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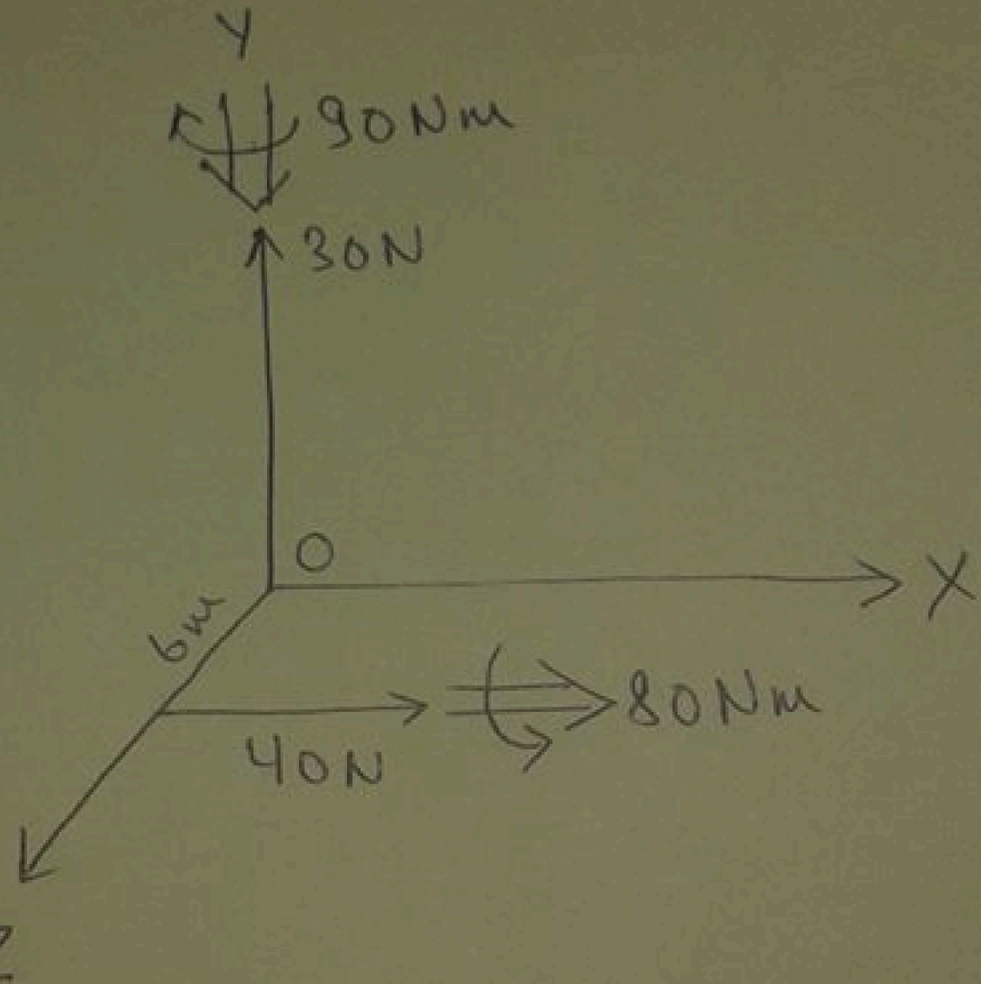
Wrench → A wrench consists of a force and a couple with the same line of action. (For theory of wrench, refer to MRD sir's manual's page 37 and 38)

Replace the two wrenches shown in the figure below by a single equivalent wrench and determine (a) the resultant force (b) the point where its axis intersects the XZ plane.



Sol^n

The figure consists of two wrenches,

i. $(30\hat{j})$ N force and $(-90\hat{j})$ Nm couple

ii. $(40\hat{i})$ N force and $(80\hat{i})$ Nm couple

Since we want to replace the two wrenches by a single equivalent wrench we proceed as follows:

i. Replace the given system of two wrenches (that is, force-couple systems) by an equivalent force-couple system at origin O:

Resultant force at origin,
$\vec{F_R} = (40\hat{i} + 30\hat{j})N$

Resultant couple at origin,
$\vec{C_R} = \vec{C_1} + \vec{C_2} + \vec{r_2} \times \vec{F_2}$
$= -90\hat{j} + 80\hat{i} + 6\hat{k} \times 40\hat{i}$
$= -90\hat{j} + 80\hat{i} + 240\hat{j}$
$\Rightarrow \vec{C_R} = (80\hat{i} + 150\hat{j})$ Nm

ii. As we know wrench consists of a force and a couple with the same line of action, so first find the line of action of $\vec{F_R}$ (that is its direction → unit vector) and then the component of $\vec{C_R}$ along $\vec{F_R}$ (let's call this component $\vec{C_F}$):

$|\vec{F_R}| = \sqrt{40^2 + 30^2} = 50$ N

unit vector, $\hat{f_R} = \frac{40}{50}\hat{i} + \frac{30}{50}\hat{j} = 0.8\hat{i} + 0.6\hat{j}$

Projection of $\vec{C_R}$ on $\vec{F_R}$ (i.e. Component of $\vec{C_R}$ along $\vec{F_R}$) is

$\vec{C_R} \cdot \hat{f_R} = C_F = (80\hat{i} + 150\hat{j}) \cdot (0.8\hat{i} + 0.6\hat{j})$

$\implies C_F = 64 + 90 = 154\ Nm$

So, $\vec{C_F} = 154 * (0.8\hat{i} + 0.6\hat{j}) = (123.2\hat{i} + 92.4\hat{j})Nm$

iii. Find the portion of the resultant couple that doesn't have the same line of action as F_R (let's call this portion $\vec{C_n}$):

$\vec{C_n} = \vec{C_R} - \vec{C_F} = (-43.2\hat{i} + 57.6\hat{j})Nm$

iv. To vanish $\vec{C_n}$, the force $\vec{F_R}$ has to be shifted parallely (that is without changing its line of action) by $(x\hat{i} + y\hat{j} + z\hat{k})m$ so that

$(x\hat{i} + y\hat{j} + z\hat{k}) \times \vec{F_R} = \vec{C_n}$

or, $(x\hat{i} + y\hat{j} + z\hat{k}) \times (40\hat{i} + 30\hat{j}) = (-43.2\hat{i} + 57.6\hat{j})$

or, $30x\hat{k} - 40y\hat{k} + 40z\hat{j} - 30z\hat{i} = -43.2\hat{i} + 57.6\hat{j}$

Equating like terms from both sides in the above equations we get,

$30x = 0 \implies x = 0$

$-40y = 0 \implies y = 0$

$40z = 57.6 \implies z = 1.44m$

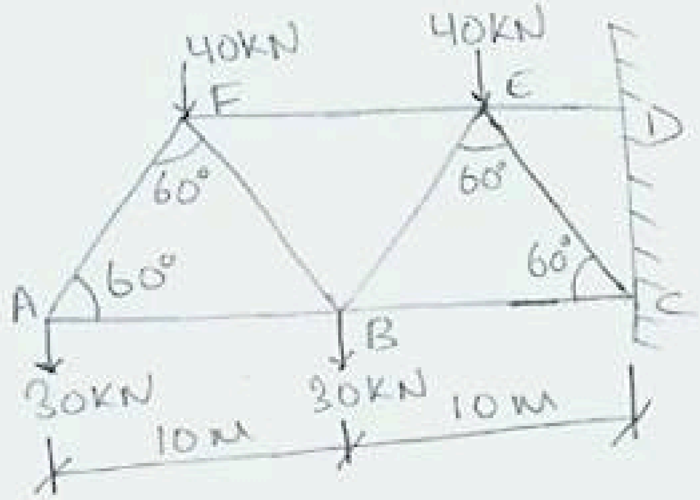
$-30z = -43.2 \implies z = 1.44m$

} we get the same z as it should be.

Now we have a single equivalent wrench that has replaced the two given wrenches.

The equivalent wrench has a force $(40\hat{i} + 30\hat{j})N$ and a couple $(123.2\hat{i} + 92.4\hat{j})\ Nm$.

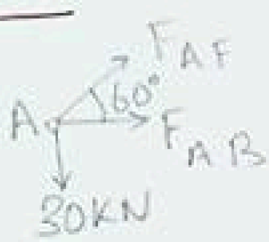
Find the member forces in the truss shown below



Solution

$\angle ABF = 60°$, $\angle CBE = 60°$, $\angle EBF = 60°$, $\angle BEF = 60°$,
$\angle BFE = 60°$, $\angle CED = 60°$

Joint 'A'



$(+\uparrow)\ \Sigma F_y = 0$

or, $-30 + F_{AF}\sin 60° = 0$

$\Rightarrow F_{AF} = 34.64\ \text{KN (T)}$

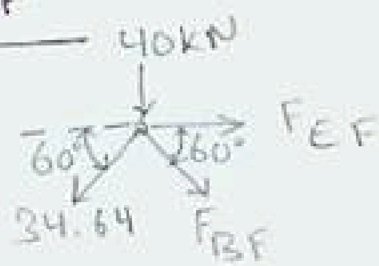
$(\xrightarrow{+})\ \Sigma F_x = 0$

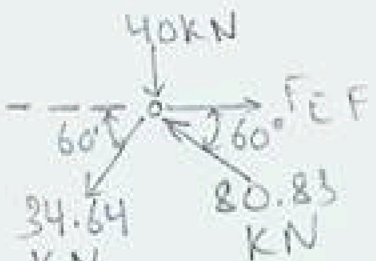
or, $F_{AB} + 34.64\cos 60° = 0$

or, $F_{AB} = -17.32\ \text{KN}$

$\Rightarrow F_{AB} = 17.32\ \text{KN (C)}$

Joint 'F'





$(+\uparrow)\ \Sigma F_y = 0$

or, $-40 - 34.64\sin 60° - F_{BF}\sin 60° = 0$

or, $F_{BF} = -80.83\ \text{KN}$

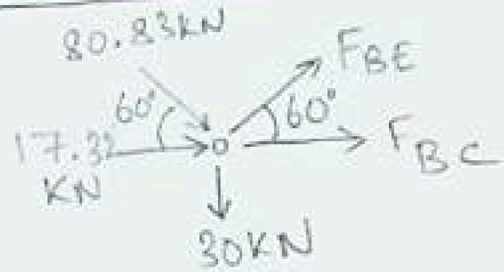
$\Rightarrow F_{BF} = 80.83\ \text{KN (C)}$

$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $-34.64\cos 60° - 80.83\cos 60° + F_{EF} = 0$

$\Rightarrow F_{EF} = 57.73\ \text{KN (T)}$

Joint 'B'



$(+\uparrow)\ \sum F_y = 0$

or, $-30 - 80.83 \sin 60° + F_{BE} \sin 60° = 0$

$\Rightarrow F_{BE} = 115.47$ KN (T)

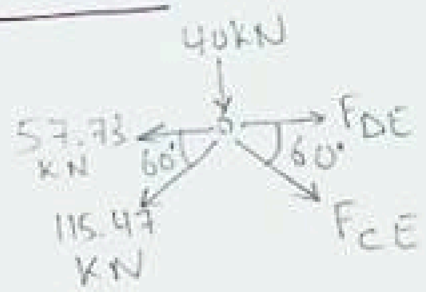
$(\overset{+}{\rightarrow})\ \sum F_x = 0$

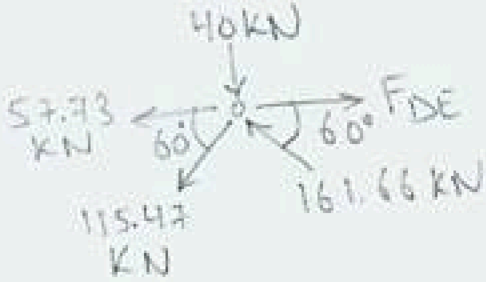
or, $17.32 + 80.83 \cos 60° + 115.47 \cos 60° + F_{BC} = 0$

or, $F_{BC} = -115.47$ KN

$\Rightarrow F_{BC} = 115.47$ KN (C)

Joint 'E'





$(+\uparrow)\ \sum F_y = 0$

or, $-40 - 115.47 \sin 60° - F_{CE} \sin 60° = 0$

or, $F_{CE} = -161.66$ KN

$\Rightarrow F_{CE} = 161.66$ KN (C)

$(\overset{+}{\rightarrow})\ \sum F_x = 0$

or, $-57.73 - 115.47 \cos 60° - 161.66 \cos 60° + F_{DE} = 0$

$\Rightarrow F_{DE} = 196.3$ kN (T)

The plane curvilinear motion of a particle is

defined in polar coordinates by $r = \frac{t^3}{4} + 3t$ and $\theta = 0.5t^2$ where 'r' is in inches, θ is in radians and 't' is in seconds. At the instant when $t = 4$ sec, determine the magnitude of velocity, acceleration and radius of curvature of the path. (2068 Chaitra, old Back. BEX; 10)

Solution

$$\dot{r} = \frac{3t^2}{4} + 3 \quad , \quad \ddot{r} = \frac{6t}{4}$$

$$\dot{\theta} = t \quad , \quad \ddot{\theta} = 1$$

$$\vec{V} = V_r \hat{e}_r + V_\theta \hat{e}_\theta$$

$$V_r = \dot{r} = \frac{3t^2}{4} + 3$$

$$V_\theta = r\dot{\theta} = \left(\frac{t^3}{4} + 3t\right) * t = \left(\frac{t^4}{4} + 3t^2\right)$$

So, $\vec{V} = \left(\frac{3t^2}{4} + 3\right)\hat{e}_r + \left(\frac{t^4}{4} + 3t^2\right)\hat{e}_\theta$

$$V = |\vec{V}| = \sqrt{V_r^2 + V_\theta^2}$$

$$= \sqrt{\left(\frac{3t^2}{4} + 3\right)^2 + \left(\frac{t^4}{4} + 3t^2\right)^2}$$

$$= \sqrt{\frac{9t^4}{16} + \frac{9t^2}{2} + 9 + \frac{t^8}{16} + \frac{3t^6}{2} + 9t^4}$$

$$\Rightarrow V = \sqrt{\frac{t^8}{16} + \frac{3t^6}{2} + \frac{153t^4}{16} + \frac{9t^2}{2} + 9}$$

$$a_t = \frac{dV}{dt} = \frac{1}{2} * \frac{1}{\sqrt{\frac{t^8}{16} + \frac{3t^6}{2} + \frac{153t^4}{16} + \frac{9t^2}{2} + 9}} \left(\frac{8t^7}{16} + \frac{18t^5}{2} + \frac{612t^3}{16} + \frac{18t}{2}\right)$$

When $t = 4$ sec,

$r = 16 + 3*4 = 28$ in, $\dot{r} = 3*4 + 3 = 15$ in/sec

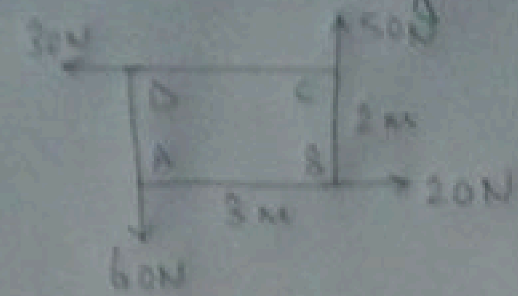
$\ddot{r} = 6$ in/sec^2

$\dot{\theta} = 4$ rad/sec , $\ddot{\theta} = 1$ rad/sec^2

$V_r = \dot{r} = 15$ in/sec

$V_\theta = r\dot{\theta} = 28 * 4 = 112$ in/sec

Determine the magnitude, direction and position of the resultant force of the forces acting on a rectangular plate shown in figure below. (2068 Chaitra, BEX; 2)



Solution

$(\xrightarrow{+})$ $(F_R)_x = \Sigma F_x$

$= -30 + 20$

$= -10\,N$

$\Rightarrow (F_R)_x = 10\,N\,(\leftarrow)$

$(+\uparrow)$ $(F_R)_y = \Sigma F_y$

$= 50 - 60$

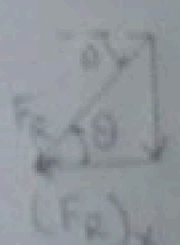
$= -10\,N$

$\Rightarrow (F_R)_y = 10\,N\,(\downarrow)$

$(\circlearrowleft +)$ $(M_R)_A = \Sigma M_A$

$= 30 * 2 + 50 * 3$

$\Rightarrow (M_R)_A = 210\,N\text{-}m\,(\circlearrowleft)$



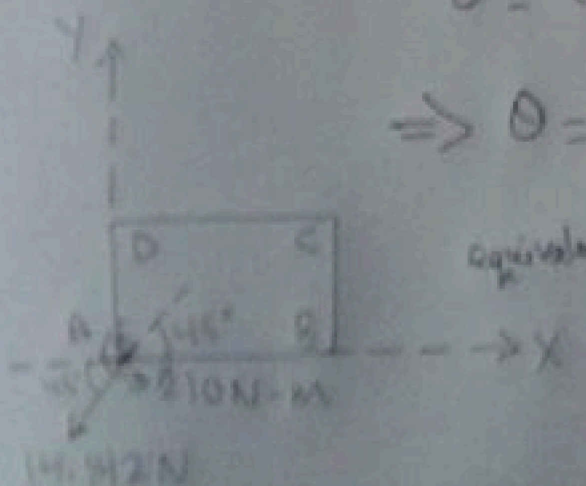
$F_R = \sqrt{(F_{Rx})^2 + (F_{Ry})^2}$

$= \sqrt{(-10)^2 + (-10)^2}$

$F_R = 10\sqrt{2}\,N = 14.142\,N$

$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right| = \tan^{-1}\left|\frac{+10}{+10}\right|$

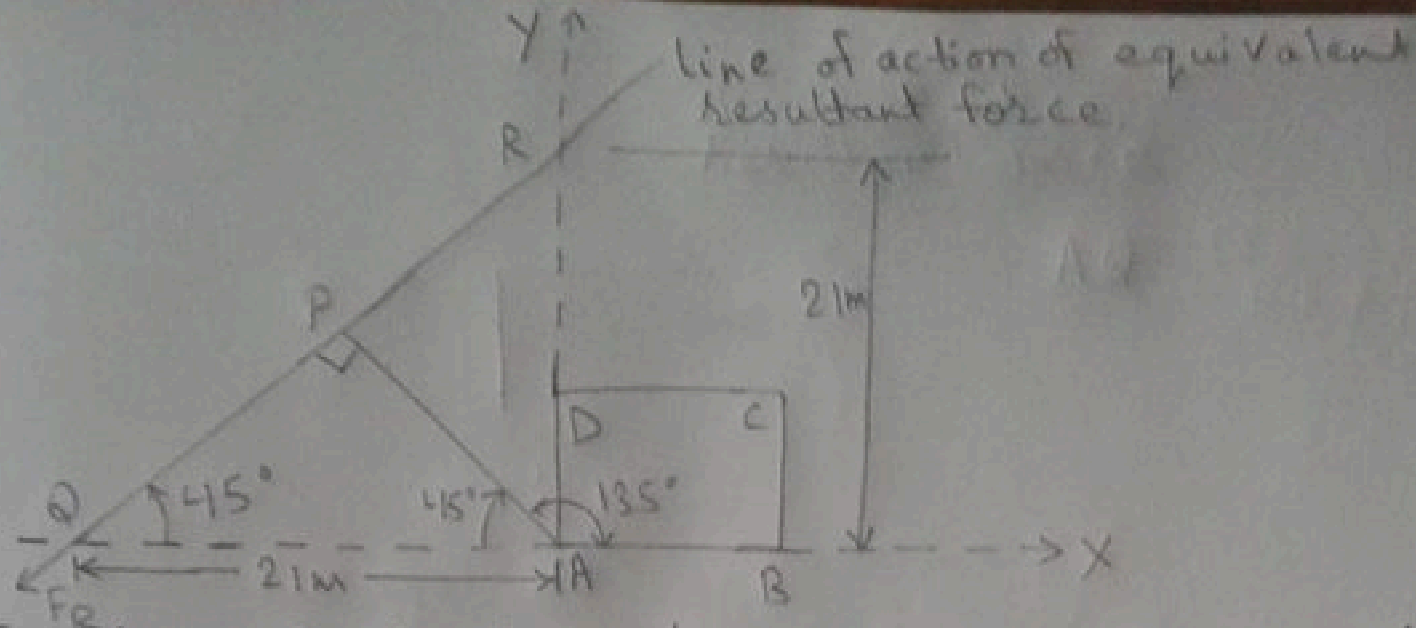
$\Rightarrow \theta = 45°$



The x and y intercept of the equivalent resultant force are:

$x = -\frac{(M_R)_A}{(F_R)_y} = -\frac{210}{10} = -21\,m$

$y = \frac{(M_R)_A}{(F_R)_x} = \frac{210}{10} = 21\,m$



Equation of the line ~~line~~ of action of equivalent resultant force is

$$\frac{x}{-21} + \frac{y}{21} = 1$$

or, $-x + y = 21$

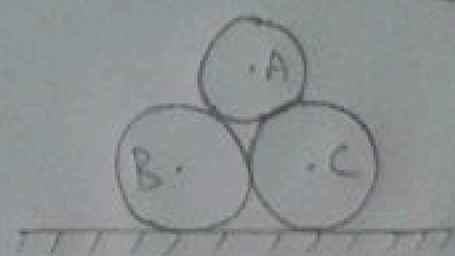
or, $\boxed{x = y - 21}$

Slope of line of action of equivalent force = 1

$$AP = \frac{(M_R)_A}{F_R} = \frac{210}{14.142} = 14.849 \text{ m}$$

Slope of AP = −1 ⇒ $\tan\theta = -1 \Rightarrow \theta = 135°$

\# Find the contact forces of the three bodies as shown in figure below. Body A has 20 cm diameter and 60 N weight and bodies B and C have 30 cm diameter and 100 N weight each. (2067 Mangsir, BCE, 10)



Solution

$AB = AC = 10 + 15 = 25\text{ cm}$

$BC = 15 + 15 = 30\text{ cm}$

ΔABC is an isosceles triangle.

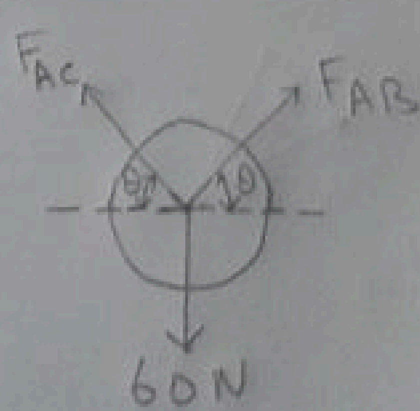
So, $\angle ABC = \angle ACB = \theta$

$AD \perp BC$ and since ΔABC is an isosceles triangle, $BD = BC = 15\text{ cm}$

In ΔADB,

$$\cos\theta = \frac{BD}{AB} = \frac{15}{25} \Rightarrow \theta = 53.13^\circ$$

FBD of 'A'



$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $-F_{AC}\cos\theta + F_{AB}\cos\theta = 0$

$\Rightarrow F_{AB} = F_{AC}$

$(+\uparrow)\ \Sigma F_y = 0$

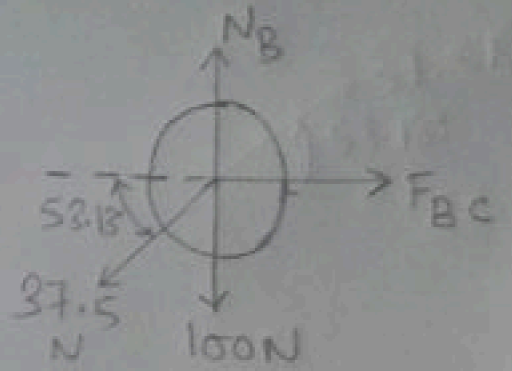
or, $F_{AB}\sin 53.13^\circ + F_{AC}\sin 53.13^\circ - 60 = 0$

or, $0.8 F_{AB} + 0.8 F_{AB} = 60$

$\Rightarrow F_{AB} = 37.5\text{ N}$

$\Rightarrow F_{AC} = 37.5\text{ N}$

FBD of 'B'



$(+\uparrow)\ \Sigma F_y = 0$

or, $N_B - 100 - 37.5 \sin 53.13° = 0$

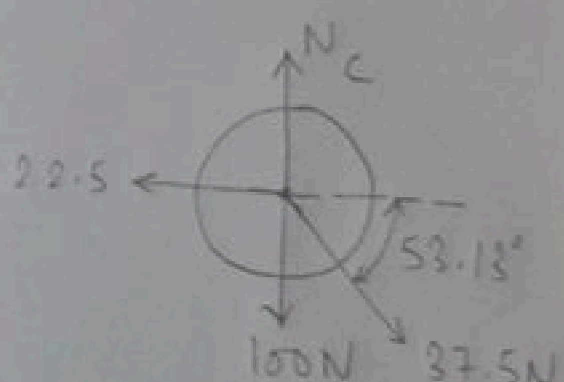
$\Rightarrow N_B = 130N$

$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $F_{BC} - 37.5 \cos 53.13 = 0$

$\Rightarrow F_{BC} = 22.5\ N$

FBD of 'C'

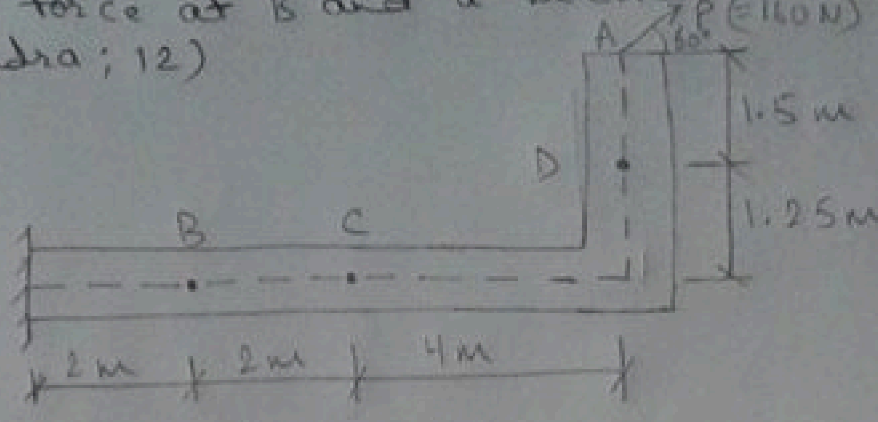


$(+\uparrow)\ \Sigma F_y = 0$

or, $N_C - 100 - 37.5 \sin 53.13° = 0$

$\Rightarrow N_C = 130N$

A 160N force P is applied at point A of a structural member. Replace P with (a) An equivalent force-couple system at C, (b) an equivalent system consisting of a vertical force at B and a second force at D
(2069 Bhadra ; 12)



Solution

a) $F_x = P\cos 60° = 160 * 0.5 = 80N\ (\rightarrow)$

$F_y = P\sin 60° = 160 * 0.866 = 138.564\ N\ (\uparrow)$

$\Sigma M_c = 138.564 * 4 - 80 * 2.75 = 334.256\ N\text{-}m$

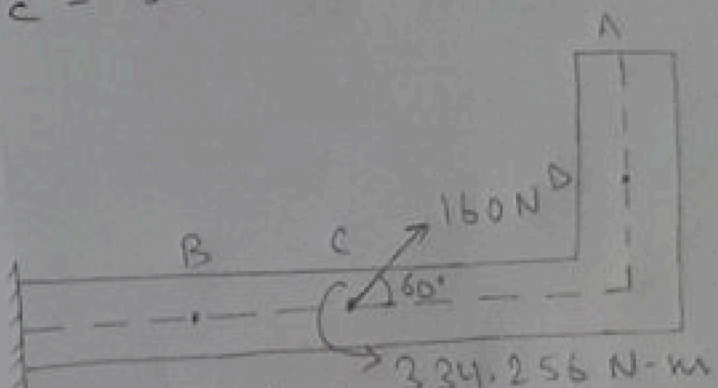


fig equivalent force-couple system at c

b) An equivalent force-couple system at D Consists of

$F_x = 160\cos 60° = 80\ N\ (\rightarrow)$
$F_y = 160\sin 60° = 138.564\ N\ (\uparrow)$
$\Sigma M_c = 80 * 1.5 = 120\ N\text{-}m$

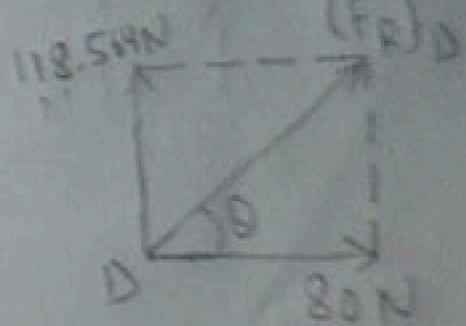
This couple moment at C may be replaced by a force of

$$\frac{120}{6} = 20N\ (\uparrow)$$

acting at B.

