

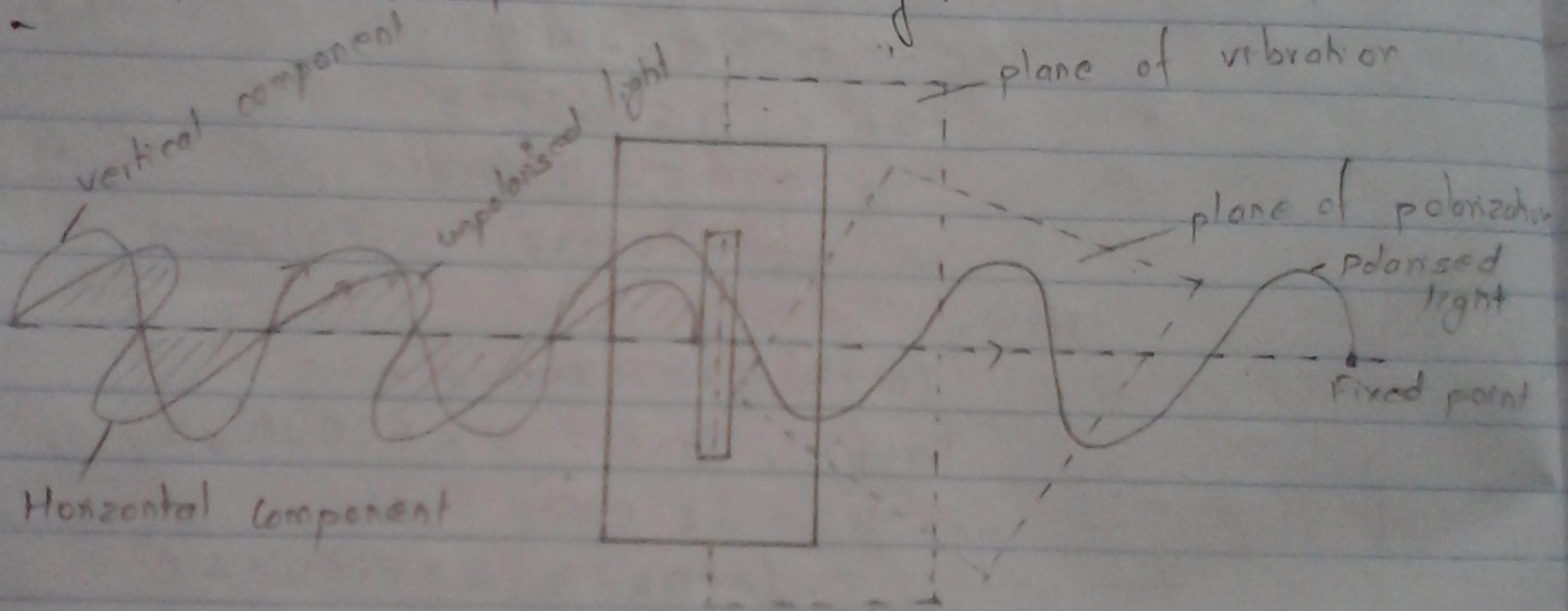


# Civinnovate

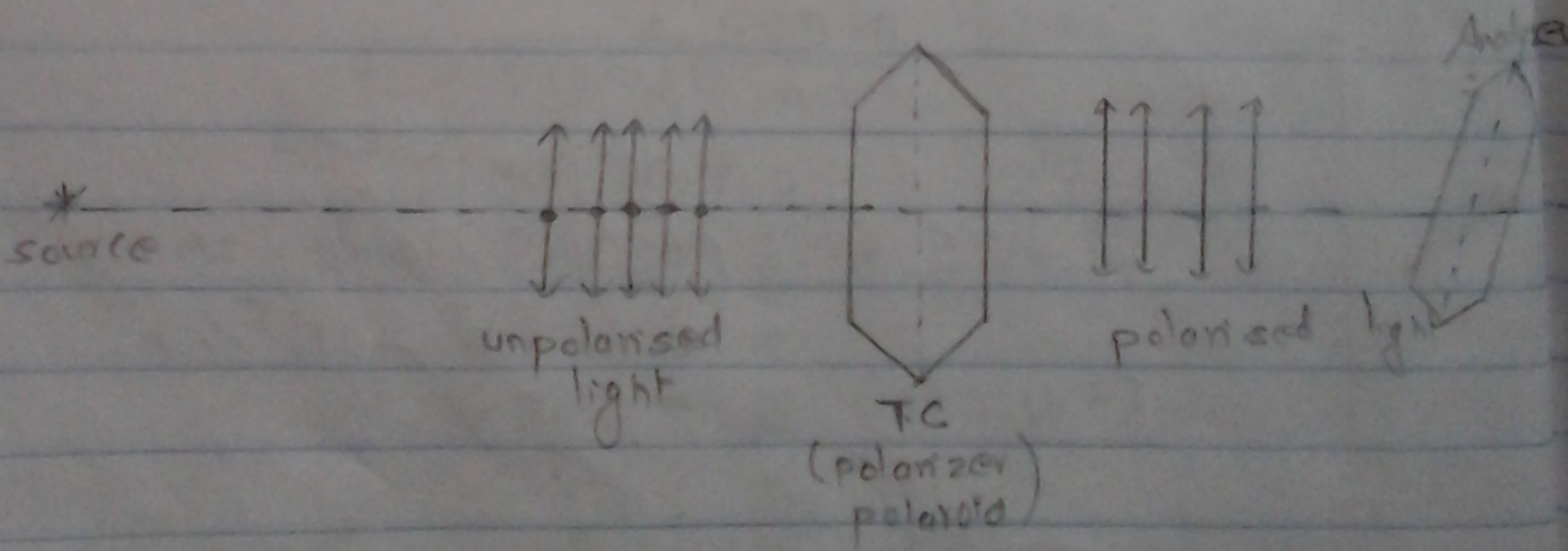
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# Polarization

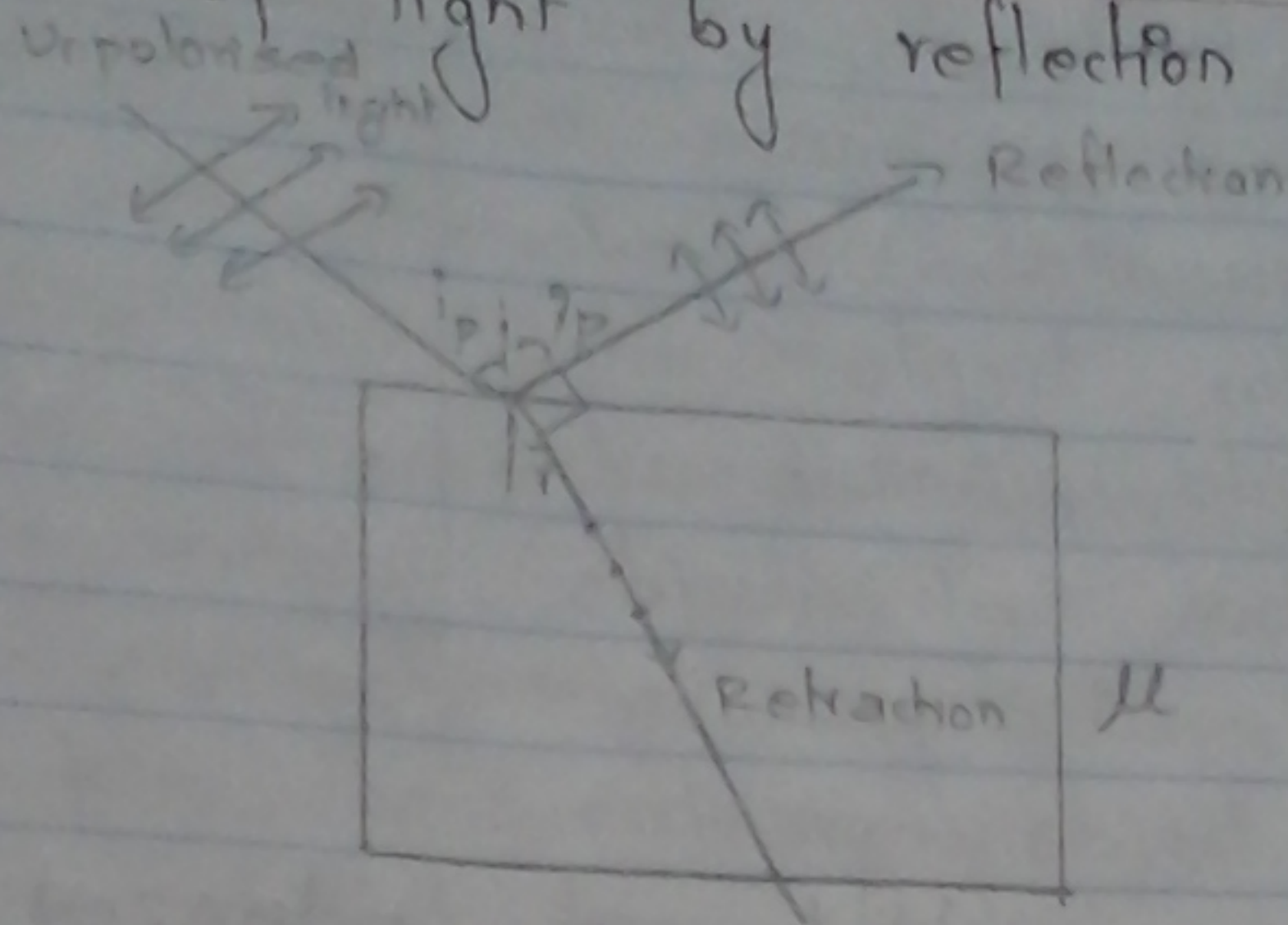
Interference and diffraction explain the properties of all waves, but the polarization explain the properties of only transverse wave. Polarization is the phenomenon of restriction of plane of vibration of transverse wave in a particular plane. The parallel plane to the axis of crystal is plane of vibrations and perpendicular to the axis or parallel to the propagation of wave is plane of polarization eg:



Symbolical representation:



## Polarization of light by reflection (Brewster's law)



The unpolarized light can completely be polarized by the phenomenon of reflection of light if the reflected and refracted of light are perpendicular to each other. Let  $\mu$  be the refractive index of the medium and  $i_p$  be the angle of polarization, the angle of incidence is said to be angle of polarization if the unpolarised light is completely polarized.

