Civinnovate

Discover, Learn, and Innovate in Civil Engineering

FM

Mechanics
(study of motion)

Kinematics — study of motion without the consideration of Basic cause of motion i.e. force.

Dynamics — study of motion with the consideration of basic cause of motion i.e. force.

$$\vec{v} = \frac{d\vec{s}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{J} = \frac{d\vec{a}}{dt} \quad \text{(jerk)}$$

Newton's Second low.

$$\vec{F_{ext}} = \frac{d}{dt}(m\vec{v}) \quad \text{Mass}$$

$$\vec{F_{ext}} = m \cdot \frac{d\vec{v}}{dt}$$

$$= m \cdot a$$

where a – acceleration (pure kinematic quantity)

Dynamic viscosity (μ) – $N \cdot s/m^2$

involving mass quanity – Dynamic.

Kinematic viscosity (ν) – m^2/sec

(no mass term. Kinematic).

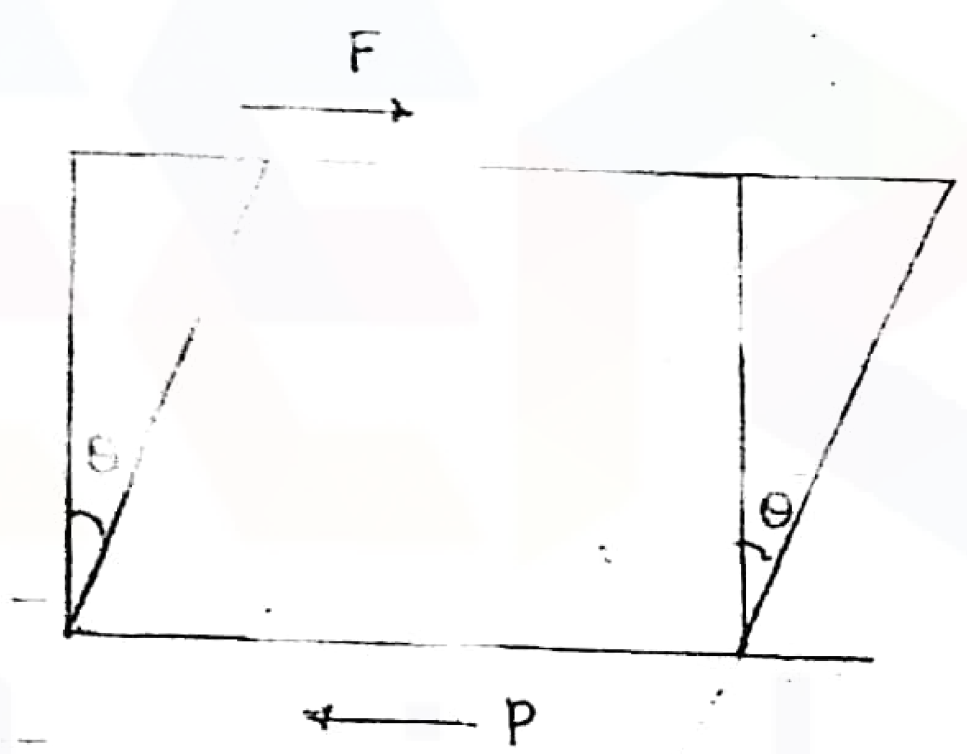
Fluid :

"liquids and gases both are having a property of continuous deformation under the action of shear or tangential forces whereas this property is not inhereted in solids. This property of the continuous deformation is known as flow property. and just because of having this flow property. liquids and gases both are kept in different category which is far away from solids, known as Fluid."

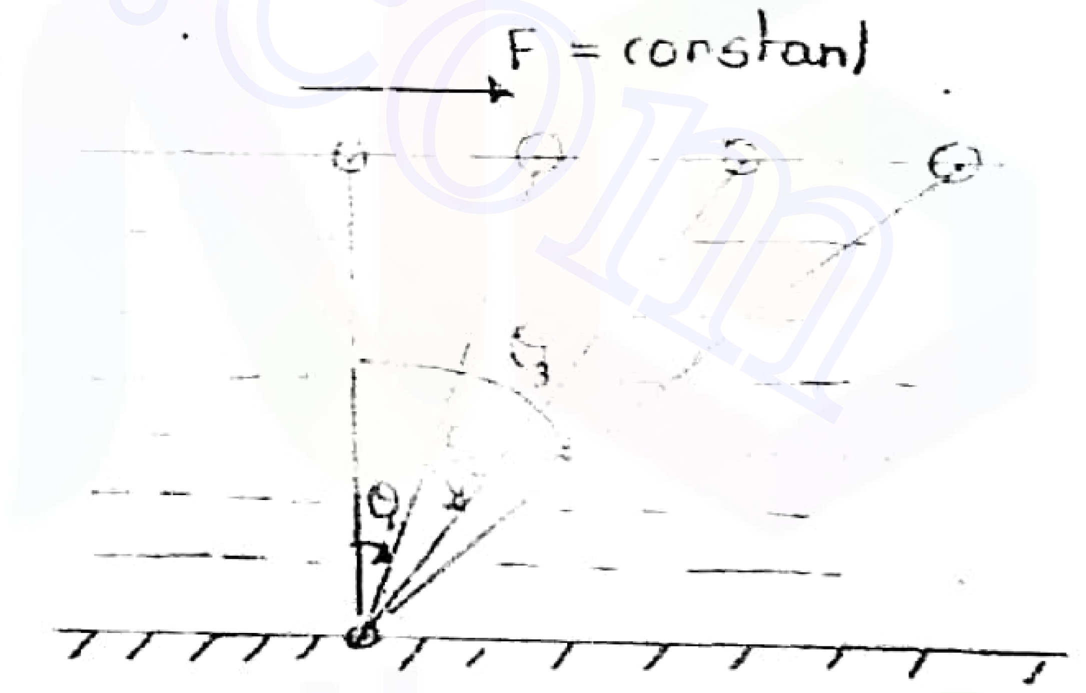
"A Fluid is substance which is having an ability to flow under the action of shear or tangential forces."

Matter – solids, liquids, gases

liquids, gases] Fluids



Solids – fixed deformation (θ)



liquids / gases – continuous deformation under action of const. force – flow

Fluid as Continuum :

"In macrosystems, the intermolecular distances in liquids and gases can be treated as negligible, as compared to the dimensions of the system. Therefore, the entire fluid mass system can be treated as the continuous distribution of mass and this continuous mass of fluid is known as Continuum."

Basic properties of Fluid :

1. Density (ρ) :

"It is defined as mass per unit volume of the substance."

$$\rho = \frac{m}{\forall} \qquad \forall - \text{volume}$$

MKS unit : kg/m^3

CGS unit : g/cm^3 or g/cc

$$1 \, g/cc = \frac{10^{-3} \, kg}{10^{-6} \, m^3} = 10^3 \, kg/m^3$$

Density of water (ρ) = $1000 \, kg/m^3$

Density of air (at atmospheric conditions) = $1.21 \, kg/m^3$

2. Specific weight :

"It is defined as the weight of the substance per unit volume."

$$\text{sp. weight} = \frac{m \cdot g}{\forall} = \rho \cdot g \quad N/m^3$$

3. Specific gravity :

"It is defined as ratio of density of fluid to the density of standard fluid."

$$(\text{Sp. gravity})_{\text{fluid}} = \frac{\text{Density of fluid}}{\text{Density of standard fluid}}$$

For liquids, standard fluid is water ($\rho = 1000 \, kg/m^3$)

For gases, standard fluid is atmospheric air ($\rho = 1.21 \, kg/m^3$)

Sometimes H_2 or CH_4 are taken as standard fluids for gases.

4. Relative density :

$$(R.D.)_{1/2} = \frac{\text{Density of } 1^{st} \text{ fluid}}{\text{Density of } 2^{nd} \text{ fluid}}$$

$$= \frac{\rho_1}{\rho_2}$$

Relative density of fluid is always defined w.r.t. the another fluid.

5. Compressibility :

$$\beta = \frac{-\left(\frac{d\forall}{\forall}\right)}{dP}$$

Mass, m = constant

$\rho \cdot \forall$ = constant

$$\rho \, d\forall + \forall \, d\rho = 0$$

$$\frac{-d\forall}{\forall} = \frac{d\rho}{\rho}$$

$$\beta = -\frac{1}{\rho}\frac{d\rho}{dP}$$

If density ρ is not changing w.r.t. P then,

$$\frac{d\rho}{dP} = 0 \qquad \beta = 0$$

∴ The fluids are called Incompressible

If density ρ is changing w.r.t. P then,

$$\frac{d\rho}{dP} \neq 0. \qquad \beta \neq 0$$

∴ The fluids are called Compressible.

(i) For liquids.
(practically compressible)

consider
Water (at 1 atm pressure) — 998 kg/m³
Water (at 100 atm pressure) — 1003 kg/m³

These are actual values of densities of water at normal temperature condition.

$$\Delta \rho = 5 \ kg/m^3$$

∴ % change in density $\approx \frac{5}{998} \times 100 \approx 0.5\%$

0.5% is very small thus considered zero i.e. $\beta = 0$

Just because, $\beta \to 0$ liquids are treated as the Incompressible fluids.

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For gases
(highly compressible)

Ideal gas equation

$$P\forall = mRT$$
$$P = \rho RT$$
$$\rho \propto P \quad \text{; at same temperature}$$

Mach number $(M_a) = \frac{V_{object}}{V_{sound}} = \frac{V_{object}}{C}$

If Mach number < 0.3, gases are treated as incompressible. for velocity of sound (= ... incompressible) ... Hertz

Note:
The reciprocal of compressibility is known as bulk modulus of elasticity (k)

$$k = \frac{1}{\beta}$$

compressibility is defined ... and then bulk modulus is defined using compressibility.

6. Isothermal compressibility of gas:

$$\beta = \frac{1}{\rho}\frac{d\rho}{dP}$$

from Ideal gas equation

$$P = \rho RT$$
$$\rho = \frac{P}{RT} \qquad \text{(T - constant for the Isothermal process)}$$
$$d\rho = \frac{1}{RT} \cdot dP$$
$$\frac{d\rho}{dP} = \frac{1}{RT}$$

$$\beta = \frac{1}{\rho} \times \frac{1}{RT}$$

$$= \frac{1}{P}$$ $\because \rho RT = P$

$$(\beta)_{Isothermal} = \frac{1}{P}$$

$$(K)_{Isothermal} = P.$$

7. Adiabatic compressibility of gas:

$$\beta = \frac{1}{\rho} \frac{d\rho}{dP}$$

Adiabatic equation.

$$P \forall^{\gamma} = \text{constant}.$$

$$P \cdot \frac{m^{\gamma}}{\rho^{\gamma}} = \text{constant}.$$

γ - adiabatic exponent.
For air (di-atomic)
$\gamma = 1.4$.

$$P \cdot \rho^{-\gamma} = \text{constant} \quad (\because m - \text{constant})$$

$$P(-\gamma)\rho^{-\gamma-1} d\rho + \rho^{-\gamma} dP = 0$$

$$\frac{d\rho}{dP} = \frac{\rho}{\gamma \cdot P}$$

$$\beta = \frac{1}{\rho} \cdot \frac{\rho}{\gamma \cdot P}$$

$$(\beta)_{Adiabatic} = \frac{1}{\gamma \cdot P}$$

$$(K)_{Adiabatic} = \gamma \cdot P.$$

Newton's failure:

velocity of sound.

in solids. $v = \sqrt{\frac{E}{\rho}}$

in liquids $c = \sqrt{\frac{K}{\rho}}$

in gases $c = \sqrt{\frac{P}{\rho}}$, which is wrong

$\because$ The atmospheric conditions on earth are adiabatic. the $(\gamma \cdot P)$ has to be used.