Since $F_y = 138.564N\ (\uparrow)$ and $20N\ (\uparrow)$ vertical force acts at B, vertical force acting at D becomes

$(F_y)_D = 118.564\ N\ (\uparrow)$



$$F_{RD} = \sqrt{80^2 + 118.564^2} = 143.029\ N$$

$$\theta = \tan^{-1}\left(\frac{118.564}{80}\right) = 56^\circ$$

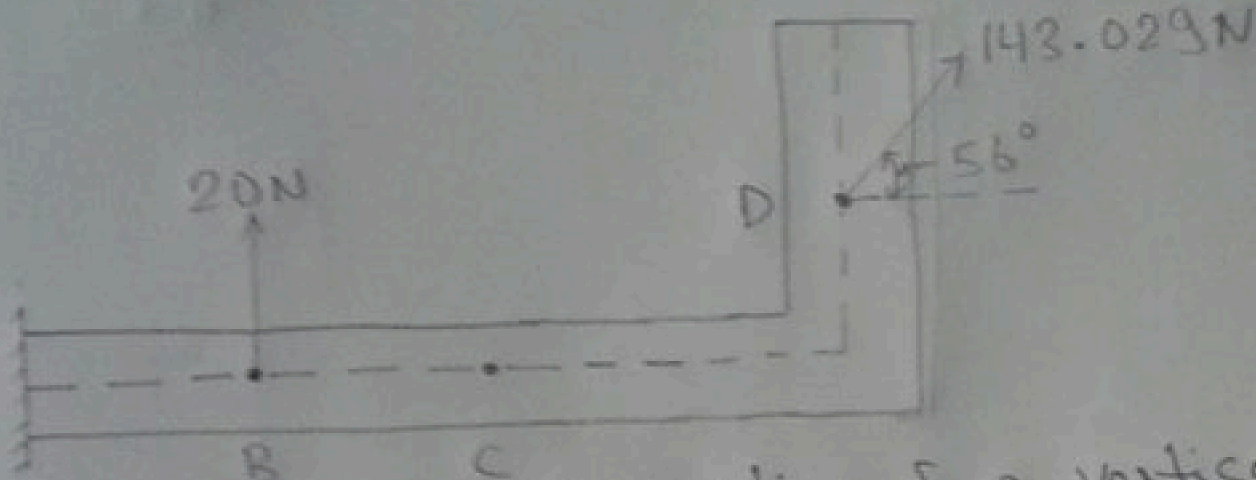
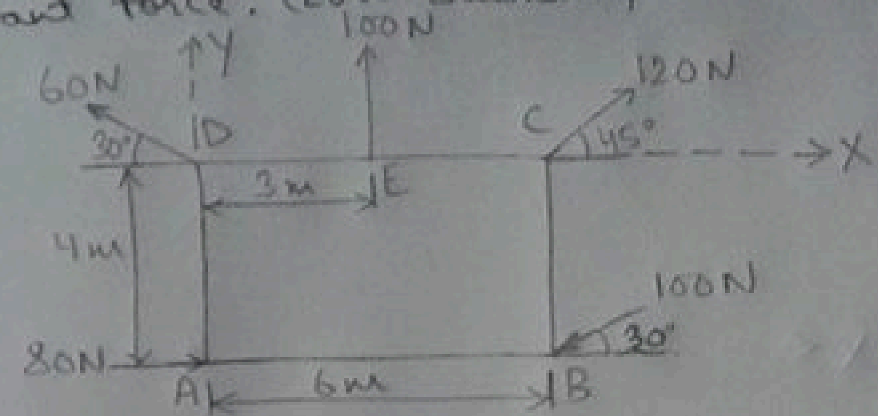


Fig. Equivalent System consisting of a vertical force at B and a second force at D.

A plate of size 6m × 4m is acted upon by a set of forces in its plane as shown in figure below. Determine the magnitude, direction and position of resultant force. (2068 Baishakh, BEX ; 12)



Solution

$(\xrightarrow{+})\ (F_R)_x = \Sigma F_x$

$= 80 - 100\cos 30° + 120\cos 45° - 60\cos 30°$

$\Rightarrow (F_R)_x = 26.29N\ (\rightarrow)$

$(+\uparrow)\ (F_R)_y = \Sigma F_y$

$= -100\sin 30° + 120\sin 45° + 100 + 60\sin 30°$

$\Rightarrow (F_R)_y = 164.85N\ (\uparrow)$

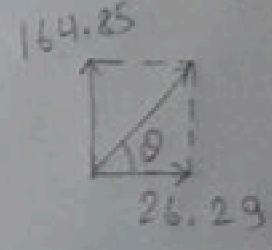
$(+\circlearrowleft)\ (M_R)_D = \Sigma M_D$

$= -80 * 4 + 100\cos 30° * 4 + 100\sin 30° * 6$

$- 120\sin 45° * 6 - 100 * 3$

$= -482.71$ N-m

$(M_R)_D = 482.71$ N-m $(\circlearrowright)$



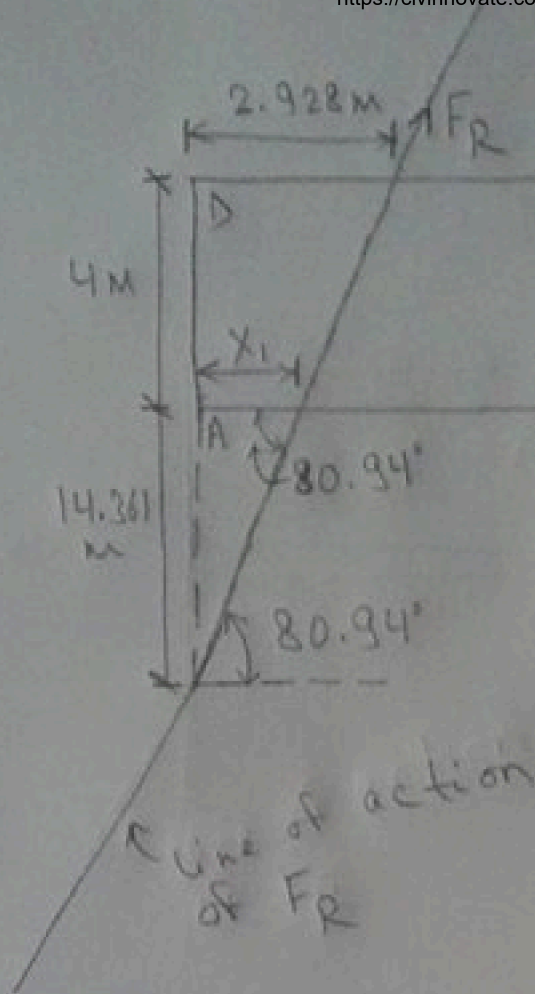
$F_R = \sqrt{26.29^2 + 164.85^2} = 166.93N$

$\theta = \tan^{-1}\left(\frac{164.85}{26.29}\right) = 80.94°$

The x and y intercept of the resultant force are

$x = \frac{(M_R)_D}{(F_R)_y} = \frac{482.71}{164.85} = 2.928m$

$y = -\frac{(M_R)_D}{(F_R)_x} = -\frac{482.71}{26.29} = -18.361$

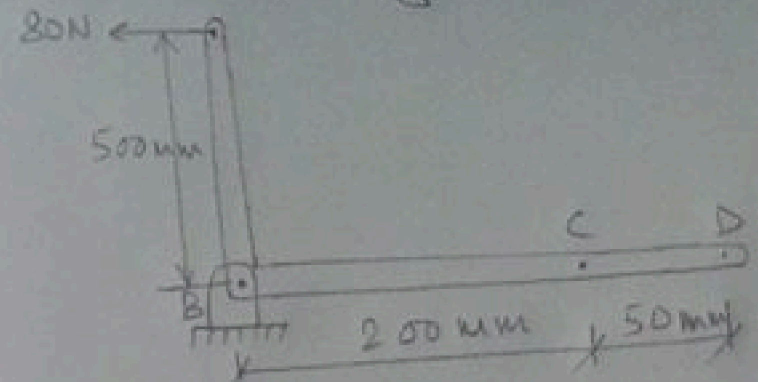


$$\tan 80.94° = \frac{14.361}{X_1}$$

$$\Rightarrow X_1 = 2.29\text{ m}$$

Hence the resultant force is 166.93 N (∠80.94°). The line of action of the resultant force intersects the edge DC of the plate at a distance 2.928m from D and the edge AB of the plate at a distance ~~of~~ 2.29m from A.

The 80N horizontal force 'P' acts on a bell crank as shown: (i) Replace P with an equivalent force couple system at B (ii) Find two vertical forces at C and D that are equivalent to the couple found in part (i). (2068 Magh; 5)



Solution

i) Equivalent force-couple system at B consist of

$F_B = 80N (\leftarrow)$

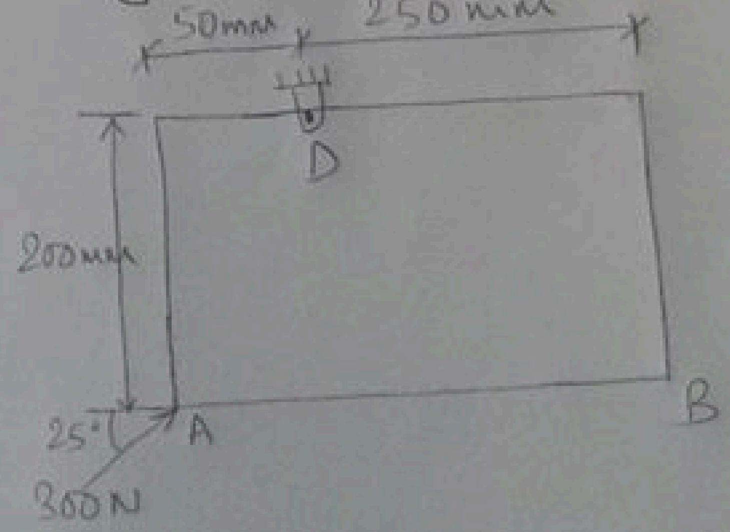
$M_B = 80 * 0.5 = 40$ N-m (↻)

ii) The ~~equivalent~~ vertical forces at C ~~and~~ and D equivalent to 40 N-m (↻) couple are

$$(F_C)_y = \frac{40}{0.05} = 800\,N\ (\downarrow)$$

$$\text{and } (F_D)_y = \frac{40}{0.05} = 800\,N\ (\uparrow)$$

A 300N force is applied at A as shown. Determine: (i) moment of 300N force about D (ii) Smallest force applied at B that creates same moment about D. (2068 Magh; 3+3)

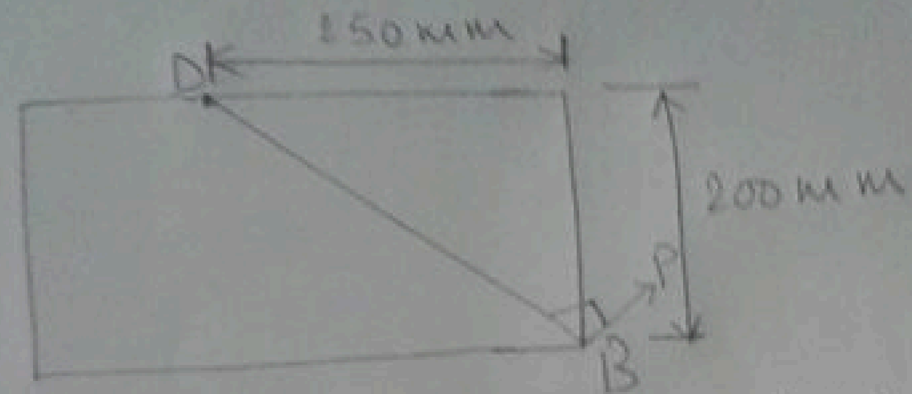


Solution

i) $M_D = 800 \cos 25° * 0.2 - 800 \sin 25° * 0.05$

$= 48.04$ N-m (↻)

ii)

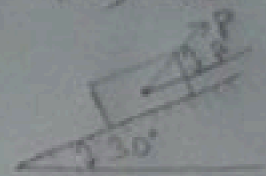


Force 'P' (at B) ~~whose~~ that is perpendicular to BD is the smallest force applied at B that creates same moment about D.

$BD = \sqrt{(0.2)^2 + (0.25)^2} = 0.320$ m

So, $P = \dfrac{48.04}{0.32} = 150$ N

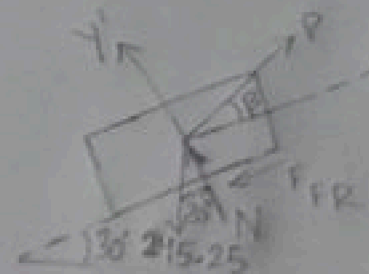
Knowing that the coeff. of friction between 25 kg block and the incline is $\mu_s = 0.25$. Determine (i) the smallest value of P required to start the block moving up the incline (ii) corresponding value of β. (2068 Bhadra; 4 marks)



Solution

$$\omega = 9.81 * 25 = 245.25\ N$$

FBD:



$F_{FR} = 0.25\,N$

$(\xrightarrow{+})\ \Sigma F_{x'} = 0$

or, $P\cos\beta - F_{FR} - 245.25\sin 30 = 0$

or, $P\cos\beta - 0.25N = 122.625$ — (i)

$(+\uparrow)\ \Sigma F_{y'} = 0$

or, $N + P\sin\beta - 245.25\cos 30 = 0$

or, $P\sin\beta + N = \cancel{245}\ 212.393$ — (ii)

Multiplying (ii) by 0.25 and adding (i) and (ii),

$$P\cos\beta - \cancel{0.25N} = 122.625$$
$$0.25P\sin\beta + \cancel{0.25N} = 53.098$$

$$P\cos\beta + 0.25P\sin\beta = 175.723$$

or, $P = \dfrac{175.723}{\cos\beta + 0.25\sin\beta}$

For P to be ~~max~~ minimum, $\cos\beta + 0.25\sin\beta$ should be maximum.

So, $\dfrac{d(\cos\beta + 0.25\sin\beta)}{d\beta} = 0$

or, $-\sin\beta + 0.25\cos\beta = 0$

or, $\sin\beta = 0.25\cos\beta$

or, $\tan\beta = 0.25$

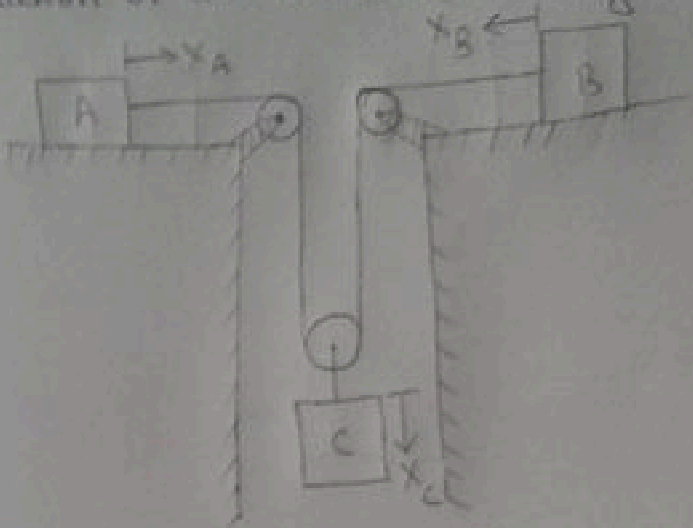
$\Rightarrow \beta = 14.036°$

when $\beta = 14.036°$, $\cos\beta + 0.25 \sin\beta = 1.03078$

So,

$\Rightarrow$ Smallest $P = \frac{175.728}{1.03078} = 170.476$ N

The coefficient of friction between blocks A and B and the horizontal surfaces are $\mu_s = 0.24$ and $\mu_K = 0.20$. Knowing that $m_A = 5$ kg, $m_B = 10$ kg and $m_c = 10$ kg, determine: (i) the tension in the cord (ii) acceleration of each block. (2068 Magh, 10)



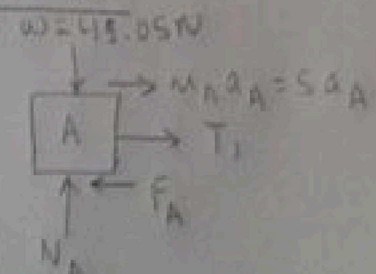
Solution

When block A ~~is~~ moves x_A distance to the right and block B moves x_B distance to the left, block C moves half the sum of distances x_A and x_B ~~to~~ downwards.

$$x_c = \frac{1}{2}(x_A + x_B) \;;\; \ddot{x}_c = \frac{1}{2}(\ddot{x}_A + \ddot{x}_B)$$

$$\Rightarrow a_c = \frac{1}{2}(a_A + a_B) \quad \text{——} \; (1)$$

FBD of 'A'



$(+\uparrow)\ \Sigma F_y = 0$

or, $N_A - 49.05 = 0$

$\Rightarrow N_A = 49.05$ N

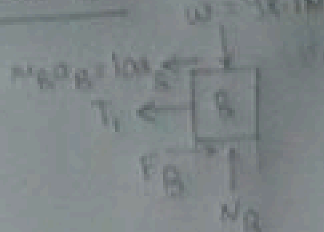
$F_A = 0.2\, N_A = 9.81$ N

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $T_1 - F_A = m_A a_A$

or, $T_1 - 9.81 = 5a_A$ —— (ii)

FBD of 'B'



$(+\uparrow)\ \Sigma F_y = 0$ or, $N_B - 98.1 = 0$

or, $\cancel{N_B = 9.81}$

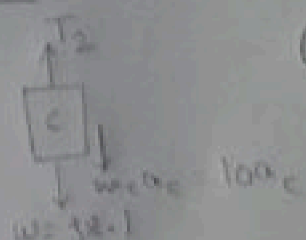
$\Rightarrow N_B = 98.1\,N$

$F_B = 0.2\,N_B = 19.62\,N$

$(+\leftarrow)\ \Sigma F_x = 0$

or, $T_1 - 19.62 = 10a_B$ — (III)

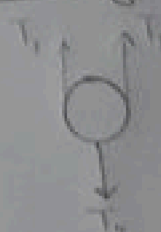
FBD of 'C'



$(+\downarrow)\ \Sigma F_y = 0$

or, $-T_2 + 98.1 = 10a_c$ — (IV)

FBD of Pulley



$(+\uparrow)\ \Sigma F_y = 0$

or, $2T_1 - T_2 = 0$

or, $T_2 = 2T_1$

Substituting T_2 from above in eqn (IV),

$-2T_1 + 98.1 = 10a_c$

Substituting a_c from eq. (I) into the above equatn,

$-2T_1 + 98.1 = 5a_A + 5a_B$

Substituting $5a_A$ from (II) into the above equation,

$-2T_1 + 98.1 = T_1 - 9.81 + 5a_B$

or, $-3T_1 + 107.91 = 5a_B$ — (V)

Multiplying eq. (V) by 2 and substracting eq. (III) from this equation,

$-6T_1 + 215.82 = 10a_B$

$T_1 - 19.62 = 10a_B$

$-\quad +\quad -$

or, $-7T_1 + 235.44 = 0$

or, $7T_1 = 235.44$

$\Rightarrow T_1 = 33.63\,N$

$$T_2 = 2T_1 = 2 \times 33.63 = 67.26\ N$$

From (iii),

$$10 a_B = 33.63 - 19.62$$
$$= 14.01$$
$$\Rightarrow a_B = 1.401\ m/sec^2\ (\leftarrow)$$

From (ii),

$$5 a_A = 33.63 - 9.81$$
$$= 23.82$$
$$\Rightarrow a_A = 4.764\ m/sec^2\ (\rightarrow)$$

From (i),

$$a_C = \frac{1}{2}(1.401 + 4.764)$$
$$\Rightarrow a_C = 3.082\ m/sec^2\ (\downarrow)$$

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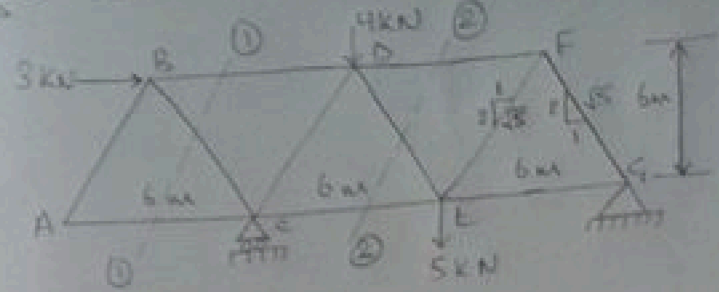
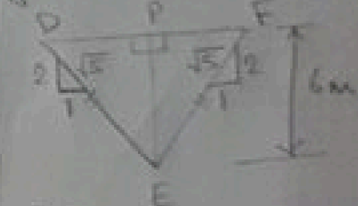
Use the method of sections to compute the force in BC, DF and CE of the warren truss loaded as shown in figure below. (2068 Chaitra, Regular, BEX ; 8 Marks)



Solution

(Warren truss → it contains a series of isosceles triangles or equilateral triangles. In this problem, the warren truss contains a series of isosceles triangles.)



DF = 6m

DP = PF = 3m

All the diagon members (i.e. the inclined members like AB, BC etc) of the truss have the same slope of 2.

$(\xrightarrow{+})\sum F_x = 0$

or, $G_x + 3 = 0$ or, $G_x = -3$ kN

$\Rightarrow G_x = 3$ kN $(\leftarrow)$

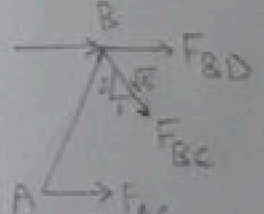
$(\circlearrowleft +)\sum M_G = 0$

or, $3 \times 6 + 12C_y - 4\times(3+6) - 5\times 6 = 0$

or, $12C_y = 48$

$\Rightarrow C_y = 4$ kN $(\uparrow)$

To get the force in BC we pass section ①-① through the truss as shown above and consider the equilibrium of the left portion.

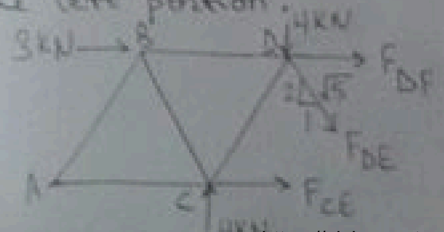


$(+\uparrow)\sum F_y = 0$

or, $- F_{BC}\times\left(\frac{2}{\sqrt{5}}\right) = 0$

$\Rightarrow F_{BC} = 0$

To compute the force in DF and CE we pass section ②-② through the truss and consider the equilibrium of the left portion.