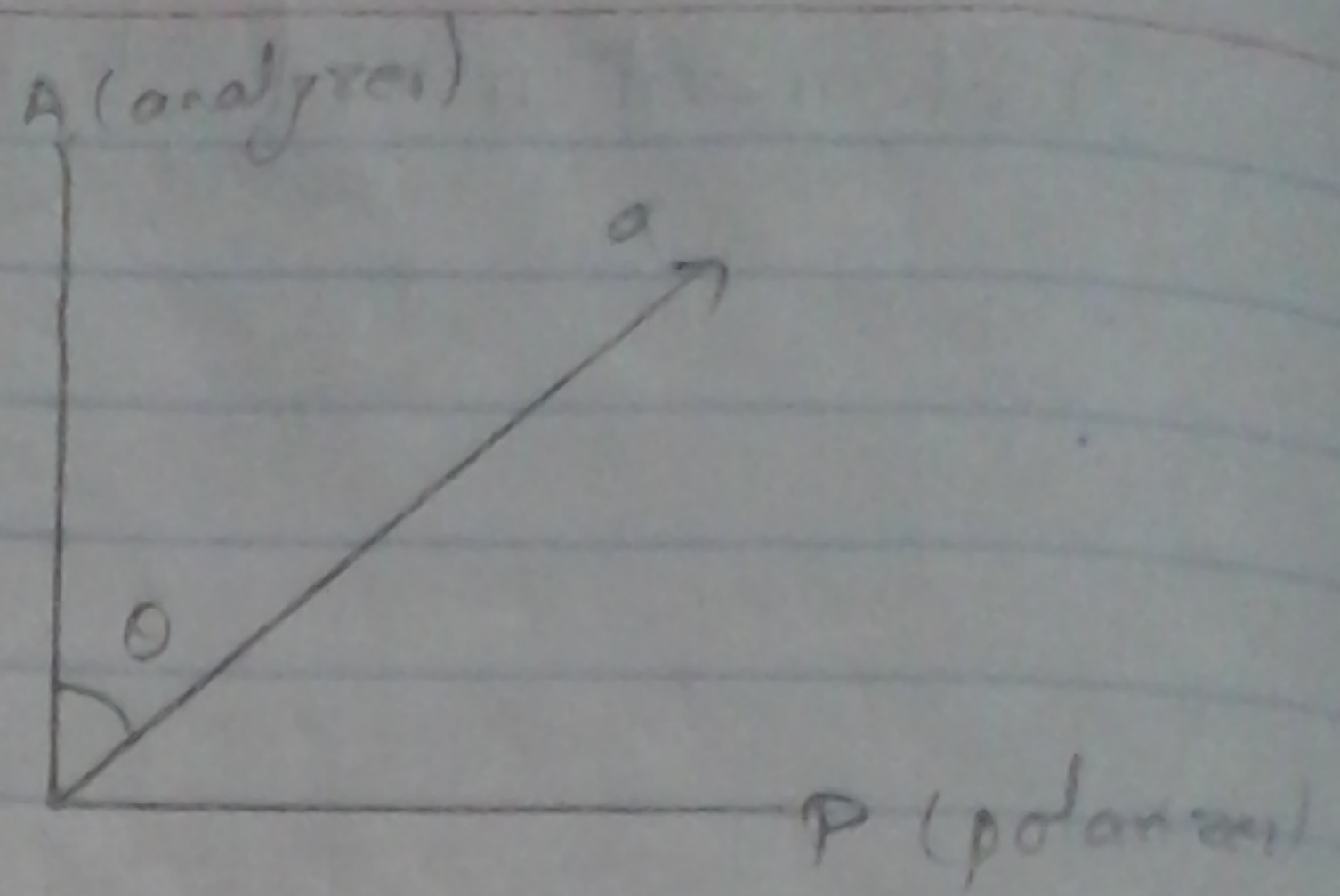
by Snell's law,

$$\begin{aligned} \mu &= \frac{\sin i}{\sin r} \\ &= \frac{\sin i_p}{\sin r} \\ &= \frac{\sin i_p}{\sin (90 - i_p)} \\ &= \frac{\sin i_p}{\cos i_p} = \tan i_p \end{aligned}$$

$$\therefore \mu = \tan i_p$$

It states that refractive index of medium is tangent of angle of polarization.

# # Malus' law :



Consider the amplitude of polarized light be 'a' so its maximum intensity  $I_0 = a^2$ . If the analyzer makes an angle  $\theta$  with the polarized light then the intensity of emerged analysed light is proportional to square of cosine of angle made by polarized light to analyzer. i.e.

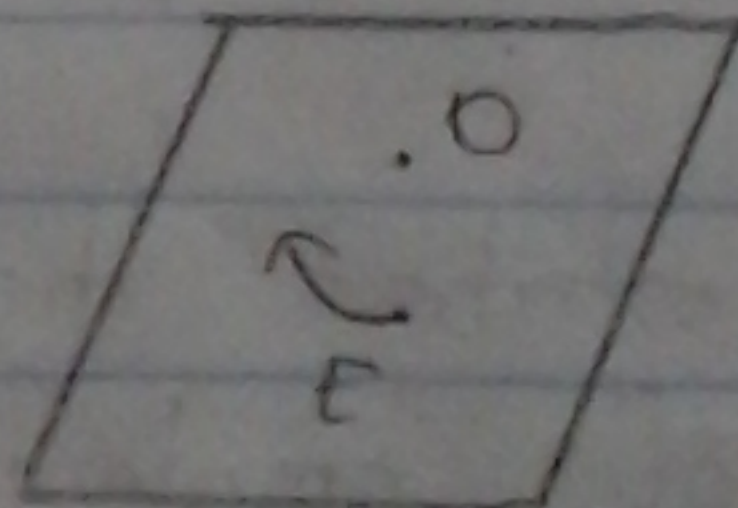
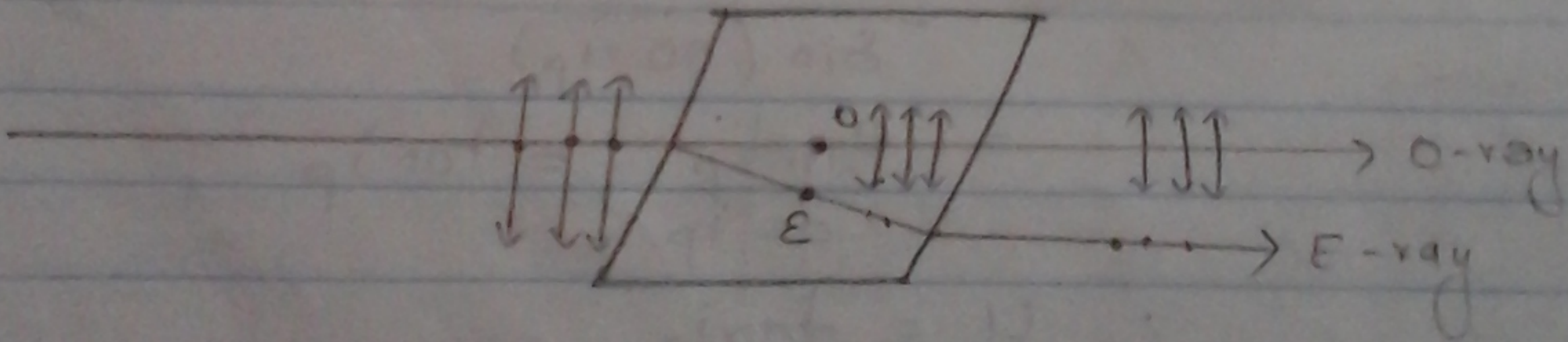
$$I \propto \cos^2 \theta$$

$$I = I_0 \cos^2 \theta$$

if  $\theta = 0$ ,  $I = I_0$  [(maximum)  $\Rightarrow$  only vertical component is emerged out]

if  $\theta = 90^\circ$ ,  $I = 0$  [(minimum)  $\Rightarrow$  all component are vanished]

# Double Refraction :



When a crystal (quartz, calcite) is placed above the two dots on the plane of paper, then rotate it continuously one dot being stationary and another begins to move. The object which moves with the crystal is extra-ordinary object and ray emitted from extra-ordinary object is extra-ordinary ray. And another remaining stationary is ordinary and light coming from the ordinary object is ordinary ray. Both rays travel equal distance in the optical axis i.e. their velocity is same in the direction of optical axis of the crystal.

This phenomenon of splitting of unpolarized light into ordinary ray and extra-ordinary ray is known as double refraction.

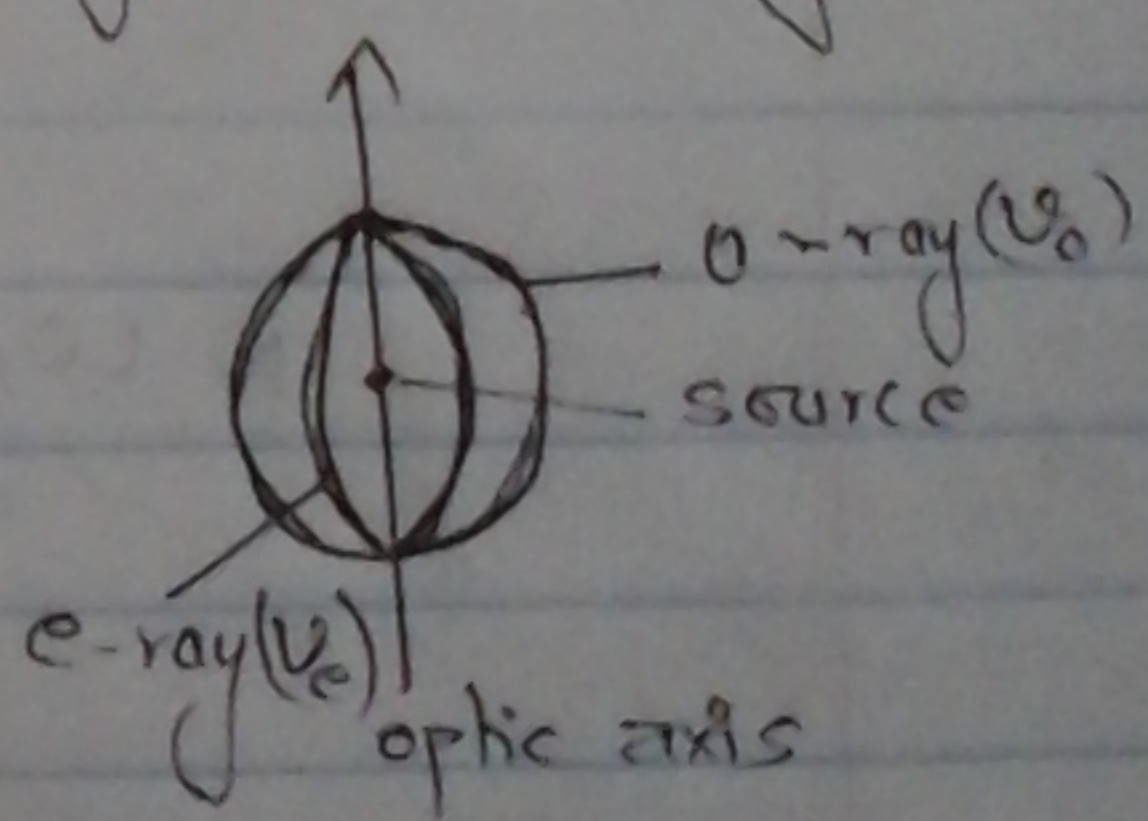
# Imp # Huygen's explanation of double Refraction:

- According to Huygen's theory of double refraction, it states that:
- ① A point of a source of monochromatic light emits the two ray o-ray and e-ray from a point and contribute the corresponding wavefront from that point.
  - ② A o-ray travels in different direction with same velocity so the emitted wavefront is spherical for o-ray.
  - ③ An e-ray travels in different direction with different velocity so the emitted wavefront from a point source is an elliptical.

Hence, the emitted polarised light follows the laws of refraction and reflection only by the due to the variation of velocity. As that velocity increased, the refractive index of decreased. Along the optical axis both wavefront due to o-ray and e-ray travel equal distance but in the perpendicular to the optic axis, the velocity of o-ray is greater or less than that of e-ray. It depends upon the nature of crystal. According to the explanation of double refraction, there are two types of crystal

## ① positive crystal (Quartz)

The velocity of o-ray is greater than that of the e-ray i.e.  $v_o > v_e$ . This type of crystal is positive crystal. By the explanation of