$\therefore$ Actual velocity of sound in gases is

$$c = \sqrt{\frac{\gamma \cdot P}{\rho}} \quad \text{(Laplace's correction)}$$

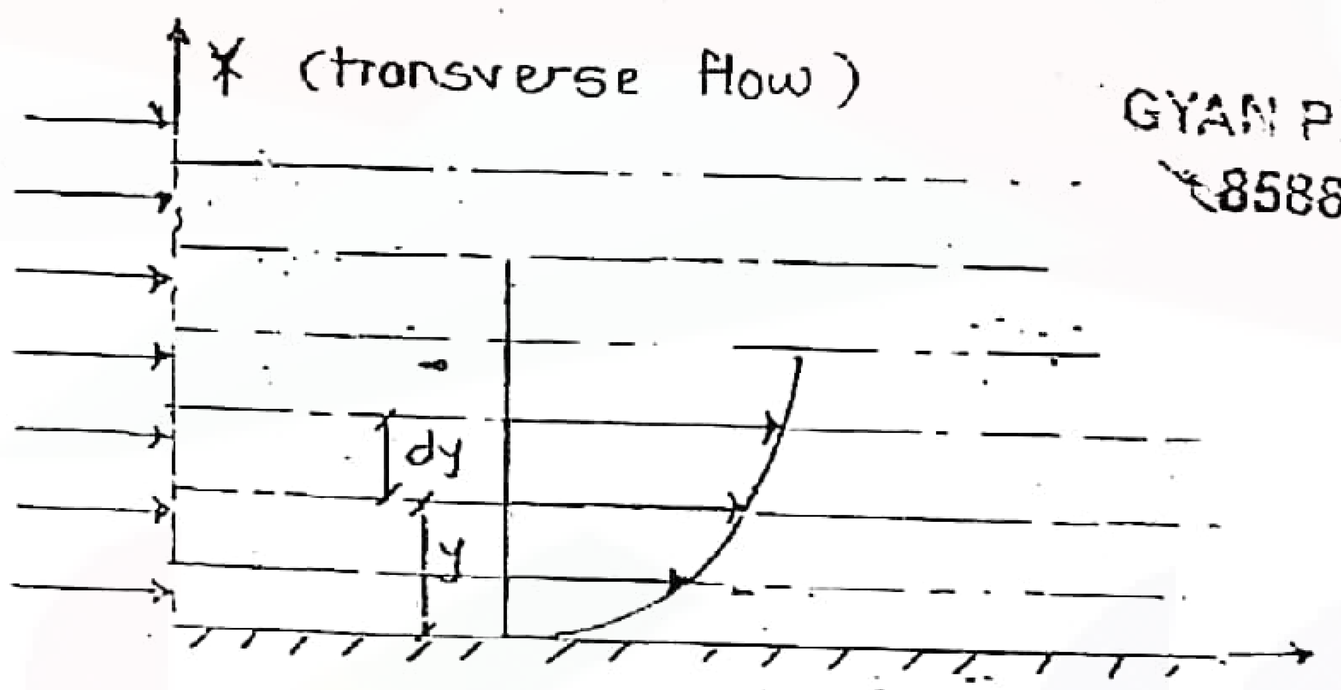
Viscosity :

"The two adjacent layers of a fluid resist the motion of each other. Such a fundamental property of fluid is known as Viscosity."

It is also known as the internal friction between the layers of the fluid.

Basic cause of viscosity :

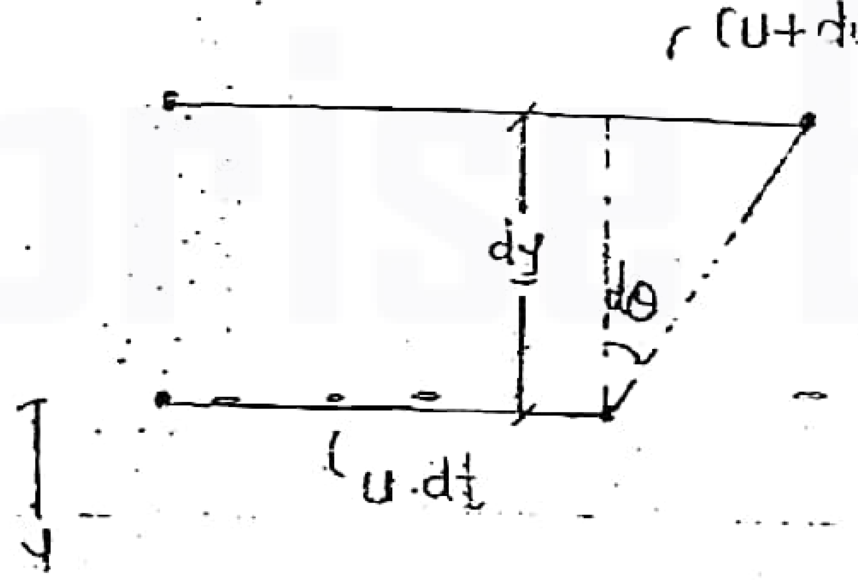
The basic cause of viscosity is the cohesive forces between the molecules of fluid i.e. cohesion.



The relative velocity of contacting layer with the surface is zero (No slip condition)

There will be the development of velocity gradient in transverse direction of flow.

velocity gradient $\left(\frac{\partial u}{\partial y}\right)$



$$\tan d\theta = \frac{du.dt}{dy}$$

$$d\theta = \frac{du.dt}{dy}$$

$d\theta$ - angular deformation (shear

$$\frac{d\theta}{dt} = \frac{du}{dy}$$

Rate of shear deformation.

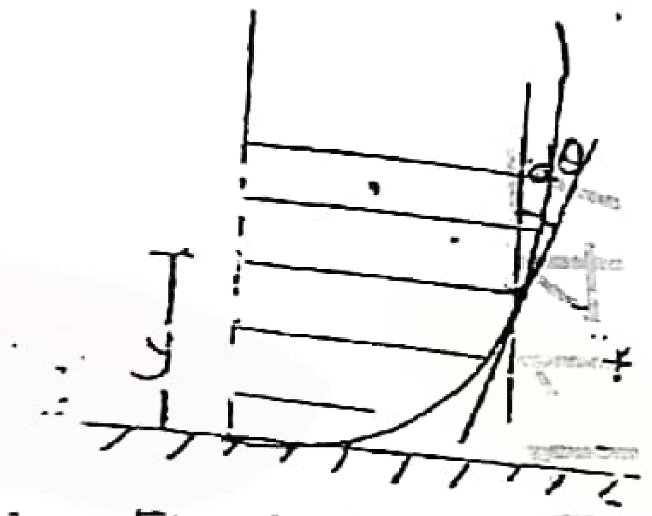


Fig. Velocity grad

Newton's law of viscosity :

The viscous shear stress between the two adjacent layers of fluid at a distance y from surface is

$$\tau \propto \frac{d\theta}{dt}$$ (only for Laminar flows)

$$\tau = \mu \frac{d\theta}{dt}$$

μ - constant (not universal) but it is property of fluid and it also depends on temperature.

$$\mu = \frac{\tau}{\left(\frac{d\theta}{dt}\right)}$$

If μ is high, $\frac{d\theta}{dt}$ is less i.e. flow is difficult

If μ is less, $\frac{d\theta}{dt}$ is high i.e. flow is easy.

It means,

μ is the direct measurement of internal resistance between the layers of fluids.

μ is called Dynamic viscosity.

Units of μ :

$$\mu = \frac{\tau}{\left(\frac{d\theta}{dt}\right)} = \frac{\tau'}{\left(\frac{du}{dy}\right)}$$

S.I. unit :

$$\frac{N\cdot s}{m^2} = Pa\cdot s.$$

$$1\ Pa.s = 1.\ \frac{N\cdot s}{m^2}$$

MKS unit :

$$1\ Pa\cdot s = 1.\ \frac{Ns}{m^2}$$

$$= 1\ \frac{kg\cdot m}{s^2}\cdot\frac{s}{m^2}$$

$$1\cdot Pa\cdot s = 1\ \frac{kg}{m\cdot s}$$

C.G.S. unit : (Poise) :

$$1\ Poise = 1\ [illegible]$$

$$\frac{10^{-3}\ kg}{10^{-2}\ m\cdot s}$$

$$= 0.1\ \frac{kg}{m\cdot s}$$

$$1\ Poise = 0.1\ Pa.s$$

Units of γ :

$$\gamma = \frac{\mu}{\rho}$$

M.K.S unit: m^2/s

CGS unit (stokes)

Effect of temperature on viscosity of fluid :

The main reason of viscosity in fluids is the cohesion. But in gases, the cohesion is almost nill. Thus, in gases, particles are in random motion (Brownian mo -given by Robert Brown)

$$(\mu)_{gas} < < < (\mu)_{liquids}$$

e.g.

$$\mu_{water} = 55\ \mu_{gas\ i.e.\ air.}$$

We know that.

$$\gamma_{gas} = \frac{\mu_{gas}}{\rho_{gas}}$$

kinematic viscosity of gas may be more than the dynamic viscosity, equal to dynamic viscosity or less than dynamic viscosity depending upon temperature and pressure conditions.

E.g. $\gamma_{air\,(1atm)} > \gamma_{water}$

Thus Absolute viscosity of gas is dynamic viscosity.

If temperature increases, thermal expansion takes place. thus, density of gas decreases

But ρ_{liq} decreases very slightly

& ρ_{gas} decrease very highly

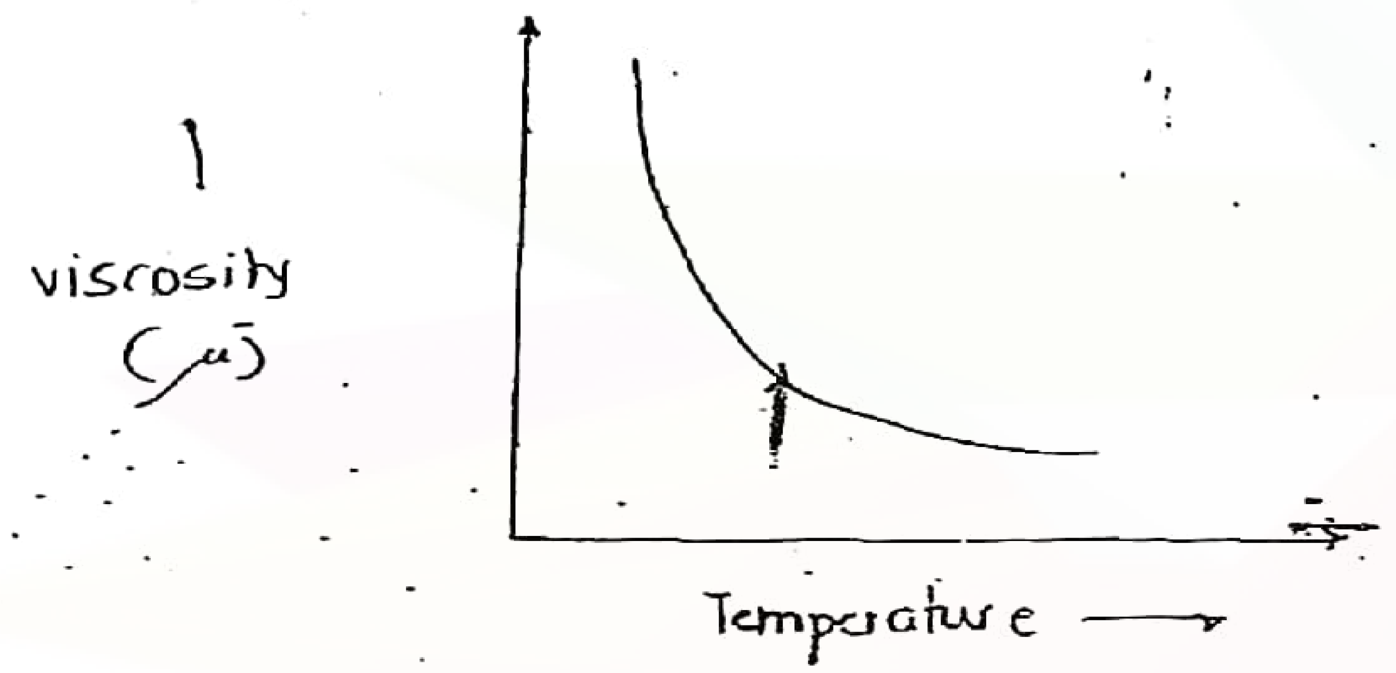
Becaus.

$$P = \rho RT.$$

$$\rho_{gas} = \frac{P}{RT}$$

For liquids:

If temperature increases cohesion decreases and dynamic viscosity also decreases



If temperature increases, kinematic viscosity also decreases but with very slight decrease.

because μ decreases

but $(\rho)_{liq}$ decreases very slightly

$$\nu_{liq} = \frac{\mu_{liq}}{\rho_{liq}}$$

Thus decrease in dynamic viscosity is more than the kinematic viscosity

For gases:

Cohesion in gases is almost zero

By Kinematic theory of gases.

$$\bar{C}_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\bar{C}_{rms} \propto \sqrt{T}$$

i.e. If temperature increases, root mean square velocity of a gas increases also.

