressure inside shell
 E = Young's Modulus of material shell.
 $\mu = 1/m$ = Poisson's ratio.
 d = Diameter of cylindrical shell.

(i) Increase in diameter.

Circumferential strain in shell

$$e_c = \frac{\text{Increase in Diameter}}{\text{Original diameter}}$$

$$e_c = \frac{\Delta d}{d} = \frac{\sigma_c}{E} = \frac{\sigma_L}{mE}$$

where,

σ_c = Circumferential or hoop stress

σ_L = Longitudinal stress.

$$e_c = \frac{1}{E} [\sigma_c - \sigma_L/m]$$

$$e_c = \frac{1}{E} \left[\frac{Pr}{t} - \frac{Pr}{2tm} \right]$$

$$e_c = \frac{1}{E} \frac{Pr}{t} \left[1 - \frac{1}{2m} \right]$$

$$= \frac{Pr}{Et} \left[1 - \frac{\mu}{2} \right] \quad \left[\because 1/m = \mu \text{ poisson's ratio} \right]$$

Circumferential strain

$$e_c = \frac{Pd}{2Et} \left[1 - \frac{\mu}{2} \right].$$

$$\text{Now increase in diameter } \Delta d = \frac{Pd^2}{2Et} \left[1 - \frac{\mu}{2} \right]$$

$$= \frac{Pd^2}{4Et} \left[2 - \mu \right]$$

ii) Increase in length (ΔL):

We know that the longitudinal strain is given by,

$$e_l = \frac{\text{Increase in length}}{\text{Original length}} = \frac{\Delta L}{L} = \frac{\sigma_L}{E} - \frac{\sigma_C}{mE}$$

$$e_l = \frac{1}{E} \left[\frac{P_n}{2t} - \frac{P_n}{tm} \right] = \frac{P_n}{Et} [1/2 - \mu]$$

$$\text{Longitudinal strain } (e_l) = \frac{P_n}{2Et} [1 - 2\mu]$$

$$e_l = \frac{Pd}{4Et} [1 - 2\mu]$$

$$\text{Now increase in length } (\Delta L) = e_l \cdot L$$

$$\text{Increase in length } (\Delta L) = \frac{PdL}{4Et} [1 - 2\mu]$$

(iii) Increase in the volume (ΔV):

We know that volumetric strain

$$e_v = \frac{\text{change in volume } (\Delta V)}{\text{Initial volume } (V)}$$

$$= \frac{V_2 - V_1}{V_1}$$

$$e_v = \frac{\pi/4 (d + \Delta d)^2 (L + \Delta L) - \pi d^2/4 L}{\pi d^2/4 L}$$

$$e_v = \frac{[d^2 + 2 \cdot d \cdot \delta d + (\delta d)^2] [1 + \delta L] - \delta^2 L}{d^2 L}$$

$$e_v = \frac{[\delta^2 L + d^2 \cdot \delta L + 2 \cdot d \cdot \delta d \cdot L + 2 \cdot d \cdot \delta d \cdot L + L \cdot \delta d^2 + \delta L \cdot \delta d^2 - d^2 L]}{d^2 L}$$

Neglecting, higher powers of δd , δd and other small quantities, we have

$$e_v = \frac{d \cdot \delta L + 2 \cdot d \cdot L \cdot \delta d}{d^2 L}$$

$$e_v = \frac{2 \cdot \delta d}{d} + \delta L/L = 2e_c + e_l$$

$$e_v = 2 \cdot \left[\frac{P_d}{2Et} (1 - \mu/2) \right] + \frac{P_d}{4Et} [1 - 2\mu]$$

$$e_v = \frac{P_d}{Et} \left[1 - \mu/2 + 1/4 (1 - 2\mu) \right]$$

$$= \frac{P_d}{Et} \left[5 - 4\mu/4 \right]$$

Change or increase in volume

$$\delta V = e_v \cdot V$$

Increase or change in

$$\text{volume } (\delta V) = \frac{P_d \cdot V}{4Et} [5 - \mu]$$

5. Determine the maximum and minimum hoop stress across the section of a pipe of 400 mm internal diameter and 100 mm thick when the pipe contains a fluid at a pressure of 8 N/mm^2 . Also sketch the radial pressure distribution and hoop stress distribution across the section.

Given data

$$\text{Internal dia } d_1 = 400 \text{ mm}$$

$$r_1 = 200 \text{ mm}$$

$$t = 100 \text{ mm}$$

$$d_2 = 400 + 2 \times 100 = 600 \text{ mm}$$

$$r_2 = 300 \text{ mm}$$

$$\text{fluid pressure } P_0 = 8 \text{ N/mm}^2$$

To find

maximum + minimum hoop stress

Solution

$$\text{B.C (I) } x = r_1 = 200 \text{ mm}; P_x = 8 \text{ N/mm}^2$$

$$\text{(II) } x = r_2 = 300 \text{ mm } P_x = 0.$$

$$8 = \frac{b}{200^2} - a = \frac{b}{40000} - a$$

$$0 = \frac{b}{300^2} - a = \frac{b}{90000} - a$$

$$\therefore 8 = \frac{b}{40000} - \frac{b}{90000} = \frac{9b - 4b}{360000}$$

$$= \frac{5b}{360000}$$

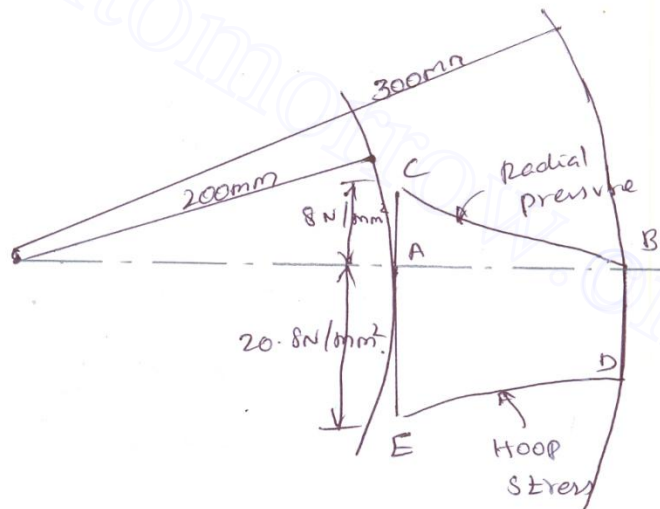
$$b = \frac{360000 \times 8}{5} = 576000$$

$$0 = \frac{576000}{90000} - a; \quad a = \frac{576000}{90000} = 6.4$$

$$\sigma_x = \frac{b}{x^2} + a = \frac{576000}{x^2} + 6.4$$

$$\text{At } x = 200 \text{ mm} \quad \sigma_{200} = \frac{576000}{200^2} + 6.4 = 14.4 + 6.4 = 20.8 \text{ N/mm}^2$$

$$x = 300 \text{ mm} \quad \sigma_{300} = \frac{576000}{300^2} + 6.4 = 6.4 + 6.4 = 12.8 \text{ N/mm}^2$$



PART C

6. A compound cylinder is made by shrinking a cylinder of external diameter 300 mm and internal diameter of 250 mm over another cylinder of external diameter 250 mm and internal diameter 200 mm. The radial pressure at the junction after shrinking

13 8 N/mm^2 . Find the final stresses set up across the section, when the compound cylinder is subjected to an internal fluid pressure of 84.5 N/mm^2 .