$(\circlearrowleft +)\sum M_D = 0$

or, $4\times 3 - 6F_{CE} = 0$

$\Rightarrow F_{CE} = 2$ kN (T)

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$(+\uparrow)\ \Sigma F_y = 0$

or, $4 - 4 - F_{DE} * \left(\frac{2}{\sqrt{5}}\right) = 0$

$\Rightarrow F_{DE} = 0$

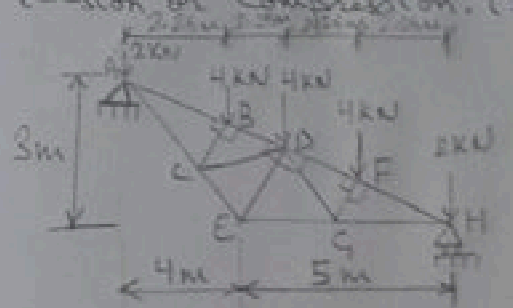
$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $3 + 2 + F_{DF} = 0$

or, $F_{DF} = -5KN$

$\Rightarrow F_{DF} = 5KN\ (C)$

Determining the force in members DE, CD, and AB for the inverted Howe roof truss. State whether each member is in tension or compression. (2068 Chaitra, old Back, BEX, 2)



(In the original question the horizontal distance betn A and B etc is given as 2 m, but this way the sum of these distances come out to be 8 m not 9 m)

Solution

$(\circlearrowleft +)\ \Sigma M_A = 0$

or, $-9H_y + 2*9 + 4*6.75 + 4*4.5 + 4*2.25 = 0$

$\Rightarrow H_y = 8KN\ (\uparrow)$

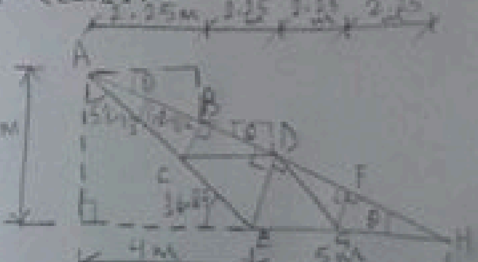
$(+\uparrow)\ \Sigma F_y = 0$

or, $A_y + 8 - 2 - 4 - 4 - 4 - 2 = 0$

$\Rightarrow A_y = 8KN\ (\uparrow)$

$(\xrightarrow{+})\ \Sigma F_x = 0$ or, $A_x + 0 =$ $\Rightarrow A_x = 0$

Let's for a moment consider the geometry of the Howe Roof truss.



$AE = \sqrt{3^2 + 4^2} = 5\ m$

$\theta = \tan^{-1}\left(\frac{3}{9}\right) = 18.43°$

$AH = \sqrt{3^2 + 9^2} = 9.487\ m$

$AD = DH = 4.743\ m$

~~(∵ ED ⊥ AH, ΔAEH is an isosceles triangle and ⊥ from the vertex of two equal sides to the 3rd side bisects the third side.)~~

$AB = BD = DF = FH = \frac{2.25}{\cos 18.43°}$

$= 2.37m$

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we have,

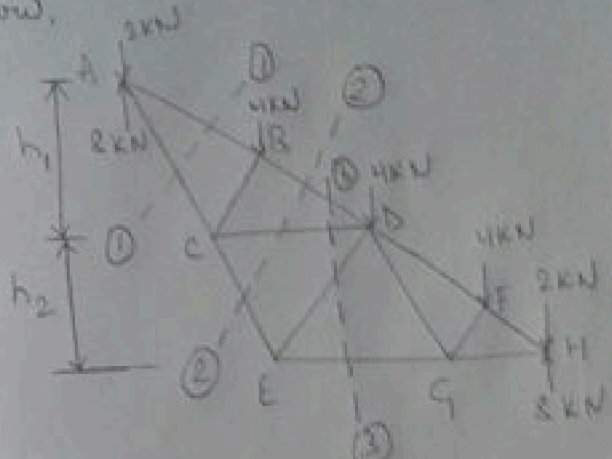
$AB = BD$, $CB \perp AD$

$AC = \sqrt{AB^2 + CB^2}$ & $CD = \sqrt{BD^2 + CB^2}$

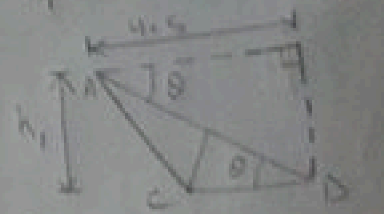
$\Rightarrow AC = CD$ Also in ΔACD, $\angle ADC = \theta = 18.43°$

Hence $CD \parallel$ to EH i.e. CD is horizontal.

Now,



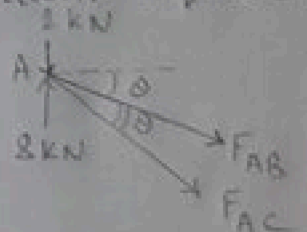
$h_1 = 4.5 \tan\theta = 1.5\,m$



$\theta = 18.43°$

$h_2 = 3 - 1.5 = 1.5\,m$

To compute force in member AB we cut the truss in two halves using section ①-① and consider the left portion's equilibrium.



$(\theta = 18.43°)$

$(\xrightarrow{+})\ \Sigma F_x = 0$ or, $F_{AB} \cos 18.43° + F_{AC} \cos 36.86° = 0$

or, $F_{AB} = -0.844 F_{AC}$ — ①

$(+\uparrow)\ \Sigma F_y = 0$ or, $-2 + 8 - F_{AB} \sin 18.43° - F_{AC} \sin 36.86° = 0$

or, $6 - 0.316 F_{AB} - 0.6 F_{AC} = 0$

Substituting F_{AB} from ① into the above equation we get,

$6 - 0.316 * (-0.844 F_{AC}) - 0.6 F_{AC} = 0$

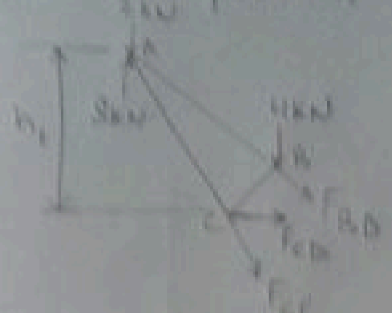
or, $6 - 0.333 F_{AC} = 0$

$\Rightarrow F_{AC} = 18\ kN\ (T)$

So, $F_{AB} = -0.844 * 18 = -15.192\ kN$

$\Rightarrow F_{AB} = 15.192\ kN\ (C)$

To compute force in member CD we cut the truss in two halves using section (2)-(2) and consider the equilibrium of left portion.

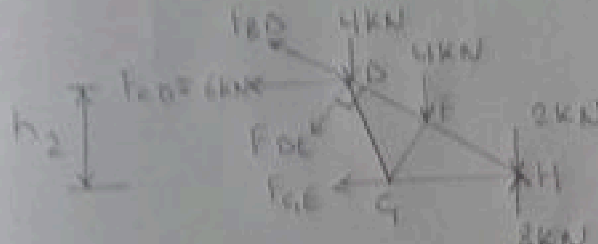


$(\circlearrowleft)\ \Sigma M_A = 0$

or, $F_{CD} * 1.5 - 4 * 2.25 = 0$

$\Rightarrow F_{CD} = 6\ KN\ (T)$

To compute force in member DE we cut the truss in two halves using section (3)-(3) and consider the equilibrium of right portion.



$(\circlearrowleft)\ \Sigma M_H = 0$

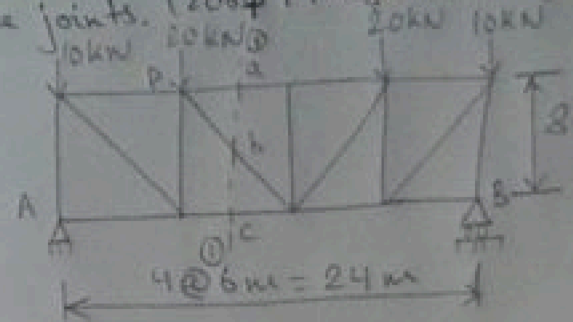
or, $F_{DE} * (2 * 2.37) + 6 * 1.5 + 4 * 4.5 + 4 * 2.25 = 0$

[Note: $DE \perp DH$, $DF = FH = 2.37$ m So $DH = 2 * 2.37$. Moment arm for F_{DE} is DH]

or, $F_{DE} = -7.6\ KN$

$\Rightarrow F_{DE} = 7.6\ KN\ (C)$

Find bar forces in members a, b and c in the truss as indicated in figure below. Shown loads are vertical at the joints. (2067 Mangsir, Regular, BCE ; 10 Marks)



Solution

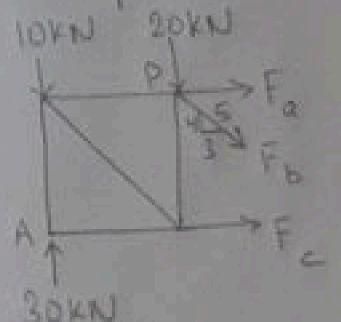
$(\circlearrowleft +)\ \Sigma M_A = 0$ or, $24B_y - 10*24 - 20*18 - 20*6 = 0$

$\Rightarrow B_y = 30\,kN\ (\uparrow)$

$(+\uparrow)\ \Sigma F_y = 0$ or, $A_y + 30 - 10 - 20 - 20 - 10 = 0$

$\Rightarrow A_x = 30\,kN\ (\uparrow)$; $A_x = 0$

Cutting the truss with section ①-① and considering the left portion.



$(\circlearrowleft +)\ \Sigma M_P = 0$

or, $8F_c + 10*6 - 30*6 = 0$

$\Rightarrow F_c = 15\,KN\ (T)$

$(+\uparrow)\ \Sigma F_y = 0$

or, $30 - 10 - 20 - F_b * \frac{4}{5} = 0$

$\Rightarrow F_b = 0$

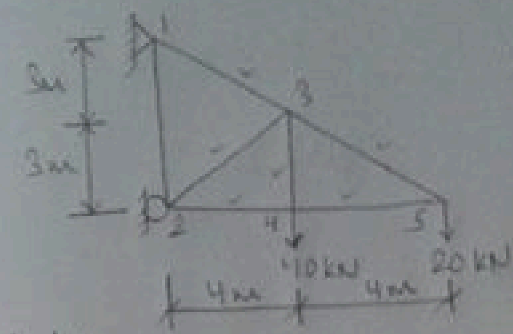
$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $F_a + F_c = 0$

or, $F_a = -15\,KN$

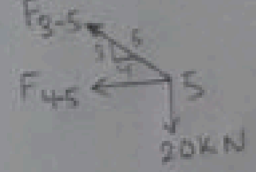
$\Rightarrow F_a = 15\,KN\ (C)$

\# Determine the member forces in the pin jointed truss shown below. (2065 Shrawan, Regular/Back, BEX 8 Marks)



Solution

Joint 5



$(+\uparrow)\ \Sigma F_y = 0$

or, $-20 + F_{3-5} * \frac{3}{5} = 0$

$\Rightarrow F_{3-5} = 33.33\text{ kN (T)}$

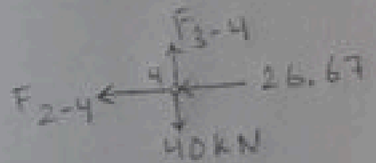
$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $-F_{4-5} - 33.33 * \frac{4}{5} = 0$

or, $F_{4-5} = -26.67\text{ kN}$

$\Rightarrow F_{4-5} = 26.67\text{ kN (C)}$

Joint 4



$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $-F_{2-4} - 26.67 = 0$

or, $F_{2-4} = -26.67\text{ kN}$

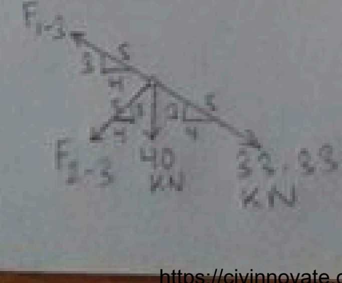
$\Rightarrow F_{2-4} = 26.67\text{ kN (C)}$

$(+\uparrow)\ \Sigma F_y = 0$

or, $F_{3-4} - 40 = 0$

$\Rightarrow F_{3-4} = 40\text{ kN (T)}$

Joint 3



$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $-F_{1-3} * \frac{4}{5} - F_{2-3} * \frac{4}{5} + 33.33 * \frac{4}{5} = 0$

or, $0.8 F_{1-3} + 0.8 F_{2-3} = 26.67$ — (1)

$(+\uparrow)\ \Sigma F_y = 0$

or, $F_{1-3} * \frac{3}{5} - F_{2-3} * \frac{3}{5} - 33.33 * \frac{3}{5} - 40 = 0$

or, $0.6 F_{1-3} - 0.6 F_{2-3} = 60$ —— (ii)

Multiplying (ii) by 4 & (i) by 3 and adding (i) & (ii)

$$2.4 F_{1-3} + \cancel{2.4 F_{2-3}} = 80$$
$$2.4 F_{1-3} - \cancel{2.4 F_{2-3}} = 240$$

or, $4.8 F_{1-3} = 320$

$\Rightarrow F_{1-3} = 66.67\text{ kN (T)}$

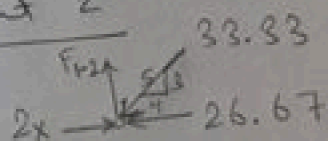
Now,

$0.6 * 66.67 - 0.6 F_{2-3} = 60$

or, $F_{2-3} = -33.33\text{ kN}$

$\Rightarrow F_{2-3} = 33.33\text{ kN (C)}$

Joint 2

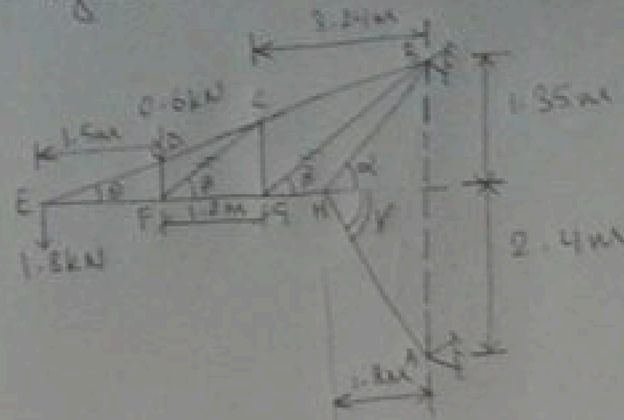


$(+\uparrow)\ \Sigma F_y = 0$

or, $F_{1-2} - 33.33 * \frac{3}{5} = 0$

$\Rightarrow F_{1-2} = 20\text{ kN (T)}$

Determine the member forces of given truss shown in figure below. (2069 Poush, BCE, 8)



Solution

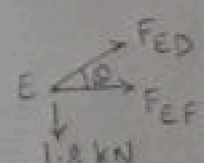
$$\theta = \tan^{-1}\left(\frac{1.35}{5.94}\right) = 12.8^\circ$$

$$\beta = \tan^{-1}\left(\frac{1.35}{3.24}\right) = 22.62^\circ$$

$$\alpha = \tan^{-1}\left(\frac{1.35}{1.8}\right) = 36.87^\circ$$

$$\gamma = \tan^{-1}\left(\frac{2.4}{1.8}\right) = 53.13^\circ$$

Joint E



$(+\uparrow)\ \Sigma F_y = 0$

or, $-1.8 + F_{ED}\sin 12.8^\circ = 0$

$\Rightarrow F_{ED} = 8.12\text{ kN (T)}$

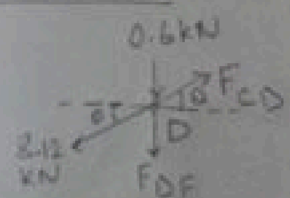
$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $F_{EF} + 8.12\cos 12.8^\circ = 0$

or, $F_{EF} = -7.92\text{ kN}$

$\Rightarrow F_{EF} = 7.92\text{ kN (C)}$

Joint D



$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $F_{CD}\cos 12.8^\circ - 8.12\cos 12.8^\circ = 0$

$\Rightarrow F_{CD} = 8.12\text{ kN (T)}$

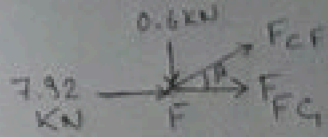
$(+\uparrow)\ \Sigma F_y = 0$

or, $-F_{DF} - 0.6 - 8.12\sin 12.8^\circ + 8.12\sin 12.8^\circ = 0$

or, $F_{DF} = -0.6\text{ kN}$

$\Rightarrow F_{DF} = 0.6\text{ kN (C)}$

Joint F



$(+\uparrow)\ \Sigma F_y = 0$

or, $-0.6 + F_{CF} \sin 22.62° = 0$

$\Rightarrow F_{CF} = 1.56\ \text{kN (T)}$

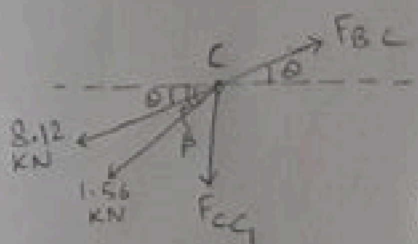
$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $F_{FG} + 1.56 * \cos 22.62° + 7.92 = 0$

or, $F_{FG} = -9.36\ \text{kN}$

$\Rightarrow F_{FG} = 9.36\ \text{kN (C)}$

Joint C



$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $F_{BC} \cos 12.8° - 8.12 \cos 12.8° - 1.56 \cos 22.62° = 0$

$\Rightarrow F_{BC} = 9.6\ \text{kN (T)}$

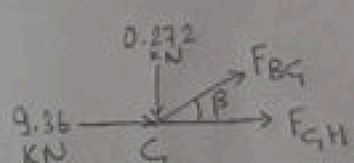
$(+\uparrow)\ \Sigma F_y = 0$

or, $F_{BC} \sin 12.8° - F_{CG} - (8.12 \sin 12.8°) - 1.56 \sin 22.62° = 0$

or, $F_{CG} = -0.272\ \text{kN}$

$\Rightarrow F_{CG} = 0.272\ \text{kN (C)}$

Joint G



$(+\uparrow)\ \Sigma F_y = 0$

or, $-0.272 + F_{BG} \sin 22.62° = 0$

$\Rightarrow F_{BG} = 0.707\ \text{kN (T)}$

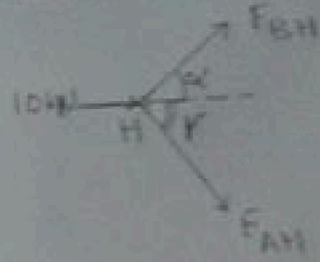
$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $F_{GH} + 0.707 \cos 22.62 + 9.36 = 0$

or, $F_{GH} = -10\ \text{kN}$

$\Rightarrow F_{GH} = 10\ \text{kN (C)}$

Joint H



$(+\uparrow)\ \Sigma F_y = 0$

or, $F_{BH} \sin 36.87^\circ - F_{AH} \sin 53.13^\circ = 0$

or, $F_{BH} = 1.333 F_{AH}$ —— (1)

$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $10 + F_{BH} \cos 36.87^\circ + F_{AH} \cos 53.13^\circ = 0$

or, $10 + 1.333 F_{AH} \cos 36.87^\circ + F_{AH} \cos 53.13^\circ = 0$

or, $F_{AH} = -6\,kN$

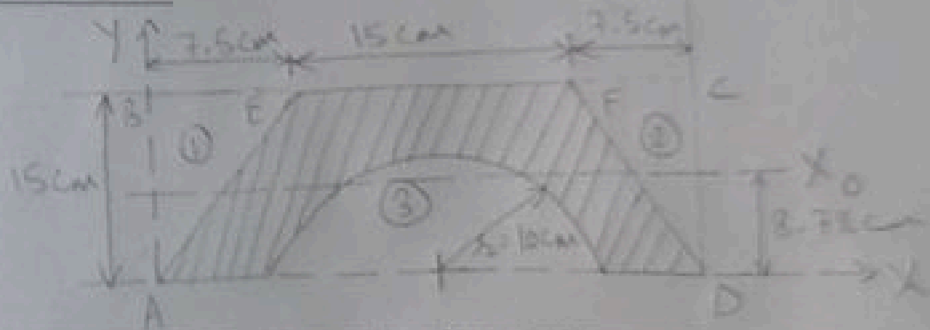
$\Rightarrow F_{AH} = 6\,kN\ (C)$

So, $F_{BH} = 1.333 F_{AH}$

$\Rightarrow F_{BH} = 8\,kN\ (C)$

Determine the moment of inertia of the hatched area about its centroidal X_0 axis. (2068 Chaitra, BEX; 12)

Solution

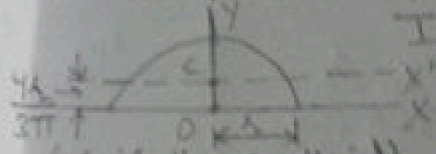


Component	Area (A)	~~$\bar{X}$~~	~~$\bar{X}A$~~	$\bar{Y}_{el}$	$A\bar{Y}_{el}$
Rectangle ABCD	450	~~15~~	~~6750~~	7.5	3375
1	− 56.25	~~2.5~~	~~−140.625~~	10	− 562.5
2	− 56.25	~~27.5~~	~~−1546.88~~	10	− 562.5
3	− 50π			$\frac{40}{3\pi}$	− 666.67
Σ	180.42				1583.33

$$\bar{Y} = \frac{\Sigma(A\bar{Y}_{el})}{\Sigma A} = \frac{1583.33}{180.42} = 8.78\text{ cm}$$

https://civinnovate.com/civil-engineering-notes/

The expression for moment of inertia of the semi-circle about x-axis is,



(c is the centroid)

$$I_x = \frac{\pi r^4}{8} \quad \cancel{\bar{I}_x}$$

$$I_x = \bar{I}_{x'} + A h^2$$

(h is the distance betn x' axis and x axis)

$$\text{or, } \bar{I}_{x'} = I_x - A h^2$$

$$\text{or, } \bar{I}_{x'} = \frac{\pi r^4}{8} - \left(\frac{\pi r^2}{2}\right) * \left(\frac{4r}{3\pi}\right)^2$$

$$\text{or, } \bar{I}_{x'} = \frac{\pi r^4}{8} - \frac{16\pi r^4}{18\pi^2}$$

$$\text{or, } \bar{I}_{x'} = \frac{\pi r^4}{8} - \frac{8r^4}{9\pi}$$

$$\Rightarrow \bar{I}_{x'} = 0.1098\, r^4$$

Now,

$$I_{x_0} = \left[\frac{1}{12} * 30 * 15^3 + (30*15)*(8.78-7.5)^2\right] \rightarrow \text{for rectangle ABCD}$$

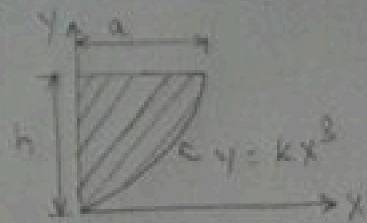
$$- \left[2*\left\{\frac{7.5*15^3}{36} + \left(\frac{1}{2}*7.5*15\right)*(10-8.78)^2\right\} \rightarrow \text{for } ① \,\&\, ②\right.$$

$$\left. + \left\{0.1098*10^4 + \left(\frac{\pi*10^2}{2}\right)*\left(8.78 - \frac{4*10}{3\pi}\right)^2\right\} \rightarrow \text{for } ③ \right]$$

$$= [8437.5 + 737.28] - [2*\{703.125 + 83.7225\} + \{1098 + 3231.77\}]$$

$$\Rightarrow I_{x_0} = 3271.32 \text{ cm}^4$$

Determine the centroid of the area shown in figure below by direct integration method. (2067 Mangsir, BCE; 10)

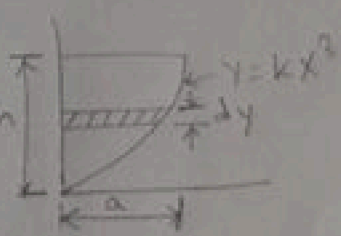


Solution

when $x = a$, $y = h$. So,

$$h = ka^3 \Rightarrow k = \frac{h}{a^3}$$

So the equation of the curve is $y = \frac{h}{a^3} x^3$



$$y = kx^3$$

$$\text{or, } \frac{dy}{dx} = 3kx^2$$

$$\text{or, } dy = 3kx^2 dx$$

$$dA = x\,dy = x * 3kx^2 dx = 3kx^3 dx$$

$$\bar{x}_{el} = \frac{x}{2}, \quad \bar{y}_{el} = y = kx^3$$

we know that,

$$\bar{X} = \frac{\int_A \bar{x}_{el}\, dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^a x * 3kx^3 dx}{\int_0^a 3kx^3 dx}$$

$$= \left(\frac{3k}{2*3k}\right) \frac{\int_0^a x^4 dx}{\int_0^a x^3 dx}$$

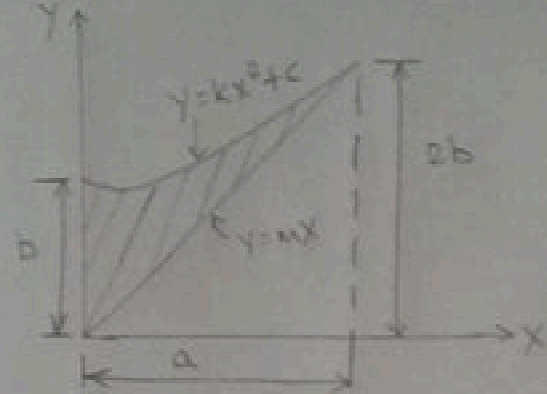
$$= 0.5 * \frac{a^5}{5} * \frac{4}{a^4}$$

$$\Rightarrow \bar{X} = 0.4a$$

Also, $\bar{Y} = \frac{\int_A \bar{y}_{el}\, dA}{\int_A dA} = \frac{3k^2 \int_0^a x^6 dx}{3k \int_0^a x^3 dx} = k * \frac{a^7}{7} * \frac{4}{a^4}$

$$\text{or, } \bar{Y} = \frac{h}{a^3} * a^3 * \frac{4}{7} = \frac{4}{7}h \Rightarrow \bar{Y} = \frac{4}{7}h$$