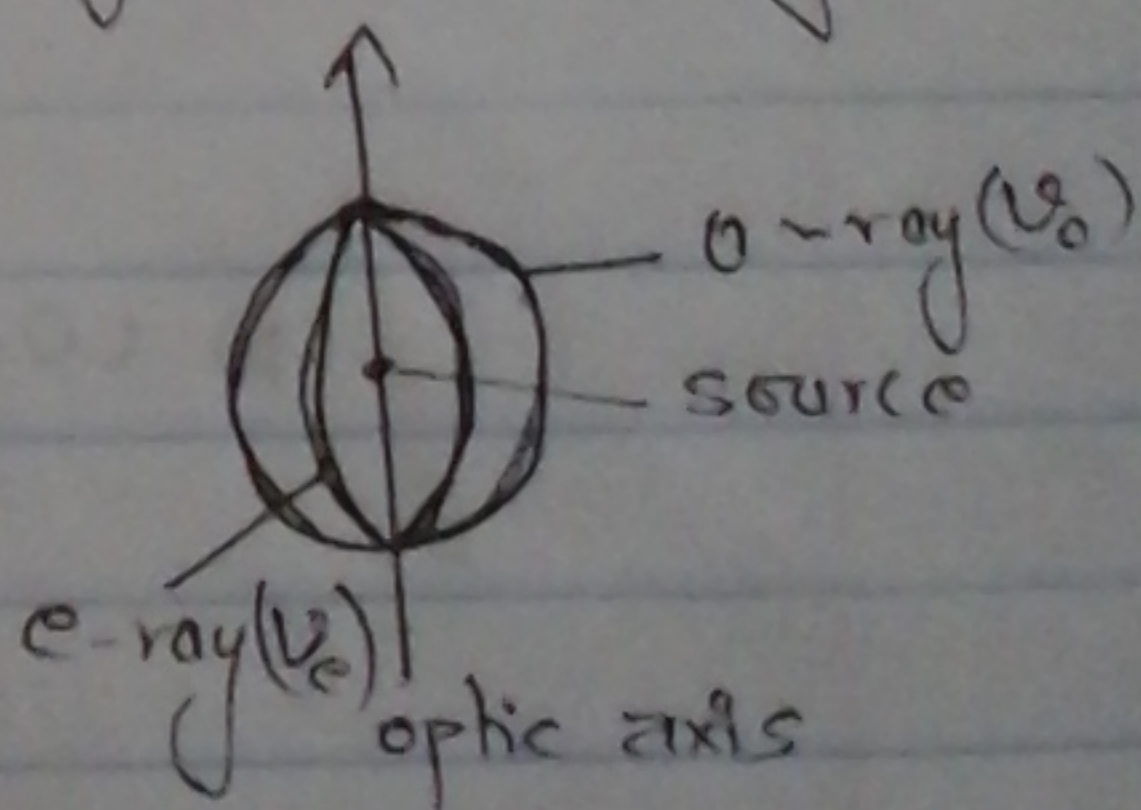


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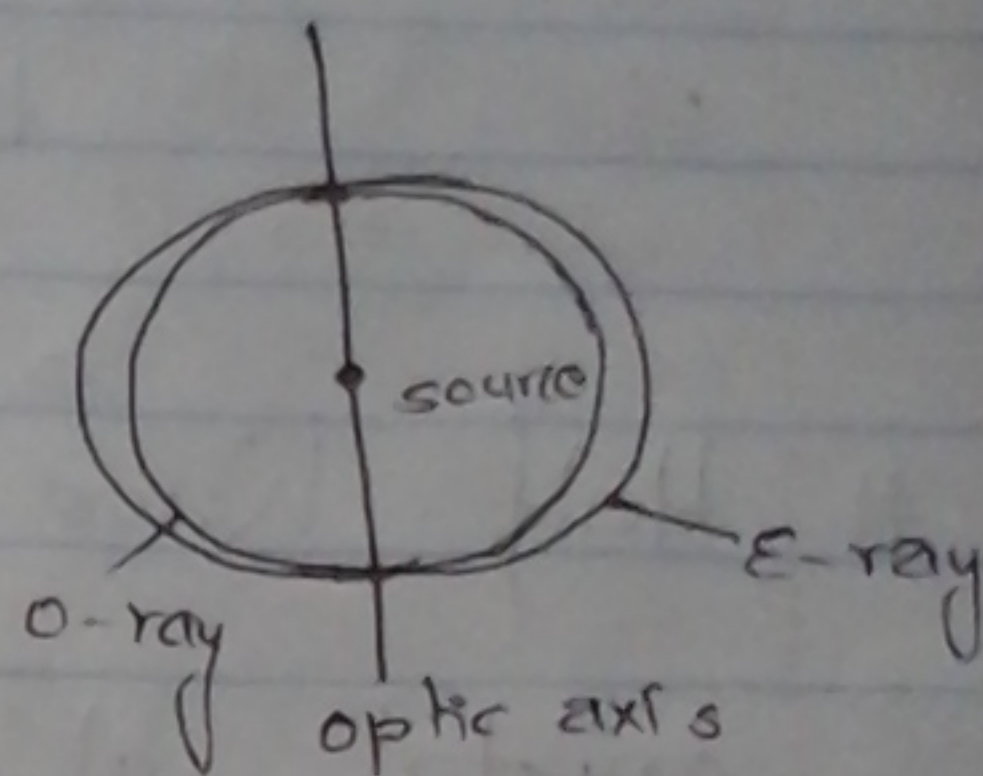
- ① positive crystal (Quartz)



The velocity of o-ray is greater than that of the e-ray i.e.  $v_o > v_e$ . This type of crystal is positive crystal. By the explanation of

double refraction on a positive crystal, the wavefront of e-ray is enclosed by o-ray and the velocity is same along the optic axis.

### ii Negative crystal (Calcite)



A doubly refraction crystal in which a speed of e-ray is greater than o-ray or refractive index of e-ray is less than o-ray is known as negative crystal.

### ## Quarter Wave plate

It is a doubly refraction crystal, cut in such a way that the path difference between e-ray and o-ray is equal to  $\lambda/4$  after emerge out from the crystal. It is uni-axial crystal.

Let 't' be the thickness of crystal and  $\mu_o$  and  $\mu_e$  are the refractive index for o-ray & e-ray resp. If the wavelength of incident light is  $\lambda$  then for quarter wave plate, the path diff betn o-ray and e-ray is  $\lambda/4$ .

For positive crystal

$$t\mu_e - t\mu_o = \lambda/4$$



$$t(\mu_e - \mu_o) = \lambda/4$$

and,

For negative crystal

$$t\mu_o - t\mu_e = \lambda/4$$

$$t(\mu_o - \mu_e) = \lambda/4$$

### # Half Wave Plate:

It is a uni-axial doubly refraction crystal cut in such a way that the path diff between e-ray and o-ray is equal to  $\lambda/2$  after emerge out from the crystal.

Let 't' be the thickness of crystal &  $\mu_o$  and  $\mu_e$  be the refractive index of o-ray & e-ray resp. If the wavelength of incident light is  $\lambda$  then for half wave plate, the path diff bet<sup>n</sup> o-ray and e-ray is  $\lambda/2$ .

For positive crystal,

$$t\mu_e - t\mu_o = \lambda/2$$

$$t(\mu_e - \mu_o) = \lambda/2$$

and,

For negative crystal,

$$t\mu_o - t\mu_e = \lambda/2$$

$$t(\mu_o - \mu_e) = \lambda/2$$

Q.1) Find the thickness of quarter wave plate when the wavelength of monochromatic light is  $5890 \text{ \AA}$ .  
 $\Rightarrow$  Soln;