Given data

External diameter = 300 mm

$$r_2 = 150 \text{ mm}$$

Internal diameter = 250 mm

$$r^* = 125 \text{ mm}$$

for Inner Cylinder

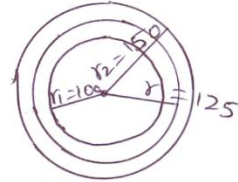
Internal diameter = 200 mm

$$r_1 = 100 \text{ mm}$$

Radial pressure $p^* = 8 \text{ N/mm}^2$

fluid pressure in the compound cylinder

$$P = 84.5 \text{ N/mm}^2$$



Solution

(i) stresses due to shrinking in the outer + inner cylinders before the fluid pressure is admitted.

for Outer cylinder

$$P_x = \frac{b_1}{x^2} - a_1 \quad + \quad \sigma_x = \frac{b_1}{x^2} + a_1$$

$$x = 150 \text{ mm}; P_x = 0$$

$$0 = \frac{b_1}{150^2} - a_1 = \frac{b_1}{22500} - a_1$$

$$x = r^* = 125 \text{ mm}, P_x = p^* = 8 \text{ N/mm}^2$$

$$8 = \frac{b_1}{125^2} - a_1 = \frac{b_1}{15625} - a_1$$

$$\therefore 8 = -\frac{b_1}{22500} + \frac{b_1}{15625} = \frac{(-15625 + 22500) b_1}{22500 \times 15625}$$

$$b_1 = \frac{8 \times 22500 \times 15625}{(-15625 + 22500)} = 409090.9$$

$$0 = \frac{409090.9}{22500} - a_1 \quad (\text{or}) \quad a_1 = \frac{409090.9}{22500} = 18.18$$

$$\sigma_x = \frac{409090.9}{x^2} + 18.18$$

$$\sigma_{150} = \frac{409090.9}{150^2} + 18.18 = 36.36 \text{ N/mm}^2$$

$$\sigma_{125} = \frac{409090.9}{125^2} + 18.18 = 44.36 \text{ N/mm}^2$$

for inner cylinder

$$P_x = \frac{b_2}{x^2} - a_2 \quad \sigma_x = \frac{b_2}{x^2} + a_2$$

$$x = r_1 = 100 \text{ mm}; P_x = 0.$$

$$0 = \frac{b_2}{100^2} - a_2 = \frac{b_2}{10000} - a_2$$

$$x = r^* \quad , \quad = 125 \text{ mm} \quad P_x = P^* = 8 \text{ N/mm}^2$$

$$8 = \frac{b_2}{125^2} - a_2 = \frac{b_2}{15625} - a_2$$

$$8 = \frac{b_2}{15625} - \frac{b_2}{10000} = \frac{-5625 b_2}{15625 \times 10000}$$

$$b_2 = \frac{8 \times 15625 \times 10000}{5625} = -222222.2$$

$$0 = \frac{-222222.2}{x^2} - a_2$$

$$a_2 = -22.22$$

$$\sigma_x = \frac{-222222.2}{x^2} - 22.22$$

$$\sigma_{125} = \frac{-222222.2}{125^2} - 22.22 = -36.22 \text{ N/mm}^2$$

$$\sigma_{100} = \frac{-222222.2}{100^2} - 22.22 = -44.44 \text{ N/mm}^2$$

(ii) Stresses due to fluid pressure alone

$$P_x = \frac{B}{x^2} - A \quad \sigma_x = \frac{B}{x^2} + A$$

$$x = 100 \text{ mm} \quad P_x = P = 84.5 \text{ N/mm}^2$$

$$x = 150 \text{ mm} \quad P_x = 0$$

$$0 = \frac{B}{150^2} - A = \frac{B}{22500} - A$$

$$87.5 = \frac{B}{10000} - \frac{B}{22500}$$

$$B = \frac{87.5 \times 10000 \times 22500}{12500} = 1521000$$

$$0 = \frac{1521000}{22500} - A ; \quad A = \frac{1521000}{22500} = 67.6$$

$$\sigma_x = \frac{1521000}{x^2} + 67.6$$

$$\sigma_{100} = \frac{1521000}{100^2} + 67.6 = 219.7 \text{ N/mm}^2$$

$$\sigma_{125} = \frac{1521000}{125^2} + 67.6 = 167.94 \text{ N/mm}^2$$

$$\sigma_{150} = \frac{1521000}{150^2} + 67.6 = 135.2 \text{ N/mm}^2$$

Inner cylinder

$$f_{100} = \sigma_{100} \text{ Shrinkage} + \sigma_{100} \text{ (Internal) Fluid pressure}$$

$$= -47.44 + 219.7 = 175.26 \text{ N/mm}^2$$

$$f_{125} = -36.22 + 167.94 = 128.72 \text{ N/mm}^2$$

Outer cylinder.

$$f_{125} = \sigma_{125} \text{ Shrinkage} + \sigma_{125} \text{ Internal fluid pressure}$$
$$= 44.36 + 164.94 = 209.3 \text{ N/mm}^2.$$

$$f_{150} = \sigma_{150} \text{ Shrinkage} + \sigma_{150} \text{ Internal fluid pressure}$$
$$= 36.36 + 135.2 = 171.56 \text{ N/mm}^2.$$

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Reg. No. :

Question Paper Code : 97029

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

Third Semester

Mechanical Engineering

CE 6306 — STRENGTH OF MATERIALS

(Common to Mechatronics Engineering, Industrial Engineering and Management,
Industrial Engineering, Manufacturing Engineering, Mechanical Engineering
(sandwich) and Material Science and Engineering)

