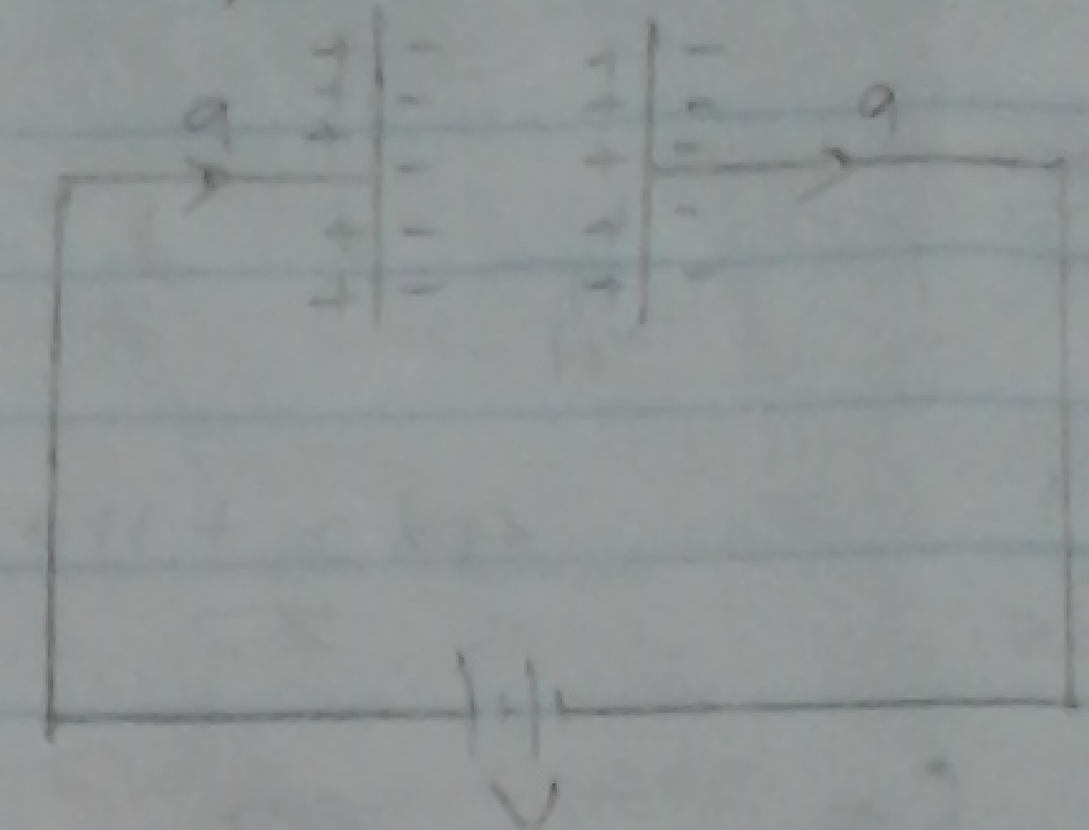
Civinnovate

Discover, Learn, and Innovate in Civil Engineering

Energy stored in Electric field (Capacitor)



During the complete charge of capacitor no further charge transfer from positive plate to negative plate. So, we have to apply external potential V to move the further charges in the capacitor. These external workdone to move the charge is the storage form of electrical energy. Hence the external workdone required to move the charge in the capacitor plate is an electrical energy stored in electric field or capacitor.

Consider V potential is applied to just move the small amount of charge dq from positive plate to negative plate at a seperation 'd' so the small workdone to move the charge dq

so, $dw = v\,dq$

at any instant of time, workdone is

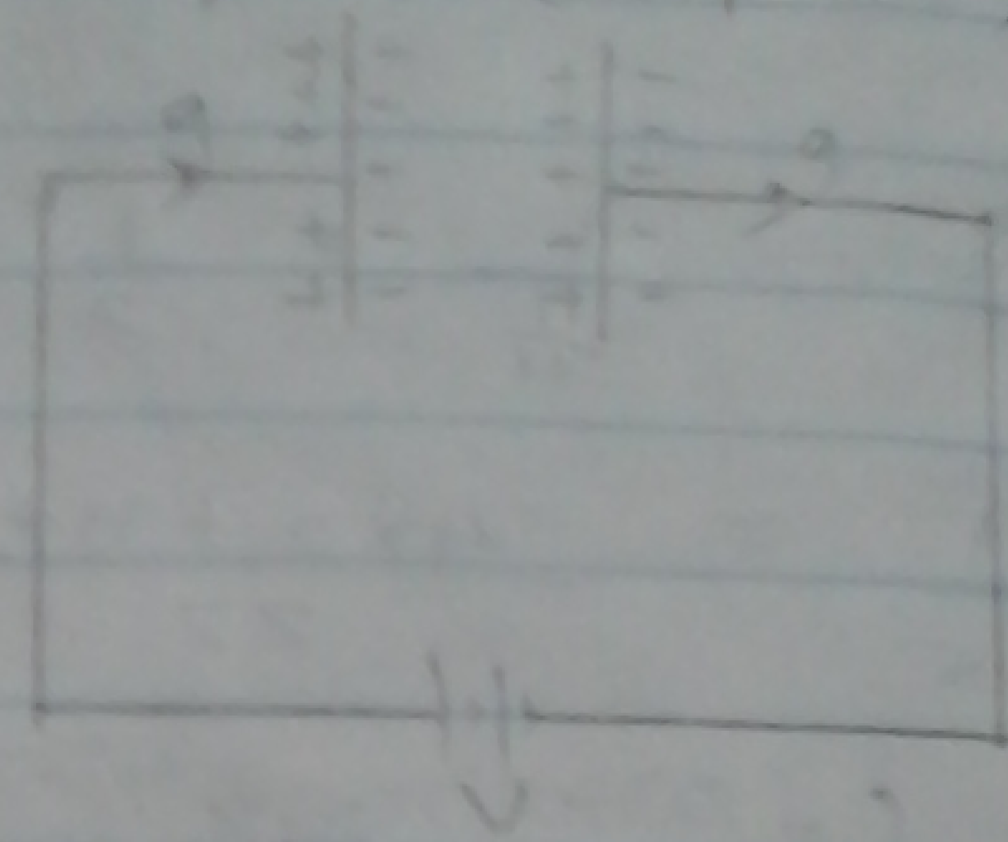
$$\Rightarrow w = \int_0^q dw$$

$$= \int_0^q v\,dq = \int_0^q \frac{q}{c}\,dq$$

$$= \frac{1}{c}\left[\frac{q^2}{2}\right]_0^q = \frac{1}{2}\frac{q^2}{c} = \frac{1}{2}cv^2$$

$\therefore$ Energy stored in capacitor $(U) = \frac{1}{2}cv^2$ . . (1)

Energy stored in Electric field (Capacitor)



During the complete charge of capacitor no further charge transfer from positive plate to negative plate. So, we have to apply external potential V to move the further charges in the capacitor. These external workdone to move the charge is the storage form of electrical energy. Hence the external workdone required to move the charge in the capacitor plate is an electrical energy stored in electric field or capacitor.

Consider V potential is applied to just move the small amount of charge dq from positive plate to negative plate at a seperation 'd' so the small workdone to move the charge dq

so, $dw = v\,dq$

at any instant of time, work done is

$$\Rightarrow w = \int_0^q dw$$

$$= \int_0^q v\,dq = \int_0^q \frac{q}{c}\,dq$$

$$= \frac{1}{c}\left[\frac{q^2}{2}\right]_0^q = \frac{1}{2}\frac{q^2}{c} = \frac{1}{2}cv^2 \quad [\because q = cv]$$

$\therefore$ Energy stored in capacitor $(U) = \frac{1}{2}cv^2$ ... (1)

For parallel plate capacitor,

$C = \frac{\epsilon_0 A}{d}$ where $A \rightarrow$ cross-sectional area of plate
$d \rightarrow$ seperation betn plates

$U = \frac{1}{2}\frac{\epsilon_0 A}{d} V^2$ ... (1)

Now, the energy density is defined as the energy stored in a capacitor at any instant of time per unit volume of capacitor. It is denoted by U_e. The electric field in between the plates is same at every point.