$\mu_o = 1.648$   
 $\mu_e = 1.486$

Since  $\mu_o > \mu_e$   
 $\therefore$  crystal is negative crystal

We have,

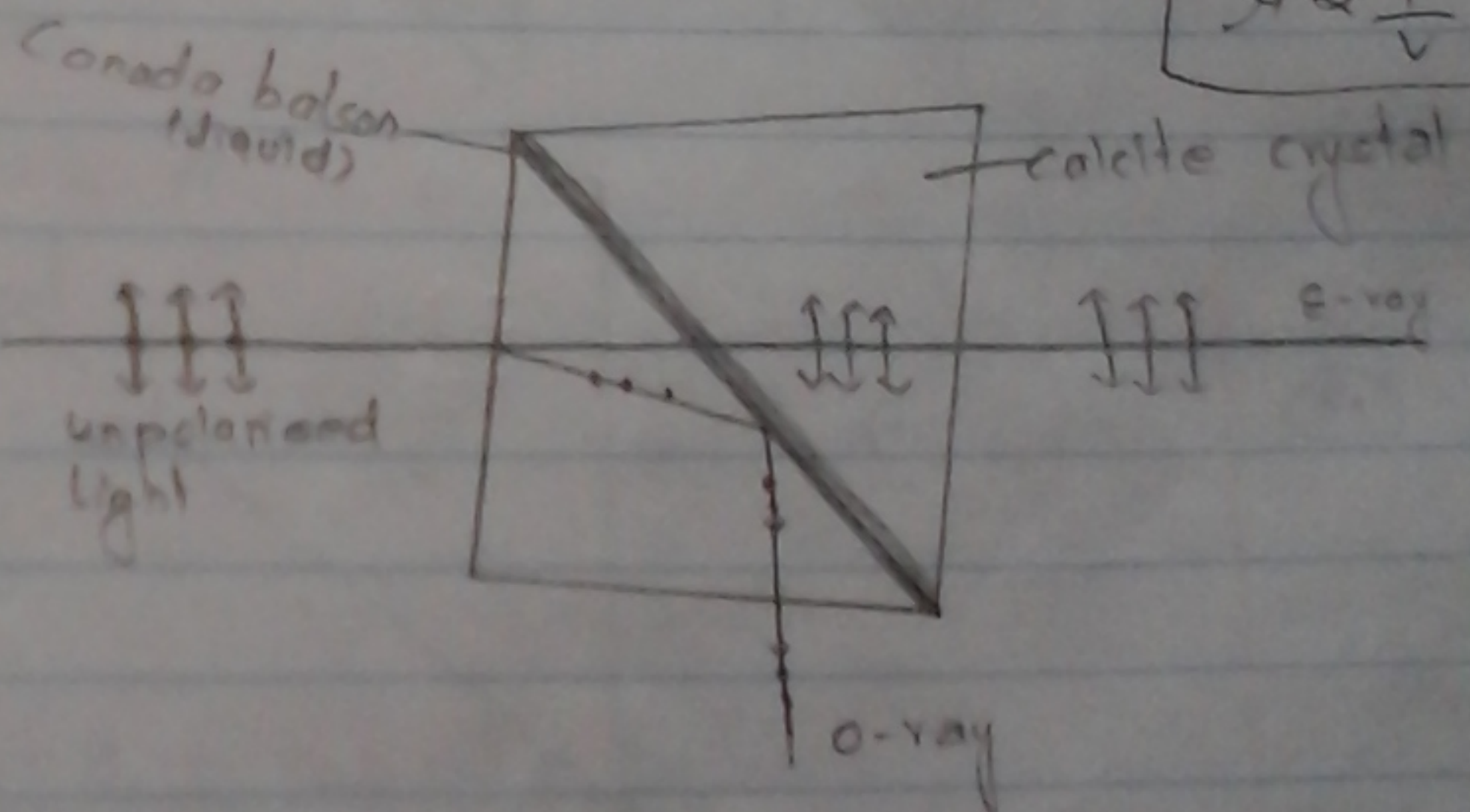
$$t (\mu_o - \mu_e) = \lambda/4$$

$$t = \frac{5890 \times 10^{-10}}{4 \times (1.648 - 1.486)}$$

$$= 9089.5 \times 10^{-10} \text{ m}$$

$$\therefore t = 9.089 \times 10^{-7} \text{ m}$$

## # Nicol Prism



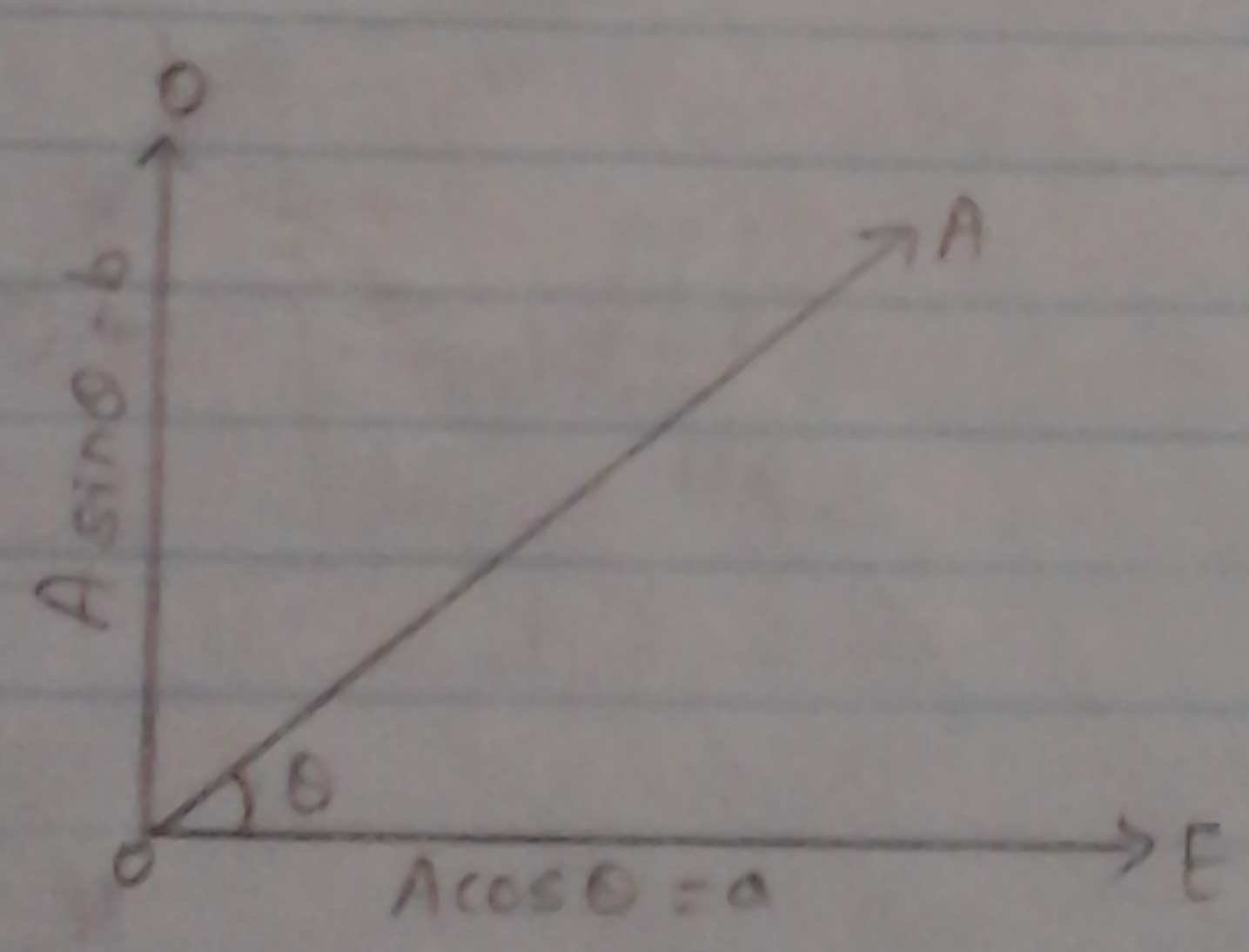
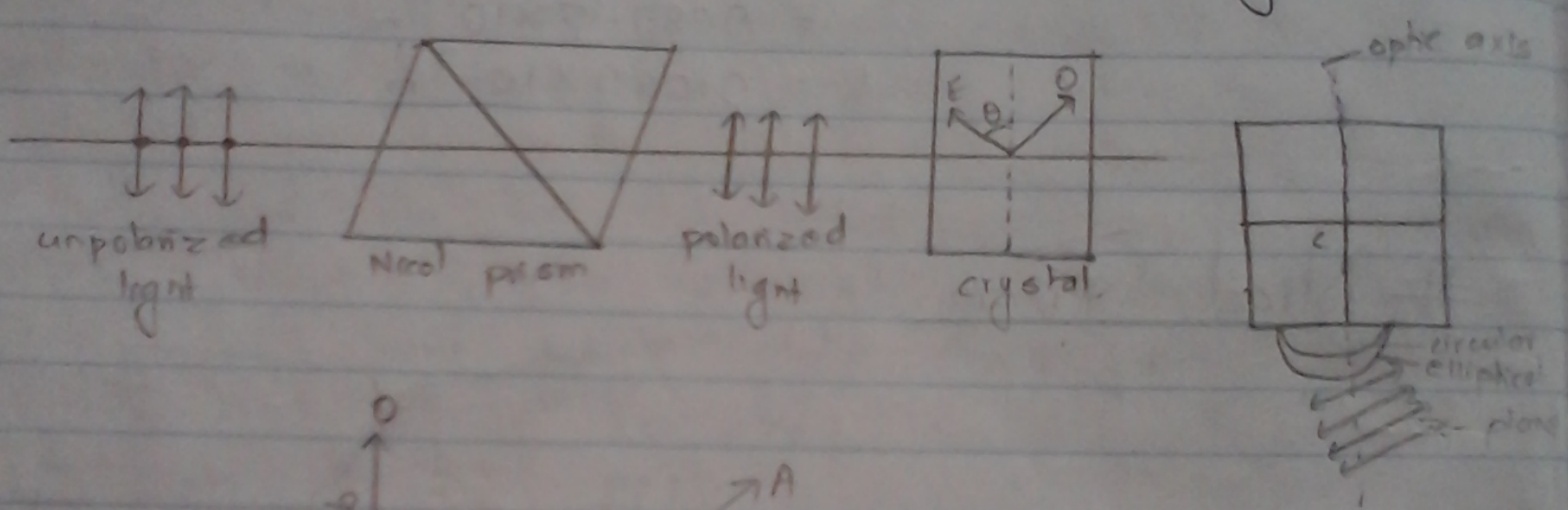
$$\mu_o = 1.658 > \mu_b = 1.55 > \mu_e = 1.486$$

It is constructed in such a way that the calcite crystal cut diagonally and cemented by the Canada balsam (liquid) whose  $\mu$  is in between O-ray and E-ray. It can be used as analyzer and polarizer.

Since the speed of e-ray is greater than that of o-ray, the o-ray is eliminated by the total internal reflection on the calcite crystal. But the e-ray is completely refracted along the direction perpendicular to optic axis. Hence the plane of vibration is in uni-direction so that the emergent ray is completely plane polarized.

$$\mu_o = 1.658 > \mu_e = 1.55 > \mu_c = 1.486$$

Imp Linear, Circular and elliptic Polarized light:

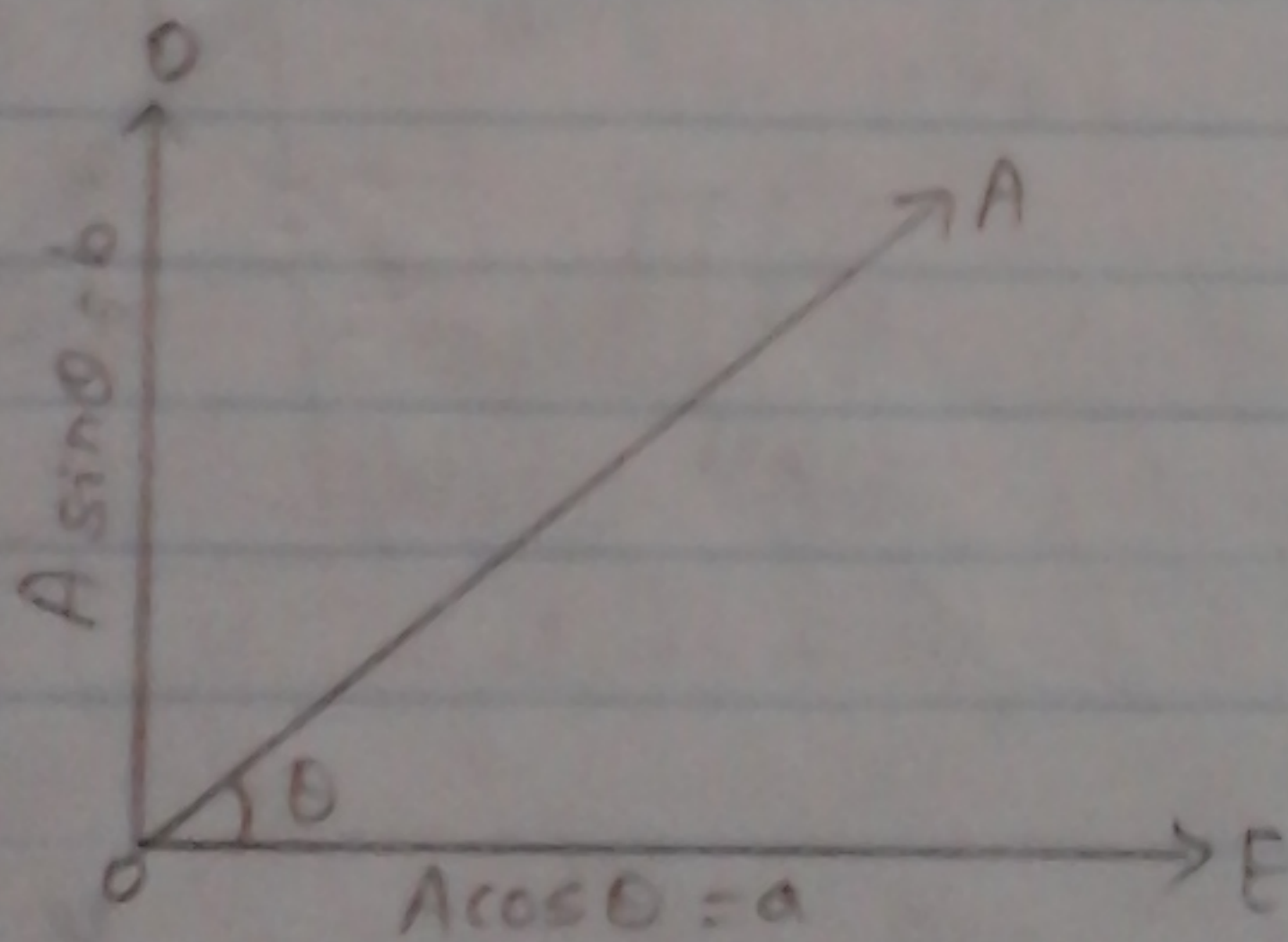
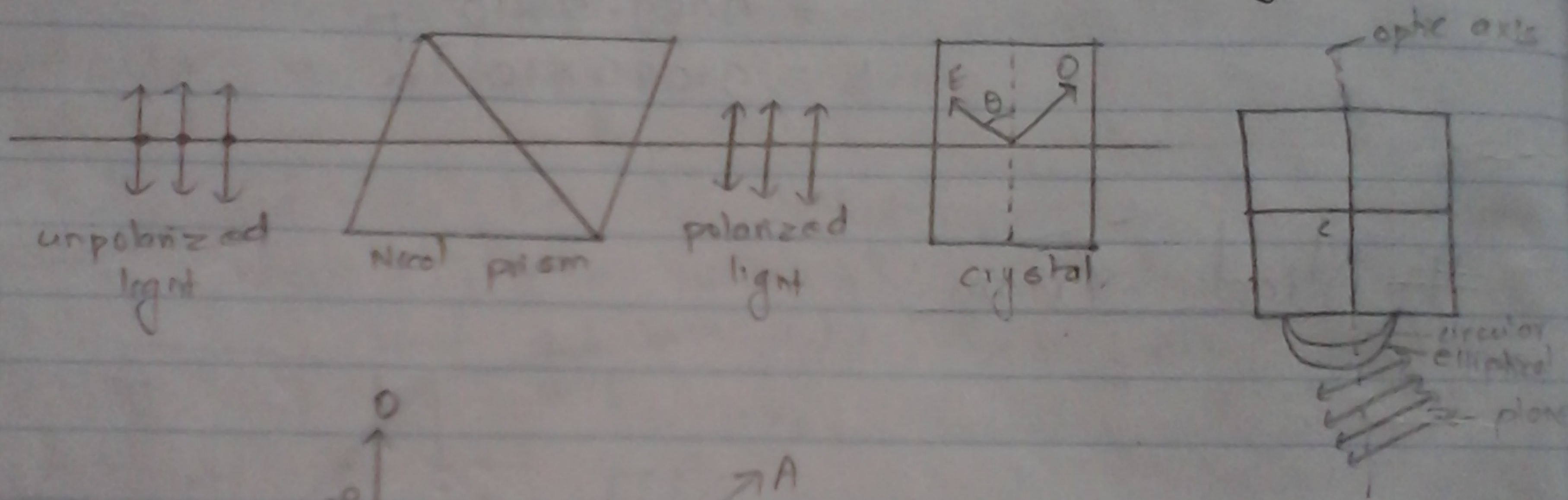


When unpolarized light monochromatic incident on the nicol prism it emerge out as a polarized light in the plane of vibration. The polarized light enters into the crystal, and then we can detect it is linear, circular or elliptical (according to