(Regulation 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Derive a relation for change in length of a bar hanging freely under its own weights.
2. Write the relationship between shear modulus and young's modulus of elasticity. **unit 1 qno 5**
3. Draw SFD for a 6 m cantilever beam carrying a clockwise moment of 6 kN-m at free end. **unit 1 Qno 11**
4. What are flitched beams?
5. What is meant by torsional rigidity? **unit3 Qno 5**
6. Differentiate open coiled and closely coiled helical springs. **unit 3 Q no 8**
7. What are the limitations of double integration method? **unit 4 Q no 9**
8. Define strain energy. **unit 4 Q no 10**
9. What is meant by circumferential stress?
10. Write down Lamé's equations.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Derive an expression for change in length of a circular bar with uniformly varying diameter and subjected to an axial tensile load 'P' (8)
- (ii) A member is subjected to point loads as shown in Fig. Q. 11(a). Calculate the force P, necessary for equilibrium if $P_1 = 45$ kN, $P_3 = 450$ kN and $P_4 = 130$ kN. Determine total elongation of the member, assuming the modulus of elasticity to be $E = 2.1 \times 10^5$ N/mm².

unit 1 Q no 3 (8)

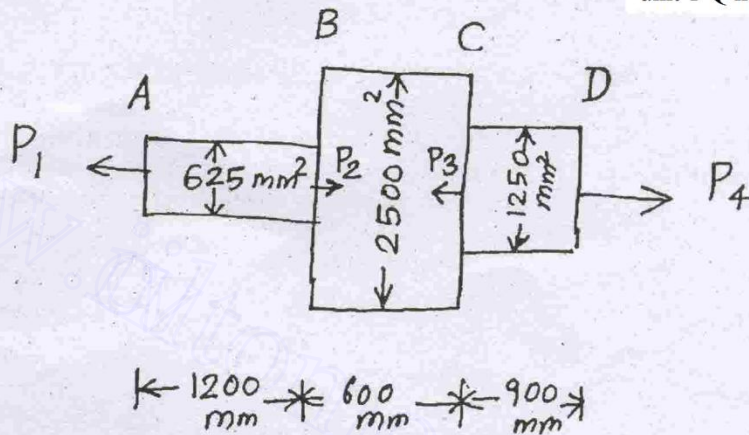


Fig. Q. 11(a)

Or

- (b) A metallic bar 300 mm (x) × 100 mm (y) × 40 mm (z) is subjected to a force of 5 kN (tensile), 6 kN (tensile) and 4 kN (tensile) along x, y and z directions respectively. Determine the change in the volume of the block. Take $E = 2 \times 10^5$ N/mm² and Poisson's ratio = 0.25. unit 1 Qno 4
12. (a) Draw SFD and BMD and find the maximum bending moment for the beam given in Fig. Q. 12(a).

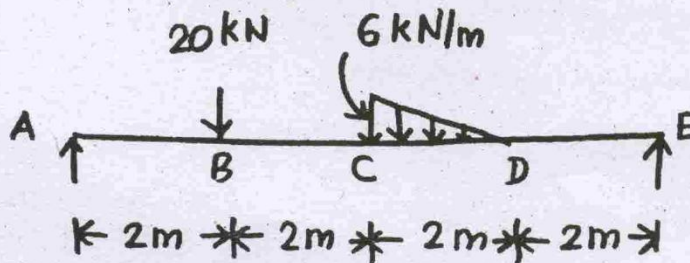


Fig. Q. 12(a)

Or

- (b) Prove that the ratio of depth to width of the strongest beam that can be cut from a circular log of diameter 'd' is 1.414. Hence calculate the depth and width of the strongest beam that can be cut out of a cylindrical log of wood whose diameter is 300 mm.

13. (a) Derive torsion equation. **unit 3 Q no 1**

Or

- (b) The stiffness of a close-coiled helical spring is 1.5 N/mm of compression under a maximum load of 60 N. The maximum shearing stress produced in the wire is 125 N/mm². The solid length of the spring (when the coils are touching) is given as 50 mm. Find **unit 3 Qno 3**

- (i) The diameter of wire
 (ii) The mean diameter of the coils
 (iii) Number of coils required.

Take $C = 4.5 \times 10^4$ N/mm².

14. (a) Determine the deflection of the beam at its mid span and also the position of maximum deflection and maximum deflection. Take $E = 2 \times 10^5$ N/mm² and $I = 4.3 \times 10^8$ mm⁴. Use Macaulay's method. The beam is given in Fig. Q. 14(a).

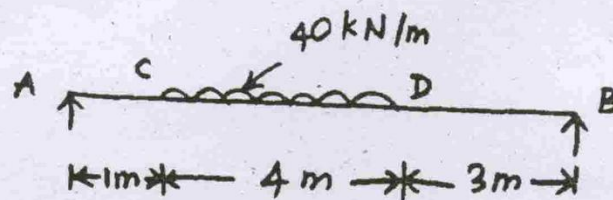


Fig. Q. 14(a)

Or

- (b) Using conjugate beam method, determine the
 (i) Slope at each end and under each load
 (ii) Deflection under each load.

for the beam given in Fig. Q. 14(b). Take $E = 2 \times 10^5$ N/mm² and $I = 10^8$ mm⁴.

unit 4 Q no 2

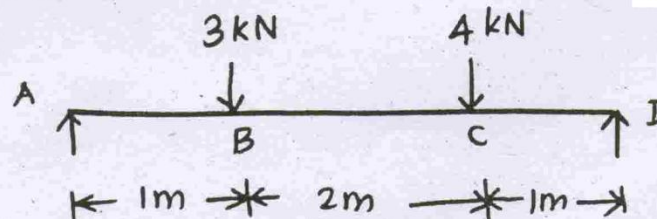


Fig. Q. 14(b)

15. (a) Derive relations for change in dimensions and change in volume of a thin cylinder subjected to internal pressure P . unit 5 Qno 4

Or

- (b) Find the thickness of metal necessary for a thick cylindrical shell of internal diameter 160 mm to withstand an internal pressure of 8 N/mm^2 . The maximum hoop stress in the section is not to exceed 35 N/mm^2 .

unit 5 Q no 3

Question Paper Code : 77058

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Third Semester

Mechanical Engineering

CE 6306 — STRENGTH OF MATERIALS

(Common to Mechatronics Engineering, Industrial Engineering and Management, Industrial Engineering, Manufacturing Engineering, Mechanical Engineering (Sandwich) Material Science and Engineering and also Common to Fourth Semester Automobile Engineering, Mechanical and Automation Engineering and Production Engineering)

(Regulation 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What do you mean by thermal stresses?
2. Draw the Mohr's circle for the state of pure shear in a strained body and mark all salient points in it.
3. Define : unit 2 Qno 3
 - (a) Shearing force and
 - (b) Bending moment.
4. What is neutral axis of a beam section? How do you locate it when a beam is under simple bending?
5. What is meant by torsional stiffness?
6. What are the uses of helical springs?
7. What are the advantages of Macaulay's method over other methods for the calculation of slope and deflection?