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Thus, the randomness in the motion of gas molecules will increase. It will introduce some additional resistance in the path of fluid flow. (i.e. viscosity)

$$\nu_{gas} = \frac{\mu_{gas}}{\rho_{gas}}$$

Due to increase in temperature dynamic viscosity of gas (μ_{gas}) and kinematic viscosity of gas (ν_{gas}) both increases but increase in ν_{gas} is more as compared to μ_{gas}

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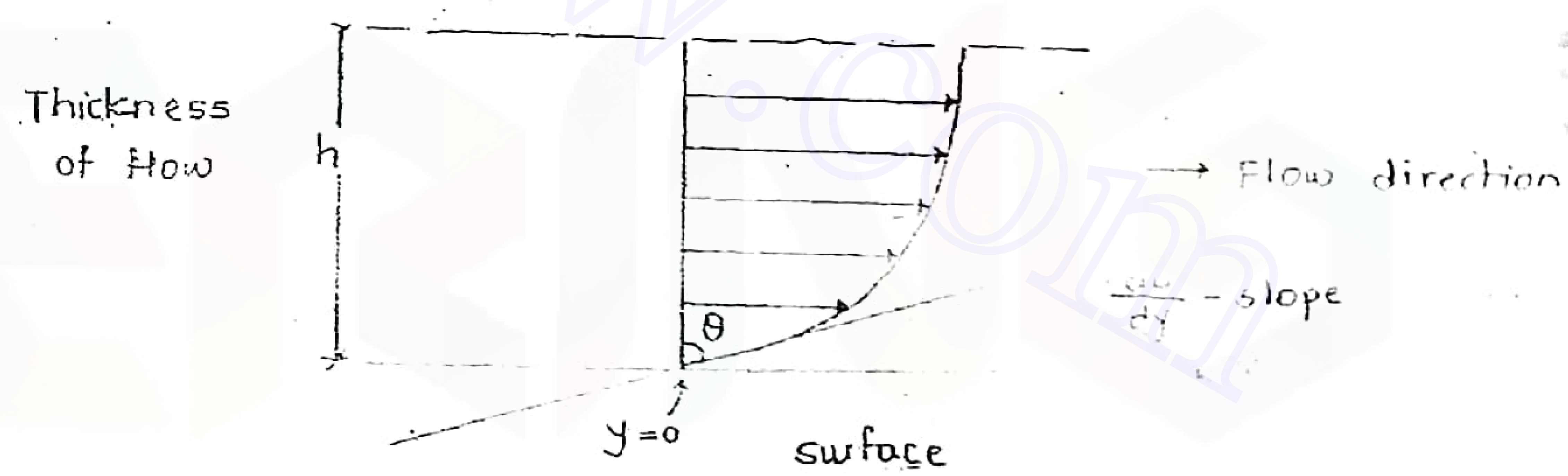
Linearisation of Newton's law of viscosity:



According to Newton's law of viscosity

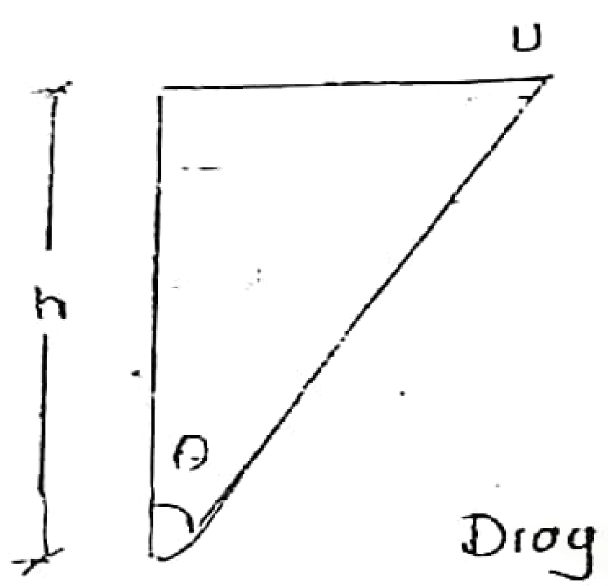
$$\tau = \mu \frac{du}{dy}$$

At surface.

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{at\ y=0}$$

To find $\left(\frac{du}{dy}\right)_{at\ y=0}$ (velocity profile is unknown)

If thickness of flow is very very small i.e. of the order of mm. Then velocity profile can be treated ...



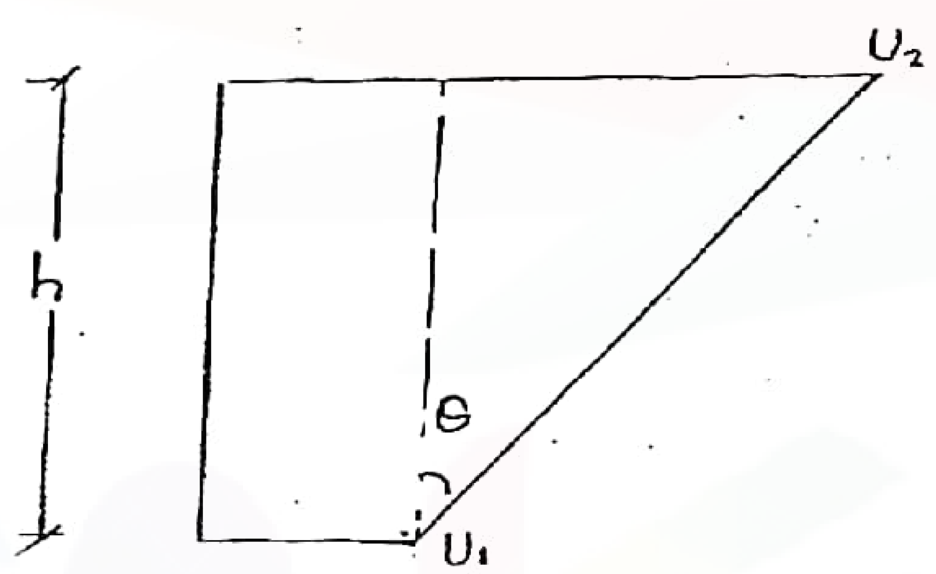
$$\tau_o = \mu\left[\frac{U-0}{h}\right]$$

$$\tau_o = \frac{\mu U}{h}$$

Drag force, $F_D = \tau_o A$

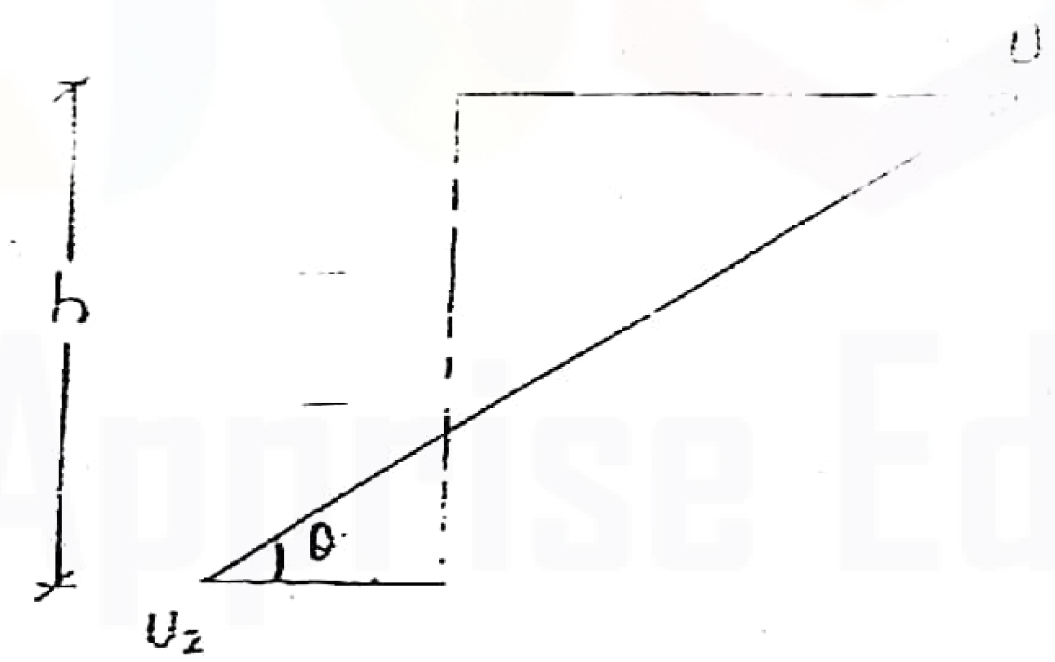
$$= \frac{\mu \cdot U}{h} \cdot A$$ (Drag force on the surface by fluid)

If plate placed on fluid is moving forward



$$\tau = \mu\left[\frac{U_2 - U_1}{h}\right]$$

If plate placed on fluid is moving in opposite direction to fluid flow



$$\tau = \mu\left[\frac{U_1 - (-U_2)}{h}\right]$$

Drag force is applied on the surface by fluid while the viscous force is force between the layers of fluids.

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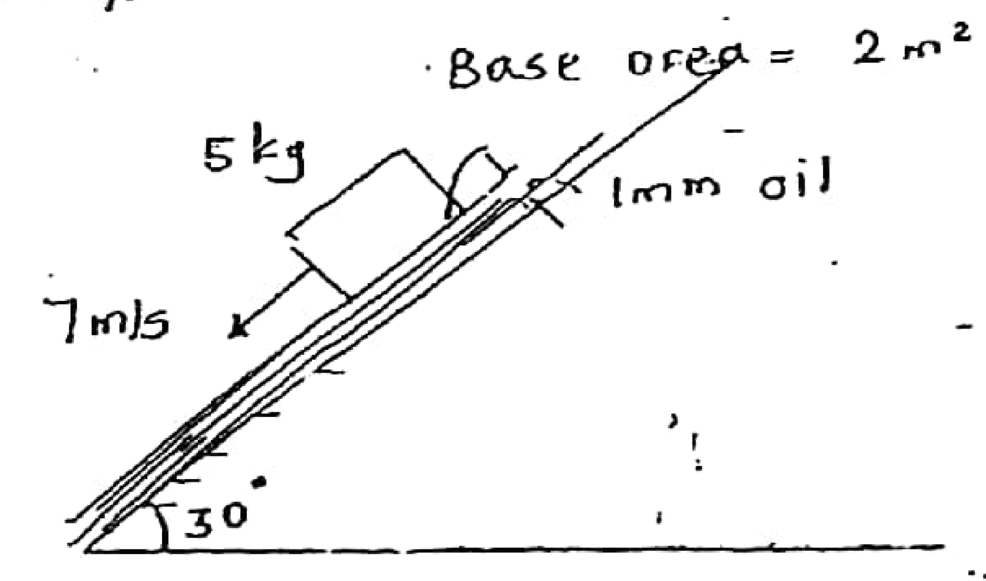
Q. Find the viscosity.