$$U_e = \frac{U}{V}$$

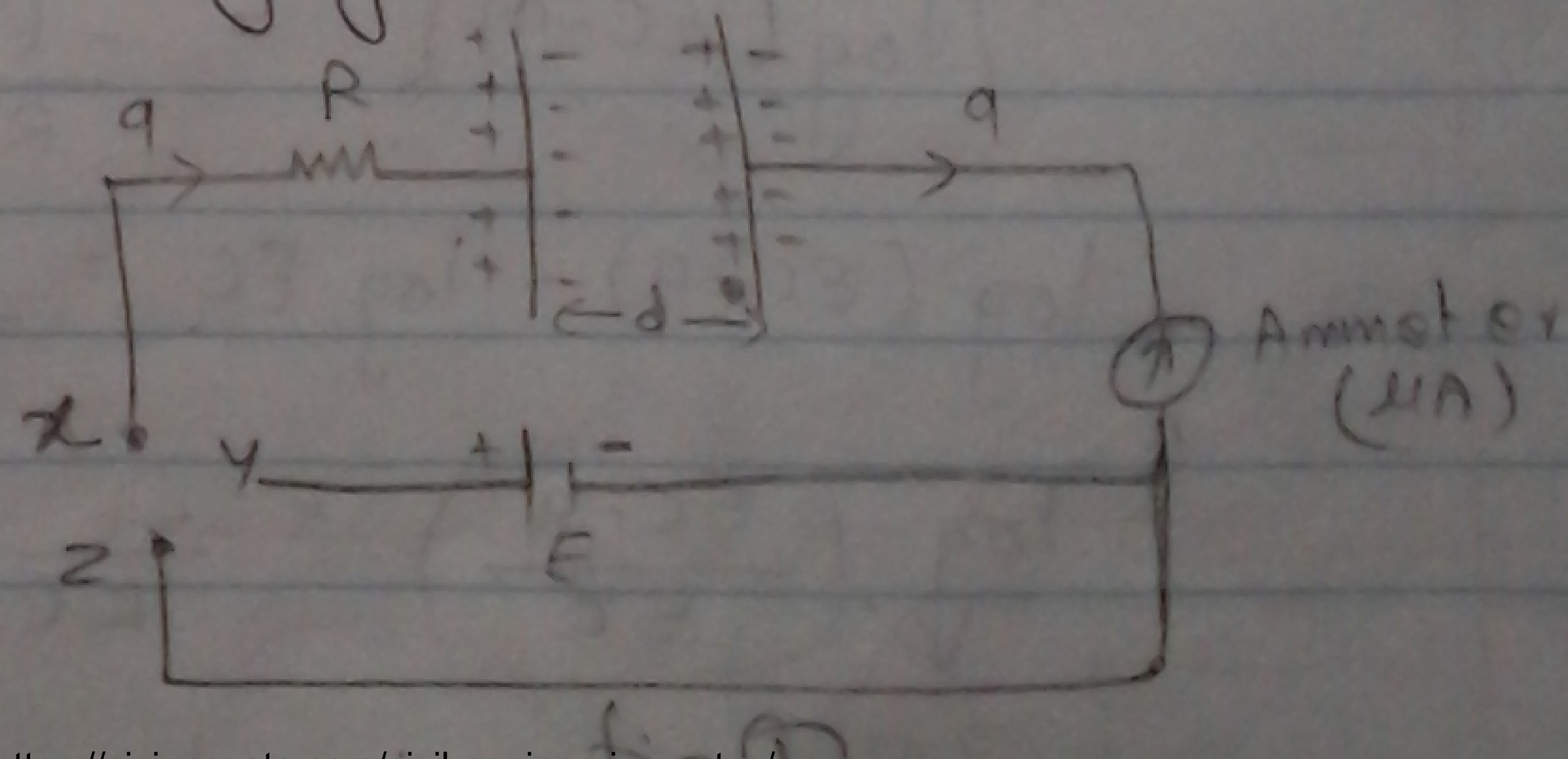
$$= \frac{\frac{1}{2}\epsilon_0 \cancel{A} \times \frac{V^2}{d}}{\cancel{A} \times d} \quad [\because V = A\,d]$$

$$= \frac{1}{2}\epsilon_0 \times \frac{V^2}{d^2} = \frac{1}{2}\epsilon_0 E^2 \quad [\because E = \frac{V}{d}]$$

$$\boxed{\therefore U_e = \frac{1}{2}\epsilon_0 E^2}$$

Hence, the energy density is directly proportional to square of uniform electric field in between the plates.

Charging and discharging of capacitor:



For parallel plate capacitor,

$$C = \frac{\epsilon_0 A}{d}$$ where $A \rightarrow$ cross-sectional area of plate, $d \rightarrow$ seperation betn plates

$$U = \frac{1}{2}\frac{\epsilon_0 A}{d} V^2 \quad \ldots (1)$$

Now, the energy density is defined as the energy stored in a capacitor at any instant of time per unit volume of capacitor. It is denoted by U_e. The electric field in between the plates is same at every point.

$$U_e = \frac{U}{V}$$

$$= \frac{\frac{1}{2}\epsilon_0 \cancel{A} \times \frac{V^2}{d}}{\cancel{A} d} \quad [\because V = A.d]$$

$$= \frac{1}{2}\epsilon_0 \times \frac{V^2}{d^2} = \frac{1}{2}\epsilon_0 E^2 \quad [\because E = \frac{V}{d}]$$

$$\boxed{\therefore U_e = \frac{1}{2}\epsilon_0 E^2}$$

Hence, the energy density is directly proportional to square of uniform electric field in between the plates.

Charging and discharging of capacitor:

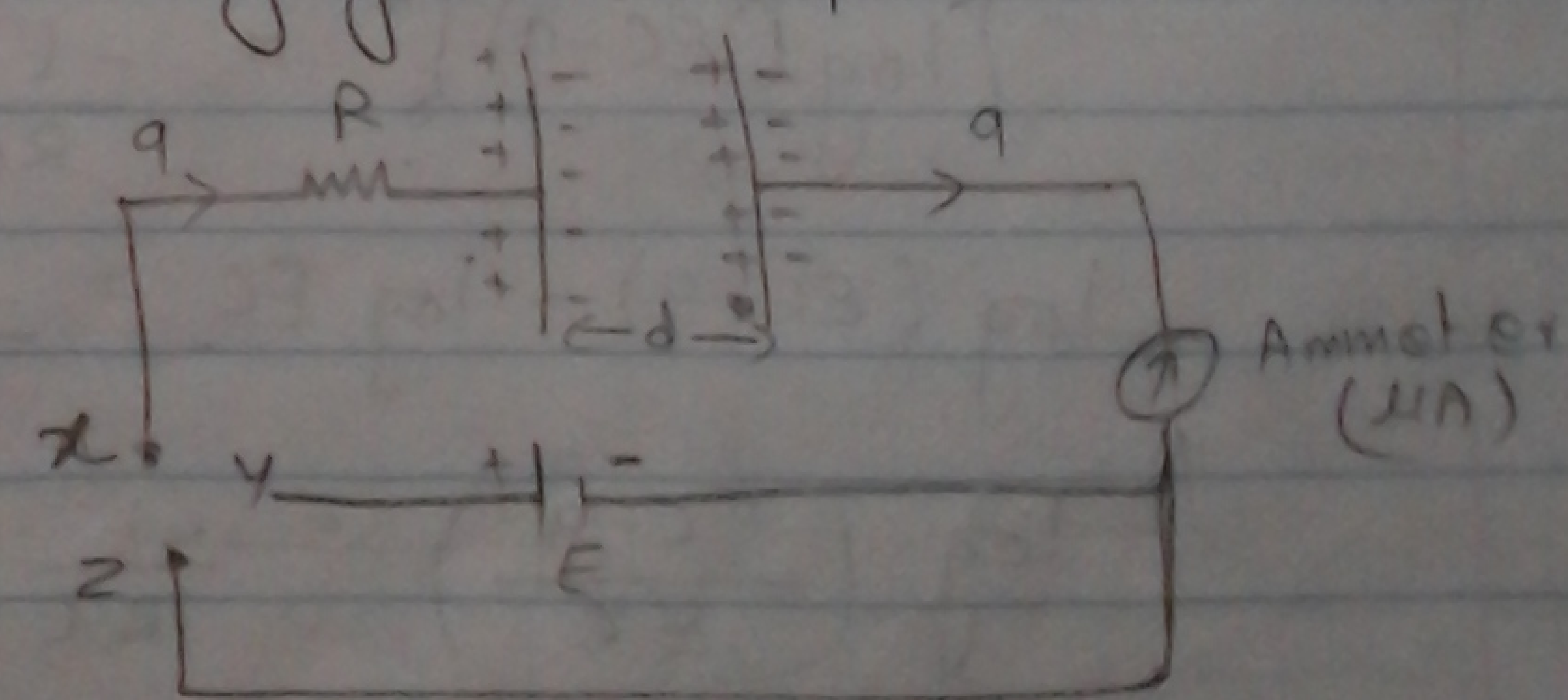


fig (1)

① Charging:
Consider a battery of emf E is connected to the capacitor's plate and resistor. The combination of RC produces the free oscillation of electrons due to the emf of the battery. The emf of battery drop into resistor and capacitor during the charging of capacitor. Therefore, mathematically,

$$E = V_R + V_C$$

$$E = IR + \frac{q}{C}$$

$$E = \frac{dq}{dt} R + \frac{q}{C}$$

$$E - q/C = \frac{dq}{dt} \times R$$

$$\text{or, } \frac{dq}{dt} = \frac{EC - q}{RC}$$

$$\frac{dq}{EC - q} = \frac{1}{RC} dt \quad \ldots (1)$$

At $t = 0$, $q = 0$ and at any instant of time t, the charge q is stored, then taking integration on both sides;

$$\int_0^q \frac{dq}{EC - q} = \int_0^t \frac{1}{RC} dt$$

$$\left[-\log(EC - q)\right]_0^q = \frac{1}{RC} t$$

$$\left[\log(EC - q)\right]_0^q = \frac{-t}{RC}$$

$$\log(EC - q) - \log EC = \frac{-t}{RC}$$

$$\log\left(\frac{EC - q}{EC}\right) = \frac{-t}{RC}$$

① Charging:
Consider a battery of emf E is connected to the capacitor's plate and resistor. The combination of RC produces the free oscillation of electrons due to the emf of the battery. The emf of battery drop into resistor and capacitor during the charging of capacitor. Therefore, mathematically,

$$E = V_R + V_c$$

$$E = IR + \frac{q}{c}$$

$$E = \frac{dq}{dt}R + \frac{q}{c}$$

$$E - q/c = \frac{dq}{dt} \times R$$

$$\text{or, } \frac{dq}{dt} = \frac{Ec - q}{RC}$$

$$\frac{dq}{Ec-q} = \frac{1}{RC}dt \quad \ldots (1)$$

At $t = 0$, $q = 0$ and at any instant of time t, the charge q is stored, then taking integration on both sides;

$$\int_0^q \frac{dq}{Ec - q} = \int_0^t \frac{1}{RC} dt$$

$$\left[-\log(EC-q)\right]_0^q = \frac{1}{RC} t$$

$$\left[\log(EC-q)\right]_0^q = \frac{-t}{RC}$$

$$\log(EC-q) - \log EC = \frac{-t}{RC}$$

$$\log\left(\frac{EC-q}{EC}\right) = \frac{-t}{RC}$$

① Charging:

Consider a battery of emf E is connected to the capacitor's plate and resistor. The combination of RC produces the free oscillation of electrons due to the emf of the battery. The emf of battery drop into resistor and capacitor during the charging of capacitor Therefore, mathematically,

$$E = V_R + V_C$$

$$E = IR + \frac{q}{C}$$

$$E = \frac{dq}{dt} R + \frac{q}{C}$$

$$E - q/C = \frac{dq}{dt} \times R$$

$$\text{or, } \frac{dq}{dt} = \frac{EC - q}{RC}$$

$$\frac{dq}{EC-q} = \frac{1}{RC} dt \quad \ldots (1)$$

At $t = 0$, $q = 0$ and at any instant of time t, the charge q is stored, then taking integration on both sides;

$$\int_0^q \frac{dq}{EC - q} = \int_0^t \frac{1}{RC} dt$$

$$\left[-\log(EC - q)\right]_0^q = \frac{1}{RC} t$$

$$\left[\log(EC-q)\right]_0^q = \frac{-t}{RC}$$

$$\log(EC - q) - \log EC = \frac{-t}{RC}$$

$$\log\left(\frac{EC-q}{EC}\right) = \frac{-t}{RC}$$

$$EC - q = EC\, e^{-t/RC}$$

$$q = EC\,(1 - e^{-t/RC})$$

or, $$q = q_0\,(1 - e^{-t/RC})$$

Since $q_0 = EC$ is maximum charge on circuit

Taking derivative w.r.t t

$$\frac{dq}{dt} = q_0\left[0 - e^{-t/RC} \times \frac{-1}{RC}\right]$$

$$\Rightarrow I = \frac{q_0}{RC}\, e^{-t/RC}$$

$$\Rightarrow \boxed{I = I_0\, e^{-t/RC}} \quad \ldots (11)$$

This is the current flow on the circuit during charging of capacitor

If $t = RC$

$$\boxed{I = \frac{1}{e} I_0 = 37\%\ \text{of}\ I_0} \quad \ldots (12)$$

$RC \rightarrow$ time period at which I decreases by 37%

The time $t = RC$ is called capacitive time constant which gives the maintainance of AC current in capacitor. At this time, 37% of maximum current is only shown in the circuit, it means the current is decreased to 37% of maximum current

Plot in graph (eqn (11) v.s d)

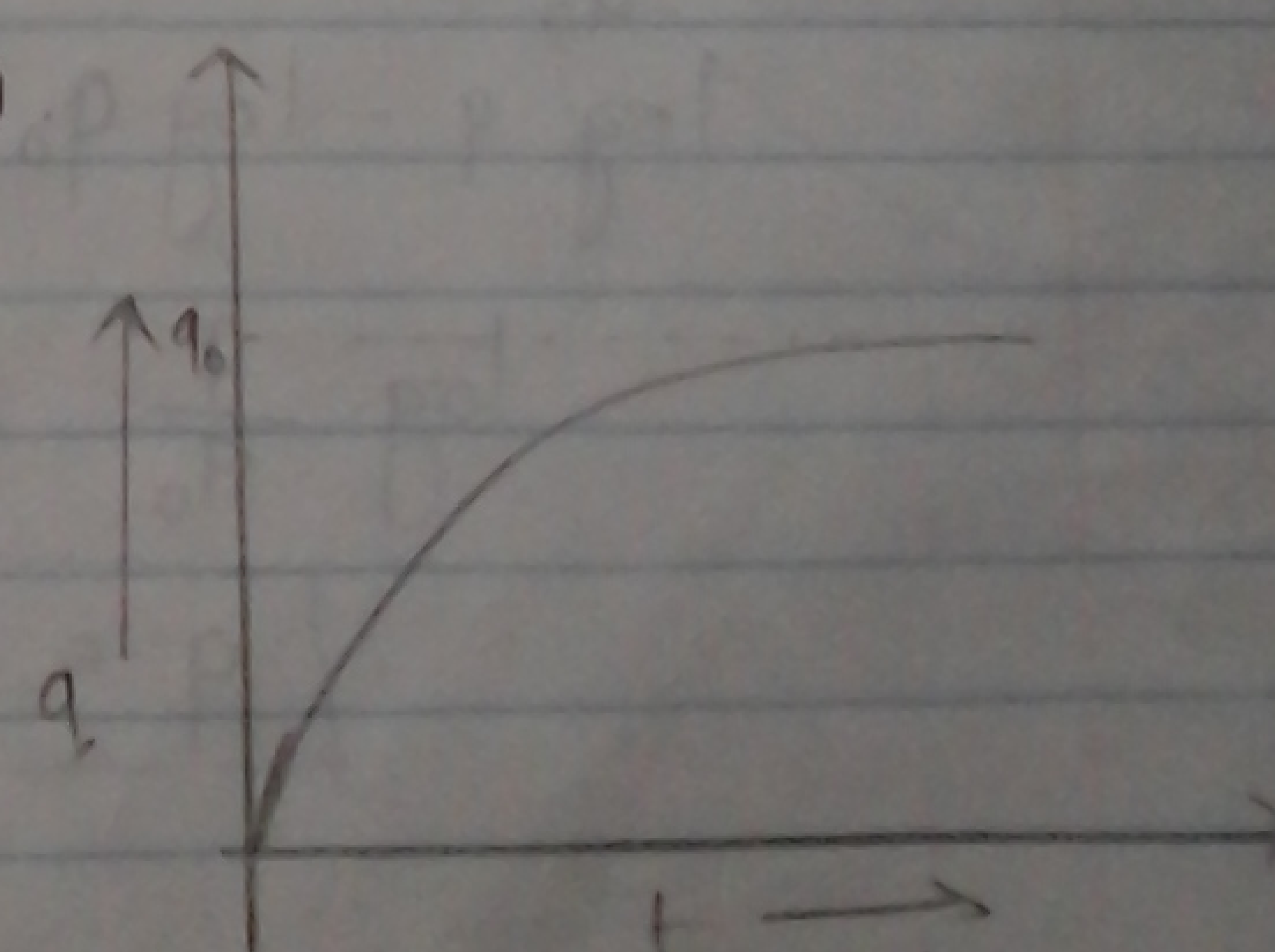


fig (1) q vs t

$$EC - q = EC\, e^{-t/RC}$$

$$q = EC\,(1 - e^{-t/RC})$$

or, $$q = q_0\,(1 - e^{-t/RC})$$

since $q_0 = EC$ is maximum charge on circuit

Taking derivative w.r.t t

$$\frac{dq}{dt} = q_0\left[0 - e^{-t/RC} \times -\frac{1}{RC}\right]$$

$$\Rightarrow I = \frac{q_0}{RC}\, e^{-t/RC}$$

$$\Rightarrow \boxed{I = I_0\, e^{-t/RC}} \quad \ldots (11)$$

This is the current flow on the circuit during charging of capacitor

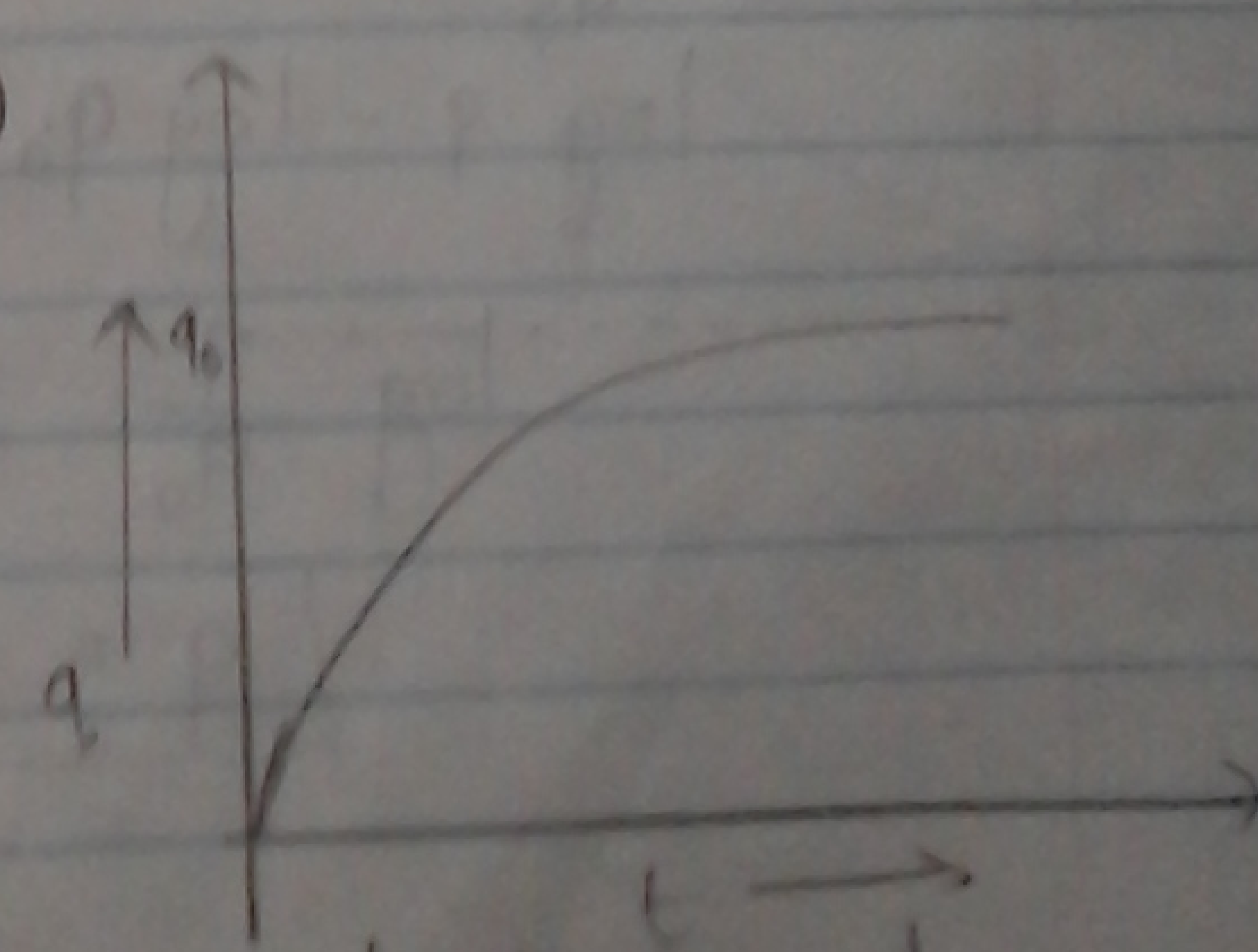
If $t = RC$

$$\boxed{I = \frac{1}{e} I_0 = 37\%\text{ of } I_0} \quad \ldots (14)$$

RC → time period at which I decreases by 63%.