8. In a cantilever beam, the measured deflection at free end was 8 mm when a concentrated load of 12 kN was applied at its mid-span. What will be the deflection at mid-span when the same beam carries a concentrated load of 7 kN at the free end?
9. Distinguish between thin and thick shells. unit 5 Qno 1
10. State the assumptions made in Lamé's theorem for thick cylinder analysis. unit 5 Qno 7

PART B — (5 × 16 = 80 marks)

11. (a) A steel rod of diameter 32 mm and length 500 mm is placed inside an aluminium tube of internal diameter 35 mm and external diameter 45 mm which is 1 mm longer than the steel rod. A load of 300 kN is placed on the assembly through the rigid collar. Find the stress induced in steel rod and aluminium tube. Take the modulus of elasticity of steel as 200 GPa and that of aluminium as 80 GPa.
- Or
- (b) At a point in a strained material the resultant intensity of stress across a vertical plane is 100 MPa tensile inclined at 35° clockwise to its normal. The normal component of intensity of stress across the horizontal plane is 50 MPa compressive. Determine graphically using Mohr's circle method :
unit 1 Q no 7(B)
- (i) The position of principal planes and stresses across them and
(ii) The normal and tangential stress across a plane which is 60° clockwise to the vertical plane.
12. (a) An overhanging beam ABC of length 7 m is simply supported at A and B over a span of 5 m and the portion BC overhangs by 2 m. Draw the shearing force and bending moment diagrams and determine the point of contra-flexure if it is subjected to uniformly distributed loads of 3 kN/m over the portion AB and a concentrated load of 8 kN at C.
- Or
- (b) Three beams have the same length, the same allowable stress and the same bending moment. The cross-section of the beams are a square, a rectangle with depth twice the width and a circle. Find the ratios of weights of the circular and the rectangular beams with respect to the square beam.
13. (a) A brass tube of external diameter 80 mm and internal diameter 50 mm is closely fitted to a steel rod of 50 mm diameter to form a composite shaft. If a torque of 10 kNm is to be resisted by this shaft, find the maximum stresses developed in each material and the angle of twist in 2 m length. Take modulus of rigidity of brass and steel as 40×10^3 N/mm² and 80×10^3 N/mm² respectively.

Or

2

77058

(b) A close-coiled helical spring is to have a stiffness of 900 N/m in compression, with a maximum load of 45 N and a maximum shearing stress of 120 N/mm². The solid length of the spring (i.e., coils touching) is 45 mm. Find :

unit 3 Qno 3

- (i) The wire diameter,
- (ii) The mean coil radius, and
- (iii) The number of coils. Take modulus of rigidity of the material of the spring as 0.4×10^5 N/mm².

14. (a) A horizontal beam of uniform section and 7 m long is simply supported at its ends. The beam is subjected to a uniformly distributed load of 6 kN/m over a length of 3 m from the left end and a concentrated load of 12 kN at 5 m from the left end. Find the maximum deflection in the beam using Macaulay's method.

Or

(b) A cantilever of span 4 m carries a uniformly distributed load of 4 kN/m over a length of 2 m from the fixed end and a concentrated load of 10 kN at the free end. Determine the slope and deflection of the cantilever at the free end using conjugate beam method. Assume EI is uniform throughout.

15. (a) A thin cylindrical shell, 2.5 m long has 700 mm internal diameter and 8 mm thickness. if the shell is subjected to an internal pressure of 1 MPa, find

- (i) The hoop and longitudinal stresses developed
- (ii) Maximum shear stress induced and
- (iii) The changes in diameter length and volume. Take modulus of elasticity of the wall material as 200 GPa and Poisson's ratio as 0.3.

Or

(b) A thick cylinder with external diameter 320 mm and internal diameter 160 mm is subjected to an internal pressure of 8 N/mm². Draw the variation of radial and hoop stresses in the cylinder wall. Also determine the maximum shear stress in the cylinder wall. unit 5 Q no 5

Reg. No. :

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Question Paper Code : 27099

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Third Semester

Mechanical Engineering

CE 6306 — STRENGTH OF MATERIALS

(Common to Mechatronics Engineering, Industrial Engineering and Management, Industrial Engineering, Manufacturing Engineering, Mechanical Engineering (Sandwich) Material Science and Engineering and also Common to Fourth Semester Automobile Engineering, Mechanical and Automation Engineering and Production Engineering)

(Regulations 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Differentiate Elasticity and Elastic Limit.
2. What is principle of super position?
3. Write the assumption in the theory of simple bending?
4. What are the types of beams?
5. The shearing stress in a solid shaft is not to exceed 40 N/mm^2 when the torque transmitted is 20000 N-m . Determine the minimum diameter of the shaft.
6. What are the various types of springs?
7. What are the methods of determining slope and deflection at a section in a loaded beam?
8. What is the equation used in the case of double integration method?
9. State the expression for maximum shear stress in a cylindrical shell.
10. Define — hoop stress and longitudinal stress.

PART B — (5 × 16 = 80 marks)

11. (a) A metallic bar 300 mm × 100 mm × 40 mm is subjected to a force of 50 kN (tensile), 6 kN (tensile) and 4 kN (tensile) along x, y and z directions respectively. Determine the change in the volume of the block. Take $E = 2 \times 10^5 \text{ N/mm}^2$ and Poisson's ratio = 0.25.

Or

- (b) A steel rod of 3 cm diameter is enclosed centrally in a hollow copper tube of external diameter 5 cm and internal diameter of 4 cm as shown in Fig-1. The composite bar is then subjected to axial pull of 45000 N. If the length of each bar is equal to 15 cm, determine: (i) The stresses in the rod and tube, and (ii) Load carried by each bar. Take E for steel = $2.1 \times 10^5 \text{ N/mm}^2$ and for copper = $1.1 \times 10^5 \text{ N/mm}^2$.

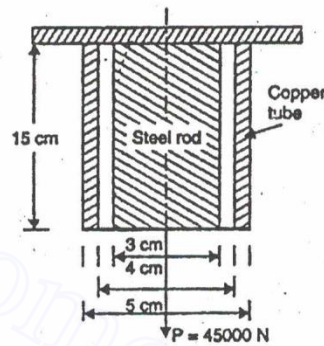


Fig. 1

12. (a) Draw the shear force and B.M diagrams for a simply supported beam of length 8 m and carrying a uniformly distributed load of 10 kN/m for a distance of 4m as shown in fig-2.

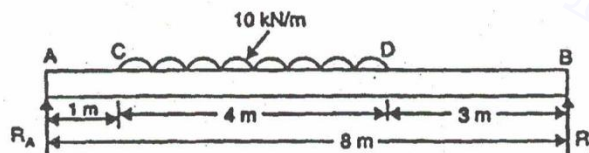


Fig. 2

Or

- (b) A steel plate of width 120 mm and of thickness 20 mm is bent into a circular arc of radius 10 m. Determine the maximum stress induced and the bending moment which will produce the maximum stress. Take $E = 2 \times 10^5 \text{ N/mm}^2$.