2 Marks



As velocity is constant, net force is zero.

$$F = 0$$

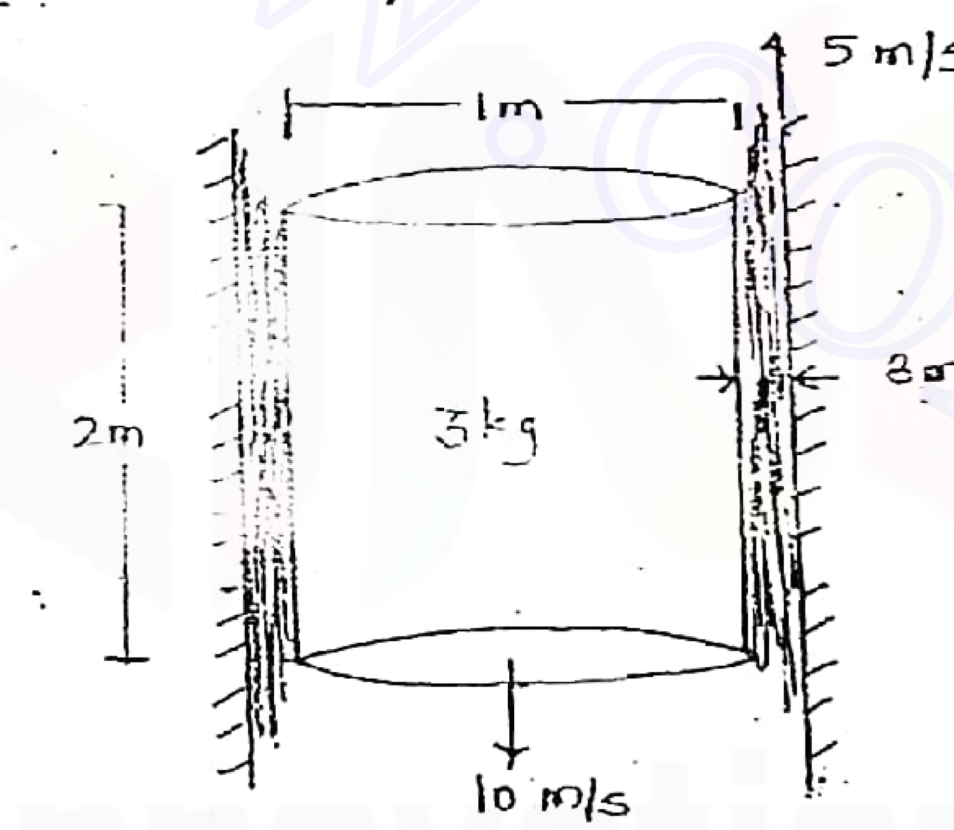
$$mg \sin\theta = F_D$$

$$5 \times 9.81 \times \sin 30^\circ = \mu\left(\frac{7-0}{1\times10^{-3}}\right) \cdot 2$$

$$\mu = 1.75 \times 10^{-3}\ \mathrm{Ns/m^2}$$

Q. Find dynamic viscosity of oil.

2 Marks



As V is constant.

$$F = 0$$

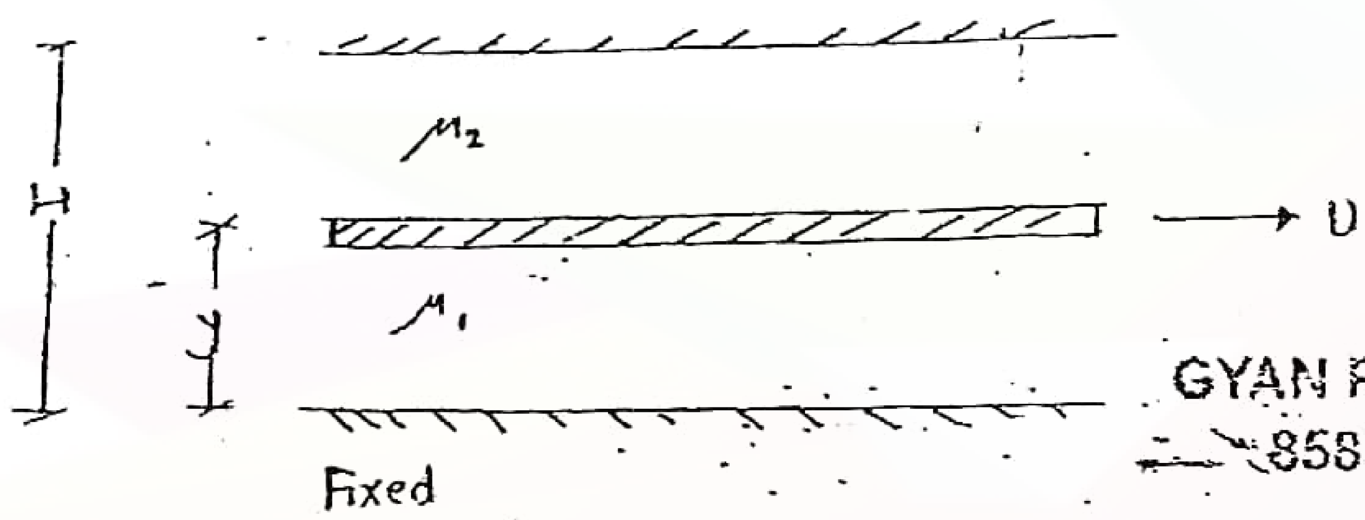
$$mg = F_D$$

$$3 \times 9.81 = \mu\left[\frac{10-(-5)}{3\times10^{-3}}\right] \times (2\pi \times 1 \times 2)$$

$$\mu = 4.68 \times 10^{-4}\ \mathrm{Ns/m^2}$$

Q. Find (i) y such that drag on moving plate by both fluids is same
(ii) y such that total drag is minimum.

10 Marks



(i) Drag on both sides is same

$$F_{D_1} = F_{D_2}$$

$$\mu_1\left[\frac{U-0}{y}\right]\cdot A = \left[\frac{U-0}{H-y}\right]\cdot \mu_2 \cdot A$$

$$\frac{\mu_1}{y} = \frac{\mu_2}{H-y}$$

$$y = \left(\frac{\mu_1}{\mu_1+\mu_2}\right)H$$

(ii) For minimum drag.

$$F_D = F_{D_1} + F_{D_2}$$

$$= \frac{\mu_1 UA}{y} + \frac{\mu_2 UA}{H-y}$$

$$F_D = UA\left[\frac{\mu_1}{y} + \frac{\mu_2}{H-y}\right]$$

for drag to be minimum,

$$(F_D)_{min} = \frac{dF_D}{dy} = 0$$

$$UA\left[-\frac{\mu_1}{y^2} + \frac{\mu_2}{(H-y)^2}\right] = 0$$

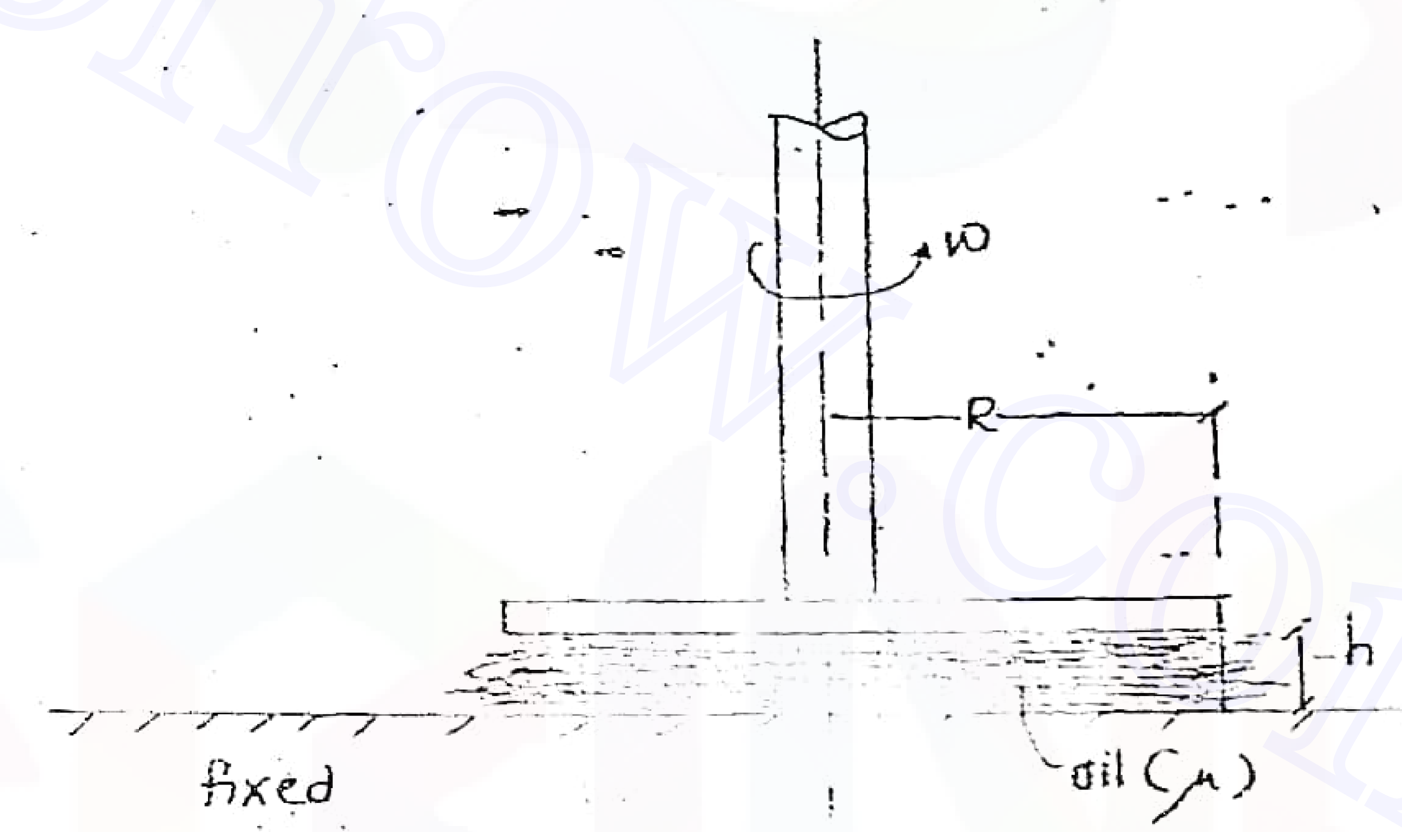
$$\frac{\mu_1}{y^2} = \frac{\mu_2}{(H-y)^2}$$

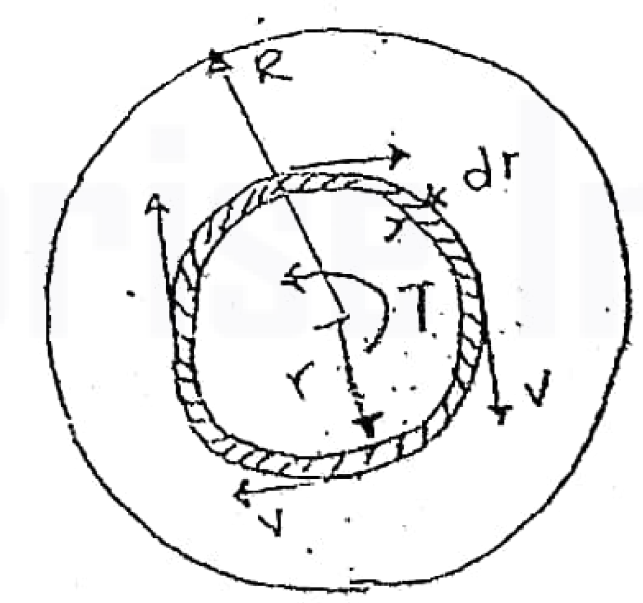
$$\frac{\sqrt{\mu_1}}{y} = \frac{\sqrt{\mu_2}}{(H-y)}$$

$$y = \left(\frac{\sqrt{\mu_1}}{\sqrt{\mu_1}+\sqrt{\mu_2}}\right)H.$$

Q. For the disc rotating with angular velocity ω, Find
(i) Total drag force
(ii) External torque required to maintain the ω const.