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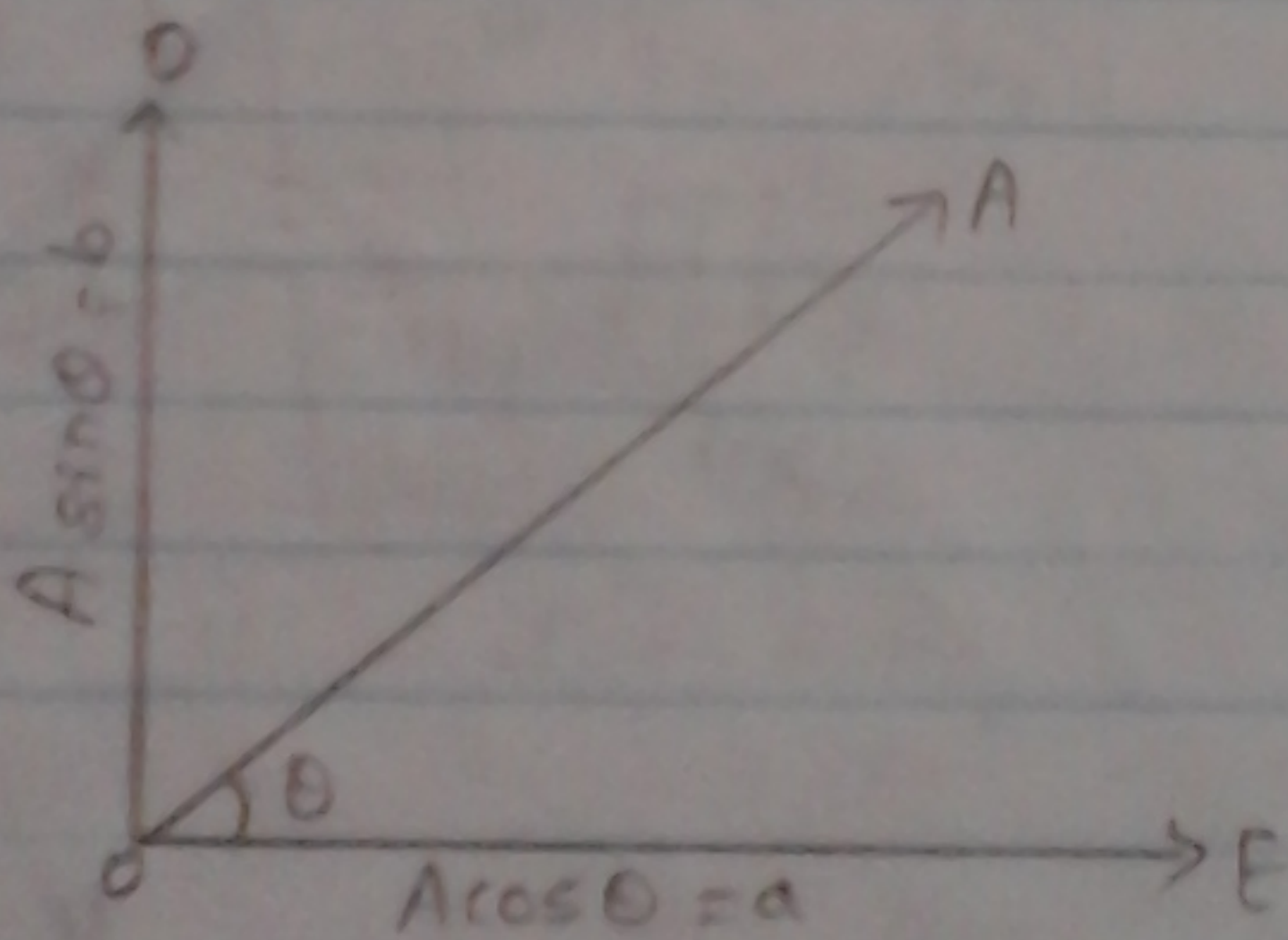
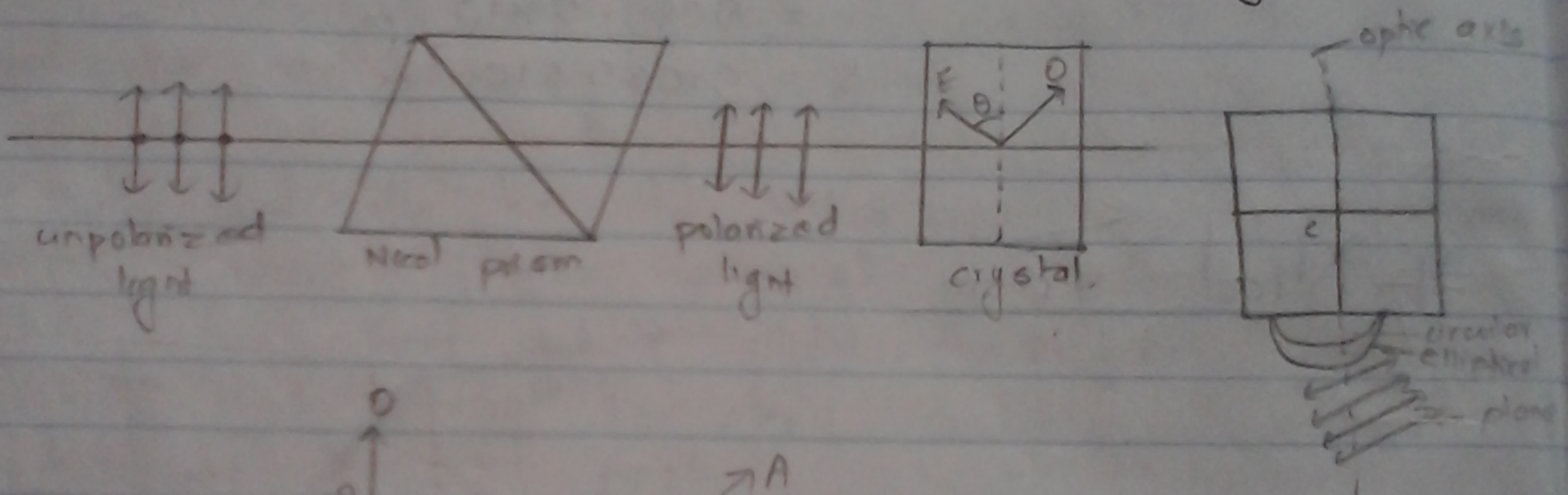


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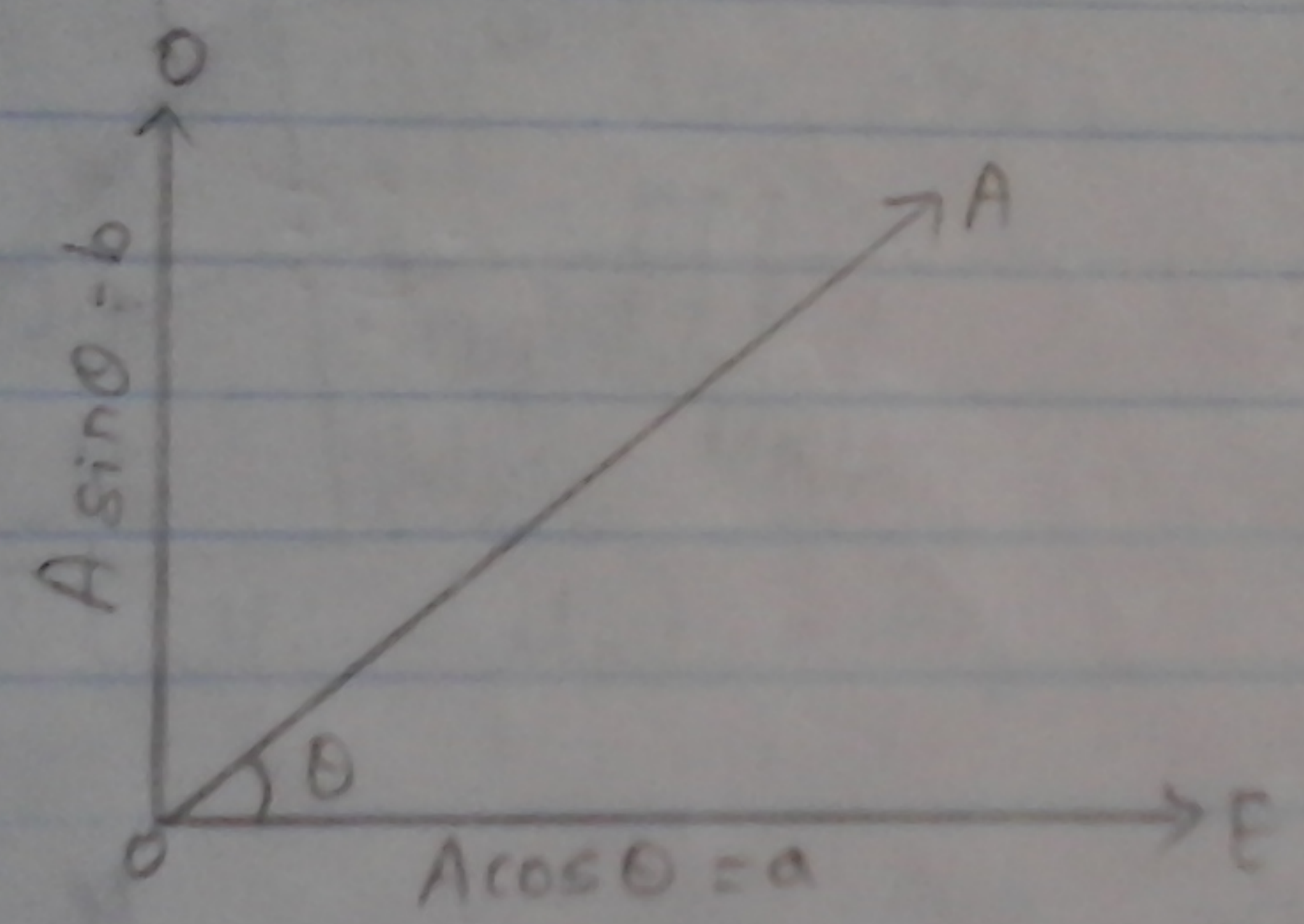
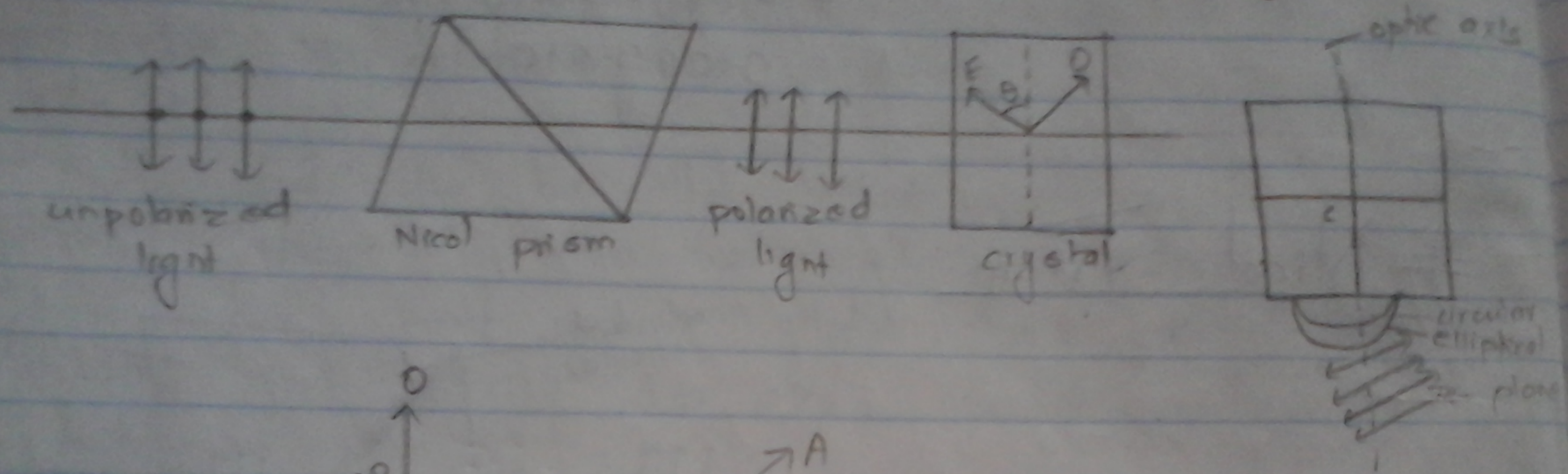
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# Linear, Circular and elliptic Polarized light:



When unpolarized light monochromatic incident on the nichol prism it emerge out as a polarized light in the plane of vibration. The polarized light enters into the crystal, and then we can get linear, circular or elliptical (according to

the nature of crystal) the nature of crystal) polarized light splits into two components O-ray and E-ray. O-ray is parallel to plane of vibration and E-ray is mutually perpendicular to O-ray as shown in fig.