Determine the ~~[illegible]~~ polar moment of inertia and polar radius of gyration of the hatched area as shown in figure below with respect to centroid.
(2069 Bhadra, BCE ; 12)



Solution

Let's first determine the value of constants k, c, and m.

when $x = 0, y = b$: $b = c$

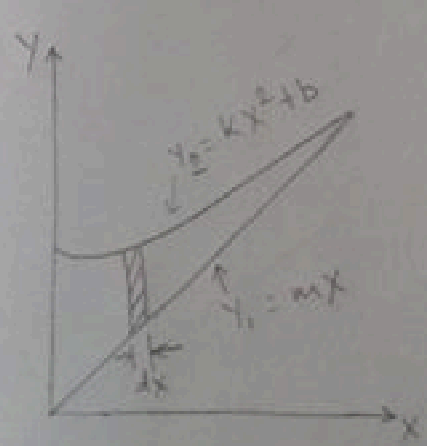
when $x = a, y = 2b$: $2b = ka^2 + b$

or, $ka^2 = b$

or, $k = \frac{b}{a^2}$

So equation of the curve is $y = \left(\frac{b}{a^2}\right)x^2 + b$

Also, $m = \frac{2b}{a}$



$$dA = (y_2 - y_1)\,dx$$

$$= \left(kx^2 + b - \frac{2b}{a}x\right)dx$$

$$dA = (kx^2 + b - mx)\,dx$$

$$\bar{x}_{el} = x, \quad \bar{y}_{el} = mx + \frac{y_2 - y_1}{2}$$

$$\text{or, } \bar{y}_{el} = \frac{2mx + kx^2 + b - mx}{2}$$

$$\text{or, } \bar{y}_{el} = \frac{mx + kx^2 + b}{2}$$

$$\bar{x} = \frac{\int_A \bar{x}_{el}\, dA}{\int_A dA} = \frac{\int_0^a x\,(kx^2+b-mx)\,dx}{\int_0^a (kx^2+b-mx)\,dx}$$

$$= \frac{k*\frac{1}{4}*a^4 + b*\frac{1}{2}*a^2 - m*\frac{1}{3}*a^3}{k*\frac{1}{3}*a^3 + b*a - m*\frac{1}{2}*a^2}$$

$$= \frac{0.25a^4*\frac{b}{a^2} + 0.5a^2b - 0.333a^3*\frac{2b}{a}}{0.333a^3*\frac{b}{a^2} + ab - 0.5a^2*\frac{2b}{a}}$$

$$= \frac{0.0833a^2b}{0.333ab}$$

$$\bar{x} = 0.25a$$

$$\bar{y} = \frac{\int_A \bar{y}_{el}\, dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^a (kx^2+b+mx)(kx^2+b-mx)\,dx}{0.333ab}$$

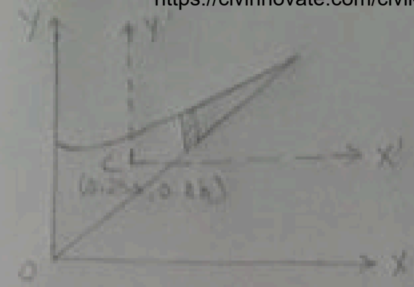
$$= \frac{0.5\int_0^a \{(kx^2+b)^2 - m^2x^2\}\,dx}{0.333ab}$$

$$= \frac{0.5\int_0^a (k^2x^4 + 2kx^2b + b^2 - m^2x^2)\,dx}{0.333ab}$$

$$= \frac{\frac{k^2}{5}*a^5 + \frac{2kb}{3}*a^3 + b^2*a - \frac{m^2}{3}*a^3}{0.667ab}$$

$$= \frac{\frac{1}{5}*\frac{b^2}{a^4}*a^5 + \frac{2b}{3}*\frac{b}{a^2}*a^3 + b^2a - \frac{1}{3}*\frac{4b^2}{a^2}*a^3}{0.667ab}$$

$$\bar{y} = 0.8b$$



$$I_y = \int_A x^2 \, dA$$

$$= \int_0^a x^2 (kx^2 + b - mx)\, dx$$

$$= \frac{k}{5} * a^5 + \frac{b}{3} * a^3 - \frac{m}{4} * a^4$$

$$= \frac{1}{5} * \frac{b}{a^2} * a^5 + \frac{ba^3}{3} - \frac{1}{4} * \frac{2b}{a} * a^4$$

$$\Rightarrow I_y = 0.0333\, ba^3$$

We know that,

$$I_y = \overline{I}_{y'} + Ah^2 \quad (\text{here } h = d_x)$$

$$\text{or, } \overline{I}_{y'} = I_y - Ah^2 = 0.0333\, ba^3 - 0.333ab * (0.25a)^2$$

$$\Rightarrow \overline{I}_{y'} = 0.012469\, ba^3 \simeq 0.0125\, ba^3$$

Now,

$$dI_x = \frac{1}{12} * dx * (y_2 - y_1)^3 + dA(\overline{y}_{el})^2$$

$$= \frac{1}{12} * dx * (kx^2 + b - mx)^3 + (kx^2 + b - mx)\, dx * \left(\frac{kx^2 + b + mx}{2}\right)^2$$

$$\text{or, } I_x = \int dI_x = \int_0^a \frac{1}{12} (kx^2 + b - mx)^3 dx + \int_0^a (kx^2 + b - mx)\left(\frac{kx^2 + b + mx}{2}\right)^2 dx$$

After a very lengthy but easy calculation we get,

$$I_x = 0.2478\, ab^3$$

We know that, (here $h = d_y$)

$I_x = \bar{I}_{x'} + Ah^2$

or, $\bar{I}_{x'} = I_x - Ah^2 = 0.2478ab^3 - 0.333ab \times (0.8b)^2$

$\Rightarrow \bar{I}_{x'} = 0.0345ab^3$

We can calculate $\bar{J}_c$ by two methods.

First Method

$\bar{J}_c = \bar{I}_{x'} + \bar{I}_{y'}$

$\Rightarrow \bar{J}_c = 0.0345ab^3 + 0.0125ba^3$

Second Method

$J_o = I_x + I_y$

or, $J_o = 0.2478ab^3 + 0.0333ba^3$

We have, $J_o = \bar{J}_c + Ah^2$ (here $h = d$)

or, $\bar{J}_c = J_o - Ah^2$

$h^2 = 0.0625a^2 + 0.64b^2$

or, $Ah^2 = 0.333ab(0.0625a^2 + 0.64b^2)$

or, $Ah^2 = 0.0208ba^3 + 0.2133ab^3$

So, $\bar{J}_c = 0.2478ab^3 + 0.0333ba^3 - 0.0208ba^3 - 0.2133ab^3$

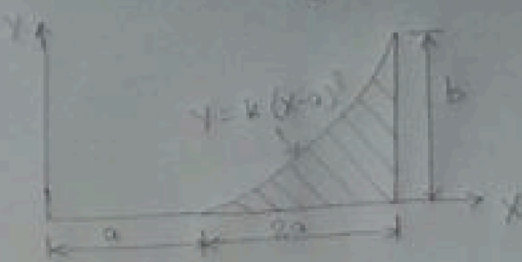
$\Rightarrow \bar{J}_c = 0.0345ab^3 + 0.0125ba^3$

Now,

Polar radius of gyration ~~[illegible]~~ with respect to centroid, $\bar{K}_c = \sqrt{\frac{\bar{J}_c}{A}} = \sqrt{\frac{(0.0345ab^3 + 0.0125ba^3)}{0.333ab}}$

$\Rightarrow \bar{K}_c = \sqrt{0.1035b^2 + 0.0375a^2}$

Determine by direct integration method, the centroid of the following shaded area. (2066 Magh, RCE; 8)



<u>Solution</u>

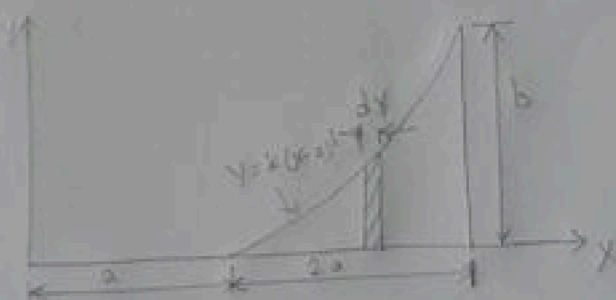
When $x = 3a$, $y = b$

$$b = k(3a-a)^3$$

or, $b = 8a^3 k$

$$\Rightarrow k = \frac{b}{8a^3}$$

Now,



$dA = y\,dx$

We have, $y = k(x-a)^3$

or, $\frac{dy}{dx} = 3k(x-a)^2$

or, $dx = \frac{dy}{3k(x-a)^2}$

$$= \frac{dy}{3k(x-a)^3} * (x-a)$$

$$= \frac{dy}{3y} * \left(\frac{y}{k}\right)^{1/3}$$

or, $dx = \frac{y^{-2/3}}{3k^{1/3}} dy$

$$dA = y\,dx = \frac{y^{1/3}}{3k^{1/3}}\,dy$$

$\bar{x}_{el} = x$

$$(x-a) = \left(\frac{y}{k}\right)^{1/3} = \left(\frac{y}{b}\right) * 2a$$

So, $x = a + 2a\left(\frac{y}{b}\right)$

$$\bar{y}_{el} = \frac{y}{2}$$

we know that,

$$\bar{x} = \frac{\int_A \bar{x}_{el}\,dA}{\int_A dA} = \frac{\int_0^b \left[\left\{a + 2a\left(\frac{y}{b}\right)\right\}\left\{\frac{y^{1/3}}{3k^{1/3}}\,dy\right\}\right]}{\int_0^b \frac{y^{1/3}}{3k^{1/3}}\,dy}$$

$$= \frac{\cancel{3k^{1/3}}}{\cancel{3k^{1/3}}} * \frac{\int_0^b \left(a y^{1/3} + \frac{2a}{b} * y^{4/3}\right)}{\int_0^b y^{1/3}\,dy}$$

$$= \frac{a * \frac{3}{4} * b^{4/3} + \frac{2a}{b} * \frac{3}{7} * b^{7/3}}{\frac{3}{4} * b^{4/3}}$$

$$\Rightarrow \bar{x} = 2.143a$$

Also,

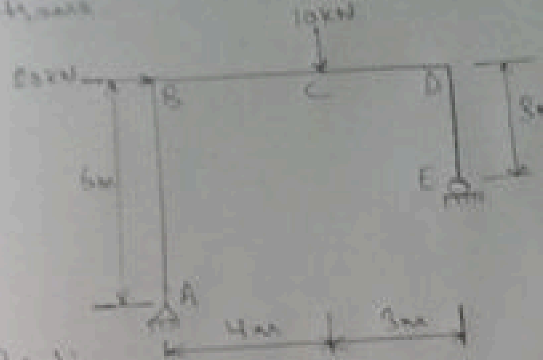
$$\bar{y} = \frac{\int_A \bar{y}_{el}\,dA}{\int_A dA} = \frac{0.5 * \int_0^b \frac{y^{4/3}}{3k^{1/3}}\,dy}{\int \frac{y^{1/3}}{3k^{1/3}}\,dy}$$

$$= \frac{\cancel{3k^{1/3}}}{\cancel{3k^{1/3}}} * 0.5 * \frac{\frac{3}{7} * b^{7/3}}{\frac{3}{4} * b^{4/3}}$$

$\cancel{\Rightarrow \bar{y} = 0.28}$

$$\Rightarrow \bar{y} = 0.286b$$

Draw the AFD, SFD, and BMD of the following frame



Solution

$(\xrightarrow{+}) \Sigma F_x = 0$ or, $20 + A_x = 0$ or, $A_x = -20\,kN$

$\Rightarrow A_x = 20\,kN\ (\leftarrow)$

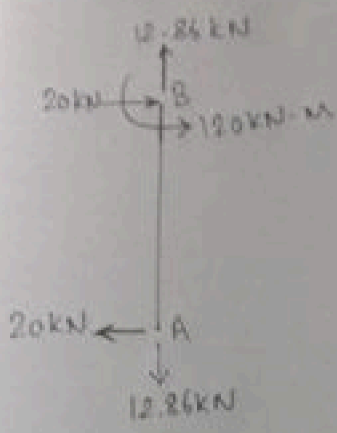
$(\circlearrowleft +)\ \Sigma M_A = 0$ or, $-7E_y + 10 \times 4 + 20 \times 6 = 0$

$\Rightarrow E_y = 22.86\,kN\ (\uparrow)$

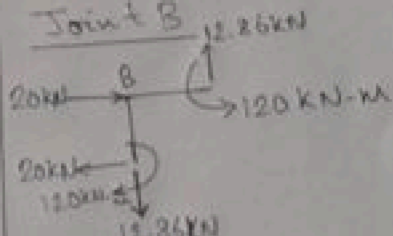
$(+\uparrow)\ \Sigma F_y = 0$ or, $A_y - 10 + 22.86 = 0$

or, $A_y = -12.86\,kN \Rightarrow A_y = 12.86\,kN\ (\downarrow)$

Member AB



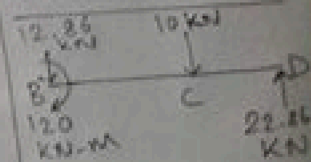
Joint B



Note: The external load 20 kN is applied at the joint B not at the member ends.

Also notice that axial force at end B of member BD is zero not 20 kN ($\leftarrow$).

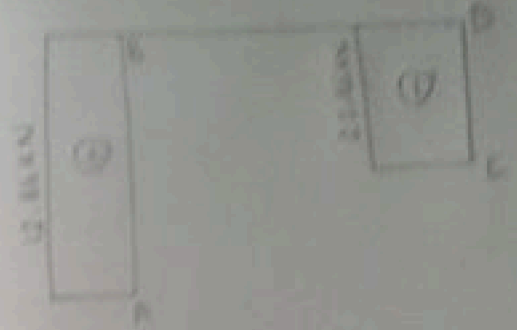
Member BD



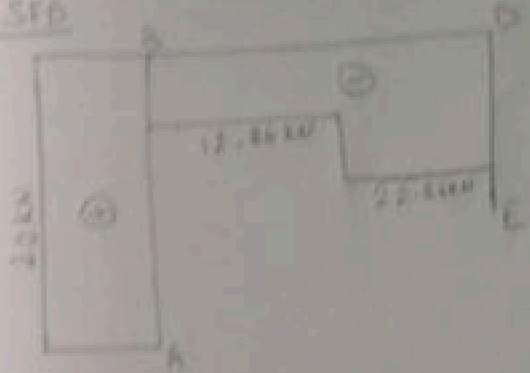
Member DE

Remember whenever point loads or moment couples are applied in a joint we need to consider the equilibrium of that joint as shown above and not proceed as before when there were no joint loads or moment couples.

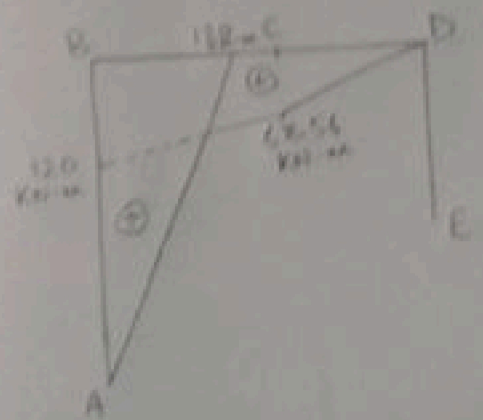
AFD



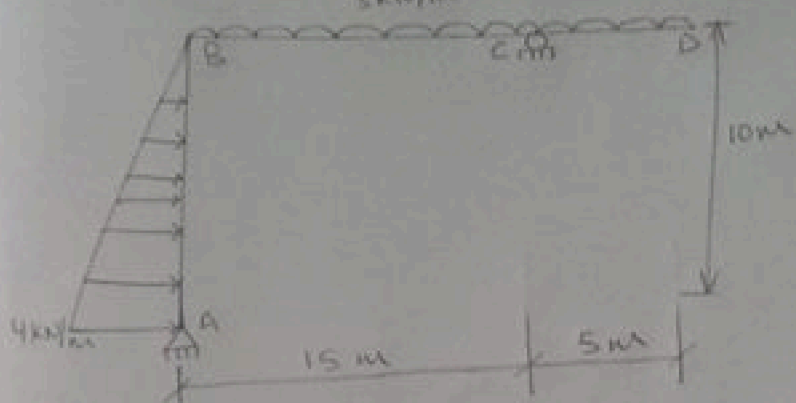
SFD



BMD



\# A Rigid frame ABCD is supported at A and C and carries a triangular load on the member AB, and a uniformly distributed load on the member BD as shown in figure below. Draw the AFD, SFD, and BMD.



Solution

$(\xrightarrow{+}) \ \Sigma F_x = 0$

or, $A_x + \frac{1}{2} * 4 * 10 = 0$

or, $A_x = -20\,kN$

$\Rightarrow A_x = 20\,kN \ (\leftarrow)$

$(\overset{+}{\circlearrowleft}) \ \Sigma M_A = 0$

or, $-15C_y + (\frac{1}{2} * 4 * 10) * \frac{10}{3} + (3 * 20) * 10 = 0$

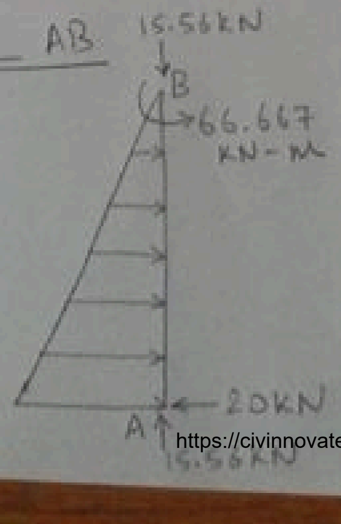
$\Rightarrow C_y = 44.44\,kN \ (\uparrow)$

$(+\uparrow) \ \Sigma F_y = 0$

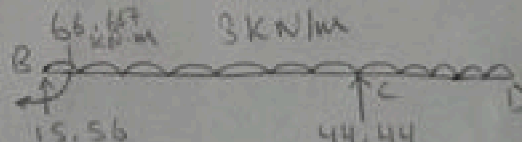
or, $A_y + 44.44 - 3 * 20 = 0$

$\Rightarrow A_y = 15.56\,kN \ (\uparrow)$

Member AB



Member ~~BD~~ BD



Since D is a free end it has no axial force, shear force and Bending moment. (You can check this fact from the FBD of BD shown above as well)

Axial force

From the FBD of member AB, we can easily see that this member is subjected to a uniform compressive force of 15.56 kN throughout its length. On the other hand member BD has no axial force.

Shear force

Member AB:

Slope of UVL $= \frac{4}{10} = 0.4$

$\frac{y}{x} = 0.4$

$y = 0.4x$

1. From B to A_{Right}

$V = \frac{1}{2} * y * x$

$= \frac{1}{2} * 0.4x * x$

$V = 0.2x^2 \quad (0 \leq x \leq 10m, \text{ B origin})$

2. A_{Left}, $V = 0.2 * 10^2 - 20 = 0$

Member BD:

1. B_{Left}, $V = 0$
2. B_{Right} to C_{Left}, $V = 15.56 - 3x$ (B origin; $0 \leq x \leq 15m$)
3. C_{Right}, $V = 15.56 - 3 * 15 + 44.444 = 15$ kN
4. D to C_{Right}, $V = 5x$ (D origin; $0 \leq x \leq 5m$)

Bending Moment

Member AB:

$M = +66.667 - (\frac{1}{2} * y * x) * (\frac{x}{3})$

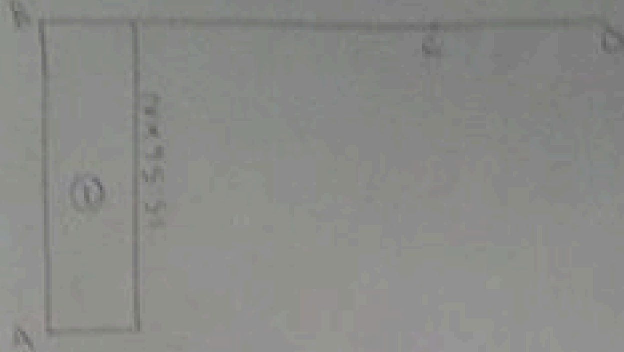
$= 66.667 - (\frac{1}{2} * 0.4x * x) * (\frac{x}{3})$

$\Rightarrow M = 66.667 - 0.06667x^3$ (B origin; $0 \leq x \leq 10m$)

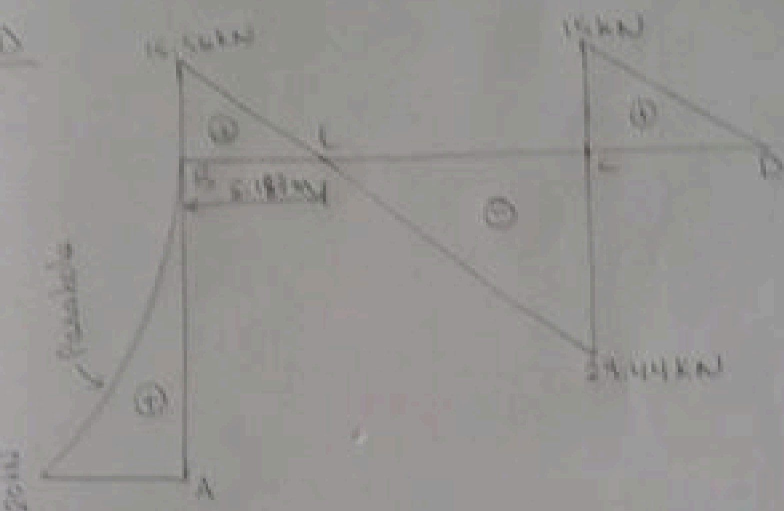
Member BD:

1. B_{Left}, $M = 0$
2. B_{Right} to C, $M = 66.667 + 15.56x - (3x) * \frac{x}{2}$
$= 66.667 + 15.56x - 1.5x^2$
(B origin, $0 \leq x \leq 15m$)
3. D to C, $M = -(3x) * \frac{x}{2} = -1.5x^2$
(D origin, $0 \leq x \leq 5m$)

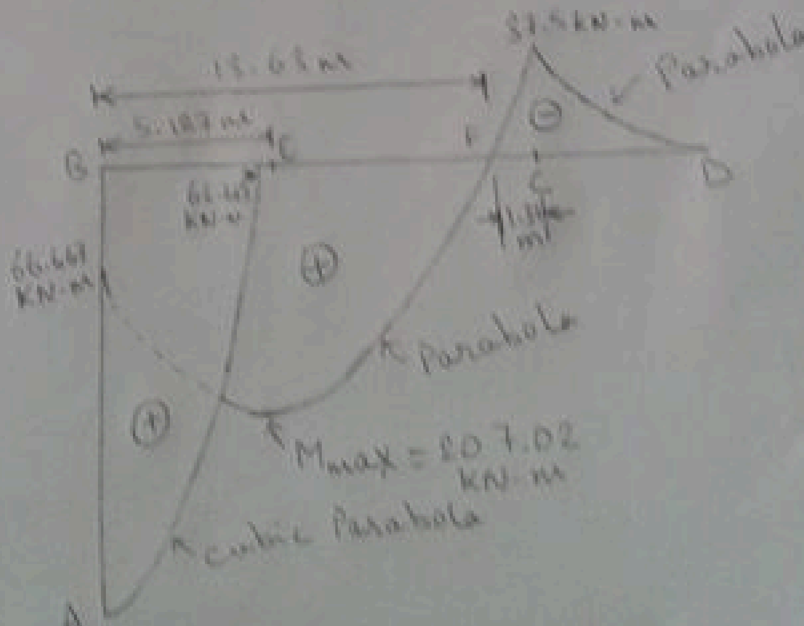
AFD



SFD

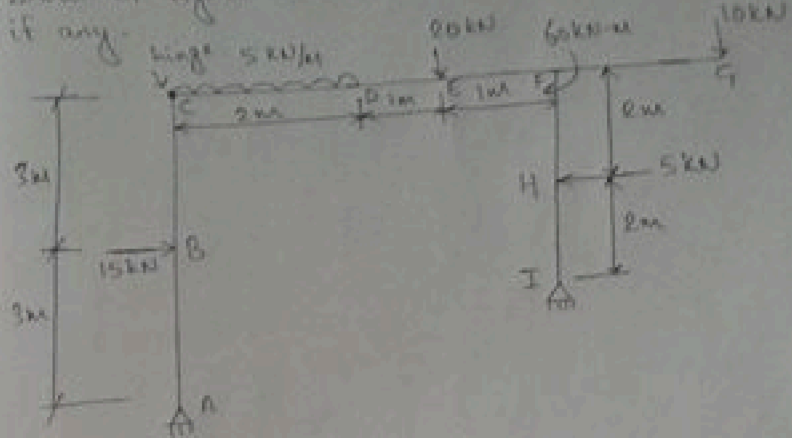


BMD



F is the point of contraflexure. Maximum +ve bending moment in Member BD occurs at E.

Draw AFD, SFD and BMD of the given frame loaded as shown in figure below. Also indicate the salient features if any.