20 Marks





(i) Differential drag force.

$$dF_D = \mu\left[\frac{r\omega - 0}{h}\right]\cdot 2\pi r\, dr$$

$$= \frac{2\pi\mu\omega}{h}\cdot r^2\, dr.$$

Total drag $= F_D$

$$= \int dF_D$$

$$= \frac{2\pi\mu\omega}{h}\int^{R} r^2\, dr$$

(ii) Resistive torque

$$d\tau_D = dF_D \cdot r$$

$$= \frac{2\pi\mu\omega}{h} \cdot r^3 \cdot dr$$

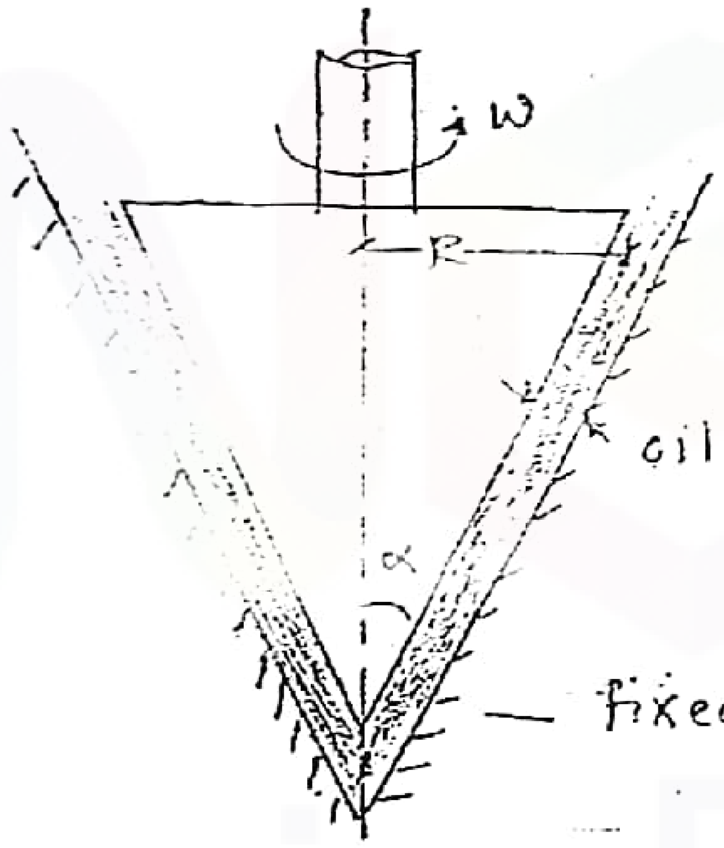
$$\tau_D = \frac{2\pi\mu\omega}{h} \int_0^R r^3 \, dr$$

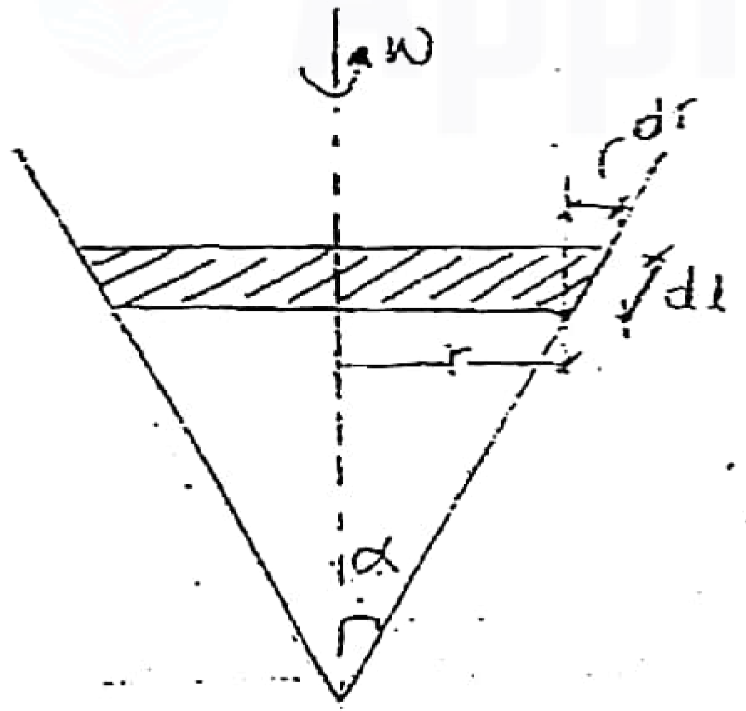
$$\tau_D = \frac{\pi\mu\omega R^4}{2h}$$

Thus the external torque required to maintain the disc in angular velocity ω is of magnitude τ_D and opposite direction to that of τ_D.

Q. Find (i) Total drage force and (ii) External torque required to maintain the ω. for

25 Marks





consider an element with radius r and slant height dl as shown in fig.

$$\sin\alpha = \frac{dr}{dl}$$

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(i) Differential drag force:

$$dF_D = \mu \left[\frac{r\omega - 0}{h}\right] \frac{2\pi(r + r + dr)}{2} \cdot dl$$

$$= \mu \cdot \left[\frac{r\omega\pi}{h}\right] \cdot 2r \cdot dl \quad \text{neglect } dr \cdot dl$$

$$= \frac{\mu\omega\pi}{h} \cdot 2r^2 \cdot \frac{dr}{\sin\alpha}$$

$$F_D = \int_0^R \frac{2\mu\omega\pi}{h \cdot \sin\alpha} \cdot r^2 \cdot dr$$

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$$= \frac{2\mu\omega\pi}{h \cdot \sin\alpha} \left[\frac{R^3}{3}\right]$$

Total drag. $F_D = \dfrac{2\mu\omega\pi R^3}{3\, h \cdot \sin\alpha}$

(ii) Resistive torque

$$d\tau_D = dF_D \cdot r$$

$$= \frac{\mu\omega\pi}{h} \cdot \frac{2r^2 \cdot dr}{\sin\alpha} \cdot r$$

$$= \frac{2\mu\omega\pi}{h \cdot \sin\alpha} \cdot r^3 \cdot dr$$

$$\tau_D = \int_0^R \frac{2\mu\omega\pi}{h \cdot \sin\alpha} \cdot r^3 \cdot dr$$

$$= \frac{2\mu\omega\pi}{h \cdot \sin\alpha} \left[\frac{R^4}{4}\right]$$

$$\tau_D = \frac{\mu\omega\pi R^4}{2h \cdot \sin\alpha}$$

The external torque required to maintain the cone in angular velocity is of magnitude τ_D and in direction opposite to that of τ_D

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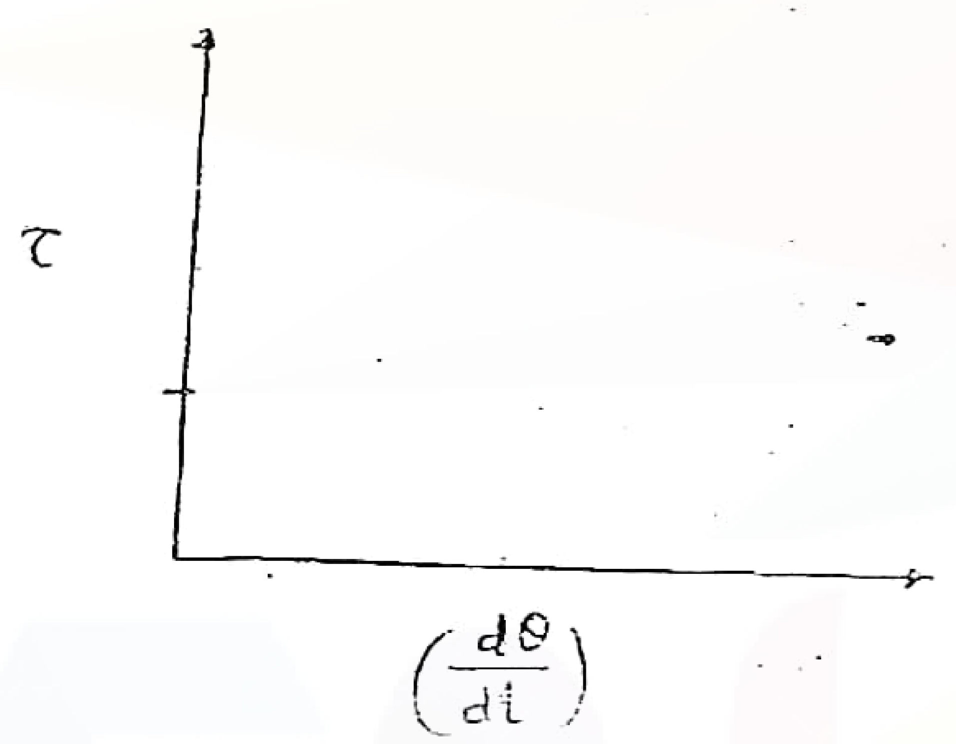
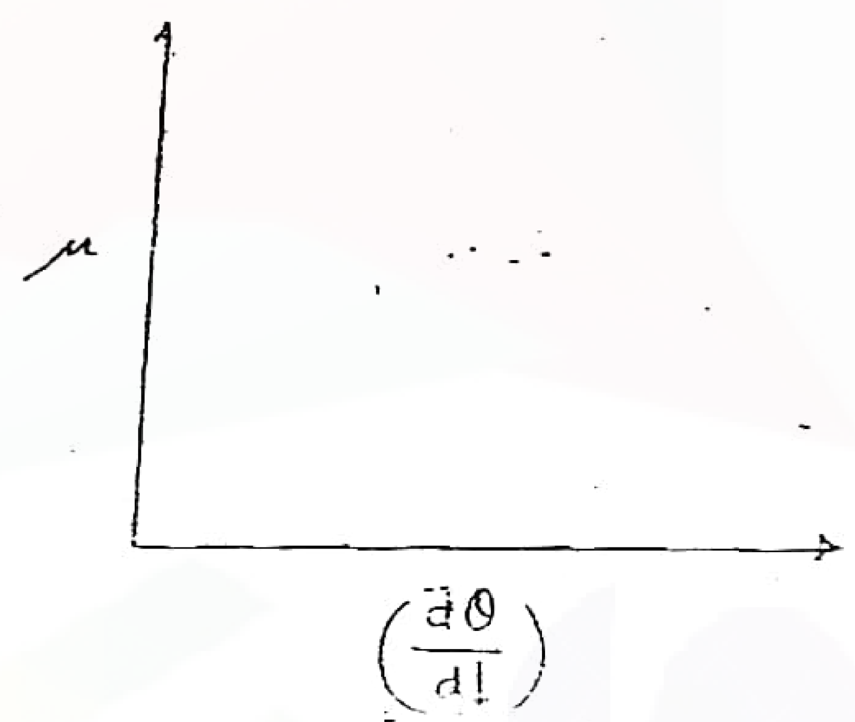
Rheology:
"It is the branch of science which deals with the studies of different types of fluid behaviour." Rehio - Fluid

1. Ideal fluids:

Ideal fluids are the one which have.

- zero viscosity (Inviscid)
- Incompressible ($\rho = 0$)
- zero surface tension effects.





(Ideal fluids have [illegible] independent on flow velocity)

2. Newtonian fluid:

They follow the Newton's law of viscosity. They are the fluids in which viscosity does not depend upon rate of shear deformation.

$$\tau = \mu \cdot \frac{d\theta}{dt}$$

μ - is constant

e.g. Water: (spreads on surface & shear stress increases)
air, petrol, oil

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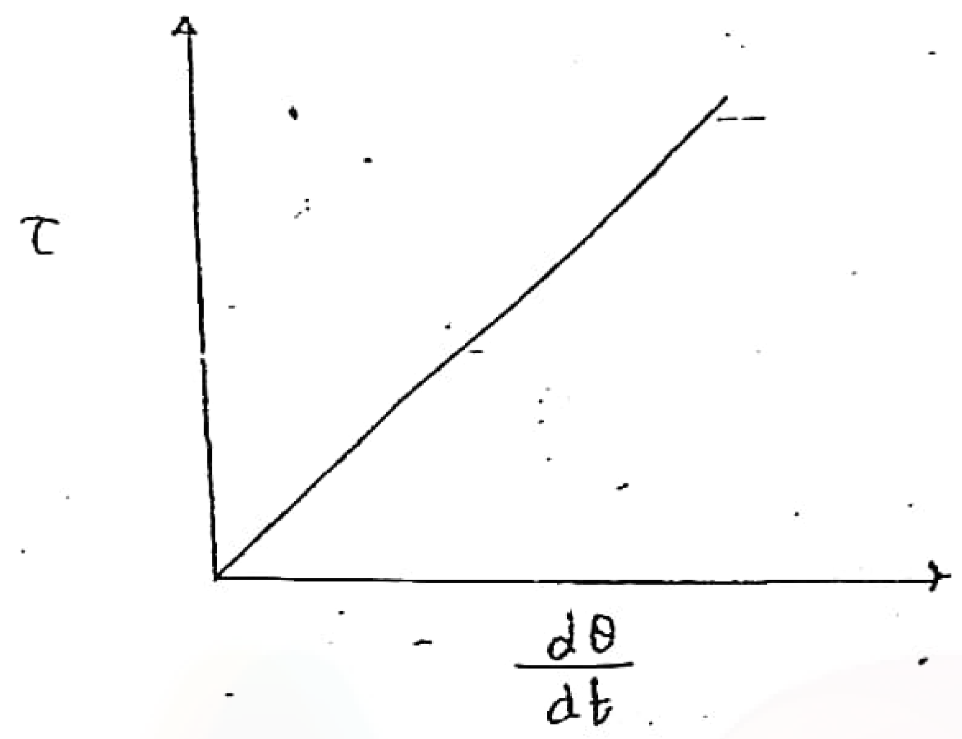
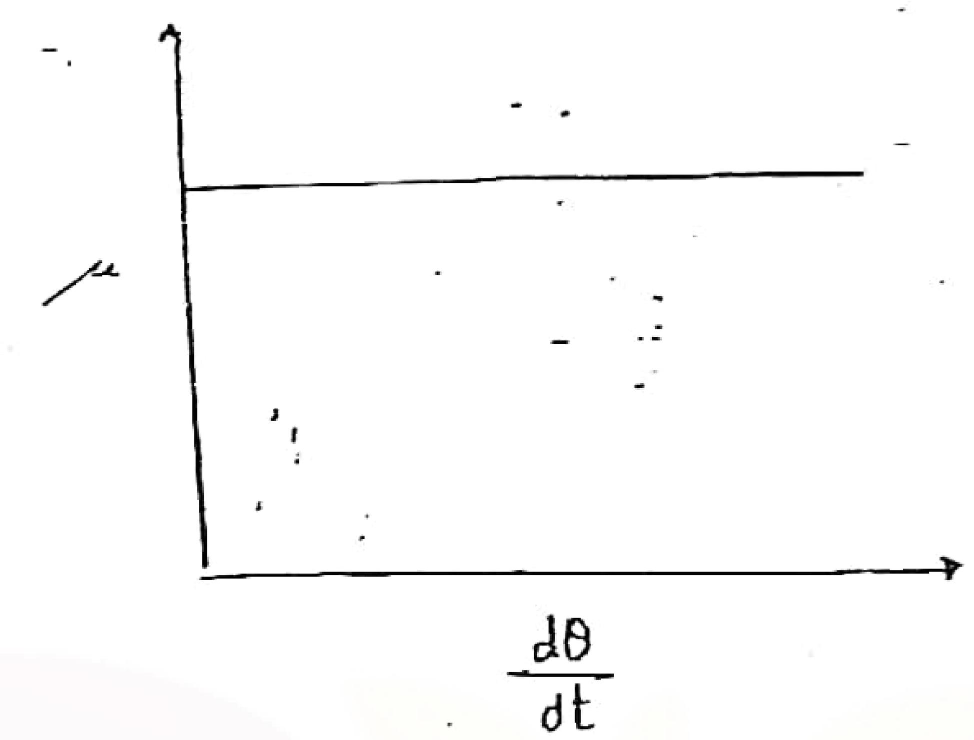