Consider 'A' be the amplitude of polarized light and the optic axis makes an angle  $\delta$  with the extraordinary ray. Draw in graph e-ray along x-axis and o-ray along y-axis as amplitude a & b resp.

Also suppose  $\delta$  is the constant phase difference between o-ray and e-ray then their displacement are

$$x = a \sin(\omega t + \delta) \quad \text{--- (i)}$$

$$\text{and } y = b \sin \omega t \quad \text{--- (ii)}$$

for e-ray and o-ray resp.

From (ii);

$$y = b \sin \omega t = A$$

$$\frac{y}{b} = \sin \omega t$$

$$\cos \omega t = \sqrt{1 - \sin^2 \omega t} = \sqrt{1 - \frac{y^2}{b^2}} \quad \text{--- (iii)}$$

Now, Taking eq. (i)

$$\frac{x}{a} = \sin \omega t \cdot \cos \delta + \cos \omega t \cdot \sin \delta$$

$$\frac{x}{a} = \frac{y}{b} \cos \delta + \sqrt{1 - \frac{y^2}{b^2}} \sin \delta \quad (\because \text{from (iii)})$$

$$\left( \frac{x}{a} - \frac{y}{b} \cos \delta \right) = \sqrt{1 - \frac{y^2}{b^2}} \sin \delta$$

Sq on both sides,

$$\left( \frac{x}{a} - \frac{y}{b} \cos \delta \right)^2 = \left( 1 - \frac{y^2}{b^2} \right) \sin^2 \delta$$

$$\frac{x^2}{a^2} - \frac{2xy}{ab} \cos \delta + \frac{y^2 \cos^2 \delta}{b^2} = \sin^2 \delta - \frac{y^2}{b^2} \sin^2 \delta$$

$$\frac{x^2}{a^2} - \frac{2xy}{ab} \cos \delta + \frac{y^2}{b^2} = \sin^2 \delta$$

Case I:  $\delta = 0$

$$\frac{x^2}{a^2} - \frac{2xy}{ab} \cos 0^\circ + \frac{y^2}{b^2} = \sin^2 0^\circ$$

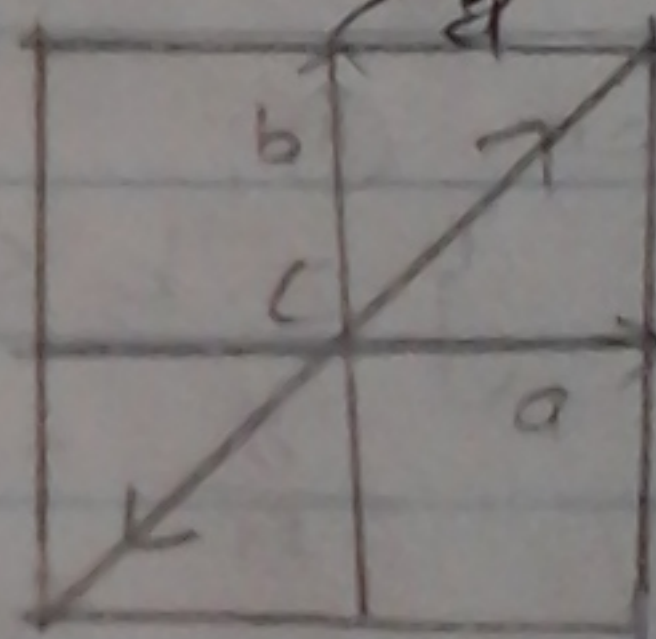
$$\frac{x^2}{a^2} - \frac{2xy}{ab} + \frac{y^2}{b^2} = 0$$