13. (a) A hollow shaft of external diameter 120 mm transmits 300 kW power at 200 r.p.m. Determine the maximum internal diameter if the maximum stress in the shaft is not to exceed 60 N/mm^2 .

Or

- (b) A closely coiled helical spring of mean diameter 20 cm is made of 3 cm diameter rod and has 16 turns. A weight of 3 kN is dropped on this spring. Find the height by which the weight should be dropped before striking the spring so that the spring may be compressed by 18 cm. Take $C = 8 \times 10^4 \text{ N/mm}^2$.

14. (a) A beam 6m long, simply supported at its ends, is carrying a point load of 50 kN at its centre. The moment of inertia of the beam is given as equal to $78 \times 10^6 \text{ mm}^4$. If E for the material of the beam = $2.1 \times 10^5 \text{ N/mm}^2$, calculate : (i) deflection at the centre of the beam and (ii) slope at the supports.

Or

- (b) A beam of length 6 m is simply supported at its ends and carries two point loads of 48 kN and 40 kN at a distance of 1 m and 3 respectively from the left support as shown Fig-3.

Using Macaulay's method find:

- (i) deflection under each load,
- (ii) maximum deflection, and
- (iii) the point at which maximum deflection occurs,

Given $E = 2 \times 10^5 \text{ N/mm}^2$ and $I = 85 \times 10^6 \text{ mm}^4$.

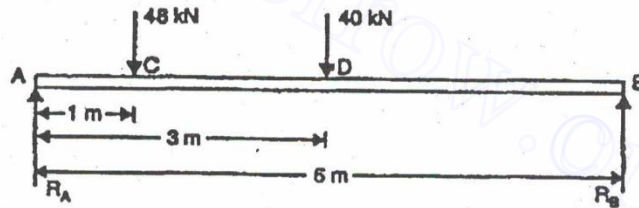


Fig. 3

15. (a) A boiler is subjected to an internal steam pressure of 2 N/mm^2 . The thickness of boiler plate is 2.6 cm and permissible tensile stress is 120 N/mm^2 . Find the maximum diameter, when efficiency of longitudinal joint is 90% and that of circumference point is 40%.

Or

- (b) Calculate: (i) the change in diameter, (ii) change in length and (iii) change in volume of a thin cylindrical shell 100 cm diameter, 1 cm thick and 5 m long when subjected to internal pressure of 3 N/mm^2 . Take the value of $E = 2 \times 10^5 \text{ N/mm}^2$ and Poisson's ratio, $\mu = 0.3$.

Reg. No.

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Question Paper Code : 57150

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2016

Third Semester

Mechanical Engineering

CE 6306 – STRENGTH OF MATERIALS

(Common to MechAtronics Engineering, Industrial Engineering and Management, Industrial Engineering, Manufacturing Engineering, Mechanical Engineering (Sandwich), Material Science and Engineering and also Common to Fourth Semester Automobile Engineering, Mechanical and Automation Engineering and Production Engineering)

(Regulations 2013)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Define principal planes.
2. Obtain the relation between E and K.
3. Discuss the fixed and hinged support.
4. What are the advantages of flitched beams ?
5. Draw and discuss the shafts in series and parallel.
6. List out the stresses induced in the helical and carriage springs.
7. How the deflection and slope is calculated for the cantilever beam by conjugate beam method ?
8. State the Maxwell's reciprocal theorem.
9. Differentiate between thin and thick cylinders.
10. Describe the Lamé's theorem.

PART - B (5 × 16 = 80 Marks)

11. (a) (i) A steel bar 20mm in diameter, 2m long is subjected to an axial pull of 50 kN. If $E = 2 \times 10^5 \text{ N/mm}^2$ and $m = 3$. Calculate the change in the (1) length, (2) diameter and (3) volume. (8)
- (ii) A mild steel bar 20mm in diameter and 40 cm long is encased in a brass tube whose external diameter is 30mm and internal diameter is 25mm. The composite bar is heated through 80°C . Calculate the stresses induced in each metal. α for steel = 11.2×10^{-6} ; α for brass = 16.5×10^{-6} per $^\circ\text{C}$. E for steel = $2 \times 10^5 \text{ N/mm}^2$ and E for brass = $1 \times 10^5 \text{ N/mm}^2$. (8)

OR

- (b) (i) Two steel rods and one copper rod, each of 20 mm diameter, together support a load of 20kN as shown in Fig. Q. 11 (b) (i). Find the stresses in the rods. Take E for steel = 210kN/mm^2 and E for copper = 110 kN/mm^2 . (8)

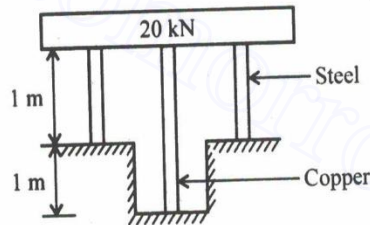


Fig. Q. 11 (b) (i)

- (ii) Direct stresses of 140N/mm^2 tensile and 100N/mm^2 compression exist on two perpendicular planes at a certain point in a body. They are also accompanied by shear stress on the planes. The greatest principal stress at the point due to these is 160 N/mm^2 . (8)
- (1) What must be the magnitude of the shear stresses on the two planes?
- (2) What will be the maximum shear stress at the point?

12. (a) Draw SFD and BMD and indicates the salient features of beam loaded Fig. Q. 12. (a) (16)

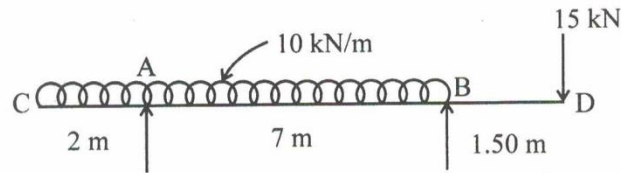


Fig. Q. 12. (a)

OR

- (b) (i) Find the dimensions of a timber joist, span 4 m to carry a brick wall 230 mm thick and 3m high if the unit weight of brickwork is 20 kN/m^3 . Permissible bending stress in timber is 10 N/mm^2 . The depth of the joist is twice the width. (8)
- (ii) A flitched beam shown in Fig. Q. 12. (b) (ii) is used as a load carrying member. Find the moment of resistance of the combined section and bending stress in steel, if $E_s = 2 \times 10^5 \text{ N/mm}^2$, $E_w = 1.25 \times 10^5 \text{ N/mm}^2$. (8)