Solution

$(\circlearrowleft +)\ \Sigma M_C^{AC} = 0$ or, $6A_x + 15*3 = 0$

$\Rightarrow A_x = -7.5\ kN = 7.5\ kN\ (\leftarrow)$

$(\xrightarrow{+})\ \Sigma F_x = 0$ or, $I_x - 7.5 + 15 - 5 = 0$

$\Rightarrow I_x = -2.5\ kN = 2.5\ kN\ (\leftarrow)$

$(\circlearrowleft +)\ \Sigma M_C = 0$ or, $\cancel{\Sigma M_C^{AC}}^{\,0} + \Sigma M_C^{Right} = 0$

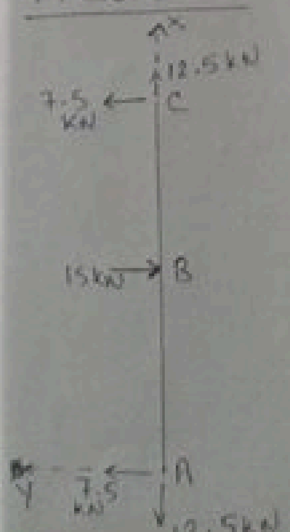
or, $-2.5*4 + 4I_y - 5*2 - 10*6 - 60 - 20*3 - (5*2)*1 = 0$

$\Rightarrow I_y = 52.5\ kN\ (\uparrow)$

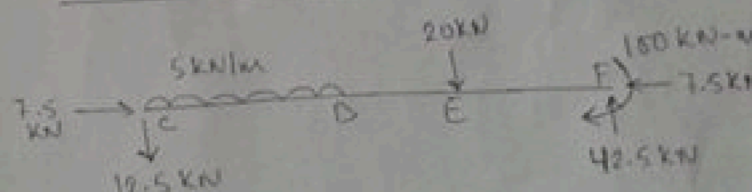
$(+\uparrow)\ \Sigma F_y = 0$ or, $A_y + 52.5 - 5*2 - 20 - 10 = 0$

$\Rightarrow A_y = -12.5\ kN = 12.5\ kN\ (\downarrow)$

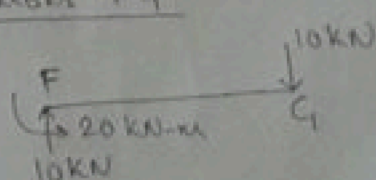
Member AC



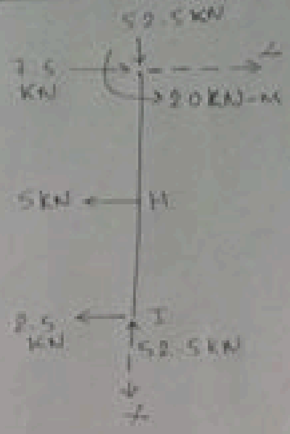
Member CF



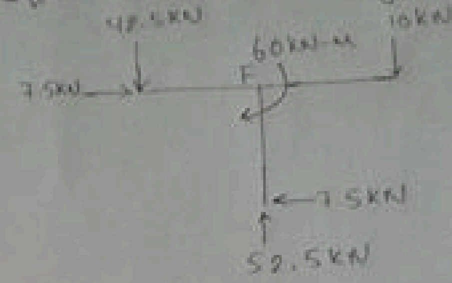
Member FG



Member FI



Equilibrium check of joint F:

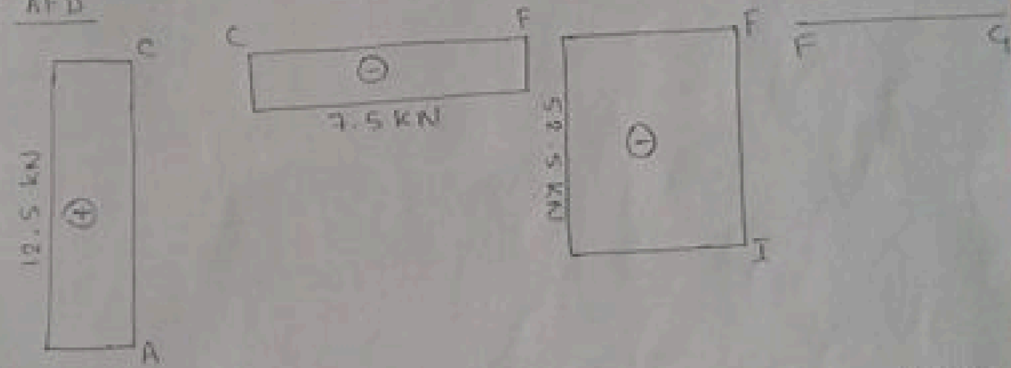


$\Sigma F_x = 0 \rightarrow$ check

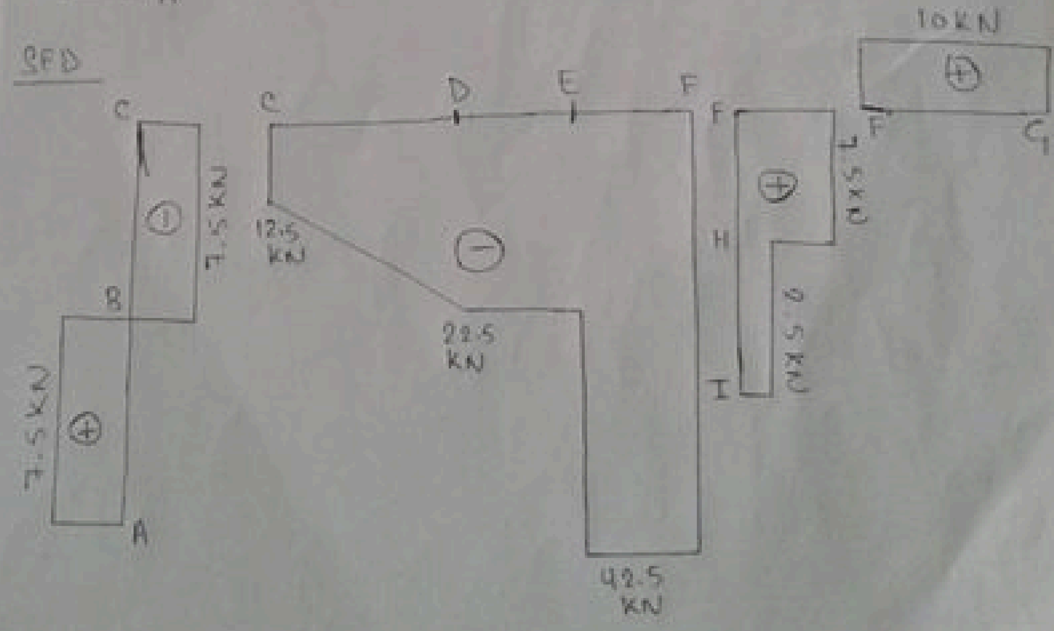
$\Sigma F_y = 0 \rightarrow$ check

$\Sigma M_F = 0 \rightarrow$ check

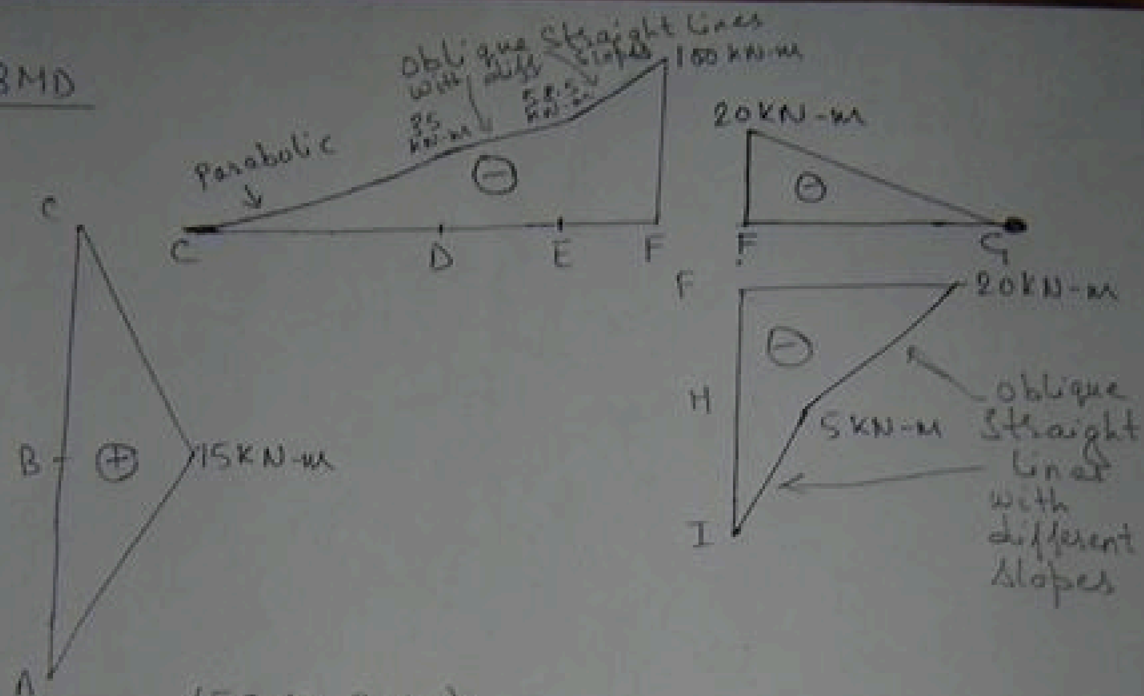
AFD



SFD



BMD



(FG is 2m)

Note : In the member CD, the expressions for BMD are as follows:

1) C to D, $M = -12.5X - (5X) * \left(\frac{X}{2}\right) = -12.5X - 2.5X^2$
(C origin, $0 \leq X \leq 2m$)

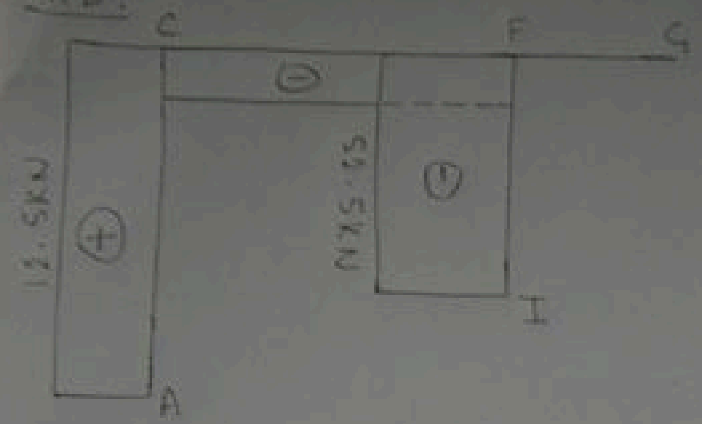
2) F_{Right}, $M = 0$

3) F_{Right} to E, $M = -100 + 42.5X$ (F origin, $0 < X \leq 1m$)

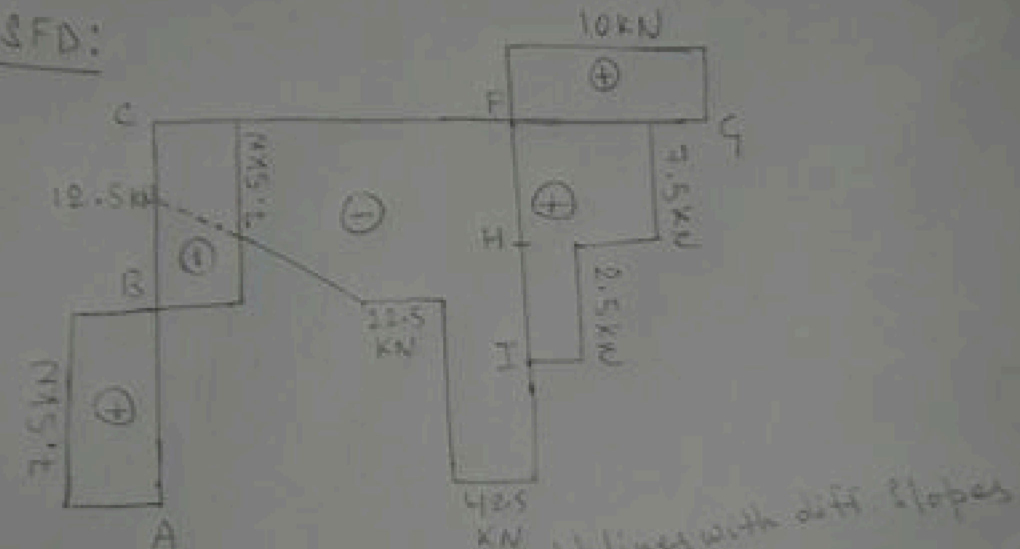
4) E to D, $M = -100 + 42.5X - 20(X-1)$
$= -80 + 22.5X$ (F origin, $1m \leq X \leq 2m$)

In the Board Exam draw the AFD, SFD, and BMD on the entire frame as shown below:

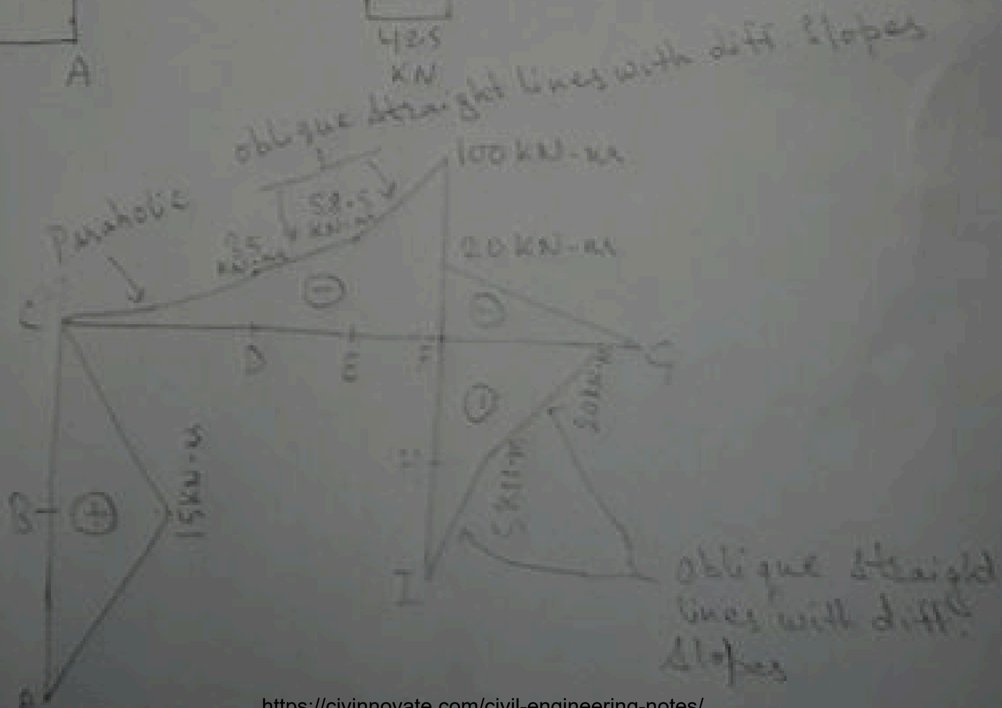
AFD:



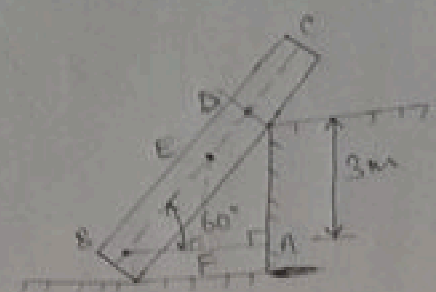
SFD:



BMD



Determine the tension in the cable AB which holds a post BC of 4 m length from sliding. The post has a mass of 9 kg. Assume all surfaces are smooth.



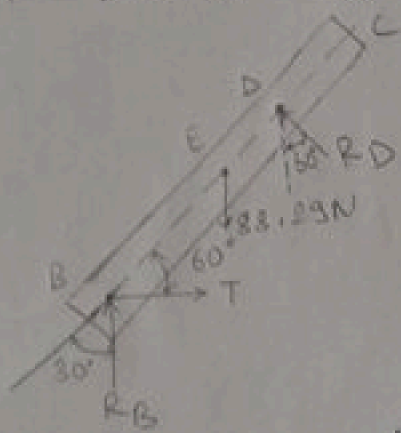
$\omega = 9.81 * 9 = 88.29\,N$

The weight of the post BC act at its C.G, E, located 2 m from B along BC (i.e. BE = 2m)

$BD = \frac{3}{\sin 60°} \Rightarrow BD = 3.464\,m$

$AB = BD \cos 60° \Rightarrow AB = 1.732\,m$

$BF = 2\cos 60° = 1\,m$; $AF = 0.732\,m$



$(\xrightarrow{+}) \Sigma F_x = 0$

or, $T - R_D \sin 60° = 0$

$\Rightarrow T = 0.866 R_D$

$(\circlearrowleft^{+}) \Sigma M_D = 0$

or, $-R_B * 1.732 + 3T + 88.29 * 0.732 = 0$

or, $-1.732 R_B + 2.598 R_D + 64.628 = 0$ — (i)

(we have substituted $T = 0.866 R_D$)

$(+\uparrow) \Sigma F_y = 0$ or, $R_B - 88.29 + R_D \cos 60° = 0$

$\Rightarrow R_B + 0.5 R_D - 88.29 = 0$ — (ii)

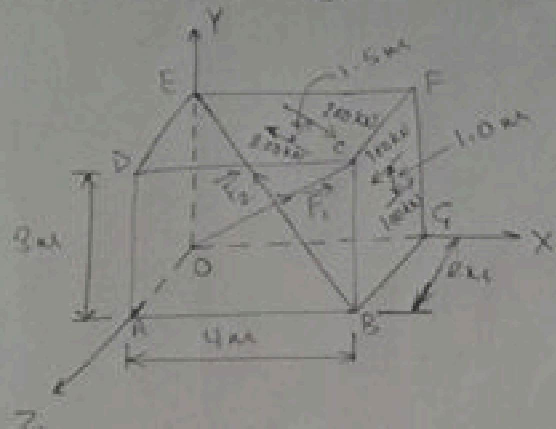
Multiplying (ii) by 1.732 and adding (i) & (ii),

$$
\begin{aligned}
-1.732 R_B + 2.598 R_D &= -64.628 \\
1.732 R_B + 0.866 R_D &= 152.918 \\
\hline
3.464 R_D &= 88.29 \\
\Rightarrow R_D &= 25.488\,KN
\end{aligned}
$$

So, $T = 0.866 R_D = 22.07\,KN$

https://civinnovate.com/civil-engineering-notes/

Find the resultant of force couple system at point O as shown in figure below. $F_1 = 500$ kN & $F_2 = 700$ kN.



$E = (0,3,0)$, $\vec{r}_E = 3\hat{j}$

$B = (4,0,2)$; $\vec{r}_{BE} = -4\hat{i} + 3\hat{j} - 2\hat{k}$; $|\vec{r}_{BE}| = \sqrt{29}$

$\hat{u}_{BE} = -\frac{4}{\sqrt{29}}\hat{i} + \frac{3}{\sqrt{29}}\hat{j} - \frac{2}{\sqrt{29}}\hat{k}$

$\Rightarrow \hat{u}_{BE} = -0.743\hat{i} + 0.557\hat{j} - 0.371\hat{k}$

$\vec{F_2} = 700(\hat{u}_{BE}) = -520.1\hat{i} + 389.9\hat{j} - 260.05\hat{k}$

$C = (4,3,2)$; $\vec{r}_{OC} = 4\hat{i} + 3\hat{j} + 2\hat{k}$; $|\vec{r}_{OC}| = \sqrt{29}$

$\Rightarrow \hat{u}_{OC} = 0.743\hat{i} + 0.557\hat{j} + 0.371\hat{k}$

$\vec{F_1} = 500(\hat{u}_{OC}) = 371.5\hat{i} + 278.5\hat{j} + 185.75\hat{k}$

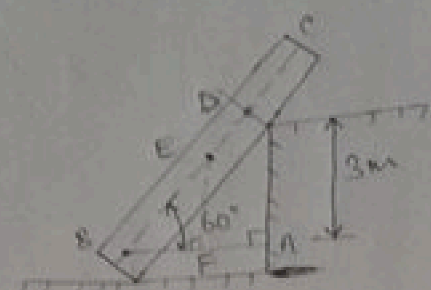
$\vec{F_R} = \vec{F_1} + \vec{F_2} = (-148.6\hat{i} + 668.4\hat{j} - 74.3\hat{k})$ kN

$\vec{M_O} = 3\hat{j} \times (-520.1\hat{i} + 389.9\hat{j} - 260.05\hat{k}) + (100 * 1)\hat{i}$
$- (200 * 1.5)\hat{j}$

$= 1560.3\hat{k} - 780.15\hat{i} - 300\hat{j} + 100\hat{i}$

$\Rightarrow \vec{M_O} = (-680.15\hat{i} - 300\hat{j} + 1560.3\hat{k})$ kN-m

Determine the tension in the cable AB which holds a post BC of 4 m length from sliding. The post has a mass of 9 kg. Assume all surfaces are smooth.



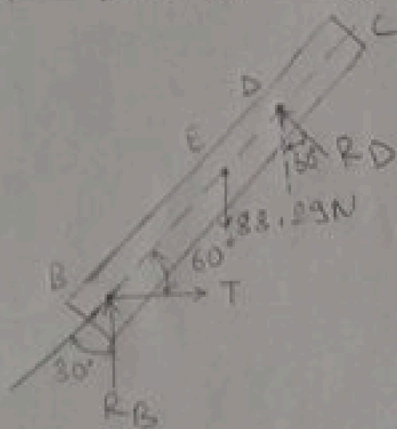
$\omega = 9.81 * 9 = 88.29\,N$

The weight of the post BC act at its C.G, E, located 2 m from B along BC (i.e. BE = 2m)

$BD = \frac{3}{\sin 60^\circ} \Rightarrow BD = 3.464\,m$

$AB = BD\cos 60^\circ \Rightarrow AB = 1.732\,m$

$BF = 2\cos 60^\circ = 1\,m$; $AF = 0.732\,m$



$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $T - R_D \sin 60^\circ = 0$

$\Rightarrow T = 0.866 R_D$

$(\circlearrowleft +)\ \Sigma M_D = 0$

or, $-R_B * 1.732 + 3T + 88.29 * 0.732 = 0$

or, $-1.732 R_B + 2.598 R_D + 64.628 = 0$ — (i)

(we have substituted $T = 0.866 R_D$)

$(+\uparrow)\ \Sigma F_y = 0$ or, $R_B - 88.29 + R_D \cos 60^\circ = 0$

$\Rightarrow R_B + 0.5 R_D - 88.29 = 0$ — (ii)

Multiplying (ii) by 1.732 and adding (i) & (ii),

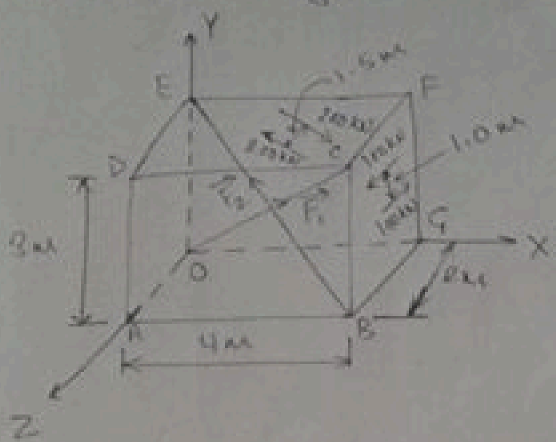
$$-1.732 R_B + 2.598 R_D = -64.628$$
$$1.732 R_B + 0.866 R_D = 152.918$$

$3.464 R_D = 88.29$

$\Rightarrow R_D = 25.488\,KN$

So, $T = 0.866 R_D = 22.07\,KN$

Q. Find the resultant of force couple system at point O as shown in figure below. $F_1 = 500\,kN$ & $F_2 = 700\,kN$.



$E = (0,3,0)$, $\vec{r}_E = 3\hat{j}$

$B = (4,0,2)$; $\vec{r}_{BE} = -4\hat{i} + 3\hat{j} - 2\hat{k}$; $|\vec{r}_{BE}| = \sqrt{29}$

$\hat{u}_{BE} = -\frac{4}{\sqrt{29}}\hat{i} + \frac{3}{\sqrt{29}}\hat{j} - \frac{2}{\sqrt{29}}\hat{k}$

$\Rightarrow \hat{u}_{BE} = -0.743\hat{i} + 0.557\hat{j} - 0.371\hat{k}$

$\vec{F_2} = 700(\hat{u}_{BE}) = -520.1\hat{i} + 389.9\hat{j} - 260.05\hat{k}$

$C = (4,3,2)$; $\vec{r}_{OC} = 4\hat{i} + 3\hat{j} + 2\hat{k}$; $|\vec{r}_{OC}| = \sqrt{29}$

$\Rightarrow \hat{u}_{OC} = 0.743\hat{i} + 0.557\hat{j} + 0.371\hat{k}$

$\vec{F_1} = 500(\hat{u}_{OC}) = 371.5\hat{i} + 278.5\hat{j} + 185.75\hat{k}$

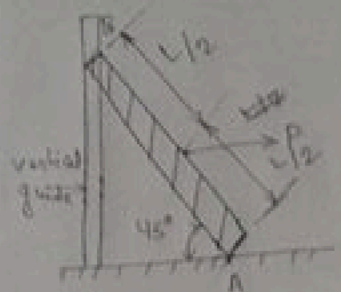
$\vec{F_R} = \vec{F_1} + \vec{F_2} = (-148.6\hat{i} + 668.4\hat{j} - 74.3\hat{k})\,kN$

$\vec{M_O} = 3\hat{j} \times (-520.1\hat{i} + 389.9\hat{j} - 260.05\hat{k}) + (100 * 1)\hat{i}$
$\quad - (200 * 1.5)\hat{j}$

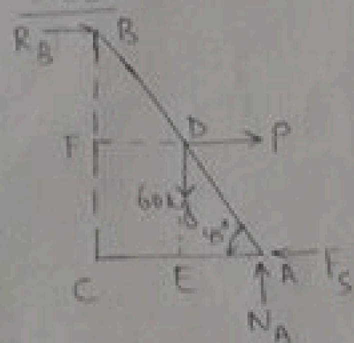
$= 1560.3\hat{k} - 780.15\hat{i} - 300\hat{j} + 100\hat{i}$

$\Rightarrow \vec{M_O} = (-680.15\hat{i} - 300\hat{j} + 1560.3\hat{k})\,kN\text{-}m$

\# A small roller on end B of uniform 60 kg bar is constructed to move in smooth vertical guides. The coefficient of friction between end A and horizontal supporting surfaces is 0.8. Determine the horizontal force P required to initiate slipping at A as shown in figure below.