3. Non-Newtonian fluids:

The fluids which don't follow Newton's law of viscosity.

$$\tau = A \left(\frac{d\theta}{dt}\right)^n$$ - non linear relation
- $n > 0$ but $n \neq 1$

A is constant

The value of n may be $n > 1$ or $n < 1$.

$$\tau = A \left(\frac{d\theta}{dt}\right)^{n-1} \frac{d\theta}{dt}$$

where $A \cdot \left(\frac{d\theta}{dt}\right)^{n-1}$ Apparent viscosity (μ_{app})

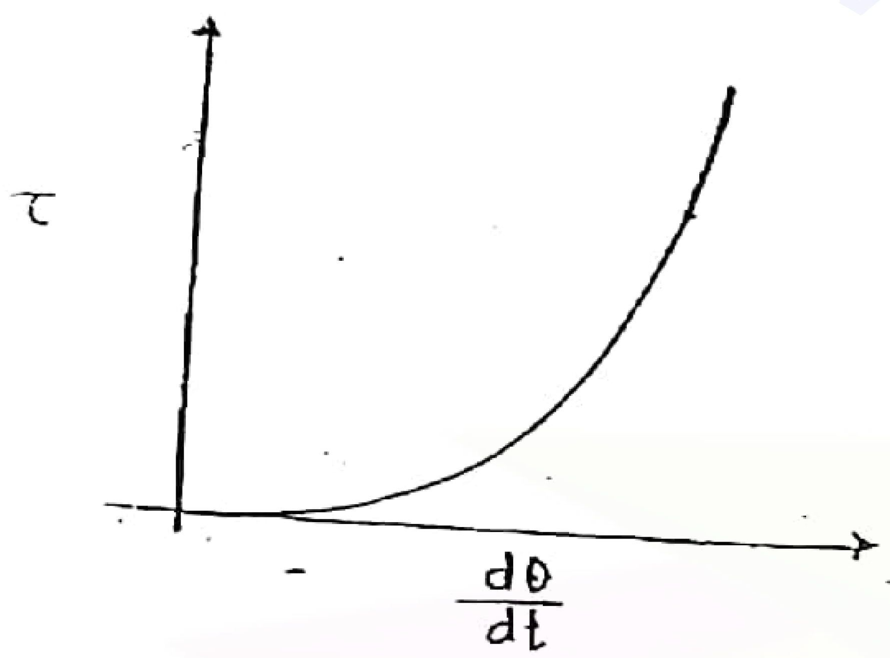
The apparant viscosity of Non-newtonian fluids depends on rate of shear deformation.

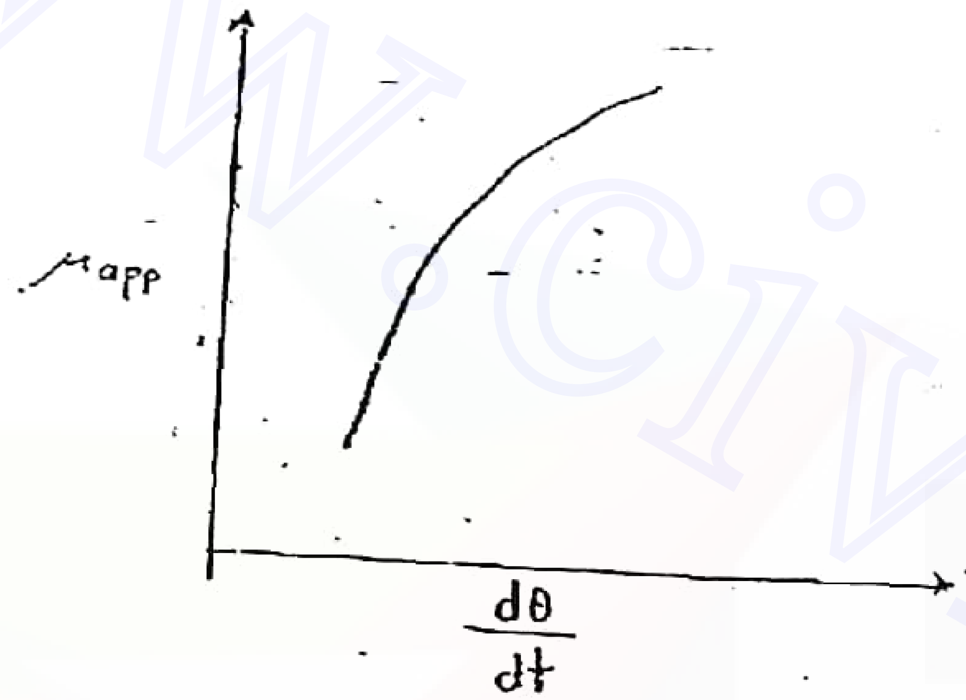
It may be of two types

(i) Di-latent fluids.
($n > 1$)

$\mu_{app.}$ increases with rate of shear deformation.

They are also called 'Shear thickning fluid'.



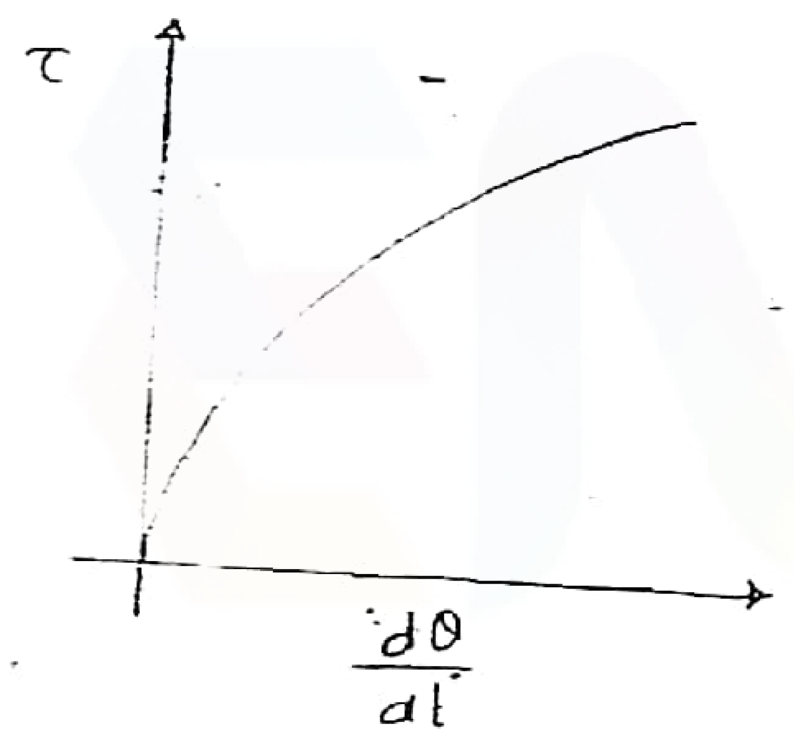


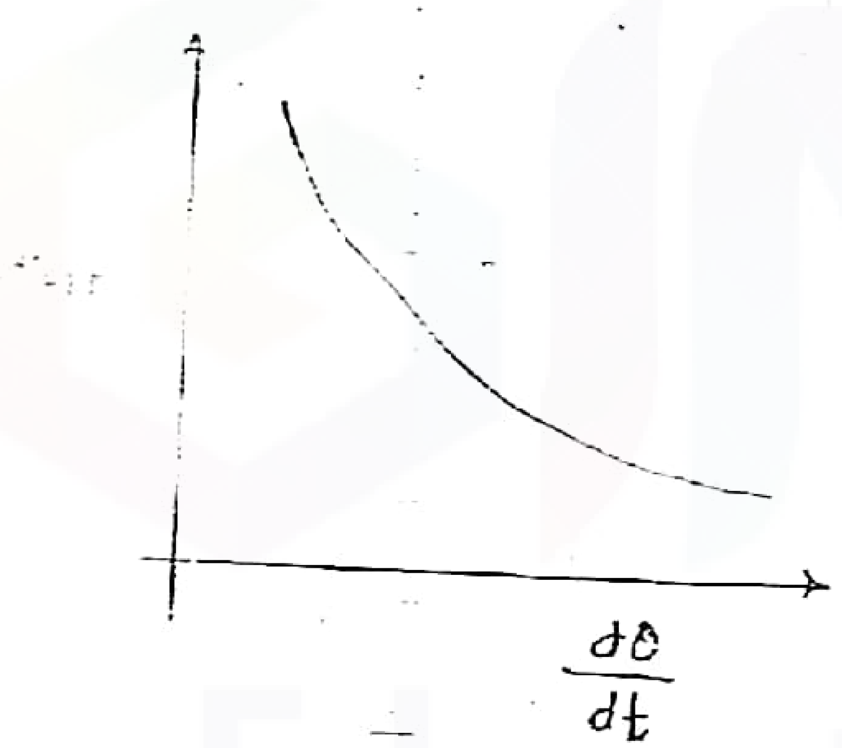
e.g. saturated sol^n of sugar in water, rice starch, sewage honey.

(ii) Pseudo plastic fluid:

$(n < 1)$

The shear deformation rate decreases the apparent viscosity. They are also called Shear Thinning fluid.