The time $t = RC$ is called capacitive time constant which gives the maintainance of AC current in capacitor. At this time, 37% of maximum current is only shown in the circuit, it means the current is decreased to 37% of maximum current

Plot in graph (eqn (11) v.s d)



fig① q vs t

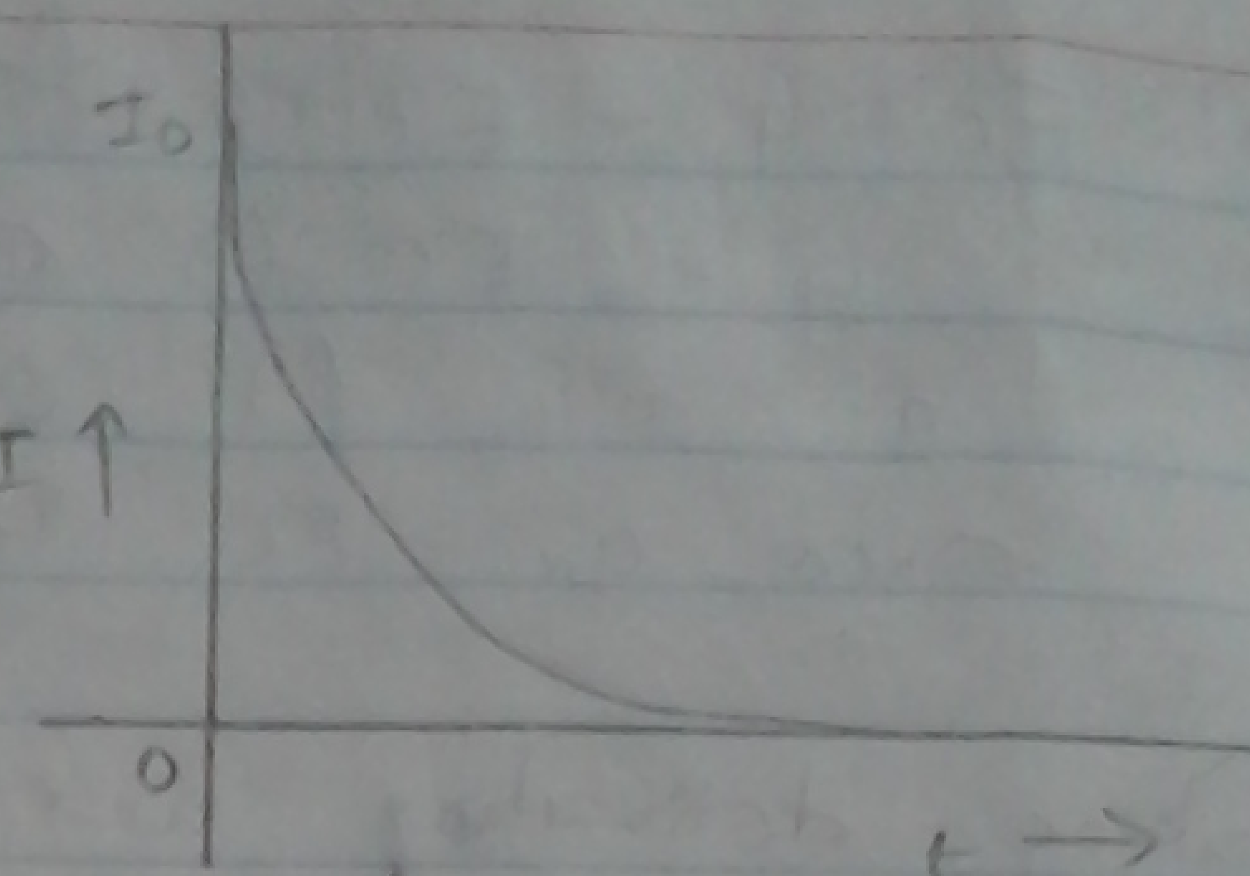


fig: I vs t

2. Discharging:
When the terminals X and Z are connected in fig①
the storage charges begins to loose from the
capacitor. Therefore,

$$V_R + V_C = 0$$

$$\Rightarrow IR + \frac{q}{C} = 0$$

$$\frac{dq}{dt} R + \frac{q}{C} = 0$$

$$\frac{dq}{q} = \frac{-1}{RC} dt$$

Taking integration at $t = 0$ to t
$q = q_0$ to q

$$\int_{q_0}^{q} \frac{dq}{q} = -\int_0^t \frac{1}{RC} dt$$

$$\log q - \log q_0 = \frac{-1}{RC} t$$

$$\log \frac{q}{q_0} = \frac{-t}{RC}$$

$$\boxed{q = q_0 e^{-t/RC}} \quad \text{... (iv)}$$

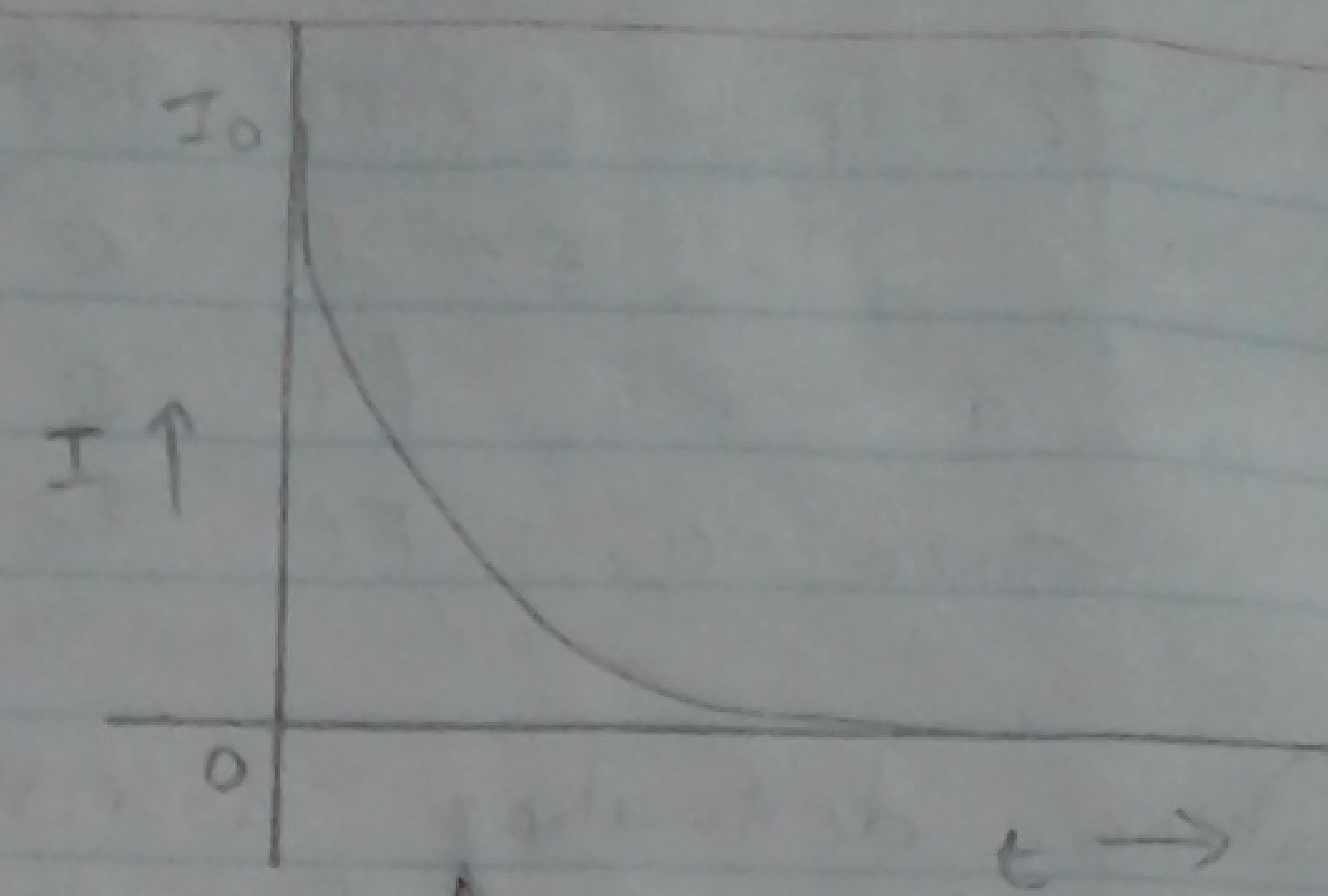


fig: I vs t

2. Discharging:

When the terminals X and Z are connected in fig(1) the storage charges begins to loose from the capacitor. Therefore,

$$V_R + V_C = 0$$

$$\Rightarrow IR + \frac{q}{C} = 0$$

$$\frac{dq}{dt} R + \frac{q}{C} = 0$$

$$\frac{dq}{q} = \frac{-1}{RC} dt$$

Taking integration at t=0 to t

q = q_0 to q

$$\int_{q_0}^{q} \frac{dq}{q} = -\int_{0}^{t} \frac{1}{RC} dt$$

$$\log q - \log q_0 = \frac{-1}{RC} t$$

$$\log \frac{q}{q_0} = \frac{-t}{RC}$$

$$\boxed{q = q_0 e^{-t/RC}} \quad \ldots \text{(iv)}$$

and

$$\frac{dq}{dt} = q_0 \left(-\frac{1}{RC}\right) e^{-t/RC}$$

$$\boxed{I = -I_0 \, e^{-t/RC}} \quad \cdots - (v)$$

The –ve sign indicates that during the discharge of capacitor, the current flow in opposite direction of the charging.