FBD



$AC = AB\cos 45° = 0.707L$

$BC = AB\sin 45° = 0.707L$

$BF = BD\sin 45° = 0.35355L$

$FC = BC - BF = 0.35355L$

$CE = FD = BD\cos 45° = 0.35355L$

$AE = AC - CE = 0.35355L$

$(+\uparrow)\ \Sigma F_y = 0$ or, $N_A - 60 = 0 \Rightarrow N_A = 60\text{ kg}$

$F_s = \mu_s N = 0.8 * 60 = 48\text{ kg}$

$(\xrightarrow{+})\ \Sigma F_x = 0$ or, $R_B + P - F_s = 0$

or, $R_B + P = 48$ — (i)

$(\circlearrowleft^{+})\ \Sigma M_A = 0$ or, $-(0.7071L)R_B - (0.35355L)P + (0.35355L) * 60 = 0$

$\Rightarrow 0.7071R_B + 0.35355P = 21.213$ — (ii)

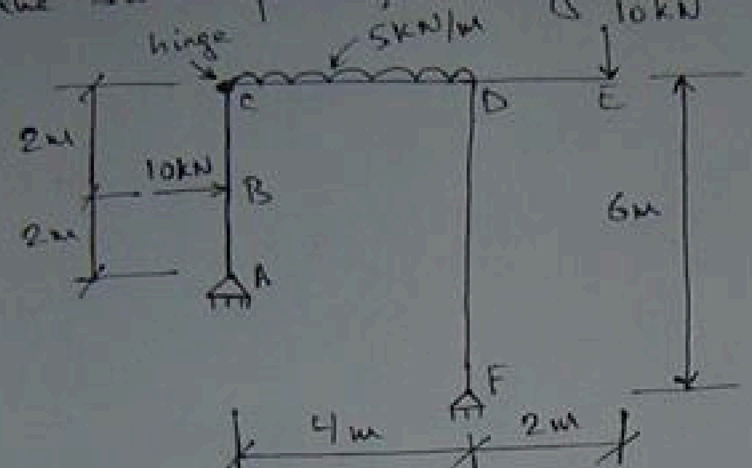
(i) * 0.7071 and (i) − (ii)

$$
\begin{array}{rcrcl}
0.7071R_B & + & 0.7071P & = & 33.941 \\
-\ 0.7071R_B & \underline{+} & 0.35355P & = & \underline{-}\ 21.213 \\
\hline
 & & 0.35355P & = & 12.728
\end{array}
$$

$\Rightarrow P = 36\text{ kg}$

2069 Poush

Draw the Axial Force, Shear Force and Bending Moment Diagram for the loaded frame supported with 2 hinge supports at A and F and internal hinge at node C as shown in figure below and also indicate the salient points, if any.



$(\circlearrowleft)\ \sum M_C^{AC} = 0$

or, $4A_x + 2*10 = 0 \Rightarrow A_x = -5\text{kN} = 5\text{kN}(\leftarrow)$

$(\xrightarrow{+})\ \sum F_x = 0$

or, $-5 + F_x + 10 = 0 \Rightarrow F_x = -5\text{kN} = 5\text{kN}(\leftarrow)$

$(\circlearrowleft)\ \sum M_A = 0$

or, $-10*2 - (5*4)*2 - 10*6 - \overset{(F_x)}{5}*2 + F_y*4 = 0$

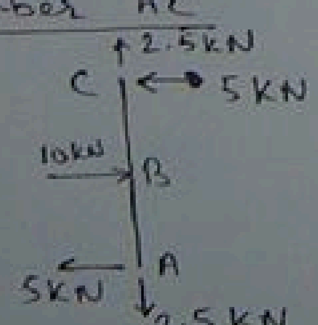
$\Rightarrow F_y = 32.5\text{ kN }(\uparrow)$

$(+\uparrow)\ \sum F_y = 0$

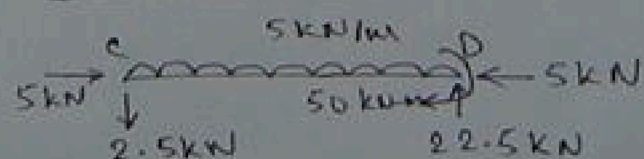
or, $A_y + 32.5 - 5*4 - 10 = 0$

$\Rightarrow A_y = -2.5\text{kN} = 2.5\text{kN }(\downarrow)$

Member AC



Member CD



Member CD:

$(+\uparrow)\ \Sigma F_y = 0$ or, $-2.5 - 5*4 + D_y^{CD} = 0$

$\Rightarrow D_y^{CD} = 22.5\ kN$

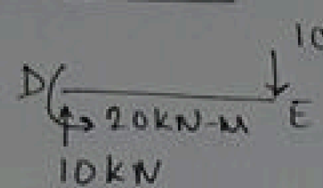
$(\circlearrowleft)\ \Sigma M_D = 0$

or, $2.5*4 + (5*4)*2 + M_D^{CD} = 0$

or, $M_D^{CD} = -50\ kN\text{-}m$

$\Rightarrow M_D^{CD} = 50\ kN\text{-}m\ (\circlearrowright)$

Member DE



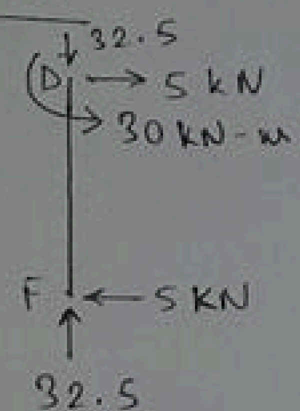
$(+\uparrow)\ \Sigma F_y = 0$

or, $-10 + D_y^{DE} = 0 \Rightarrow D_y^{DE} = 10\ kN$

$(\circlearrowleft)\ \Sigma M_D = 0$

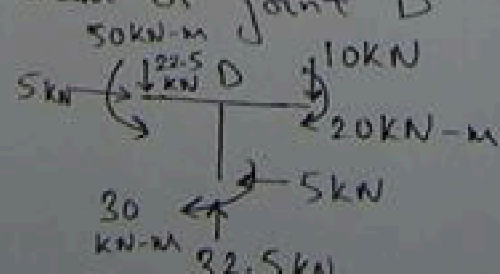
or, $-10*2 + M_D^{DE} = 0 \Rightarrow M_D^{DE} = 20\ kN\text{-}m$

Member DF



Here we have calculated the member forces at end D of member DF by using the reactions at support F.

We can check our calculation by considering the equilibrium of joint D

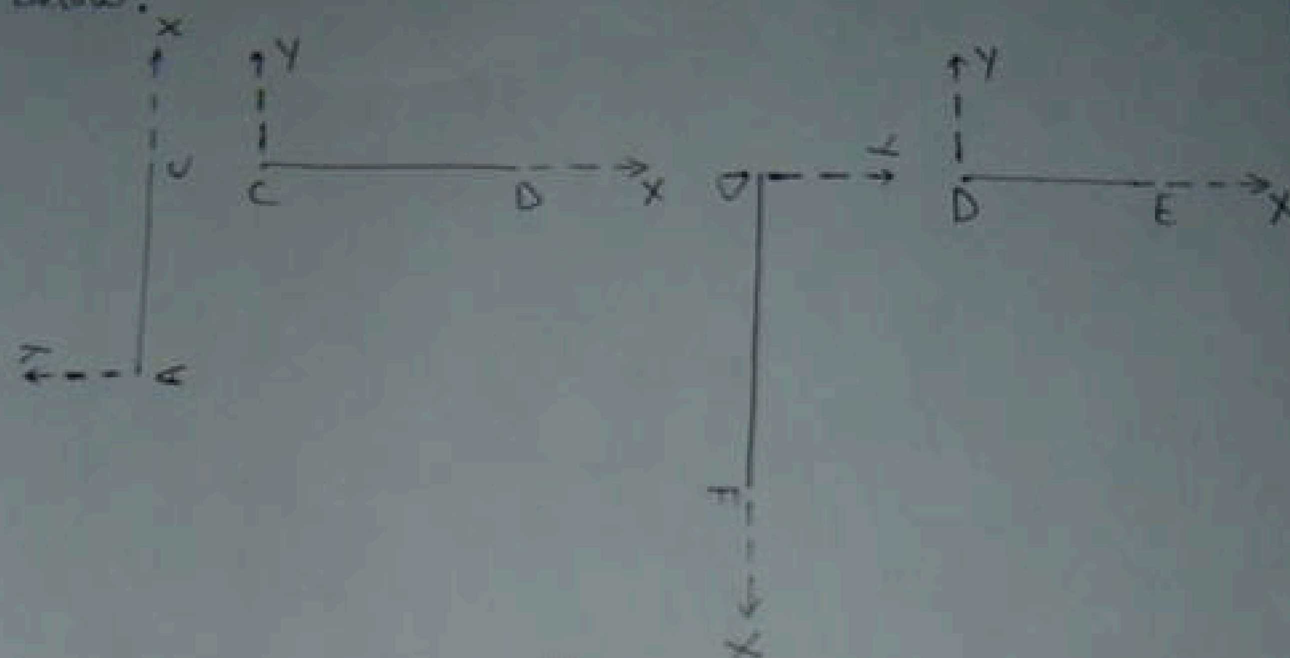


$\Sigma F_x = 0 \rightarrow$ check

$\Sigma F_y = 0 \rightarrow$ check

$\Sigma M_D = 0 \rightarrow$ check

The coordinate system for each member is shown below:



Axial Force

Member AC:

1) A_{left}, $Q = 0$
2) A_{Right} to C_{left}, $Q = 2.5\,kN$
3) C_{Right}, $Q = 0$

Member CD:

1) C_{left}, $Q = 0$
2) C_{Right} to D_{left}, $Q = -5\,kN$
3) D_{Right}, $Q = 0$

There is no axial force in member DE.

Member DF:

1) D_{left}, $Q = 0$
2) D_{Right} to F_{left}, $Q = -32.5\,kN$
3) F_{Right}, $Q = \cancel{32.5}\ 0$

Shear Force

Member AC:

1) A_{left}, $V = 0$
2) A_{Right} to B_{left}, $V = 5\ kN$
3) B_{Right}, $V = 5 - 10 = -5\ kN$
4) B_{Right} to C_{left}, $V = -5\ kN$
5) B_{Right}, $V = 0$

Member CD:

1) C_{left}, $V = 0$
2) C_{Right} to D_{left}, $V = -2.5 - 5x$ $(0 < x < 4m)$ (C origin)
3) D_{Right}, $V = 0$

Member DE:

1) D_{left}, $V = 0$
2) D_{Right} to E_{left}, $V = 10\ kN$
3) E_{Right}, $V = 0$

Member DF:

1) D_{left}, $V = 0$
2) D_{Right}, $V = 5\ kN$
3) D_{Right} to F_{left}, $V = 5\ kN$
4) F_{Right}, $V = 0$

Bending moment

Member AC:

1) A to B, $M = 5X$ $(0 \le X \le 2m,\ A \text{ origin})$
2) B to C, $M = 5X - 10(X - 2)$
$= -5X + 20$ $(2m \le X \le 4m,\ A \text{ origin})$

Member CD:

1) C to D_{left}, $M = -2.5X - (5X)\left(\frac{X}{2}\right)$
$= -2.5X - 2.5X^2$ $(0 \le X < 4m,\ C \text{ origin})$
2) D_{Right}, $M = 0$

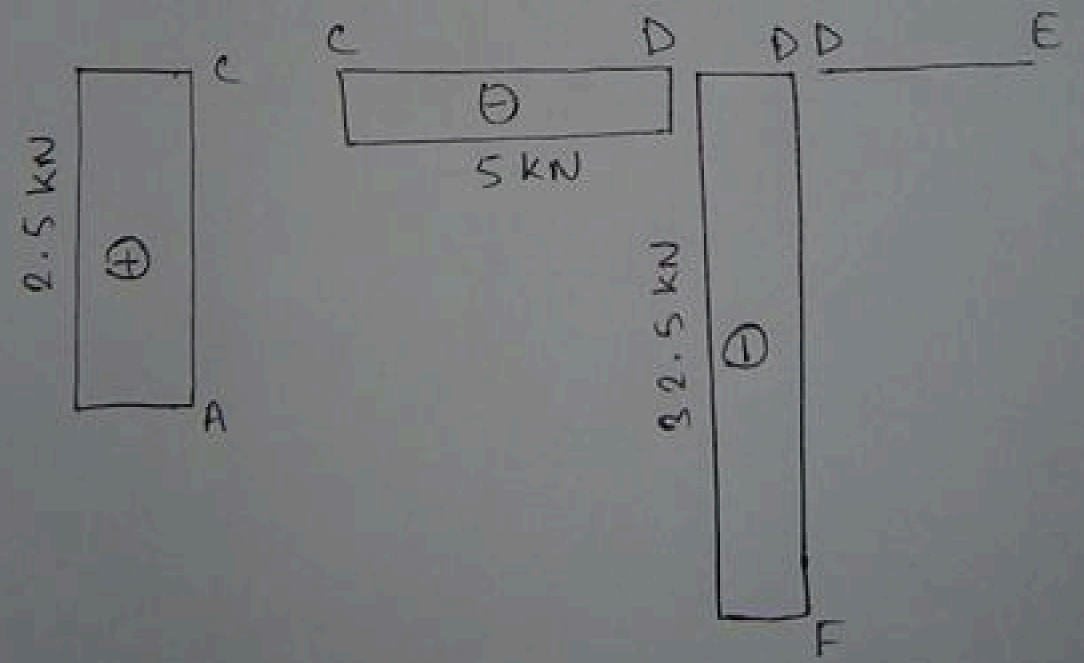
Member DE:

1) E to D_{Right}, $M = -10X$ $(0 \le X \le 2m,\ E \text{ origin})$
2) D_{left}, $M = 0$

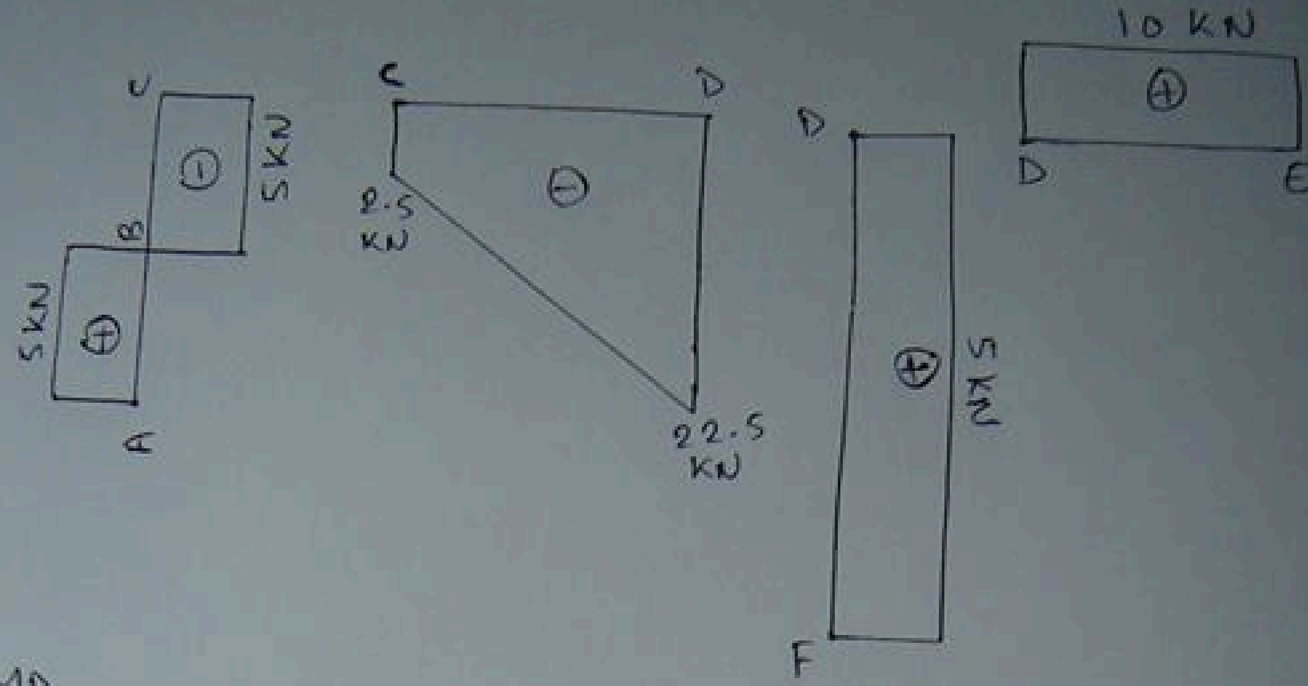
Member DF:

1) F to D_{Right}, $M = -5X$ $(0 \le X < 6m,\ F \text{ origin})$
2) D_{left}, $M = 0$