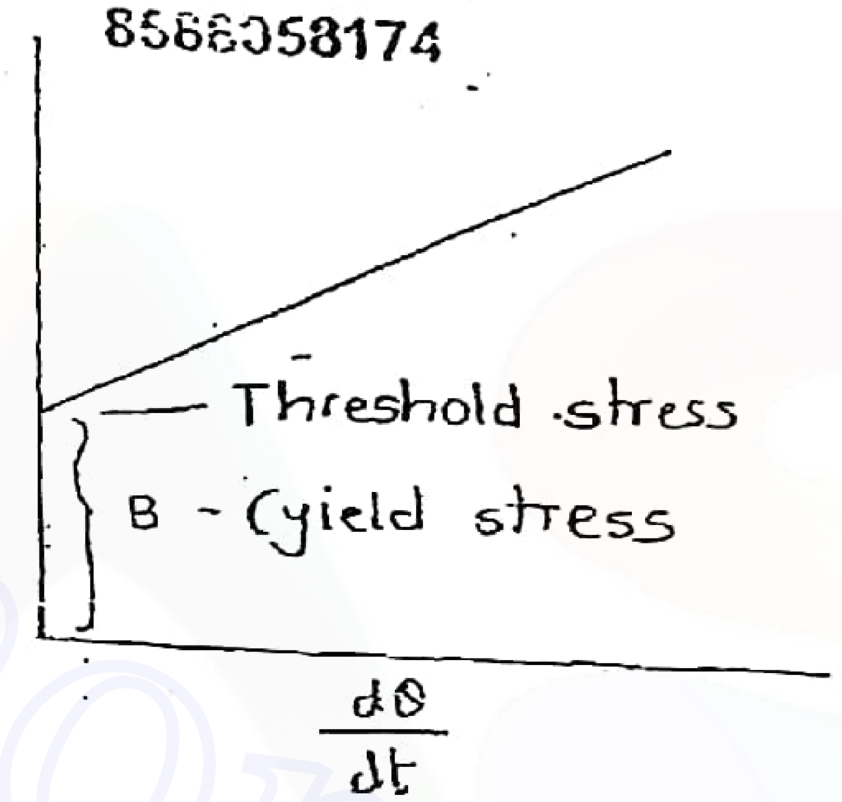


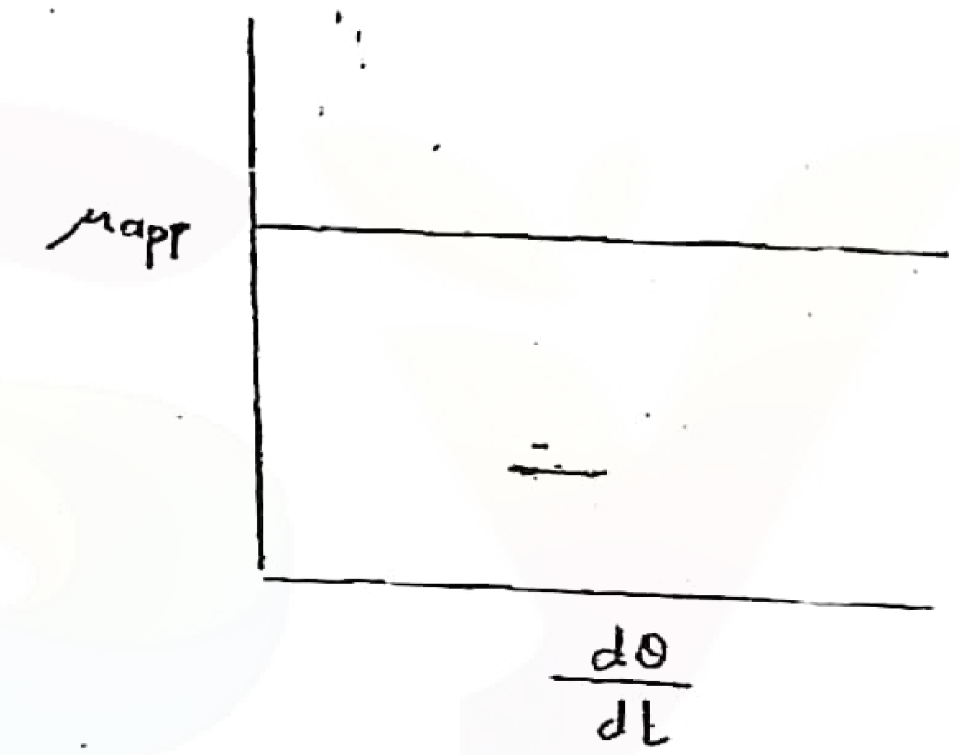
e.g. Milk, blood.

4. Ideal Bingham plastic fluid:

$$\tau = A \cdot \left(\frac{d\theta}{dt}\right) + B$$

A, B are positive constants.





e.g. Tooth paste, Hair gel. Cosmetic creams.

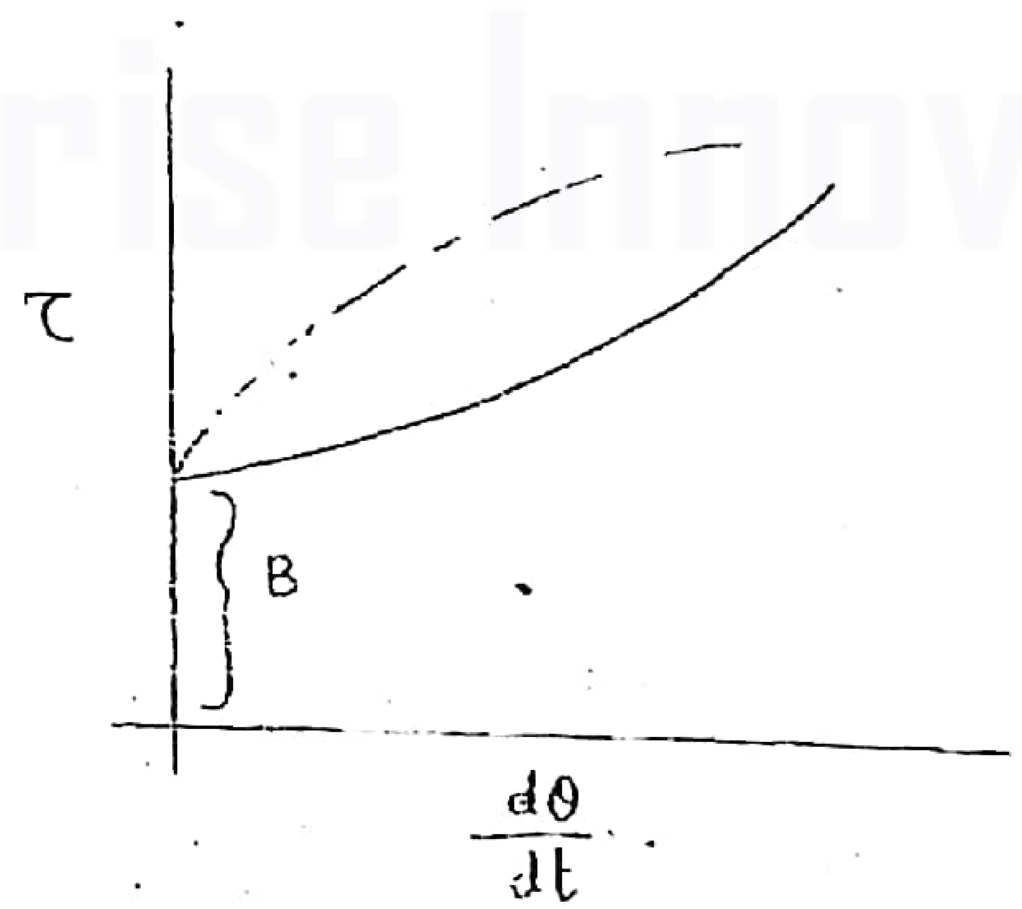
We need to overcome yield stress to make these fluids flow by applying shear stress.

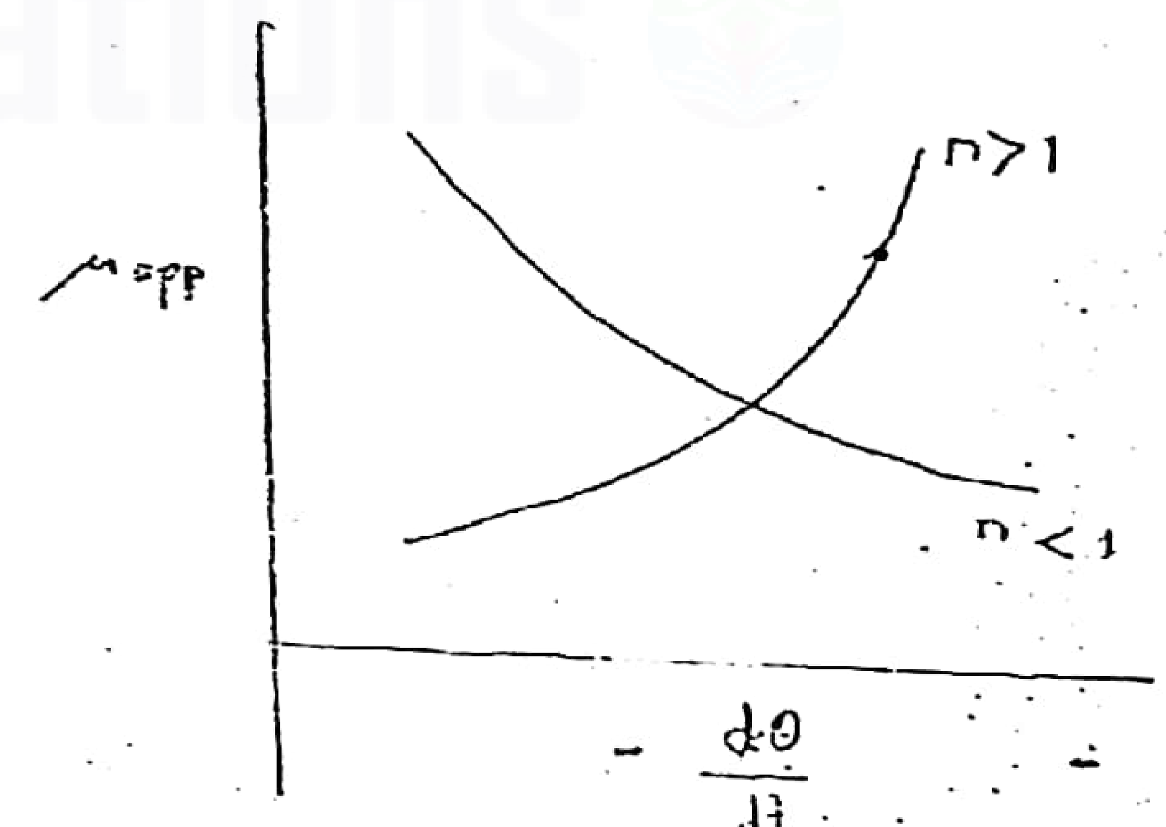
5 Thixotropic fluid.

$$\tau = A \cdot \left(\frac{d\theta}{dt}\right)^n + B$$

$n \neq 1$

n may b $n > 1$, $n < 1$





The viscosity of thixotropic fluids also depends upon the time.

If μ increases with time, the fluids are called grade 1

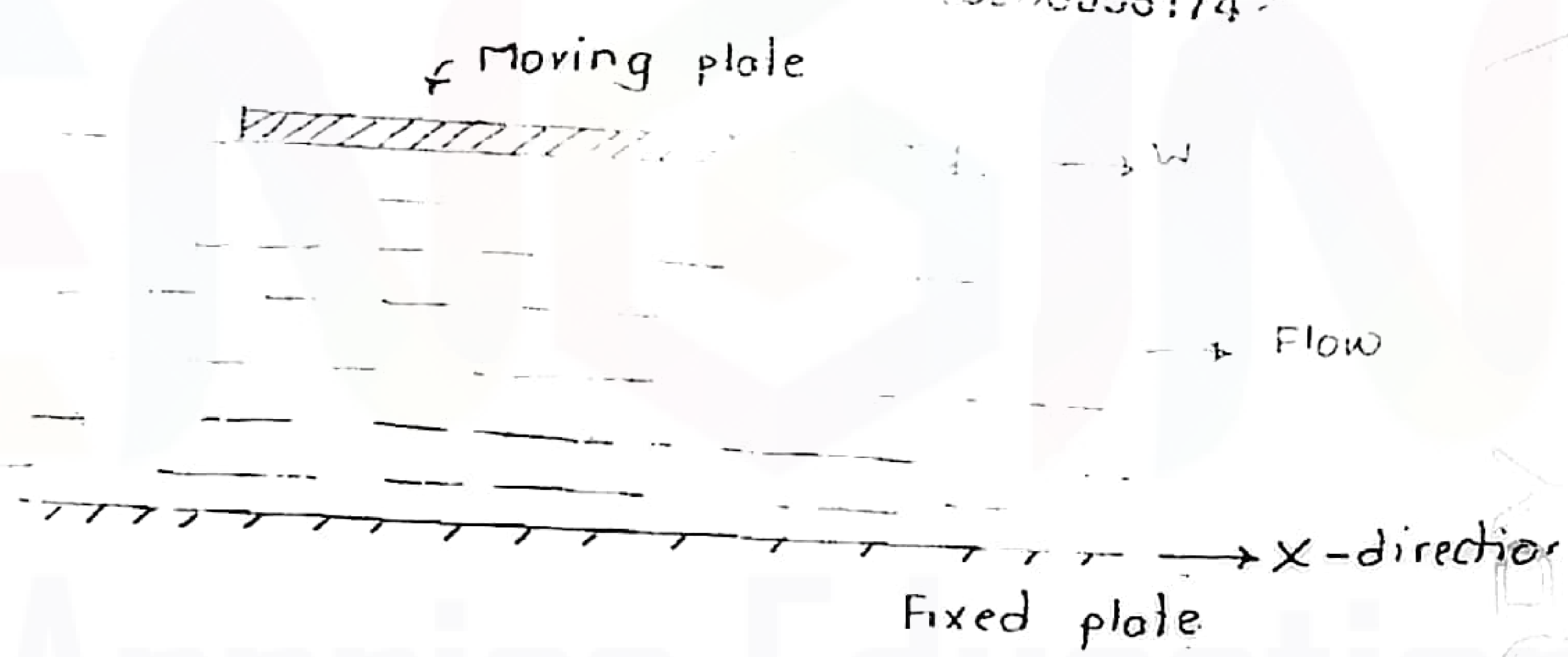
If μ decreases with time, the fluids are called grade 2. (also called as Rheopectic fluids)

e.g. Printer's ink, paints, pediatric drugs, chemotropic drugs, glucose in powder form.