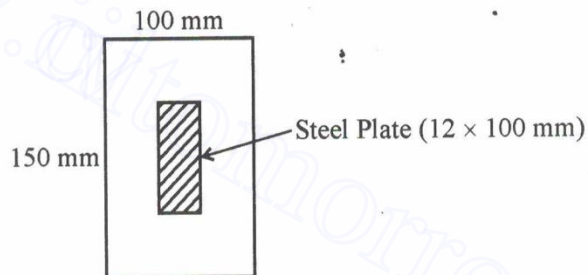


Fig. Q. 12. (b) (ii)

13. (a) A solid circular shaft 200mm in diameter is to be replaced by a hollow shaft the ratio of external diameter to internal diameter being 5:3. Determine the size of the hollow shaft if maximum shear stress is to be the same as that of a solid shaft. Also find the percentage savings in mass. (16)

OR

- (b) (i) A closely coiled helical spring made from round steel rod is required to carry a load of 1000 Newton for a stress of 400 MN/m^2 , the spring stiffness being 20 N/mm . The diameter of the helix is 100mm and G for the material is 80 GN/m^2 . Calculate (1) the diameter of the wire and (2) the number of turns required for the spring. (8)

- (ii) A spiral spring is made of 10 mm diameter wire has 20 close coils, each 100 mm mean diameter. Find the axial load the spring will carry if the stress is not to exceed 200 N/mm^2 . Also determine the extension of the spring. Take $G = 0.8 \times 10^5 \text{ N/mm}^2$. (8)

14. (a) A simply supported beam subjected to uniformly distributed load of $w \text{ kN/m}$ for the entire span. Calculate the maximum deflection by double integration method. (16)

OR

- (b) A simply supported beam AB of span 5m carries a point of 40 kN at its centre. The value of moment of inertia for the left half is $2 \times 10^8 \text{ mm}^4$ and for the right half portion is $4 \times 10^8 \text{ mm}^4$. Find the slopes at the two supports and deflection under the load. Take $E = 200 \text{ GN/m}^2$. (16)

15. (a) (i) A cylindrical vessel is 2 m diameter and 5 m long is closed at ends by rigid plates. It is subjected to an internal pressure of 4 N/mm^2 . If the maximum principal stress is not to exceed 210 N/mm^2 , find the thickness of the shell. Assume $E = 2 \times 10^5 \text{ N/mm}^2$ and Poisson's ratio = 0.3. Find the changes in diameter, length and volume of the shell. (12)
- (ii) A spherical shell of 1.50 m internal diameter and 12 mm shell thickness is subjected to pressure of 2 N/mm^2 . Determine the stress induced in the material of the shell. (4)

OR

- (b) (i) A spherical shell of internal diameter 1.2 m and of thickness 12 mm is subjected to an internal pressure of 4 N/mm^2 . Determine the increase in diameter and increase in volume. Take $E = 2 \times 10^5 \text{ N/mm}^2$ and $\mu = 0.33$. (8)
- (ii) A steel cylinder of 300 mm external diameter is to be shrunk to another steel cylinder of 150 mm internal diameter. After shrinking the diameter at the junction is 250 mm and radial pressure at the common junction is 40 N/mm^2 . Find the original difference in radii at the junction. (8)
- Take $E = 2 \times 10^5 \text{ N/mm}^2$.

Reg. No. :

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Question Paper Code : 80197

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Third Semester

Mechanical Engineering

CE 6306 — STRENGTH OF MATERIALS

(Common to Mechatronics Engineering, Industrial Engineering and Management
Agriculture Engineering, Industrial Engineering, Manufacturing Engineering
Mechanical Engineering (Sandwich), Materials Science and Engineering and also
Common to Fourth Semester Automobile Engineering, Mechanical and Automation
Engineering and Production Engineering)

(Regulations 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define Young's Modulus.
2. What do you mean by principal planes and principal stresses?
3. Draw the shear force diagram and bending moment diagram for the cantilever beam carries uniformly varying load of zero intensity at the free end and w kN/m at the fixed end.
4. List out the assumptions used to derive the simple bending equation.
5. Define torsional rigidity.
6. What is a spring? Name the two important types of springs.
7. List out the methods available to find the deflection of a beam.
8. State Maxwell's reciprocal theorem.
9. Name the stresses develop in the cylinder.
10. Define radial pressure in thin cylinder.

PART B — (5 × 13 = 65 marks)

11. (a) (i) A compound tube consists of a steel tube 140 mm internal diameter and 160 mm external diameter and an outer brass tube 160 mm internal diameter and 180 mm external diameter. The two tubes are of same length. The compound tube carries an axial compression load of 900 kN. Find the stresses and the load carried by each tube and the amount of its shortens. Length of each tube is 140 mm. Take E for steel as 2×10^5 N/mm² and for brass 1×10^5 N/mm². (10)

- (ii) Two members are connected to carry a tensile force of 80 kN by a lap joint with two number of 20 mm diameter bolt. Find the shear stress induced in the bolt. (3)

Or

- (b) (i) A point in a strained material is subjected to the stress as shown in fig. Q.11(b)(i). Locate the principle plane and find the principle stresses. (7)

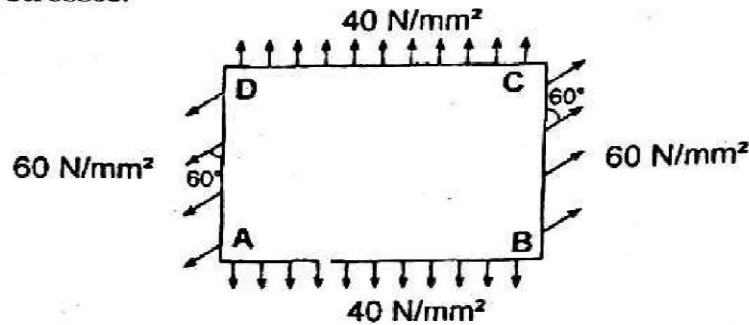


Fig. Q. 11(b)(i)

- (ii) A steel rod of 20 mm diameter passes centrally through a copper tube of 50 mm external diameter and 40 mm internal diameter. The tube is closed at the end by rigid plates of negligible thickness. The nuts are tightened lightly on the projecting parts of the rod. If the temperature of the assembly is raised by 50°C , calculate the stresses developed in copper and steel. Take E for steel as $2 \times 10^5 \text{ N/mm}^2$ and copper as $1 \times 10^5 \text{ N/mm}^2$ and α for steel and copper as 12×10^{-6} per $^{\circ}\text{C}$ and 18×10^{-6} per $^{\circ}\text{C}$. (6)
12. (a) (i) A simply supported beam AB of length 5 m carries point loads of 8 kN, 10 kN and 15 kN at 1.50 m, 2.50, and 4.0 m respectively from left hand support. Draw shear force diagram and bending moment diagram. (8)
- (ii) A cantilever beam AB of length 2 m carries a uniformly distributed load of 12 kN/m over entire length. Find the shear stress and bending stress, if the size of the beam is 230 mm \times 300 mm. (5)