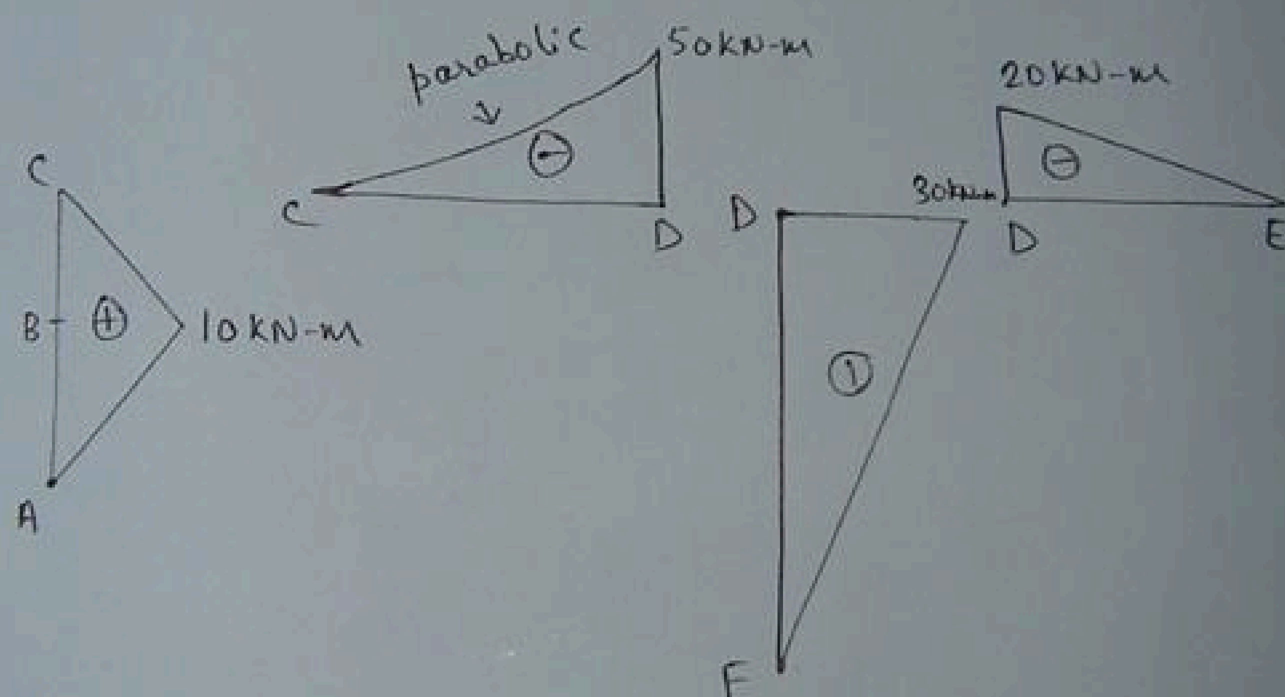
AFD



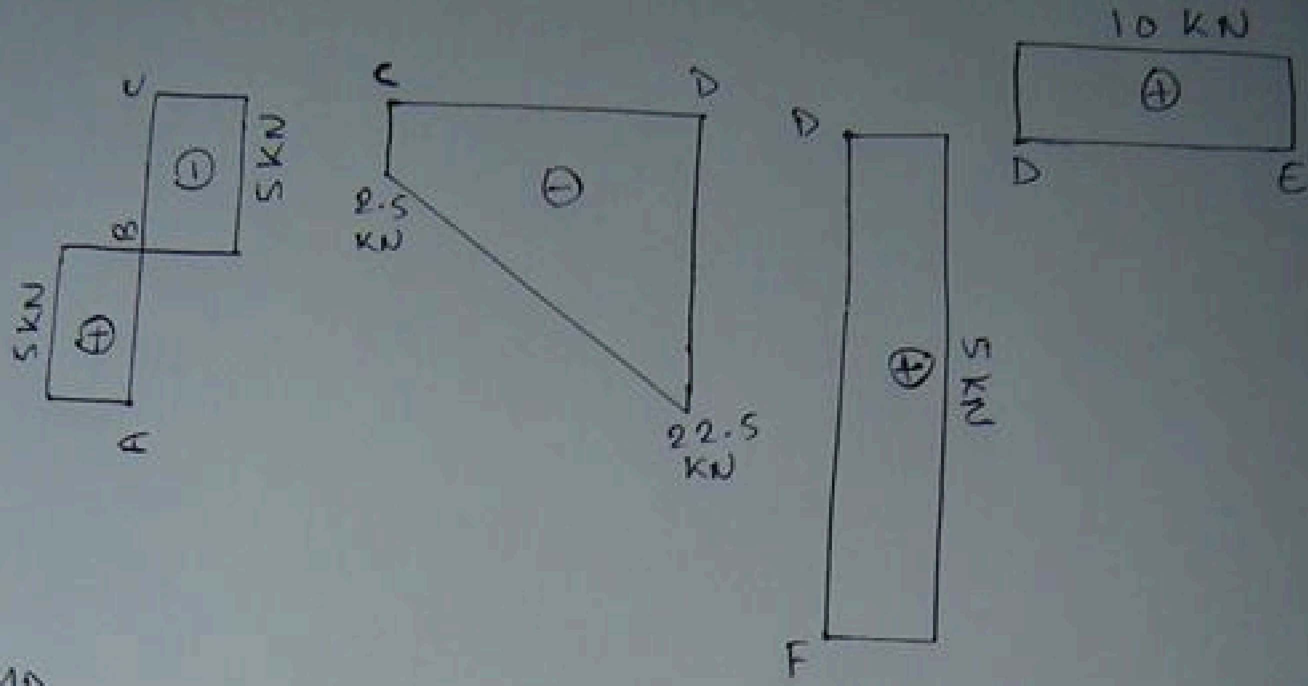
SFD


C
B
A
5KN
5KN
C
D
2.5 KN
22.5 KN
D
F
5KN
10 KN
D
E

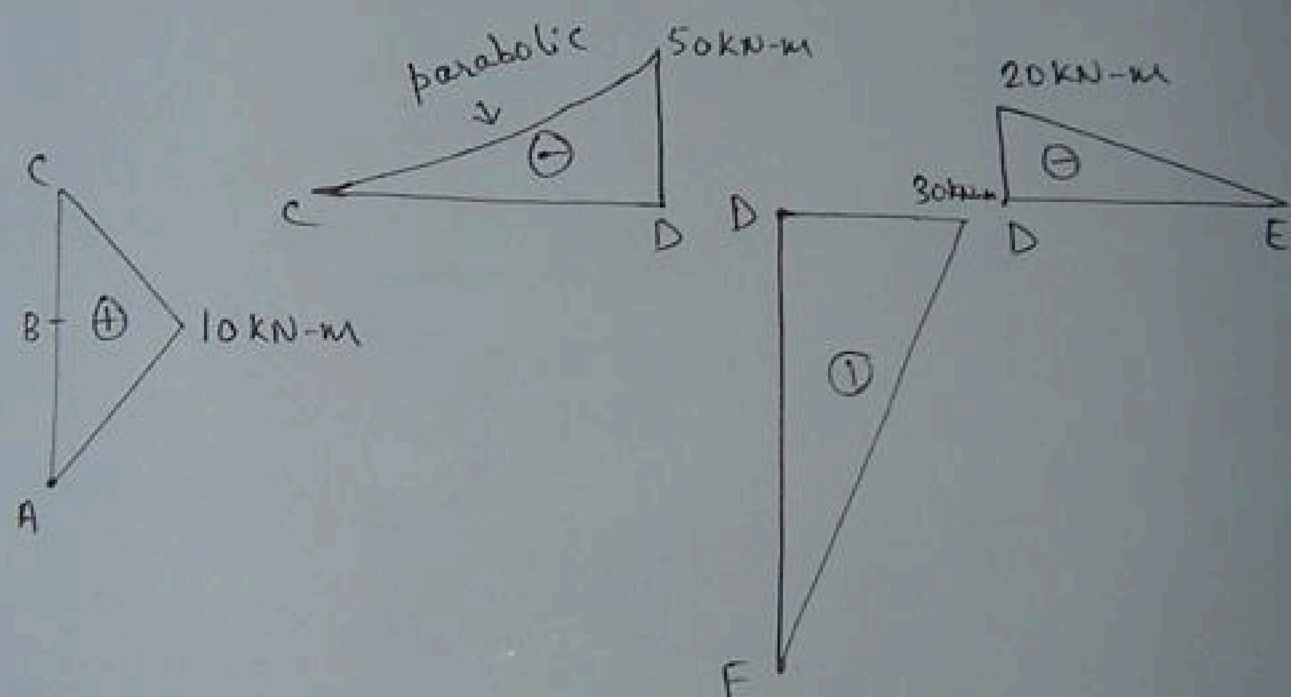
BMD


C
B
A
10KN-m
parabolic
50KN-m
C
D
20KN-m
30KN-m
D
E
D
F

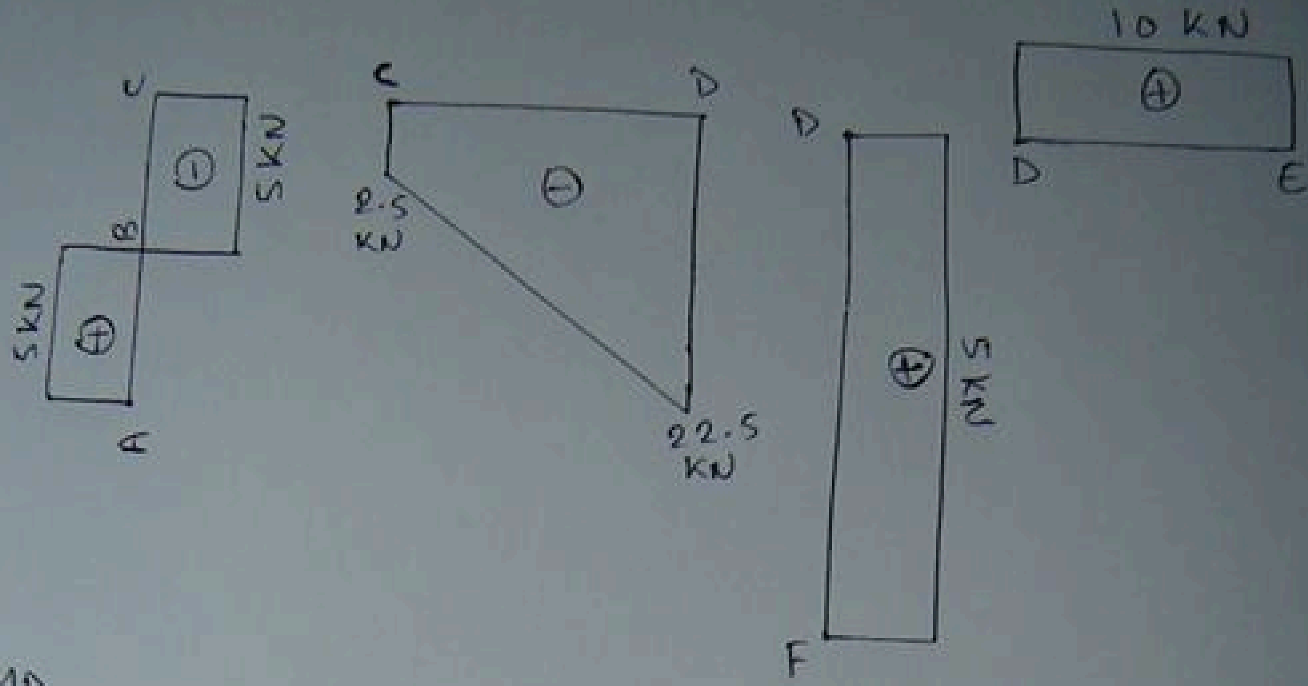
SFD


C
B
A
5KN
5KN
C
D
2.5 KN
22.5 KN
D
F
5KN
10 KN
D
E

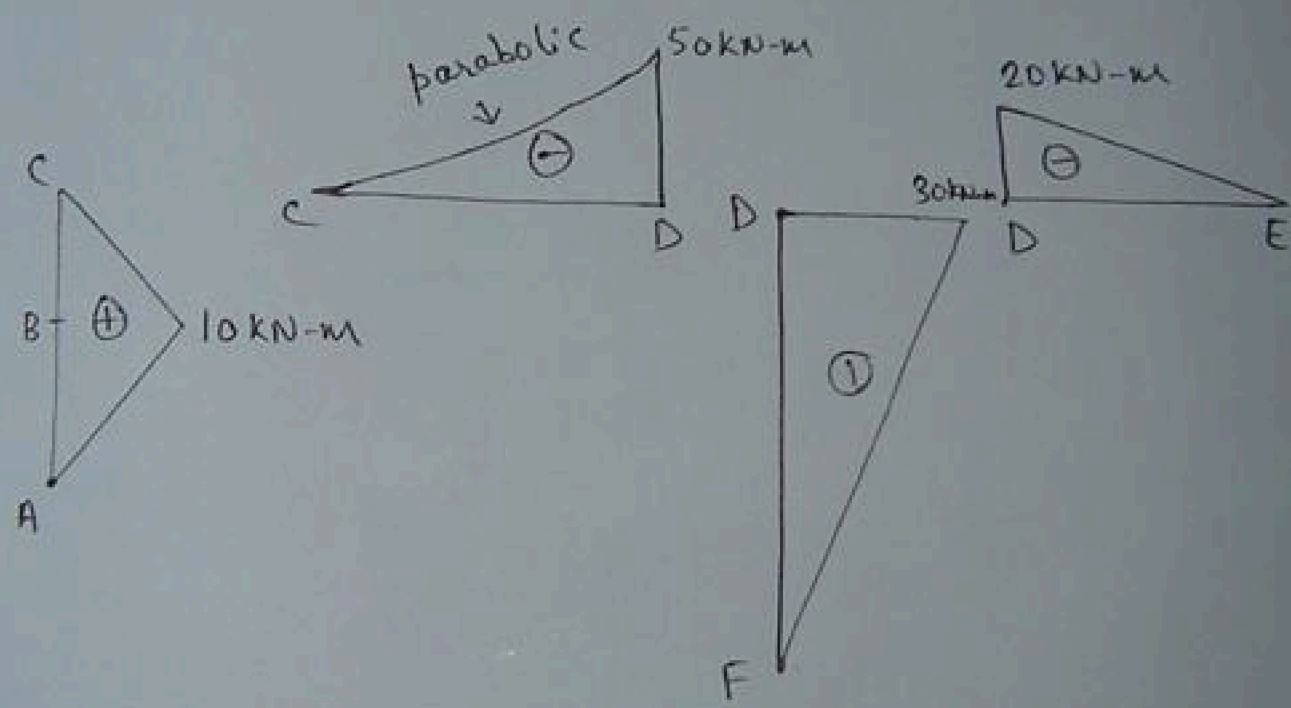
BMD


C
B
A
10KN-m
parabolic
50KN-m
C
D
20KN-m
30KN-m
D
E
D
F

SFD

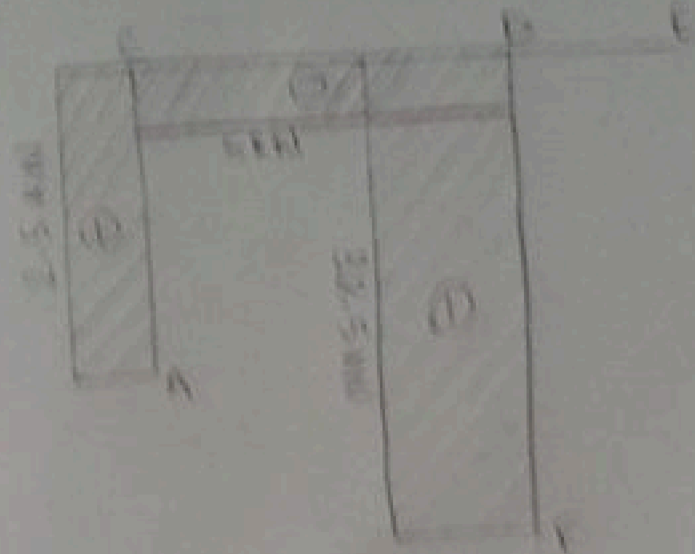

C
B
A
5KN
5KN
C
D
2.5 KN
22.5 KN
D
F
5KN
10 KN
D
E

BMD

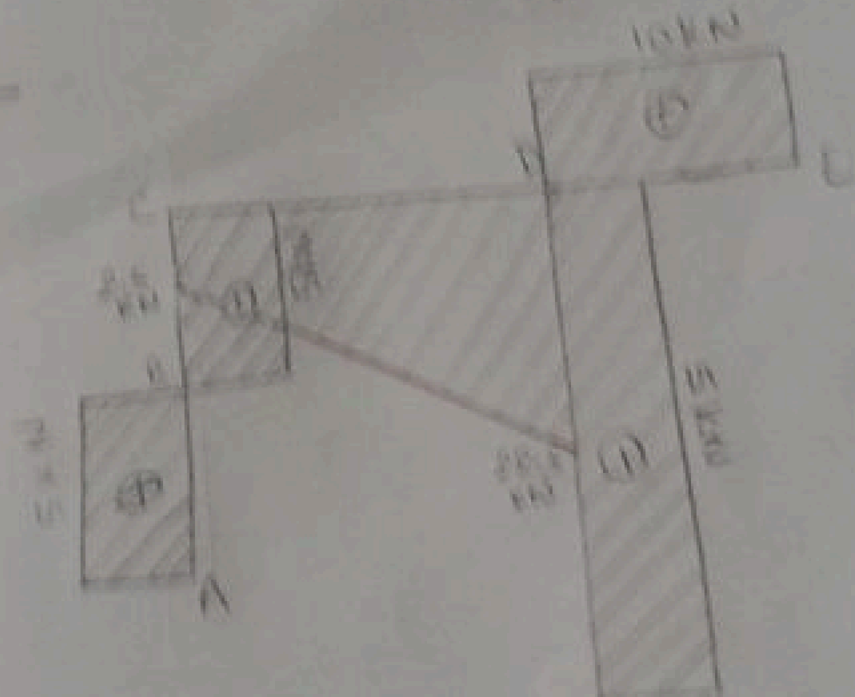

C
B
A
10KN-m
parabolic
50KN-m
C
D
20KN-m
30KN-m
D
E
D
F

For the Board frame draw the AFD, SFD, and BMD on the entire frame as shown below!

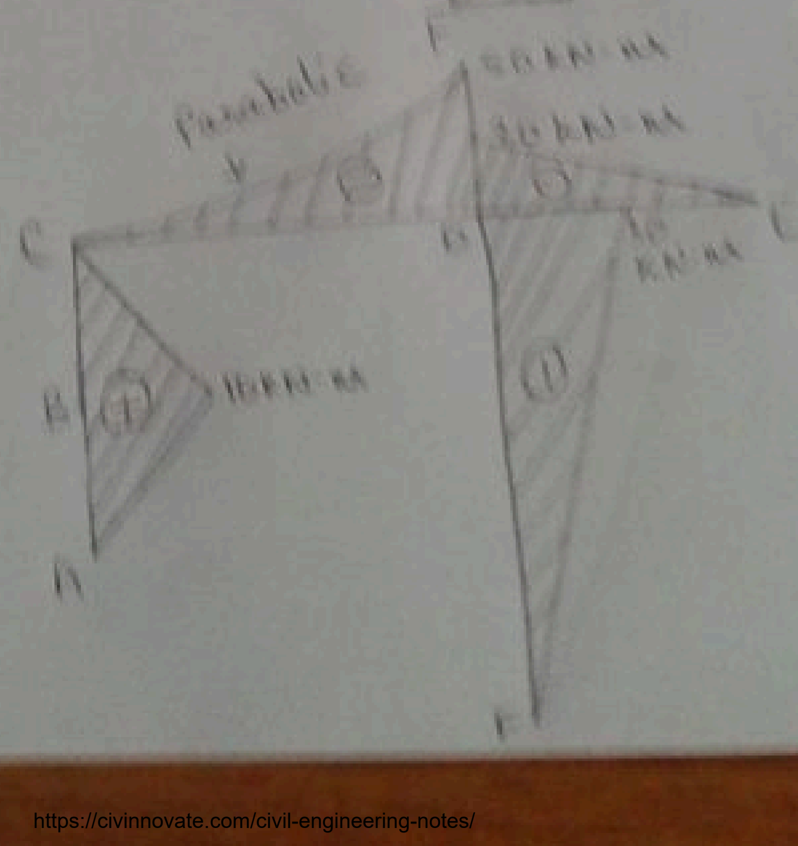
AFD



SFD

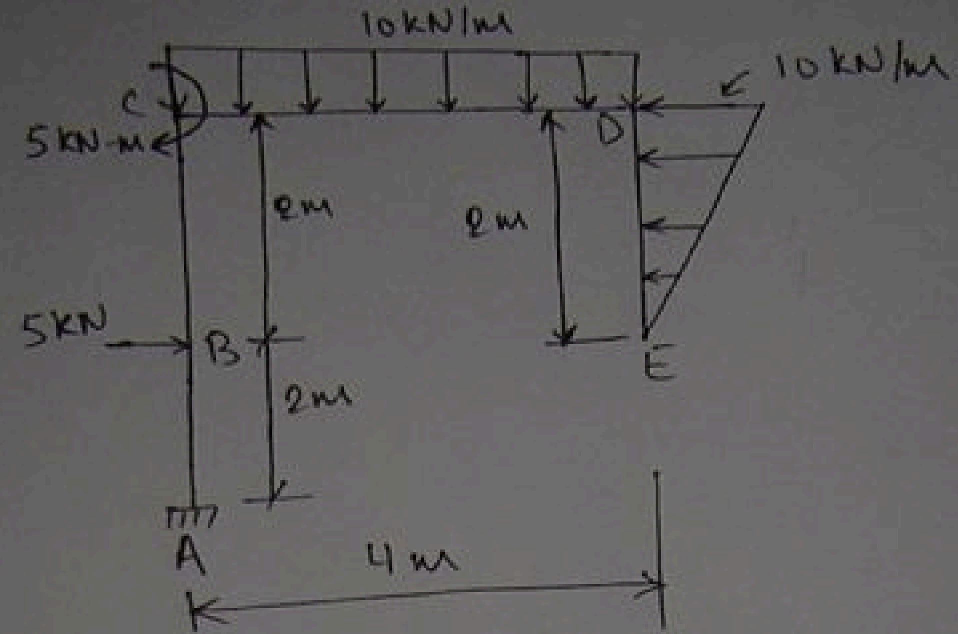


BMD



2065 Shrawan

Draw BMD, SFD, and AFD for the frame shown below.



$(\xrightarrow{+}) \Sigma F_x = 0$

or, $A_x + 5 - 0.5 * 10 * 2 = 0$

$\Rightarrow A_x = 5\,kN (\rightarrow)$

$(+\uparrow) \Sigma F_y = 0$

or, $A_y - 10 * 4 = 0$

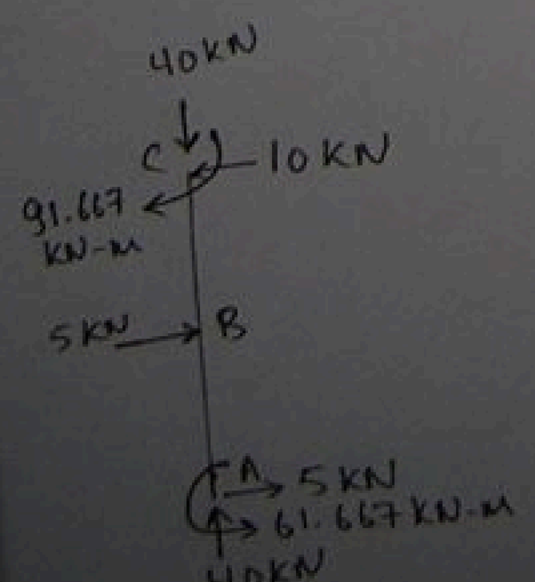
$\Rightarrow A_y = 40\,kN$

$(\circlearrowright +) \Sigma M_A = 0$

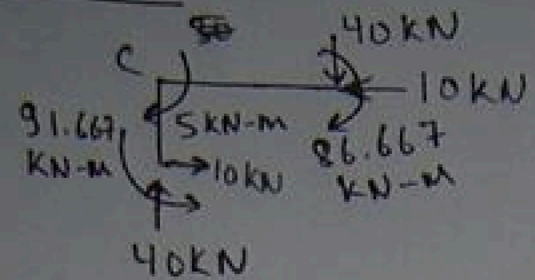
or, $M_A - 5 * 2 - 5 - 10 * 4 * 2 + (0.5 * 10 * 2) * (2 + \frac{4}{3}) = 0$

$\Rightarrow M_A = 61.667\,kN\text{-}m\ (\circlearrowleft)$

Member AC



Joint C

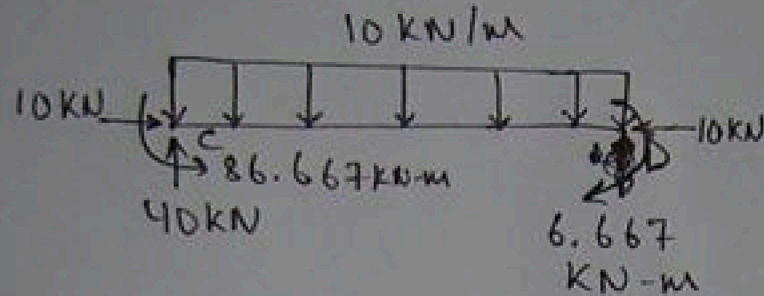


$\Sigma M_c = 0$

or, $M_c^{CD} + 5 - 91.667 = 0$

$\Rightarrow M_c^{CD} = 86.667$ KN-m

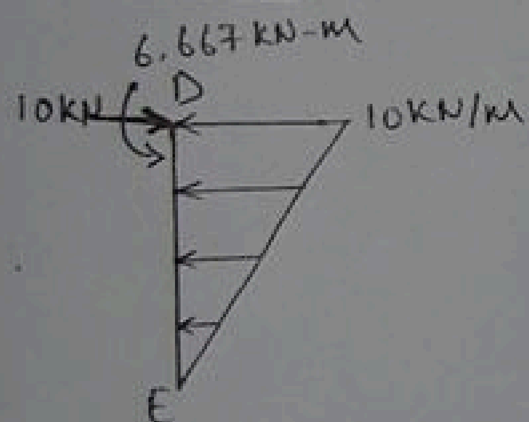
Member CD



$D_y^{CD} = 0$, $D_x^{CD} = -10$KN

$M_D^{CD} = -6.667$ KN-m

Member DE



$(\circlearrowright)\ \Sigma M_E = 0$

or, $M_c^{DE} + 6.667 + (0.5*10*2)*\left(\frac{4}{3}\right) - 10*2 = 0$

$\Rightarrow M_c^{DE} = 0$

Axial force

Member AC:

1) A_{left}, $Q = 0$
2) A_{Right} to C_{left}, $Q = -40$KN
3) C_{Right}, $Q = 0$

Member CD:

1) C_{left}, $Q = 0$
2) C_{Right} to D_{left}, $Q = -10$KN
3) D_{Right}, $Q = 0$

Member DE doesn't have axial force.

(Note: we have followed the usual co-ordinate system we have used in the past.)

Shear Force

Member AC:

1) A_{left}, $V = 0$
2) A_{Right} to B_{left}, $V = -5kN$
3) B_{Right} to C_{left}, $V = -10kN$
4) C_{Right}, $V = 0$

Member CD:

1) C_{left}, $V = 0$
2) C_{Right} to D, $V = 40 - 10X$ (C origin, $0 < X \leq 4m$)

Member DE:

1) E to D_{Right}, $V = \frac{1}{2} * X * 5X$

$= 2.5X^2$ (E origin, $0 \leq X < 2m$)

(The slope of UVL is 5)

2) D_{left}, $V = 0$

Bending moment

Member AC:

1) A_{left}, $M = 0$
2) A_{Right} to B, $M = -61.667 - 5X$ (A origin, $0 < X \leq 2m$)
3) B~~Right~~ to C_{left}, $M = -61.667 - 5X - 5(X-2)$

$= -51.667 - 10X$ (A origin, $2 \leq X < 4m$)

4) C_{Right}, $M = 0$

Member CD:

1) C_{left}, $M = 0$
2) C_{Right} to D_{left}, $M = -86.667 + 40X - 10X\left(\frac{X}{2}\right)$

$= -86.667 + 40X - 5X^2$

(C origin, $0 < X < 4m$)

3) D_{Right}, $M = 0$

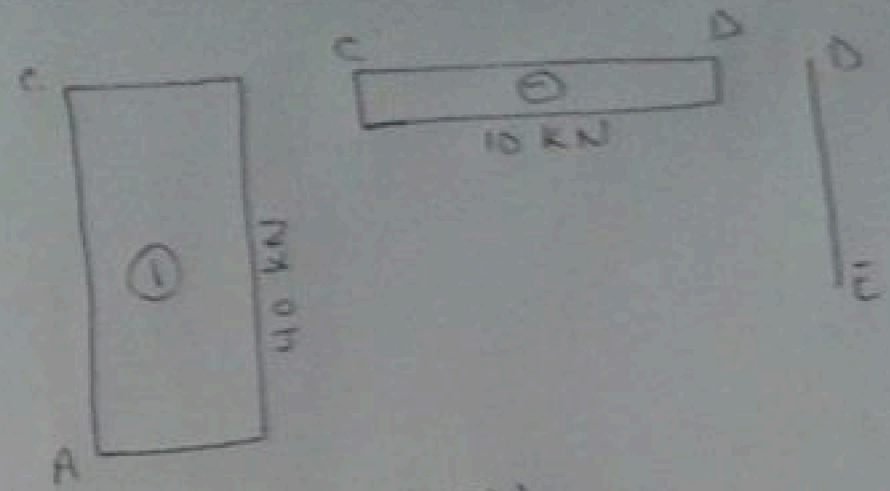
Member DE:

1) E to D_{Right}, $M = -\left(\frac{1}{2} * X * 5X\right) * \frac{X}{3} = -0.833X^3$

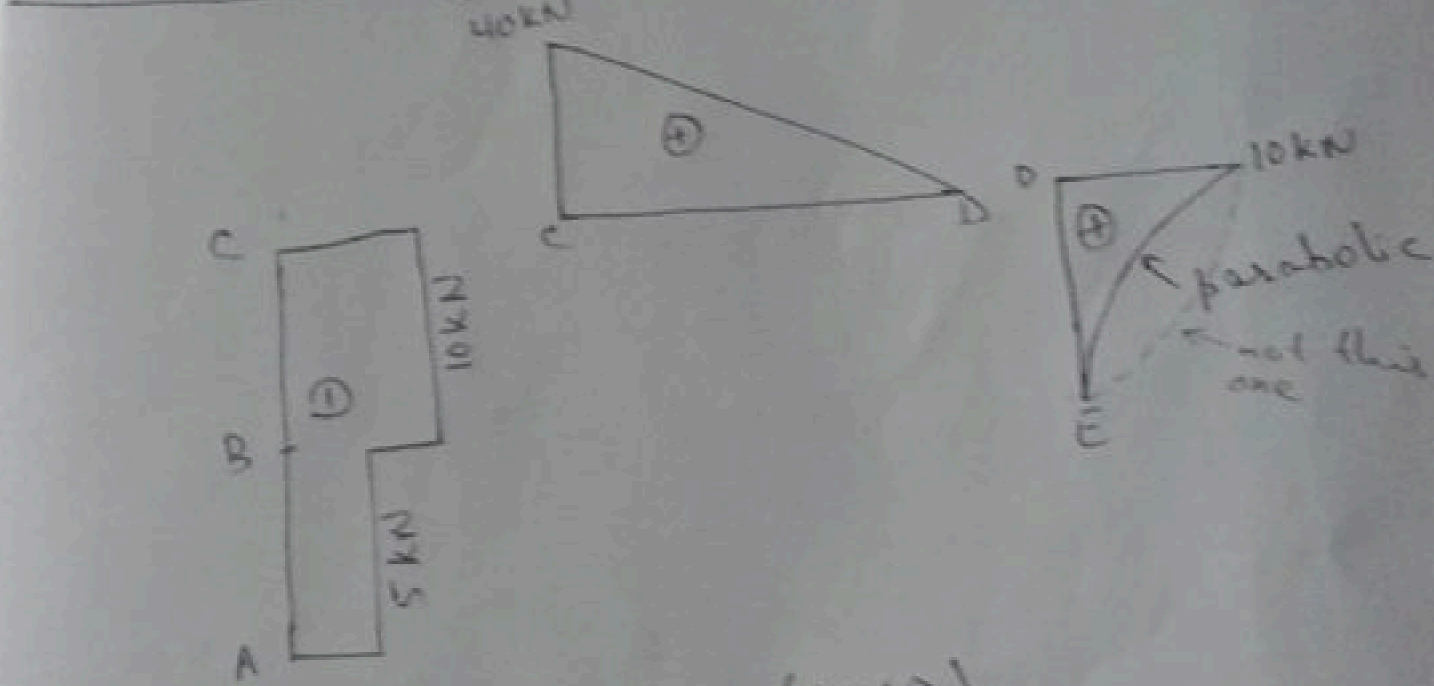
(E origin, $0 \leq X < 2m$)

2) D_{left}, $M = 0$

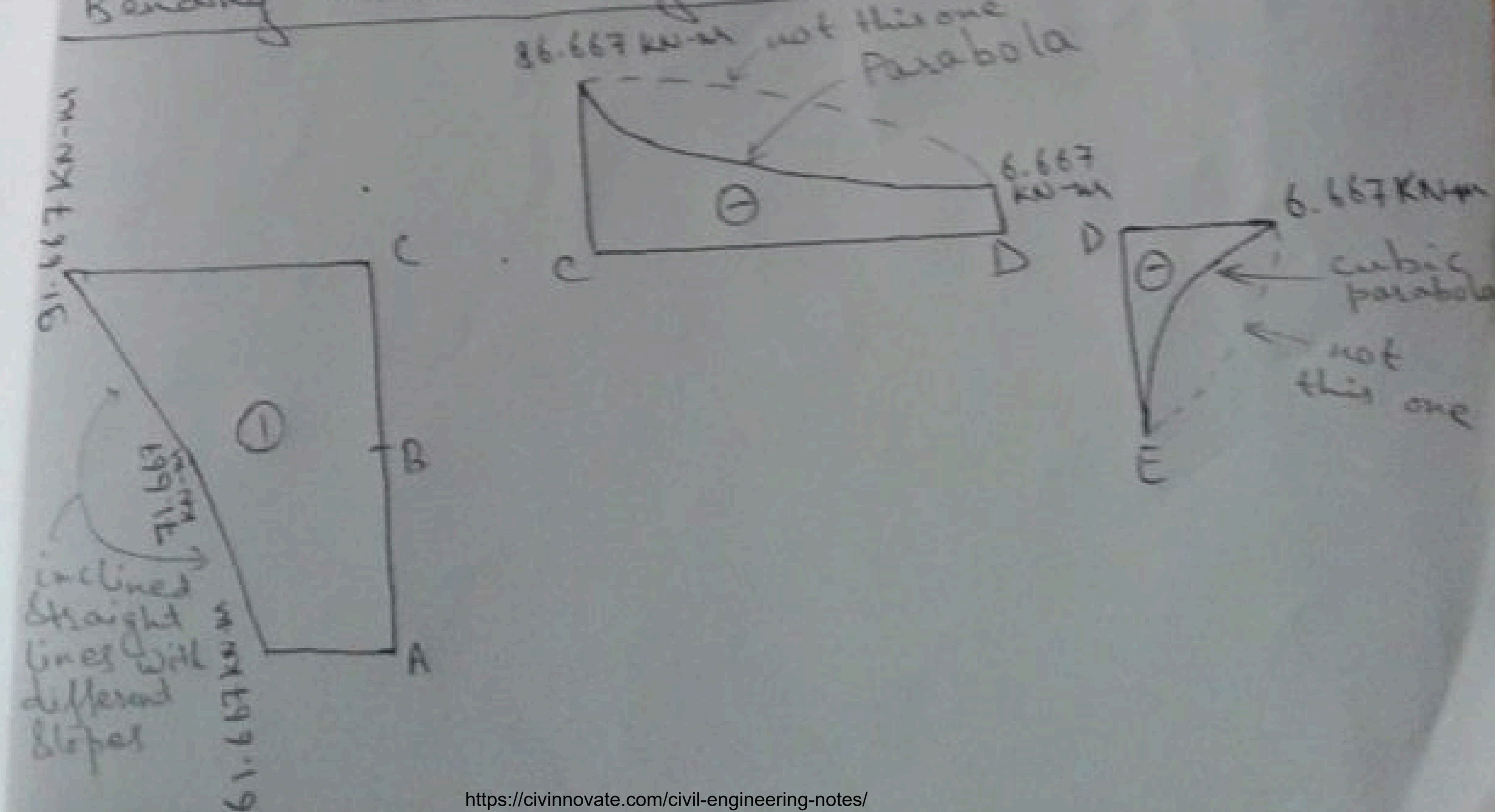
Axial Force Diagram (AFD)



Shear Force Diagram (SFD)



Bending Moment Diagram (BMD)



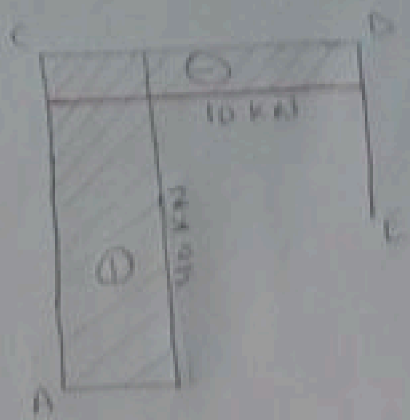
Note: The SFD for member DE is like the one shown by the solid line and not the dotted line. This is because slope of SFD at any point on a member is equal to the intensity of the UDL or UVL at that point. For member DE value of UVL at end E is zero, so the slope of SFD at end E is also zero. At end D value of UVL is $-10 kN/m$, so the slope of SFD at end D is -10. Hence the SFD looks like the one shown by the solid line.