$$\left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0$$

$$\frac{x}{a} = \pm \frac{y}{b}$$

$$y = +\frac{b}{a}x$$

Eq<sup>n</sup> of st. line having slope  $b/a$



The polarized light

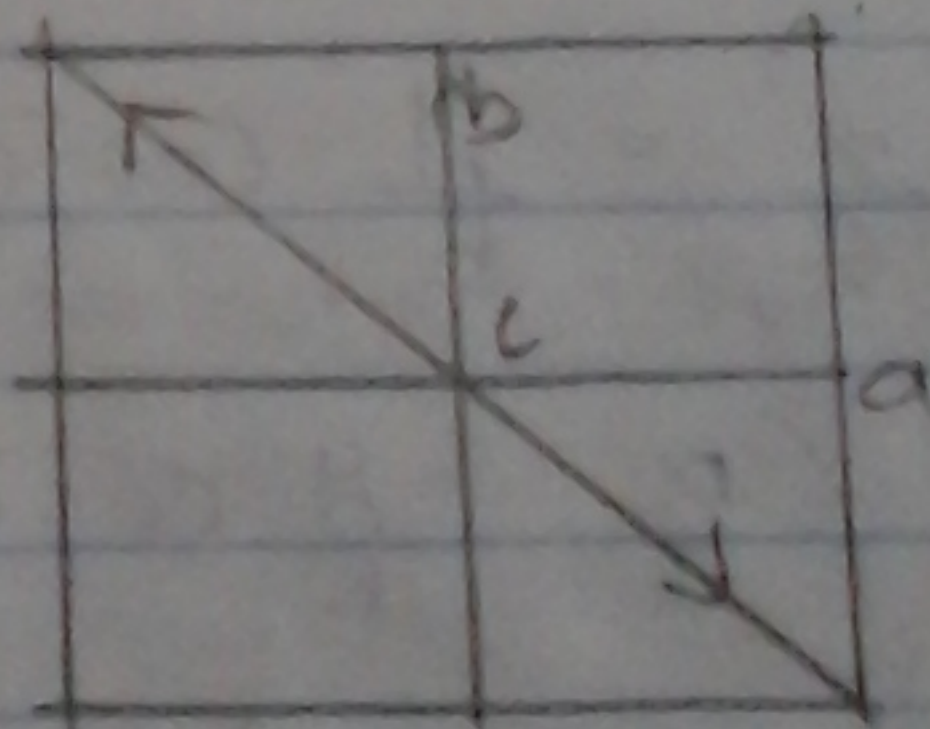
~~Case II:  $\delta = \pi$~~

Case I II:  $\delta = \pi$

$$\frac{x^2}{a^2} + \frac{2xy}{ab} + \frac{y^2}{b^2} = 0$$

$$\frac{x}{a} = -\frac{y}{b}$$

$$y = -\frac{b}{a}x$$

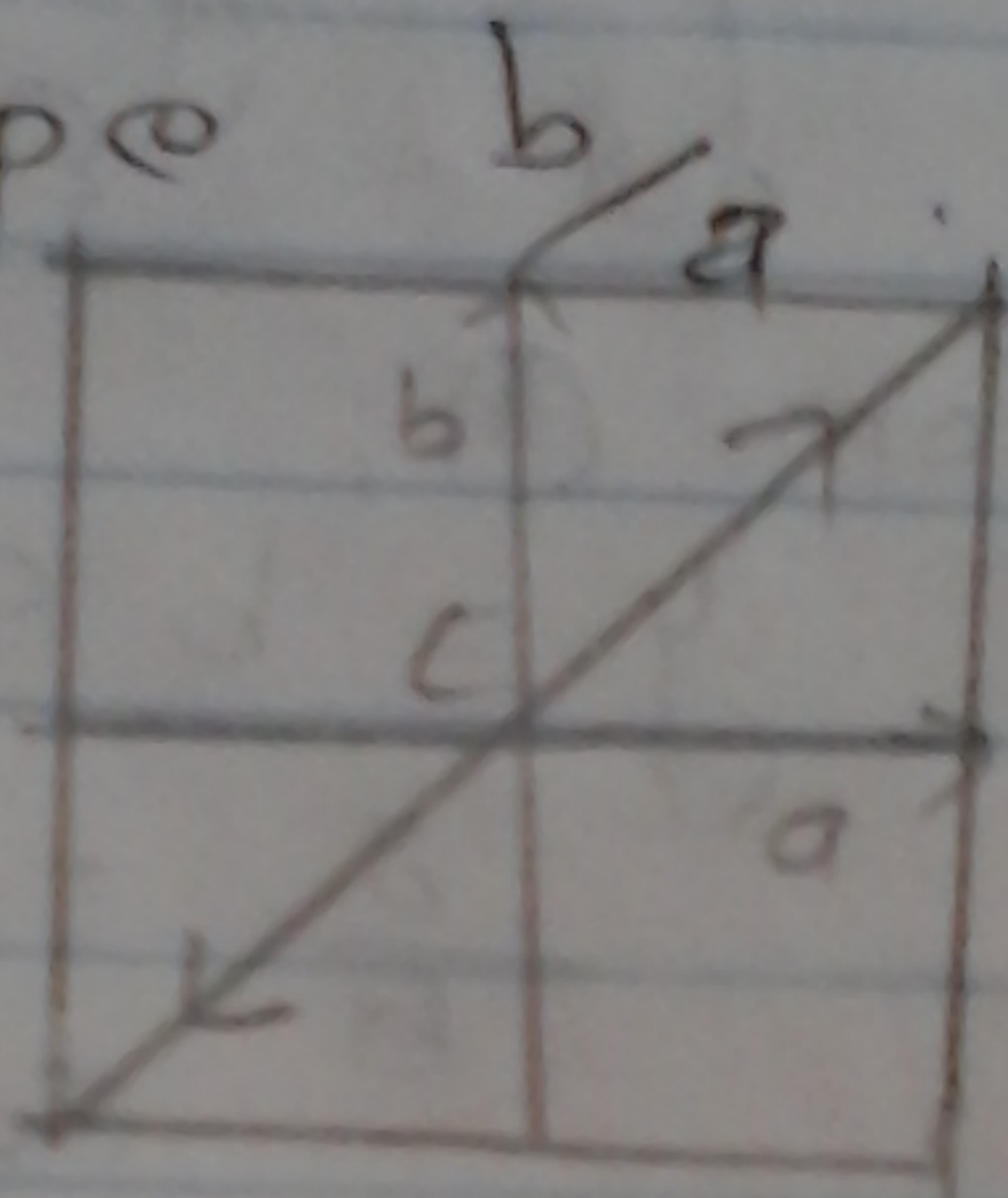


The polarized light

If  $\delta = 0, \pi, 2\pi \dots$  is linear

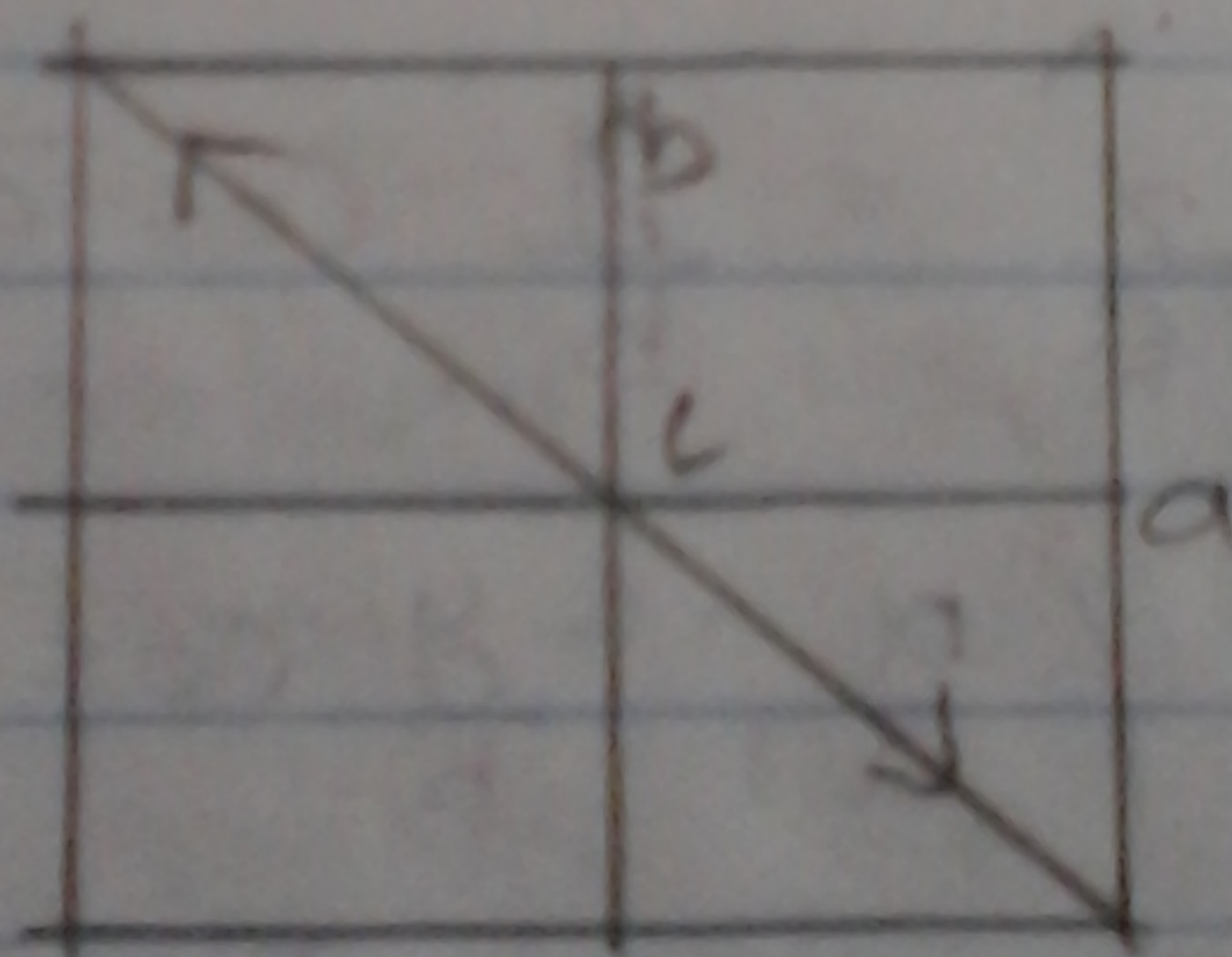


having slope  $\theta$



The polarized light

= 0



The polarized light

linear

Case II

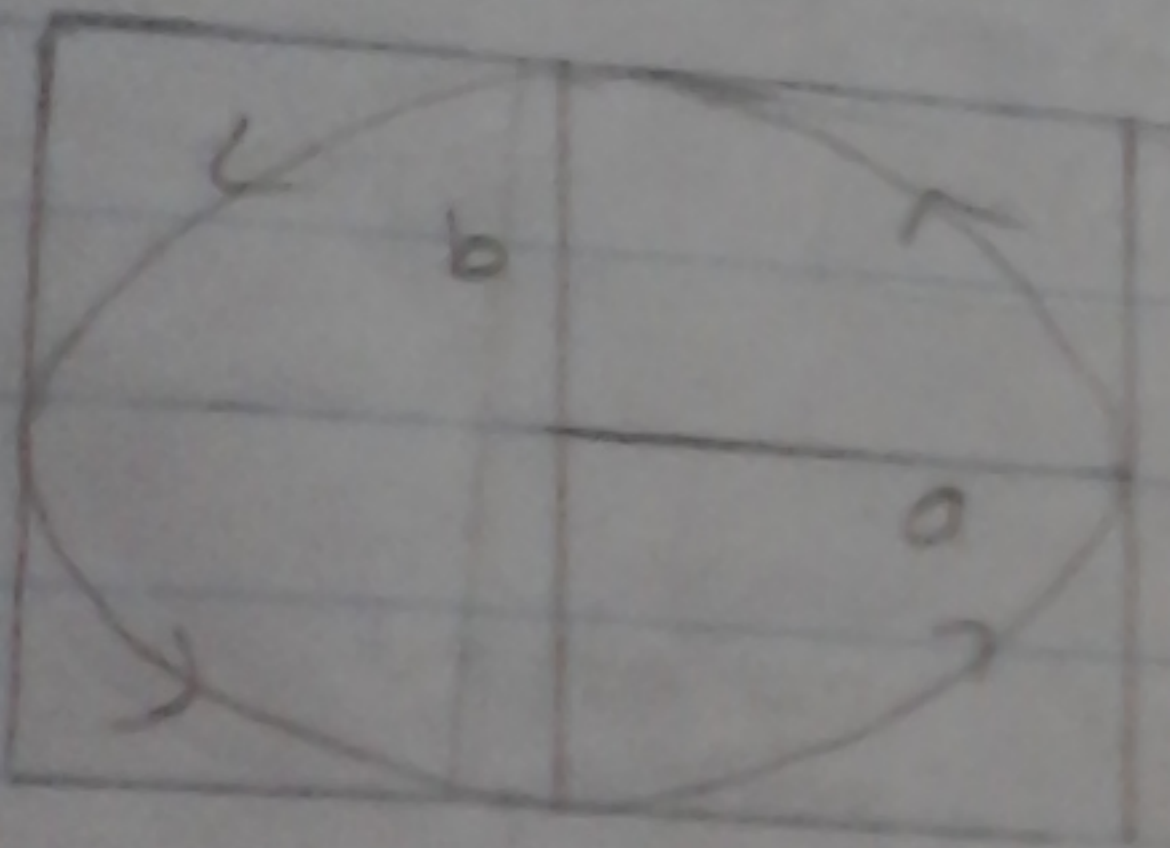
$$\frac{x^2}{a^2} - \frac{2xy}{ab} \cos \theta + \frac{y^2}{b^2} = \sin^2 \theta$$

Case 2:

$$\theta = \pi/2$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

eqn of ellipse



$$\theta = 3\pi/2$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

eqn of ellipse

At  $\theta = \pi/2, 3\pi/2, \dots$   
The polarized light is elliptical

If  $a = b$  or  $\theta = 45^\circ$

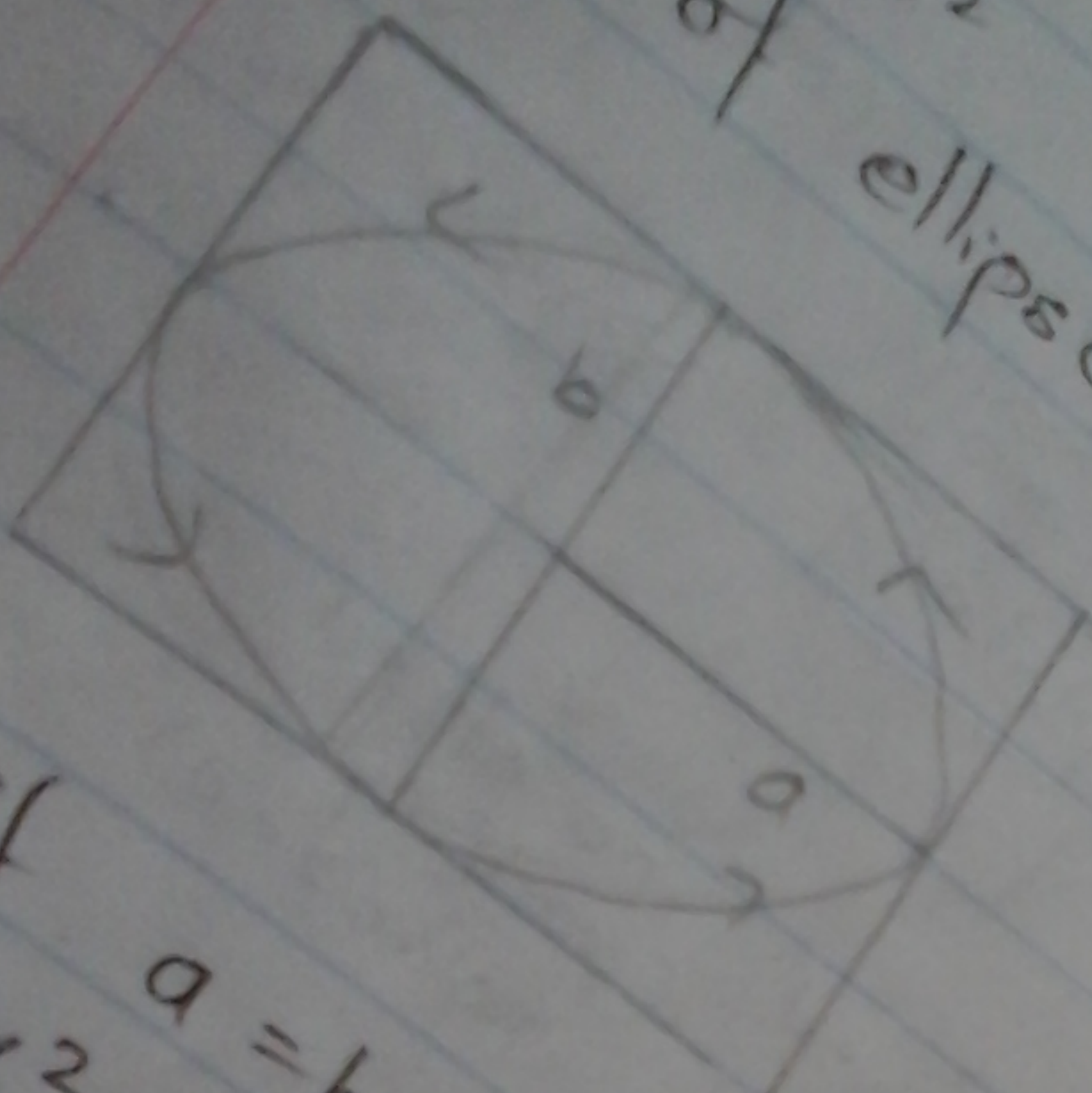
$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$$

$$x^2 + y^2 = a^2$$



circular polarized

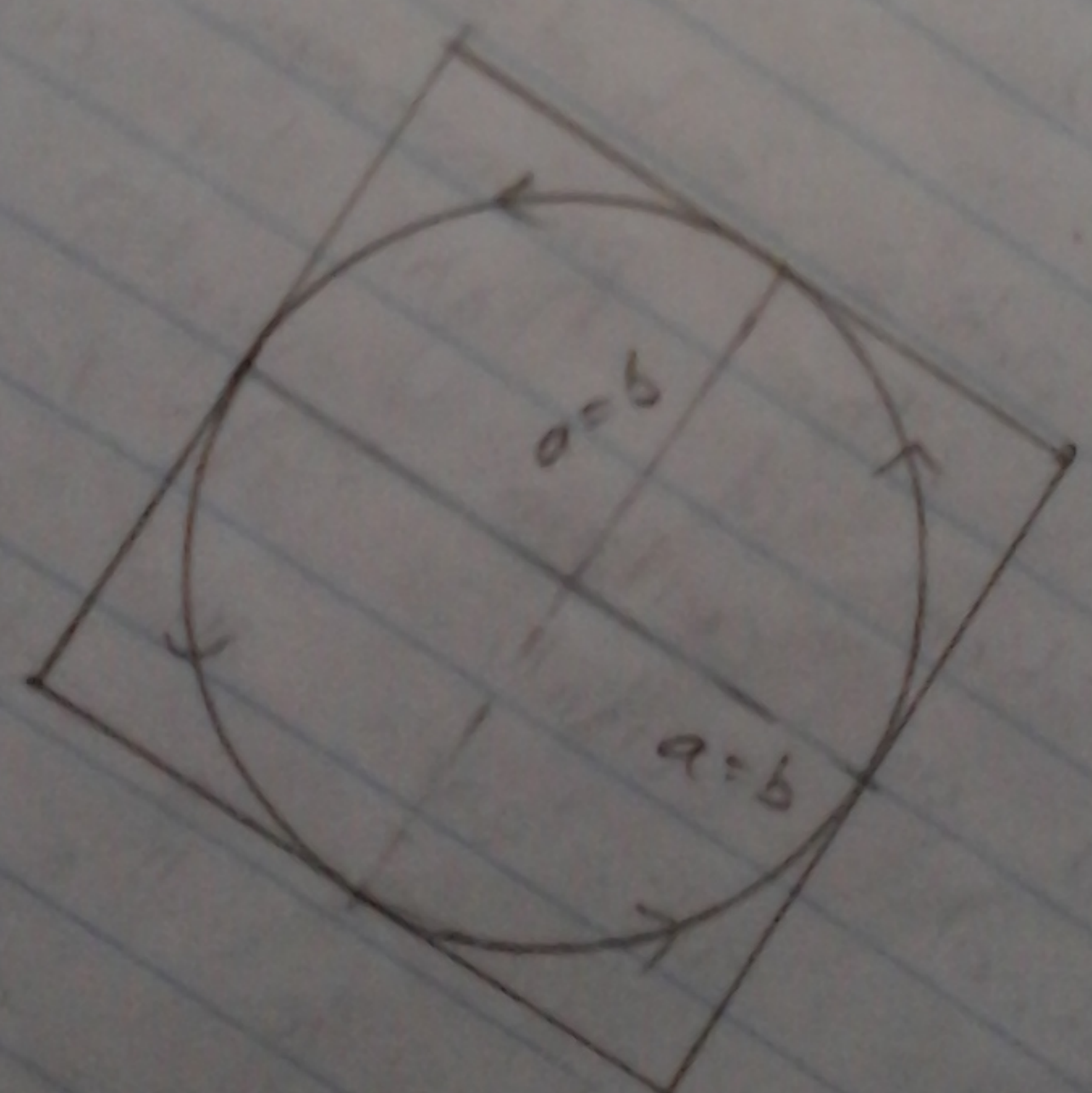
Eqn of ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