Or

- (b) (i) Construct the SFD and BMD for the beam shown in fig. Q. 12(b)(i). (6)

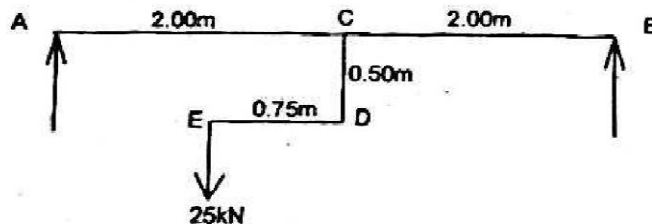


Fig. Q. 12(b)(i)

- (ii) Two timber joist are connected by a steel plate, are used as beam as shown in fig. Q. 12(b)(ii). Find the load W if, the permissible stresses in steel and timber are 165 N/mm^2 and 8.5 N/mm^2 respectively. (7)

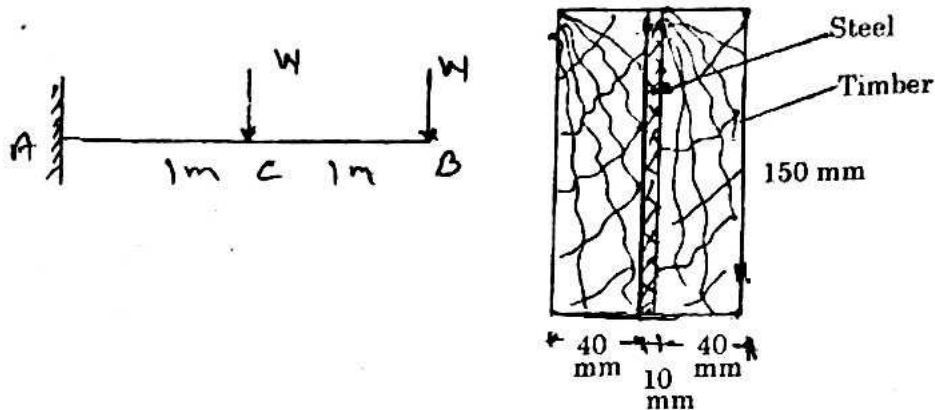


Fig. Q. 12(b)(ii) Cross section

13. (a) (i) A solid shaft has to transmit the Power 105 kW at 2000 r.p.m. The maximum torque transmitted in each revolution exceeds the mean by 36% . Find the suitable diameter of the shaft, if the shear stress is not to exceed 75 N/mm^2 and maximum angle of twist is 1.5° in a length of 3.30 m and $G = 0.80 \times 10^5 \text{ N/mm}^2$. (8)
- (ii) A laminated spring carries a central load of 5200 N and it is made of 'n' number of plates, 80 mm wide, 7 mm thick and length 500 mm . Find the numbers of plates, if the maximum deflection is 10 mm . Let $E = 2.0 \times 10^5 \text{ N/mm}^2$. (5)

Or

- (b) (i) A stepped solid circular shaft is built in at its ends and subject to an externally applied torque T at the shoulder as shown in fig. Q.13(b)(i). Determine the angle of rotation θ of the shoulder section when T is applied. (7)

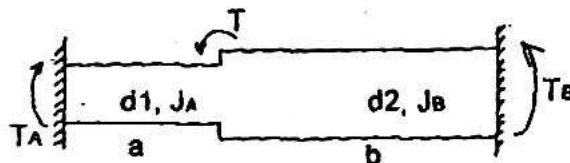


Fig. Q.13(b)(i)

- (ii) A closed coiled helical spring is to be made out of 5 mm diameter wire 2 m long so that it deflects by 20 mm under an axial load of 50 N . Determine the mean diameter of the coil. Take $C = 8.1 \times 10^4 \text{ N/mm}^2$. (6)
14. (a) Cantilever of length l carrying uniformly distributed load $w \text{ kN}$ per unit run over whole length. Derive the formula to find the slope and deflection at the free end by double integration method. Calculate the deflection if, $w = 20 \text{ kN/m}$, $l = 2.30 \text{ m}$ and $EI = 12000 \text{ kN m}^2$. (13)

Or

- (b) (i) Derive the formula to find the deflection of a simply supported beam with point load W at the centre by moment area method. (8)
- (ii) A simply supported beam of span 5.80 m carries a central point load of 37.50 kN, find the maximum slope and deflection, let $EI = 40000 \text{ kN m}^2$. Use conjugate beam method. (5)
15. (a) Calculate Change in diameter, Change in length and Change in volume of a thin cylindrical shell 100 cm diameter, 1 cm thick and 5 m long when subjected to internal pressure of 3 N/mm^2 . Take the value of $E = 2 \times 10^5 \text{ N/mm}^2$ and Poisson's ratio = 0.30. (13)

Or

- (b) Calculate the thickness of metal necessary for a cylindrical shell of internal diameter 160 mm to with stand an internal pressure of 25 MN/m^2 , if maximum permissible shear stress is 125 MN/m^2 . (13)

PART C — ($1 \times 15 = 15$ marks)

16. (a) The intensity of resultant stress on a plane AB (Fig. Q. 16(a)) at a point in a material under stress is 8 N/mm^2 and it is inclined at 30° to the normal to that plane. The normal component of stress on another plane BC at right angles to plane AB is 6 N/mm^2 . Determine the following :
- (i) The resultant stress on the plane BC
- (ii) The principal stresses and their directions
- (iii) The maximum shear stresses. (15)

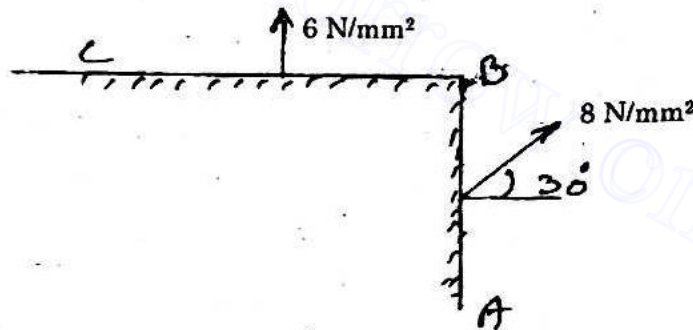


Fig. Q. 16(a)

Or

- (b) A water tank vertical wall is stiffened by vertical beam, and the height of the tank is 8 m. The beams are spaced at 1.5 m centre to centre. If the water reaches the top of the tank, calculate the maximum bending moment on a vertical beam. Sketch the shear force and bending moment diagrams. Unit weight of water = 9.8 kN/m^3 . (15)