At $t = RC$

$$q = q_0 e^{-1}$$

$$q = \frac{q_0}{e} = 37\% \text{ of } q_0$$

Also,

$$I = -\frac{I_0}{e}$$

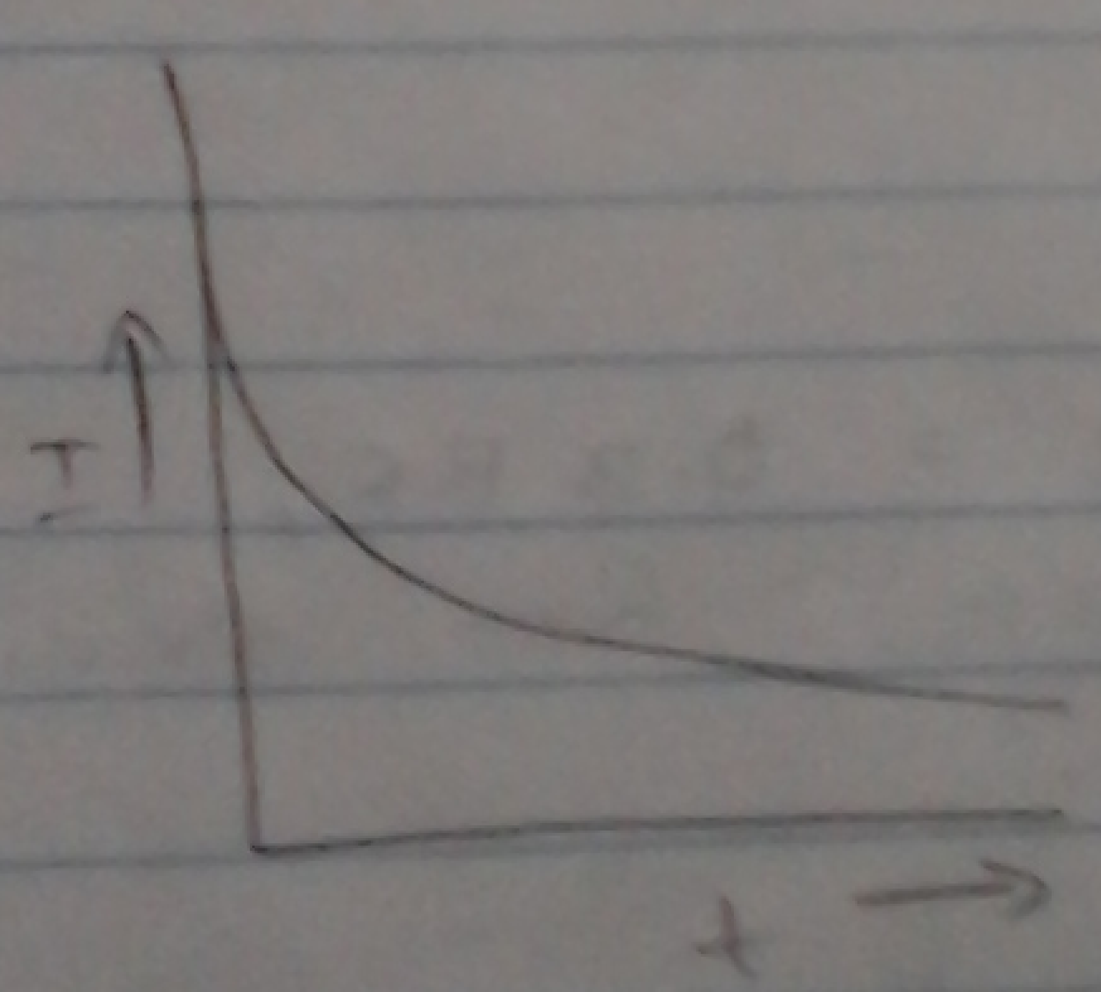
$$= 37\% \text{ of } I_0$$

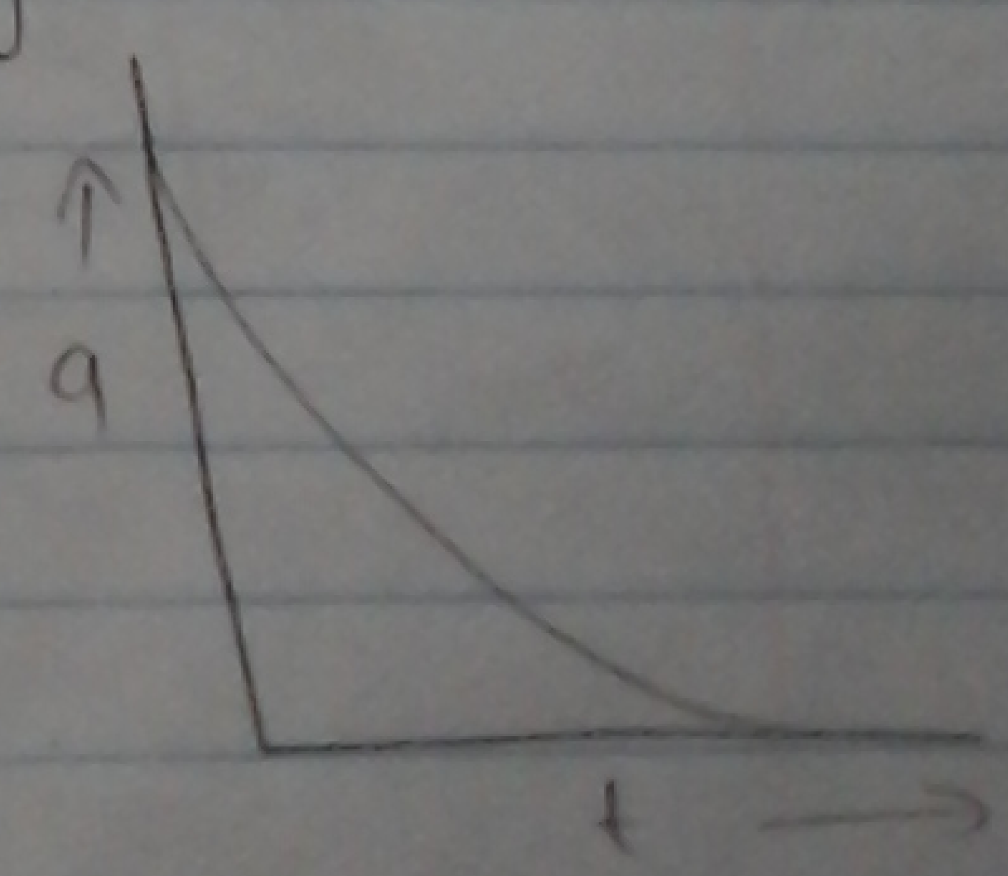
For charging, at $t = RC$

$q = 63\%$ of q_0 is stored

For discharging $q = 37\%$ of q_0 is discharged

$\therefore$ time for discharging is greater.





and

$$\frac{dq}{dt} = q_0 \left(-\frac{1}{RC}\right) e^{-t/RC}$$

$$\boxed{I = -I_0 \, e^{-t/RC}} \quad \ldots \text{(V)}$$

The −ve sign indicates that during the discharge of capacitor, the current flow in opposite direction of the charging.