If  $a = b$  or  $\theta = 45^\circ$   
 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$   
 $x^2 + y^2 = a^2$

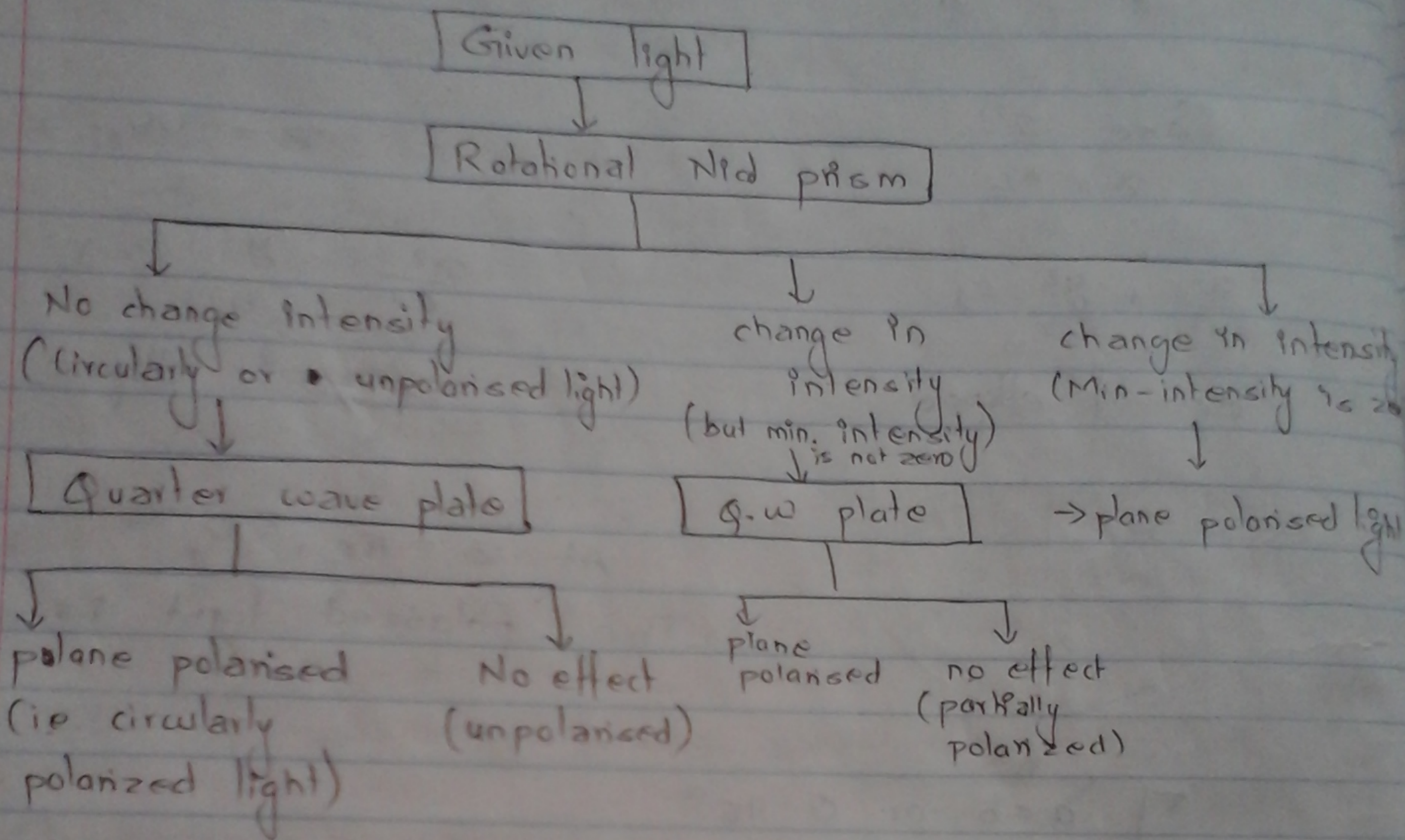
The At  $\theta = \pi/2, 3\pi/2$  -  
polarized light is ellip

Eqn of ellip  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



Circular

# # Detection of linear, circular and elliptically polarised light:



## Optically active substance:

The phenomenon of rotation of plane of polarization of light by certain angle when it incident on certain crystal or solution is called optical activity. It is the property of crystal, such substances are called optically active substance. Eg: sugar sol<sup>n</sup>, sugar crystal, silicon crystal.

According to the property of optical activity, the substances are categorised in

(1) Dextro rotatory (Right handed rotatory)

(2) The substance which produce clockwise rotation or towards right are called dextro rotatory.

Laevo Rotatory (left handed rotatory)  
 The substance which produce anti-clockwise rotation or toward left are called laevo rotatory.

# Specific - rotation:

The specific rotation 'S' is defined as the rotation produced by a deci-meter long column of liquid containing 1 gm of active substance in 1cc of the solution where a plane polarized light is incident on it.

$$S \cdot L \cdot C = \theta$$

$$S = \frac{\theta}{L \cdot C}$$

where  $\theta$  is angle of rotation  
 L is length of column (cm)  
 C is concentration of solution

Q. A 200mm long tube containing 48cc of sugar solution produced an optical rotation of  $11^\circ$  when placed on a saccharimeter. If the specific rotation of sugar sol<sup>n</sup> is  $66^\circ$ . Calculate the quantity of sugar content in the tube in the form of sol<sup>n</sup>.

⇒ sol<sup>n</sup>;

$$S = 66^\circ$$

$$\theta = 11^\circ$$

$$L = 200 \times 10^{-3}$$

$$C = \frac{m}{48} = \frac{\text{mass}}{\text{volume}}$$

$$s = \frac{100}{LC}$$

$$66 = \frac{10 \times 11}{200 \times 10^{-2} \times \frac{m}{48}}$$

$$m = \frac{110 \times 48}{66 \times 200}$$

$$= 4 \text{ gms}$$

### # Half - Shaded Polarimeter :

A polarimeter is a device for measuring the angle through which the plane of polarization rotates by an optical active substance. A Laurent half shade polarimeter consists two Nicol Prisms as an analyzer and polarizer rotating about a common axis. It also consists a circular half-shade plate, objective, eye piece and a 10cm long tube. Fig ① represents the arrangement of different parts of the polarimeter.

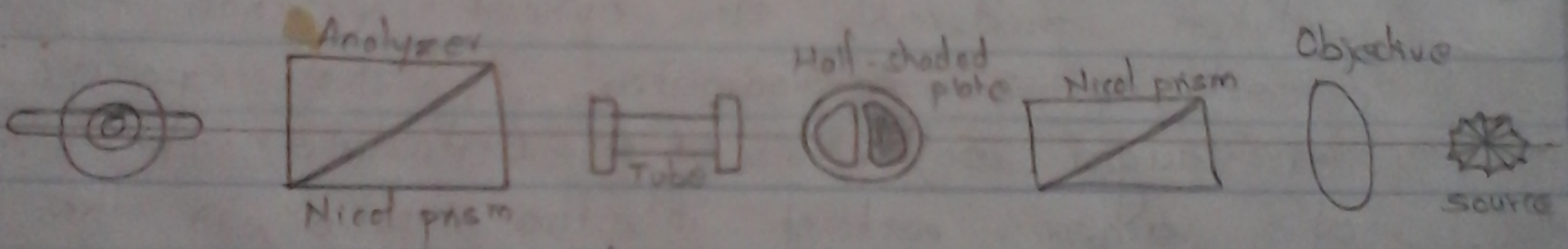


fig ① a.

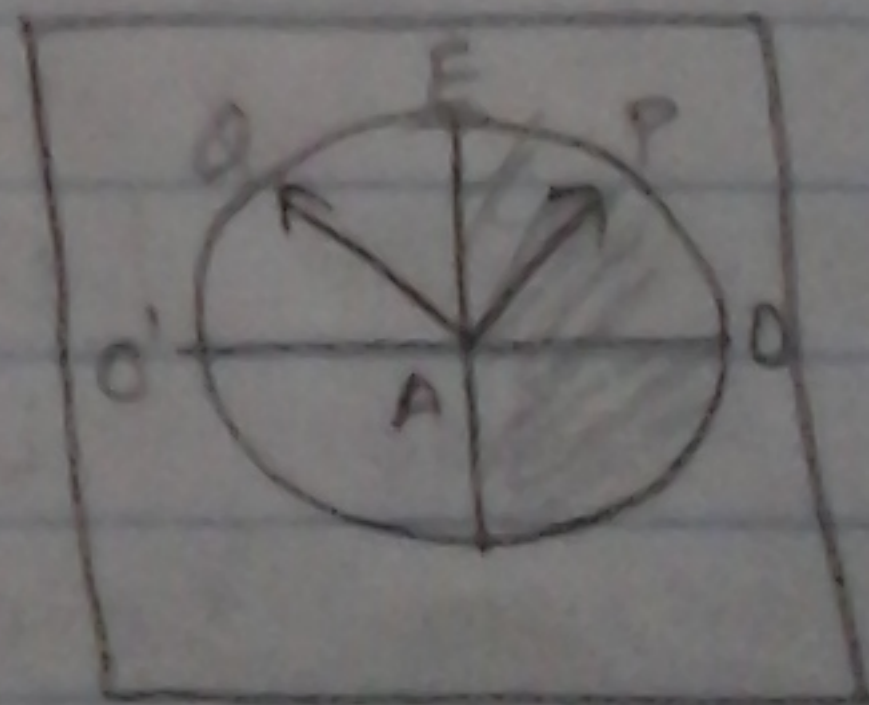


fig ② b.

The incident light from source centres into the Nicol prism through objective lens. The plane of vibration of the polarized light will come out of the polarizer and fall on the half-shade plate. The half-shade plate is a semi-circular quartz plate cemented with the glass plate. The quartz plate is half wave thickness of the glass plate is so choose that it absorbs the same amount of light as is done by quartz plate. Let the incident vibration on the half ~~remains~~<sup>shade</sup> along  $OP$  fig 1.6. On passing light through the glass half, the vibrations remain along  $AP$  but on passing through quartz half these will split up into extraordinary  $AE$  and ordinary components  $AO$ . On passing through the quartz plate  $x$  is these vibrations. Thus  $AO$  suffers a phase difference by  $\pi$  vibration will advance in phase by  $\pi$  and will take along  $AO'$ . So the resultant vibration will be along  $AQ$  with angle  $\theta$  as shown in fig 1.6

If the principal section of analyzing Nicol is parallel to  $OAO'$ , both halves of the field of view will be equally illuminated.

A slight rotation of the analyzer from this setting either in clockwise or anticlockwise direction causes one component greater than other and then either the quartz appears brighter than the glass or vice-versa.



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