The BMD for member CD is the one shown by the solid line & not the dotted line. This is because slope of BMD at any point on a member is equal to the value of shear force at that point. For member CD value of shear force at end C is $+40 kN$ and at end D it's zero. So the slopes of BMD at C and D are $+40$ and zero respectively. Hence the BMD looks like the one shown by solid line. Similarly, the BMD for member DE is the one shown by the solid line and not the dotted line. In this case slope of BMD at end E is zero and at end D is $+10$ because shear force at these two ends are zero and $+10$ respectively.

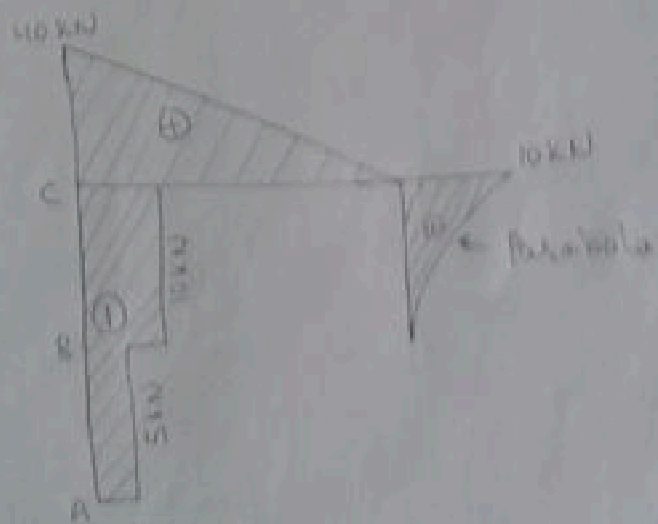
Note 2: If you are confused by the method described above, you can use the equations for Shear Force and Bending Moment to get their values at mid-point (if possible one or two more points besides the mid-point), with the values known at the ends and mid-point (and maybe some other points) you can draw the shape of the parabola.

In the frame above draw the AFD, SFD, and BMD ~~taken~~ as shown below:

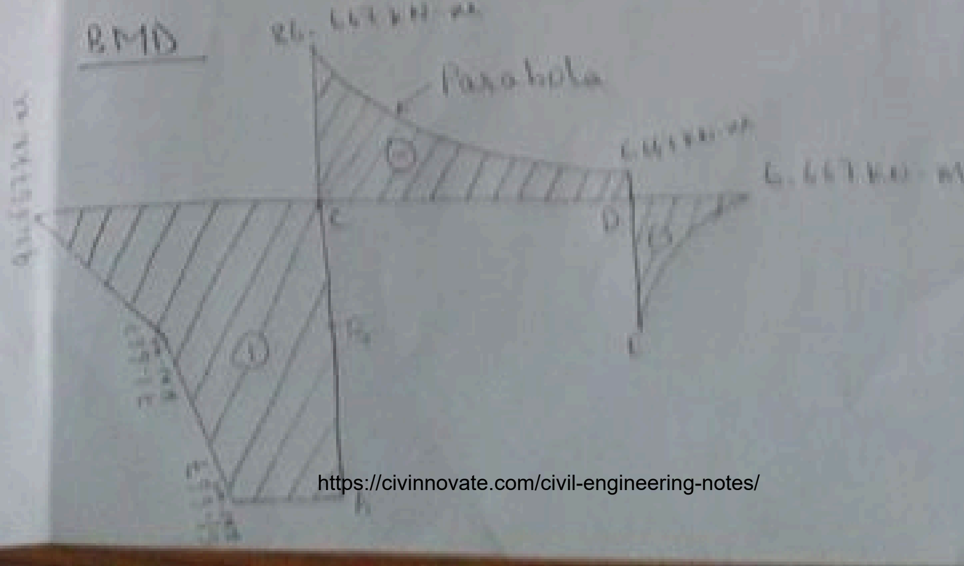
AFD



SFD



BMD



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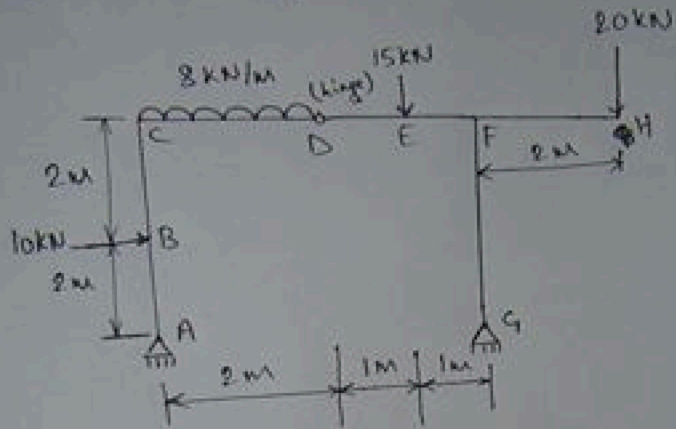
\# Draw the axial force, shear force and bending moment diagram of the given frame. Also show the salient features.



$(\circlearrowleft +)\ \Sigma M_A = 0$

or, $4G_y - 10*2 - (8*2)*1 - 15*3 - 20*6 = 0$

$\Rightarrow G_y = 50.25\ kN\ (\uparrow)$

$(+\uparrow)\ \Sigma F_y = 0$

or, $A_y + 50.25 - 8*2 - 15 - 20 = 0$

$\Rightarrow A_y = 0.75\ kN\ (\uparrow)$

$(\circlearrowleft +)\ \Sigma M_D^{ACD} = 0$

or, $-0.75*2 + A_x*4 + 10*2 + (8*2)*1 = 0$

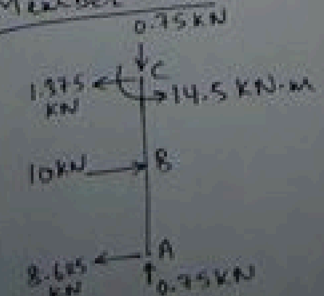
$\Rightarrow A_x = -8.625\ kN = 8.625\ kN\ (\leftarrow)$

$(\xrightarrow{+})\ \Sigma F_x = 0$

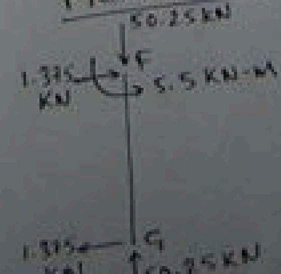
or, $G_x - 8.625 + 10 = 0$

$\Rightarrow G_x = -1.375\ kN = 1.375\ kN\ (\leftarrow)$

Member AC



Member FG



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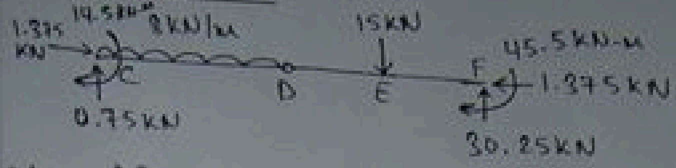
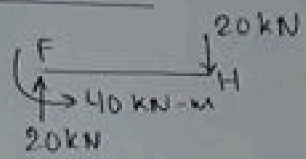
Member CF



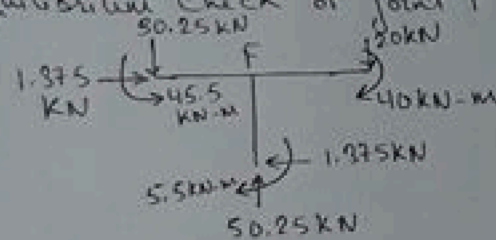
(a) $\Sigma M_D^{DF} = 0$ or, $30.25 * 2 - 15 * 1 + M_F^{CF} = 0$

$\Rightarrow M_F^{CF} = -45.5\,kN\text{-}m$

Member FH



Equilibrium check of joint F

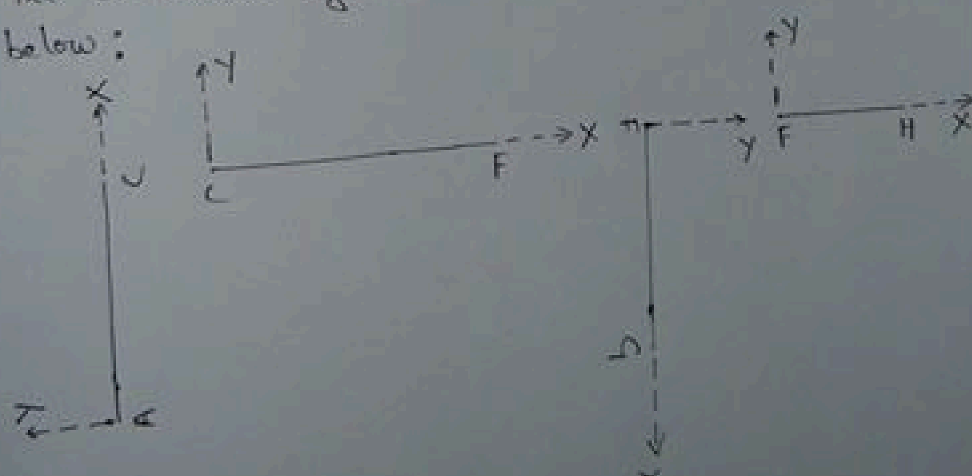


$\Sigma F_x = 0 \rightarrow$ check

$\Sigma F_y = 0 \rightarrow$ check

$\Sigma M_F = 0 \rightarrow$ check

The co-ordinate system for each member is shown below:



Axial Force

Member AC:

1) A_{left}, $Q = 0$
2) A_{Right} to C_{left}, $Q = -0.75\,KN$
3) C_{Right}, $Q = 0$

Member FG:

1) F_{left}, $Q = 0$
2) F_{Right} to G_{left}, $Q = -50.25\,KN$
3) G_{Right}, $Q = 0$

Member CF:

1) C_{left}, $Q = 0$
2) C_{Right} to F_{left}, $Q = -1.375\,KN$
3) F_{Right}, $Q = 0$

Member FH has no axial force.

Shear Force

Member AC:

1) A_{left}, $V = 0$
2) A_{Right} to B_{left}, $V = \cancel{5\,KN}\ 8.625\,KN$
3) B_{Right} to C_{left}, $V = 8.625 - 10 = -1.375\,KN$
4) C_{Right}, $V = 0$

Member FG:

1) F_{left}, $V = \cancel{1.375\,KN}\ 0$
2) F_{Right} to G_{left}, $V = 1.375\,KN$
3) G_{Right}, $V = 0$

Member CF:

1) C_{left}, $V = 0$
2) C_{Right} to D, $V = 0.75 - 8x$ (C origin, $0 \leq x \leq 2m$)
3) F_{Right}, $V = 0$
4) F_{left} to E_{Right}, $V = -30.25\,KN$
5) E_{left} to D, $V = -30.25 + 15 = -15.25\,KN$

Member FH:

1) F_{left}, $V = 0$
2) F_{Right} to H_{left}, $V = 20\ kN$
3) H_{Right}, $V = 0$

Bending Moment

Member AC:

1) A to B, $M = 8.625X$ (A origin, $0 \le X \le 2m$)
2) B to C_{left}, $M = 8.625X - 10(X-2)$
$= 20 - 1.375X$ (A origin $2m \le X \le 4m$)
3) C_{Right}, $M = 0$

Member FG:

1) G to F_{Right}, $M = -1.375X$ (G origin, $0 \le X \le 4m$)
2) F_{left}, $M = 0$

Member CF:

1) C_{left}, $M = 0$
2) C_{Right} to D, $M = 14.5 + 0.75X - 4X^2$ (C origin, $0 \le X \le 2m$)
3) F_{Right}, $M = 0$
4) F_{left} to E, $M = +30.25X - 45.5$ (F origin, $0 \le X \le 1m$)
5) E to D, $M = -45.5 + 30.25X - 15(X-1)$
$= -30.5 + 15.25X$ (F origin, $1 \le X \le 2m$)

Member FH:

1) H to F_{Right}, $M = -20X$ (H origin, $0 \le X \le 2m$)
2) F_{left}, $M = 0$

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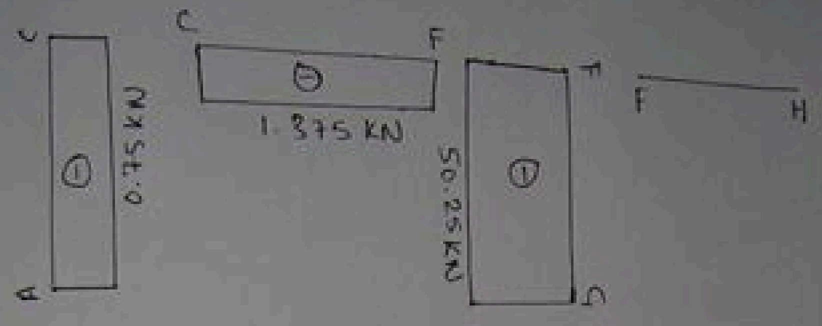
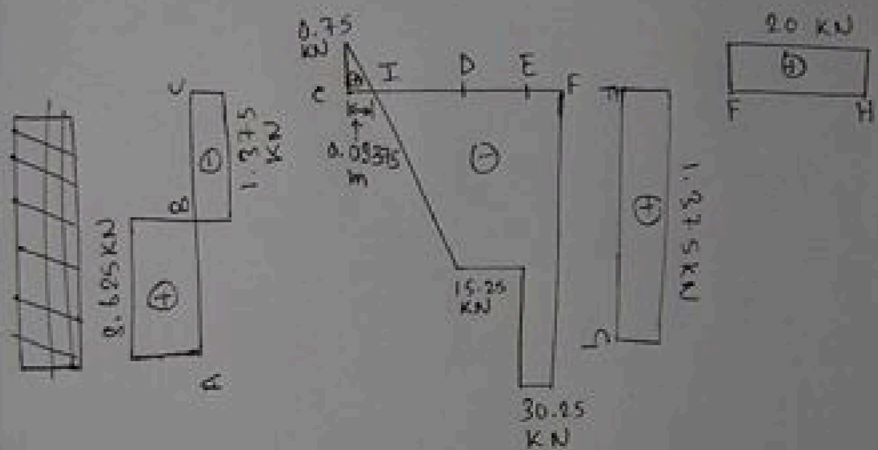
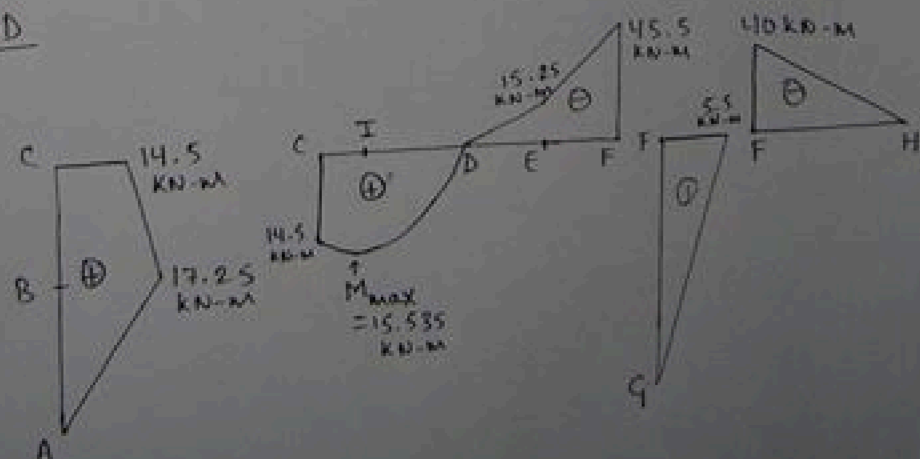
AFD


C
A
0.75 KN
C
F
1.375 KN
F
G
50.25 KN
F
H

SFD

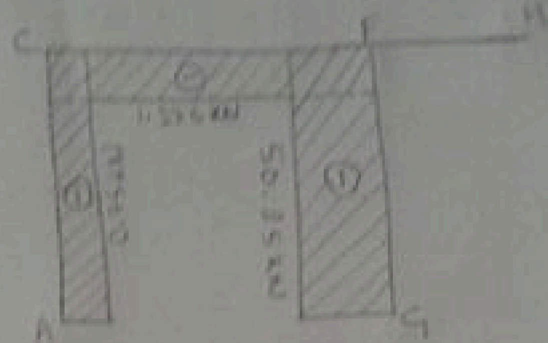

C
B
A
8.625 KN
1.375 KN
0.75 KN
C
I
D
E
F
0.09375 m
15.25 KN
30.25 KN
F
G
1.375 KN
20 KN
F
H

BMD

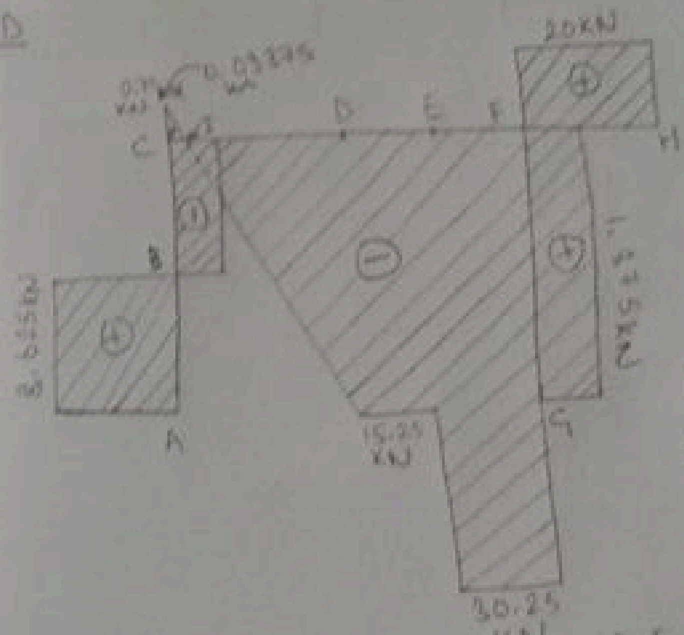

C
B
A
14.5 KN-M
17.25 KN-M
C
I
D
E
F
14.5 KN-M
M max =15.535 KN-M
15.25 KN-M
45.5 KN-M
F
G
5.5 KN-M
40 KN-M
F
H

In the Board Exam draw the AFD, SFD, and BMD on the entire frame as shown below:

AFD

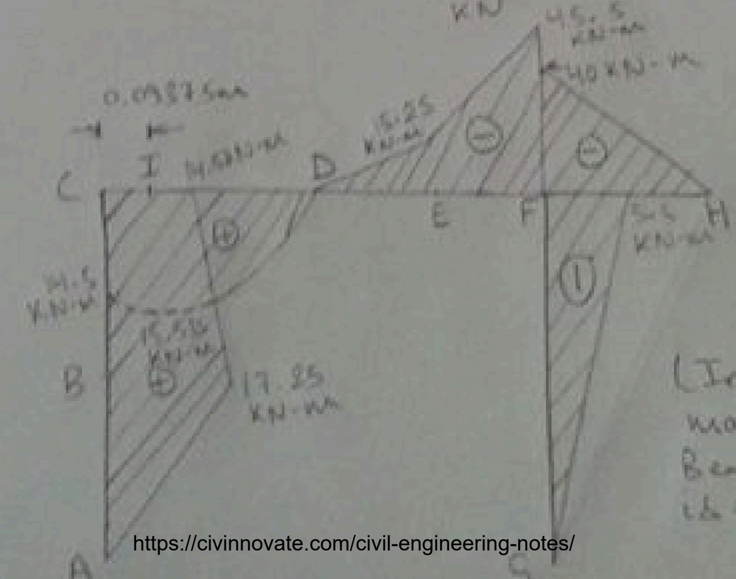


SFD



(In member CF Shear Force is zero at I i.e. 0.09375 m from C.)

BMD



(In member CF maximum +ve Bending moment is at I.)

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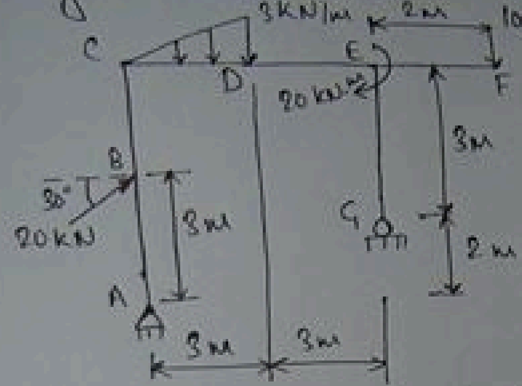
\# Calculate and draw the axial force, shear force, and bending moment diagram with its salient features for the given frame as shown in figure below.



$A_x = 20\cos30° = 17.32$ kN (←)

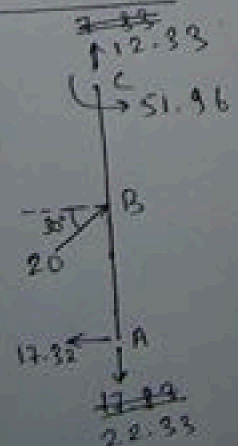
$(\circlearrowleft +)\ \Sigma M_A = 0$ or, $-(20\cos30)*3 - (\frac{1}{2}*3*3)*2 - 20 - 10*8 + 6G_y = 0$

$\Rightarrow G_y = 26.83$ kN (↑)

$(+\uparrow)\ \Sigma F_y = 0$ or, $26.83 - 10 - \frac{1}{2}*3*3 + 20\,\cancel{\cos}\sin30 + A_y = 0$

or, $A_y = \frac{\cancel{17.83}}{-22.33}$ kN $\Rightarrow A_y = \frac{\cancel{17.83}}{22.33}$ kN (↓)

Member AC



$(+\rightarrow)\ \Sigma F_x = 0$

or, $-17.32 + 2\cos30° + C_x^{AC} = 0$

$\Rightarrow C_x^{AC} = 0$

$(+\uparrow)\ \Sigma F_y = 0$

or, $-\frac{22.33}{\cancel{17.83}} + 20\sin30° + C_y^{AC} = 0$

$\Rightarrow C_y^{AC} = \cancel{7.33\ kN}\ 12.33$ kN

$(\circlearrowleft +)\ \Sigma M_C = 0$

or, $-17.32 * 5 + (20\cos30°)*2 + M_C^{AC} = 0$

$\Rightarrow M_C^{AC} = 51.96$ kN-m (↺)


Everything was impossible
until someone did it...

JAY NEPAL

जय नेपाल

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