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Question Paper Code : 71551

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Third Semester

Mechanical Engineering

CE 6306 — STRENGTH OF MATERIALS

(Common to Mechatronics Engineering, Industrial Engineering and Management, Agriculture Engineering, Industrial Engineering, Manufacturing Engineering, Mechanical Engineering (Sandwich), Materials Science and Engineering and also Common to Fourth Semester Automobile Engineering, Mechanical and Automation Engineering and Production Engineering)

(Regulations 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Derive a relation for change in length of a bar hanging freely under its own weight.
2. What does the radius of Mohr's circle refer to?
3. Draw shear force diagram for a simply supported beam of length 4 m carrying a central point load of 4 kN.
4. Prove that the shear stress distribution over a rectangular section due to shear force is parabolic.
5. Draw shear stress distribution of a circular section due to torque.
6. What is meant by spring constant?
7. Write down the equation for the maximum deflection of a cantilever beam carrying a central point load 'W'.
8. Draw conjugate beam for a double side over hanging beam.
9. How does a thin cylinder fail due to internal fluid pressure?
10. State Lamé's equations.

PART B — (5 × 13 = 65 marks)

11. (a) The bar shown in fig.Q.11(a) is subjected to a tensile load of 160 kN. If the stress in middle portion is limited to 150 N/mm^2 , determine the diameter of the middle portion. Find also the length of the middle portion if the total elongation of the bar is to be 0.2 mm . Young's modulus is $2.1 \times 10^5 \text{ N/mm}^2$.

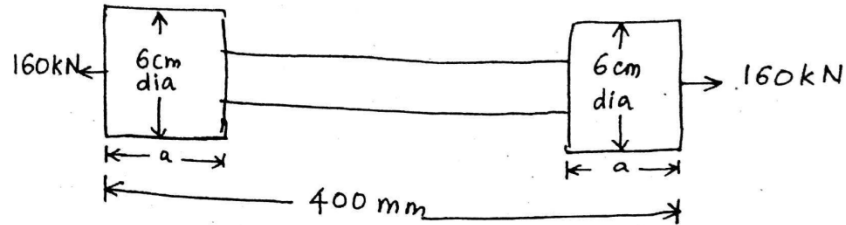


fig.Q.11(a)

Or

- (b) A bar of 30 mm diameter is subjected to a pull of 60 kN . The measured extension on gauge length of 200 mm is 0.1 mm and change in diameter is 0.004 mm . Calculate :
- Young's modulus
 - Poisson's ratio and
 - Bulk modulus.
12. (a) Draw shear force diagram and bending moment diagram for the beam given in fig.Q.12(a)

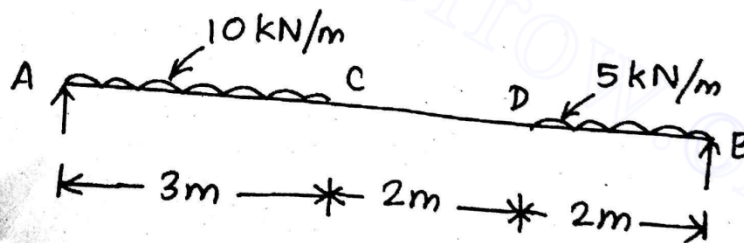


fig.Q.12(a)

Or

- (b) A beam of square section is used as a beam with one diagonal horizontal. The beam is subjected to a shear force F , at a section. Find the maximum shear in the cross section of the beam and draw shear stress distribution diagram for the section.

13. (a) A hollow shaft, having an inside diameter 60% of its outer diameter, is to replace a solid shaft transmitting in the same power at the same speed. Calculate percentage saving in material, if the material to be is also the same.

Or

- (b) Derive a relation for deflection of a closely coiled helical spring subjected to an axial compressive load 'W'.
14. (a) Determine the deflection at its mid point and maximum deflection for the beam given in fig.Q.14(a). Use Macaulay's method.
 $E = 2 \times 10^5 \text{ N/mm}^2$, $I = 4.3 \times 10^8 \text{ mm}^4$.

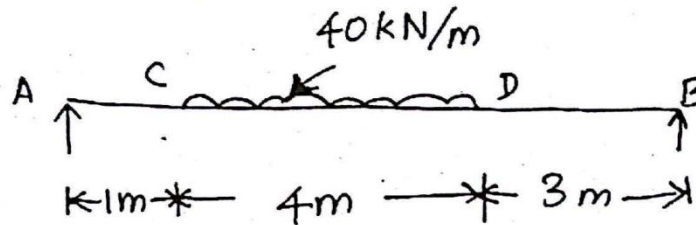


fig.Q.14(a)

Or

- (b) Determine the slope at the two supports and deflection under the loads. Use conjugate beam method. $E = 200 \text{ GN/m}^2$, I for right half is $2 \times 10^8 \text{ mm}^4$, I for left half is $1 \times 10^8 \text{ mm}^4$ the beam is given in fig.Q.14(b).

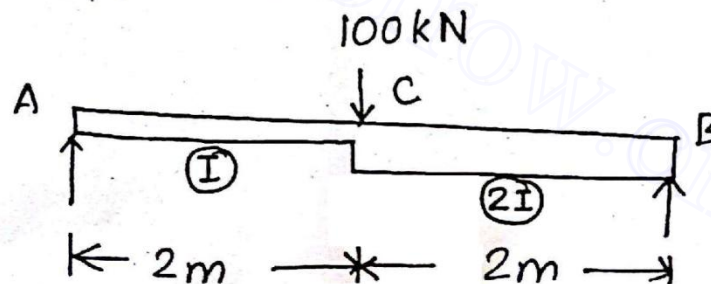


fig.Q.14(b)

15. (a) Derive a relation for change in volume of a thin cylinder subjected to internal fluid pressure.

Or

- (b) Determine the maximum and minimum hoop stress across the section of a pipe of 400 mm internal diameter and 100 mm thick, when the pipe contains a fluid at a pressure of 8 N/mm^2 . Also sketch the radial pressure distribution and hoop stress distribution across the section.

PART C — (1 × 15 = 15 marks)

16. (a) (i) Draw stress strain curve for mild steel and explain the salient points on it. (7)
- (ii) Derive a relation for change in length of a circular bar with uniformly varying diameter, subjected to an axial tensile load 'W'. (8)

Or

- (b) A water main of 500 mm internal diameter and 20 mm thick is full. The water main is of cast iron and is supported at two points 10 m apart. Find the maximum stress in the metal. The cast iron and water weigh 72000 N/m^3 and 10000 N/m^3 respectively.

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