At $t = RC$

$$q = q_0 e^{-1}$$

$$q = \frac{q_0}{e} = 37\% \text{ of } q_0$$

Also,

$$I = -\frac{I_0}{e}$$

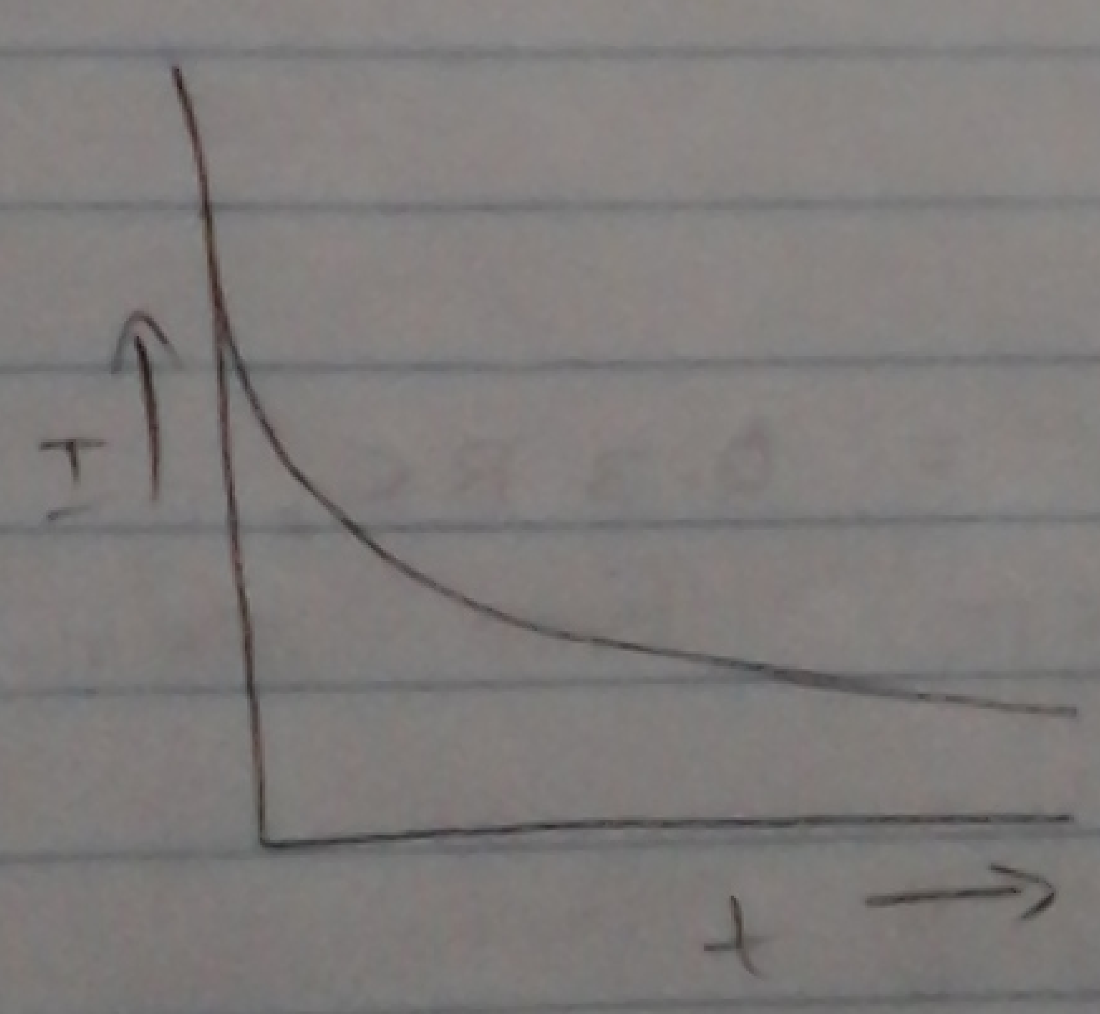
$$= 37\% \text{ of } I_0$$

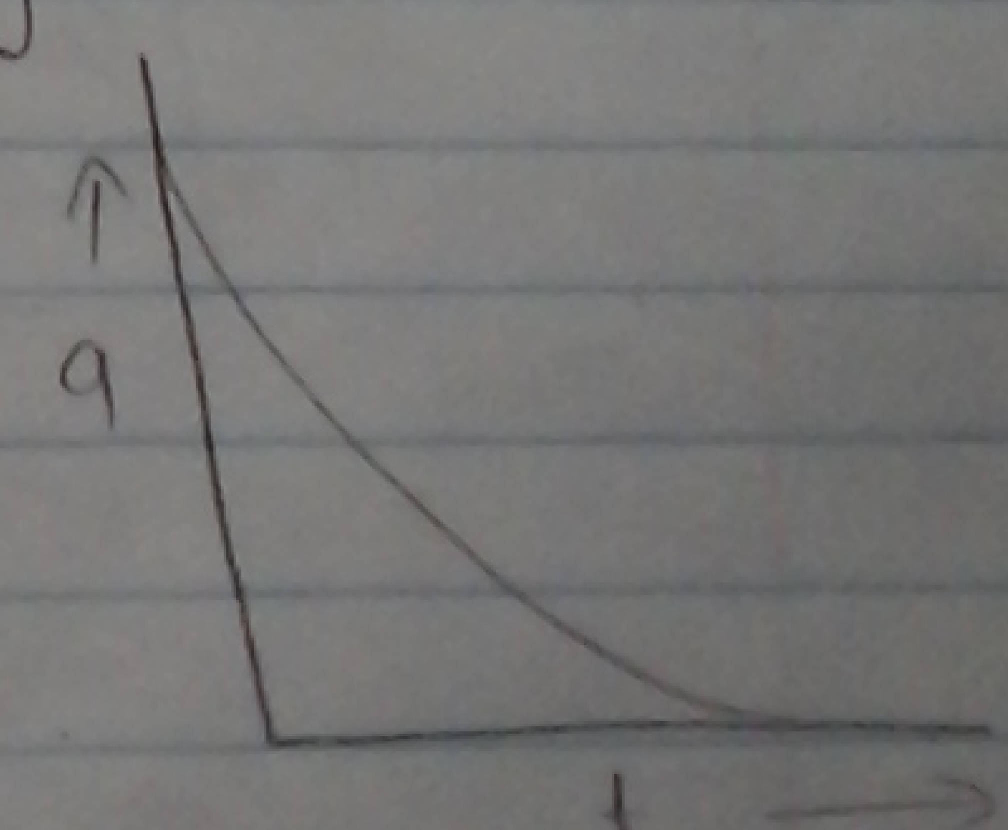
For charging, at $t=RC$

$q = 63\%$ of q_0 is stored

For discharging, $q = 37\%$ of q_0 is discharged

∴ time for discharging is greater.





Q.1) A capacitor of capacitance C is discharged through a resistor of resistance R, after how many times constant is the stored energy 1/4 of it's initial value?

⇒ Soln;

$$\text{Energy stored}(\varepsilon) = \frac{1}{2}\frac{q^2}{C}$$

$$U = \frac{1}{4}U_0$$

At $t = t_1$

$$\varepsilon_1 = \frac{1}{4} \times \varepsilon$$

$$= \frac{1}{4} \times \frac{1}{2}\frac{q^2}{C} = \frac{1}{8}\frac{q^2}{C} = \frac{1}{2} \times \frac{1}{C} \times \left(\frac{q}{2}\right)^2$$

Now,

$$q = q_0 e^{-t/RC}$$

$$\frac{q}{2} = q e^{-t/RC}$$

We have;

$$q^2 = \frac{1}{4} q_0^2$$

$$(q_0 e^{-t/RC})^2 = \frac{1}{4} q_0^2$$

$$e^{-t/RC} = \frac{1}{4}$$

$$\frac{2t}{RC} = \log 4$$

$$t = \frac{1}{2}\log 4 \times RC = 0.3\,RC$$

Dielectric Constant:

The polar and non-polar insulator are the dielectric material like glass, rubber etc. Dielectric constant of the material is defined as the ratio of capacitance of capacitor with the presence of dielectric material to the capacitance without dielectric material in between the parallel plates. It is denoted by k and unitless i.e.

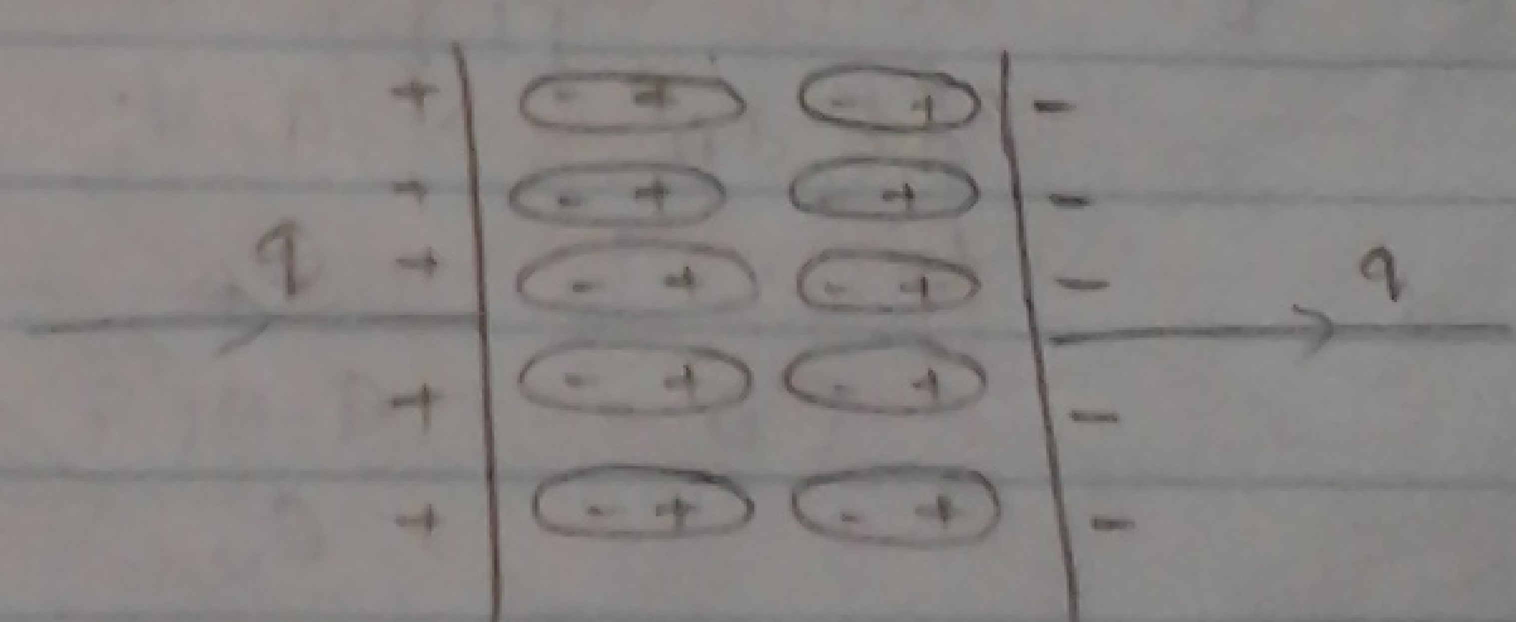
$$k = \frac{C}{C_0} = \frac{qC}{qC_0}$$

$$k = \frac{q/C_0}{q/C}$$

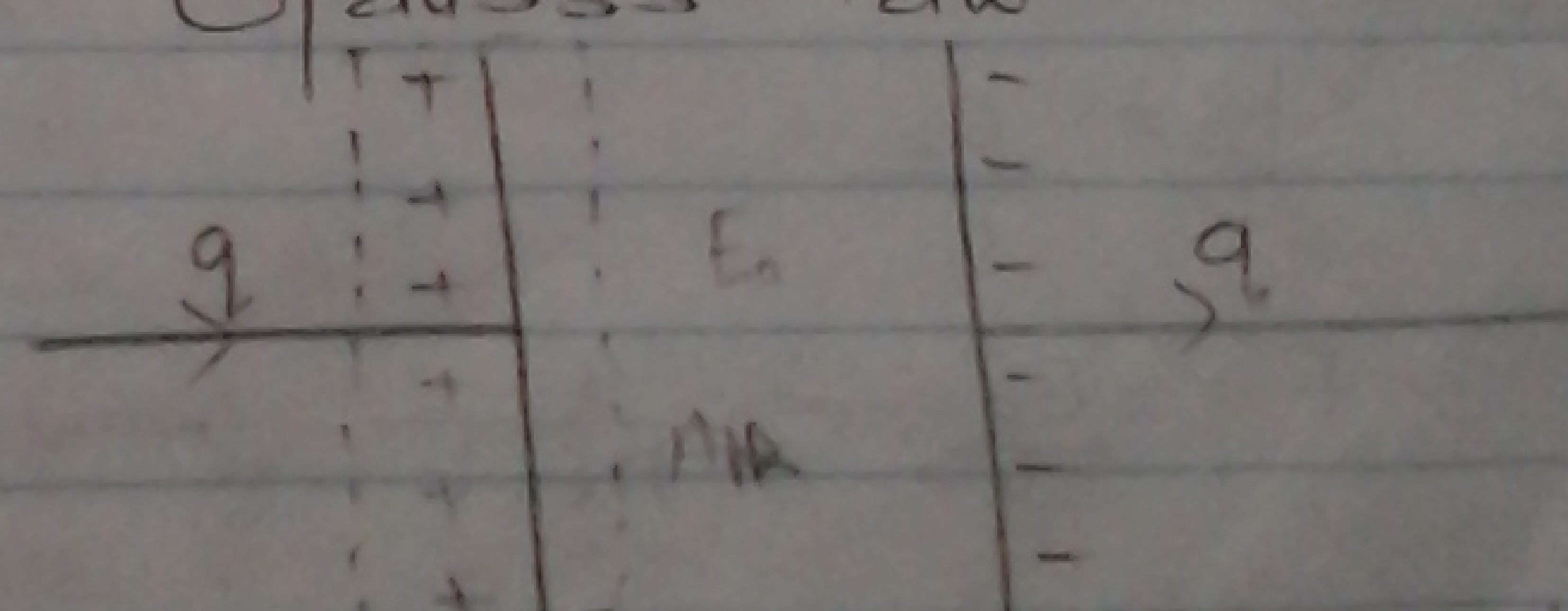
$$k = \frac{E_0}{E}$$

i.e. $k = \dfrac{\text{Electric field without dielectric material}}{\text{Reduced electric field with dielectric material}}$

It's value is always greater than 1, so the capacitance of capacitor is increased by the factor k if the dielectric material is introduced in between the plates.



Imp
Dielectric and Gauss's law



fig① Capacitance without dielectric material

Dielectric Constant:

The polar and non-polar insulator are the dielectric material like glass, rubber etc. Dielectric constant of the material is defined as the ratio of capacitance of capacitor with the presence of dielectric material to the capacitance without dielectric material in between the parallel plates. It is denoted by k and unitless. i.e

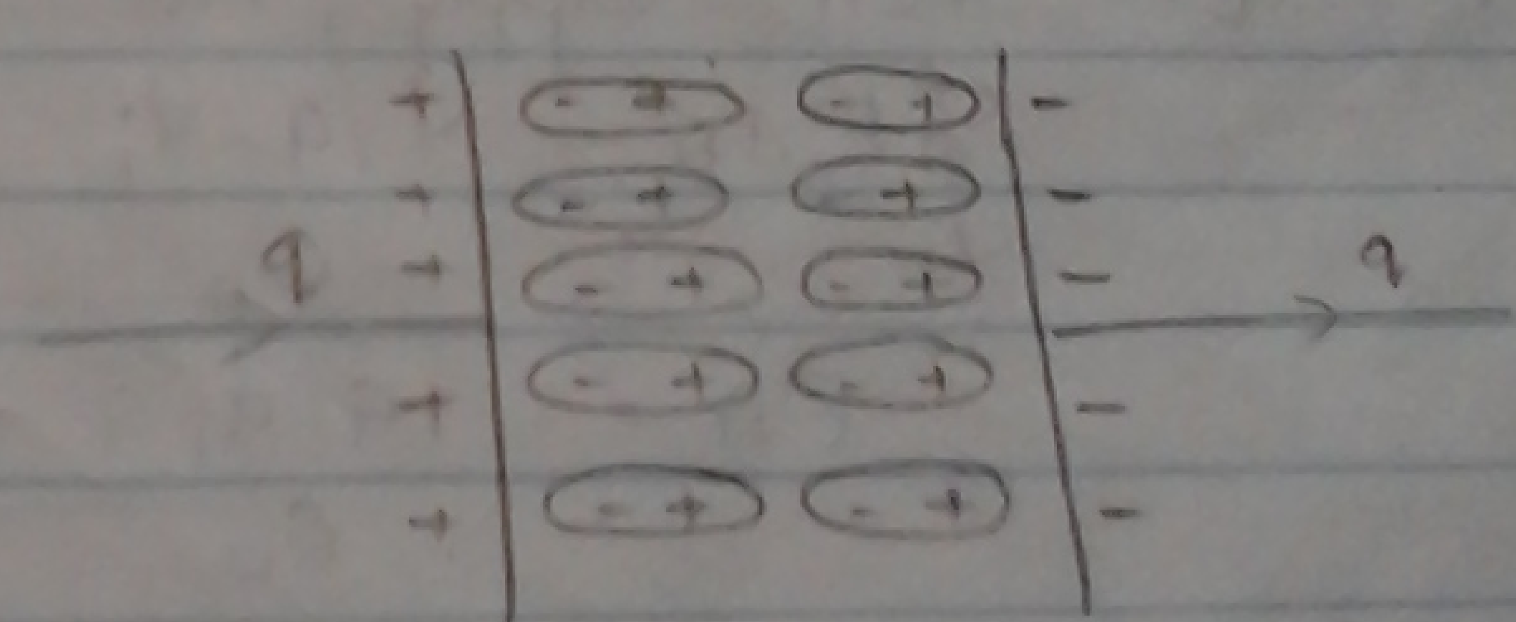
$$k = \frac{C}{C_0} = \frac{q/V}{q/V_0}$$

$$k = \frac{V_0}{V}$$

$$k = \frac{E_0}{E}$$

i.e $k = \dfrac{\text{Electric field without dielectric material}}{\text{Reduced electric field with dielectric material}}$

It's value is always greater than 1, so the capacitance of capacitor is increased by the factor k if the dielectric material is introduced in between the plates.



Imp

Dielectric and Gauss's law:

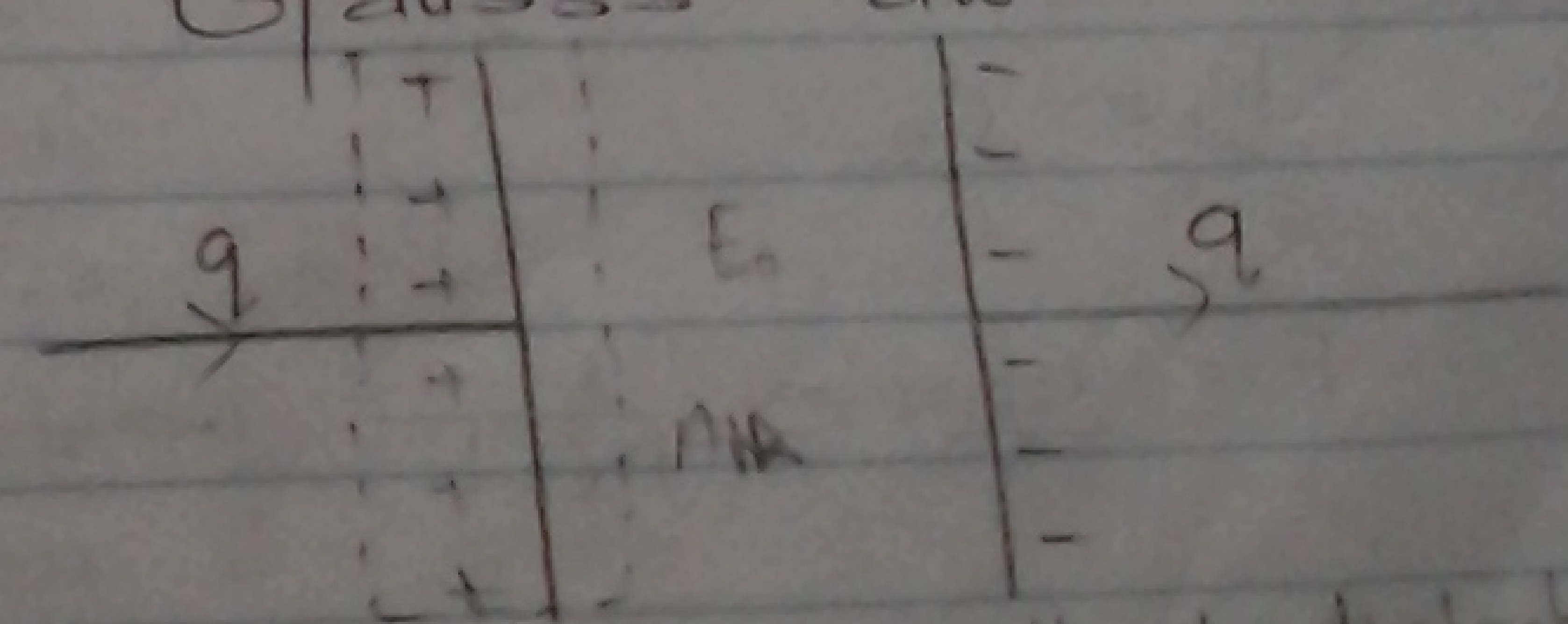


fig (1) Capacitance without dielectric material

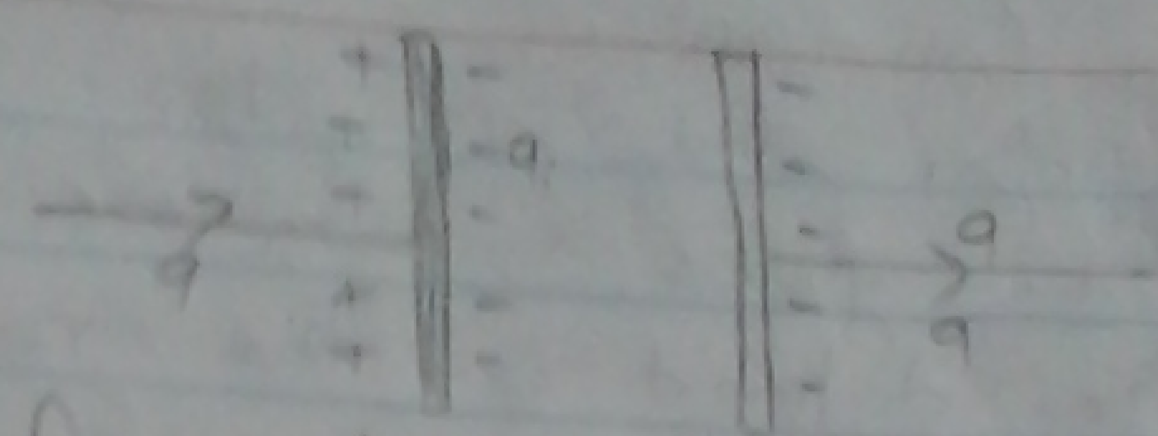


fig ⓐ Capacitor with dielectric material

Consider 'q' amount of charge be applied into the plates of capacitor without presence of any dielectric material then the uniform electric field is produced in between them ~~say~~ i.e E_0. Now using Gauss's law

$$\oint E_0 \, dA = \frac{q}{\epsilon_0} \quad \cdots \text{(a)}$$

$$E_0 = \frac{q}{A\epsilon_0} \quad \cdots \text{(i)}$$

where A is the cross-sectional area of plate.

When the dielectric material is introduced in between the plates and apply the same charge q. then certain charges are induced in the surface of parallel plate. These charges are responsible to polarize the opposite charge on next plate so that a dielectric materials the electric field is reduced to E. Then apply the Gauss's law

$$\oint E \, dA = \frac{q - q_i}{\epsilon_0}$$

$$EA = \frac{q - q_i}{\epsilon_0}$$

$$E = \frac{q - q_i}{A\epsilon_0} = \frac{q}{A\epsilon_0} - \frac{q_i}{A\epsilon_0} \quad \cdots \text{(ii)}$$

Since

$$k = \frac{E_0}{E}$$

$$E = \frac{E_0}{k} = \frac{q}{A\epsilon_0 k} \quad \cdots \text{(iii)}$$

From (ii) & (iii)

$$\frac{q}{A\epsilon_0 k} = \frac{q}{A\epsilon_0} - \frac{q_i}{A\epsilon_0}$$

$$\frac{q}{k} = q - q_i$$

$$\Rightarrow k = \frac{q}{q - q_i}$$

$$k = \frac{1}{1 - q_i/q}$$

Since $q_i < q$

$$\Rightarrow k > 1$$

Again, from eqn (ii)

$$E = \frac{q}{A\epsilon_0} - \frac{q_i}{A\epsilon_0}$$

$$\Rightarrow \frac{E_0}{k} = \frac{q}{A\epsilon_0} - \frac{q_i}{A\epsilon_0}$$

$$\Rightarrow \frac{q}{A\epsilon_0} = \frac{E_0}{k} + \frac{q_i}{A\epsilon_0}$$

$$\Rightarrow \boxed{\frac{q}{A} = \epsilon_0 E + \frac{q_i}{A}}$$

or,

$$\boxed{\vec{D} = \epsilon_0 \vec{E} + \vec{P}}$$

where $\vec{D}$ = Displacement vector $= \dfrac{\text{total free charge}}{\text{cross-sectional area}}$

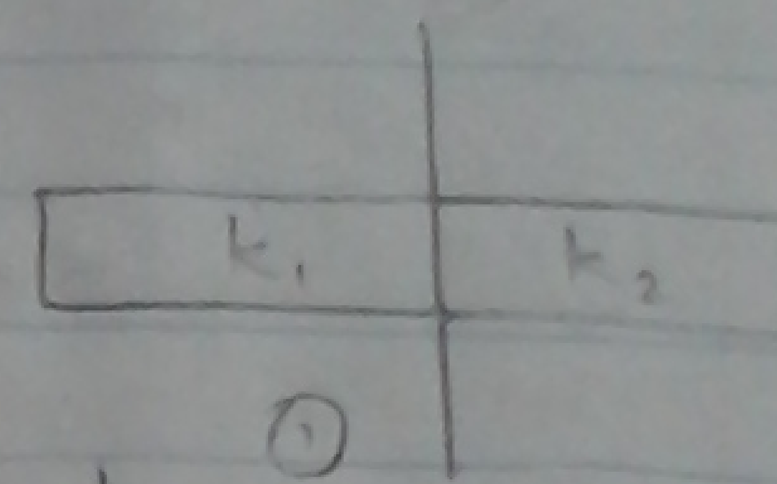
E = Electric field due to presence of dielectric material.

$\vec{P}$ = Polarization vector

$= \dfrac{\text{Total induced free charge}}{\text{Area of plate}}$

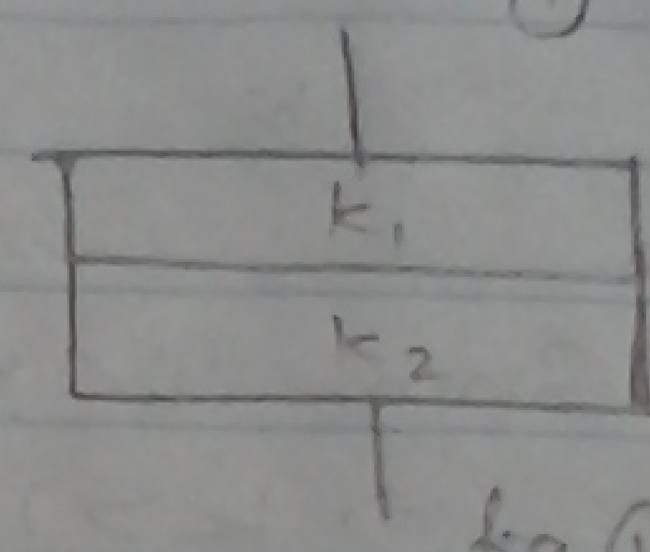
N

Q. 1) A parallel plate capacitor is filled with two dielectric material of dielectric constant k_1 & k_2 so that the capacitance is given by (i) $C = \frac{\epsilon_0 A}{d}\left(\frac{k_1+k_2}{2}\right)$



(i) in paralle

(ii) $C = \frac{2\epsilon_0 A}{d}\left(\frac{k_1 k_2}{k_1+k_2}\right)$



in series

fig (ii)

(i)

$k_1 = \frac{C_1}{C_0}$

$k_2 = \frac{C_2}{C_0}$ $\qquad C_1 = \frac{k_1 \epsilon_0 A/2}{d}$

$C_1 = C_0 k_1$ $\qquad C_2 = \frac{k_1 \epsilon_0 A/2}{d}$

$C_2 = C_0 k_2$

C_1 & C_2 are in parallel

$\therefore C = C_1 + C_2$

$= C_0 k_1 + C_0 k_2$

$= C_0(k_1 + k_2)$

$= \frac{\epsilon_0 A}{2d}(k_1 + k_2)$

(ii)

$C_1 = C_0 k_1 \Rightarrow C_1 = \frac{k_1 \epsilon_0 A/2}{d}$ $\qquad \therefore C_0 = \frac{\epsilon_0 A}{2d}$

$C_2 = C_0 k_2 \Rightarrow C_2 = \frac{k_2 \epsilon_0 A/2}{d}$

C_1 & C_2 are in series

$\therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{C_0 \cdot C_0 k_1 \cdot k_2}{C_0 k_1 + C_0 k_2}$$

$$= \frac{C_0^2 k_1 k_2}{C_0 (k_1 + k_2)}$$

$$= C_0 \times \left(\frac{k_1 k_2}{k_1 + k_2} \right)$$

$$= \frac{2 \epsilon_0 A}{d} \left(\frac{k_1 k_2}{k_1 + k_2} \right)$$

Q. A parallel plate capacitor has a capacitance of 100 $\mu\mu F$ ~~A~~ of plate area of 100 cm^2 mica dielectric at 50V p.d. calculate.

i) E in the mica

ii) q in the plate

iii) induced surface charge

⇒ Soln;

$$C = 100 \mu\mu F = 100 \times 10^{-12} F$$

$$A = 100 \times 10^{-4} m^2$$

$$V = 50V$$

$$q = CV = 100 \times 10^{-12} \times 50 = 5 \times 10^{-9}$$

$$E = \frac{q}{\epsilon_0 A} = \frac{5 \times 10^{-9}}{100 \times 10^{-4} \times 8.85 \times 10^{-12}}$$

$$= 5.88 \times 10^{4}$$

$$\therefore E = \frac{1}{A \epsilon_0} (q - q_i)$$

$$5.88\times10^{4} = \frac{1}{100\times10^{-4}\times 8.85\times10^{-12}}\left(5\times10^{-9} - q_i\right)$$

$$q\left(5\times10^{-9} - q_i\right) = 5.88\times10^{4}\times100\times10^{-4}\times8.85\times10^{-12}$$

https://civinnovate.com/civil-engineering-notes/

Civinnovate

Discover, Learn, and Innovate in Civil Engineering