Visco-elastic fluids:

This is a fluid which shows the property of the elasticity upto certain limits. Beyond that they are continuously deformed under shear force.

e.g. melted rubber



shear stress on upper plate will be in → −ve X-direction

shear stress on contacting layer with upper plate will be → +ve X direction

shear stress on contacting layer with lower plate will be → −ve X

shear stress on lower plate will be in → +ve X-direction

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Note:

At a constant temperature, if pressure is increased,

μ_{liq} → doesn't change

ν_{liq} → doesn't change

μ_{gas} → $\because C_{rms} = \sqrt{\frac{3RT}{M}}$ T-constant

$\therefore$ No change in μ_{gas}

$\because \rho \propto P$ $\therefore \nu_{gas} = \frac{\mu_{gas}}{\rho_{gas}}$

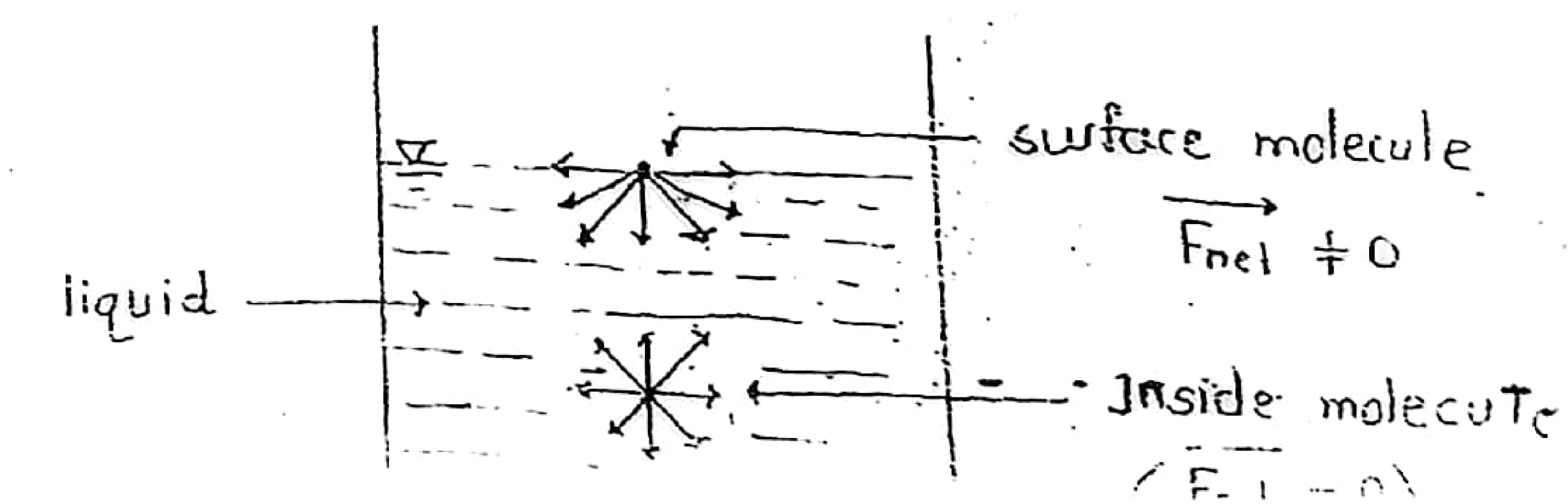
ν_{gas} decreases with increase in pressure.

Surface tension:

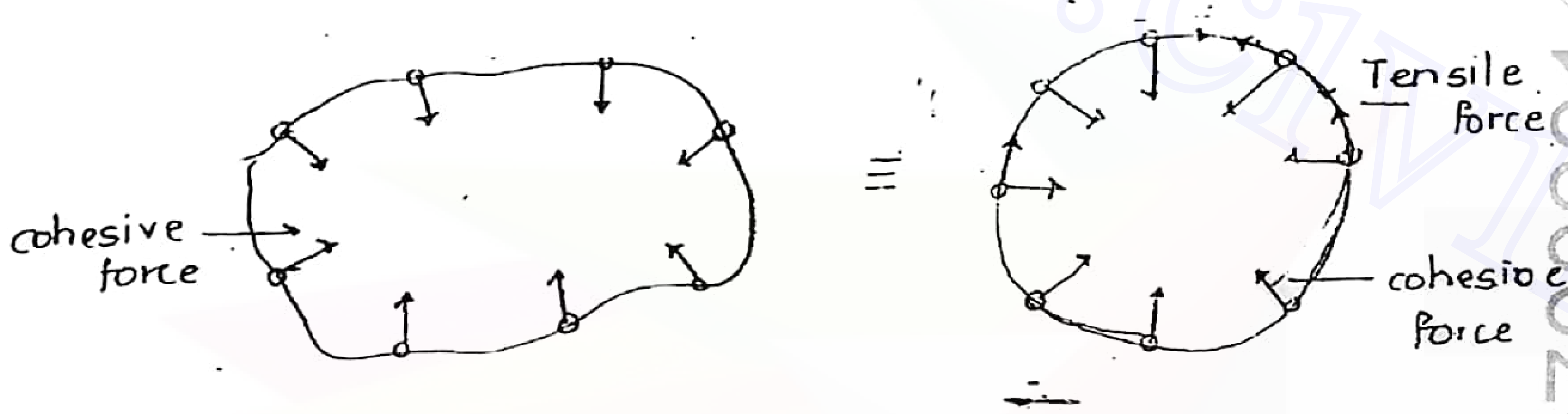
"Every fluid is having property of minimising its surface area upto its maximum extent. Such a property of fluid is known as surface tension". The only reason of this property is the cohesion i.e. cohesive force between the molecules of fluid.

Therefore it is mainly the property ... because the cohesion in the gases is almost nill.

For the same volume, sphere has the minimum surface area ($4\pi r^2$) amongst the all geometric shapes.



The surface molecule is pulled by the other molecules in inward direction.

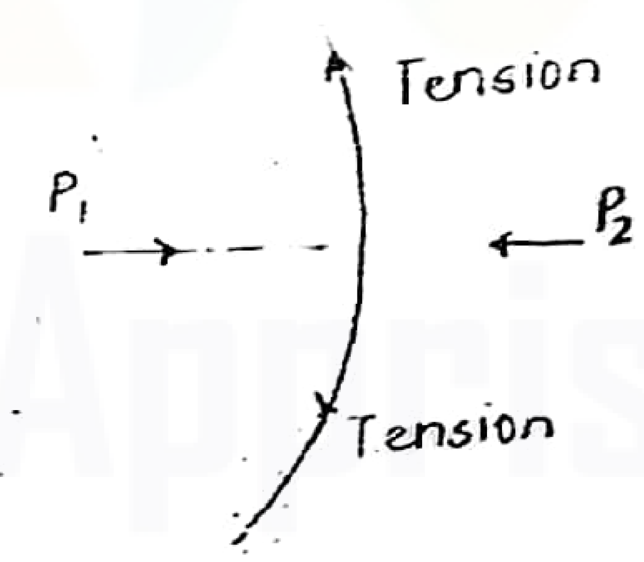


Mathematically, surface tension is defined as the force per unit length of free surface.

$$\sigma = \frac{F}{l} \quad N/m$$

where F is the tensile force developed at the surface due to cohesion.

i.e. cohesive force is cause
surface tension is the effect

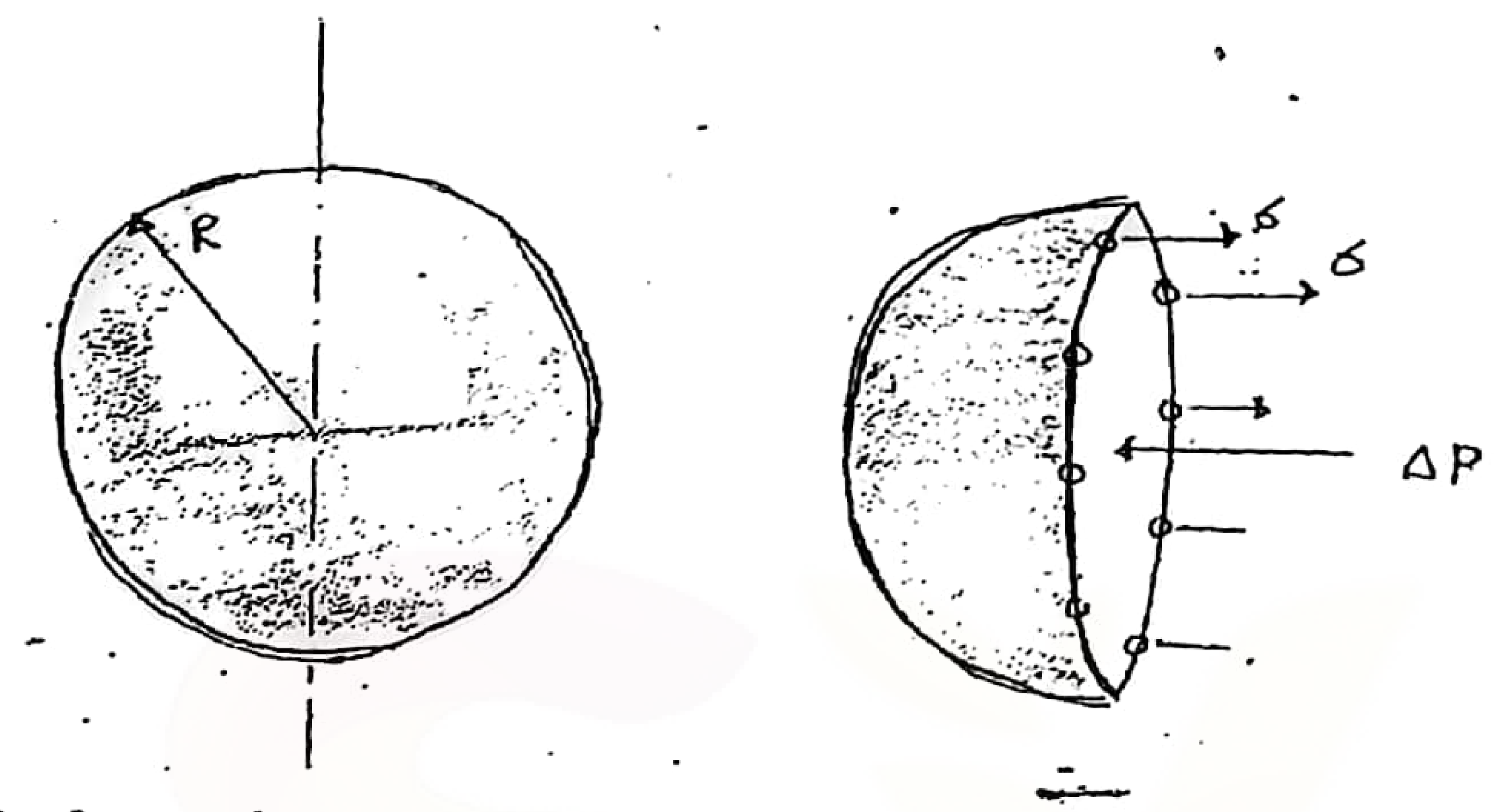


$P_1 > P_2$

Excess pressure on concave side is.

$$(P_1 - P_2) = \Delta P.$$

Drop formation :



surface tension force (σ) acting on $2\pi R$.
Excess pressure (ΔP) acting on inner concave.

$$\Delta P \cdot \pi \cdot R^2 = \sigma \cdot 2\pi R.$$

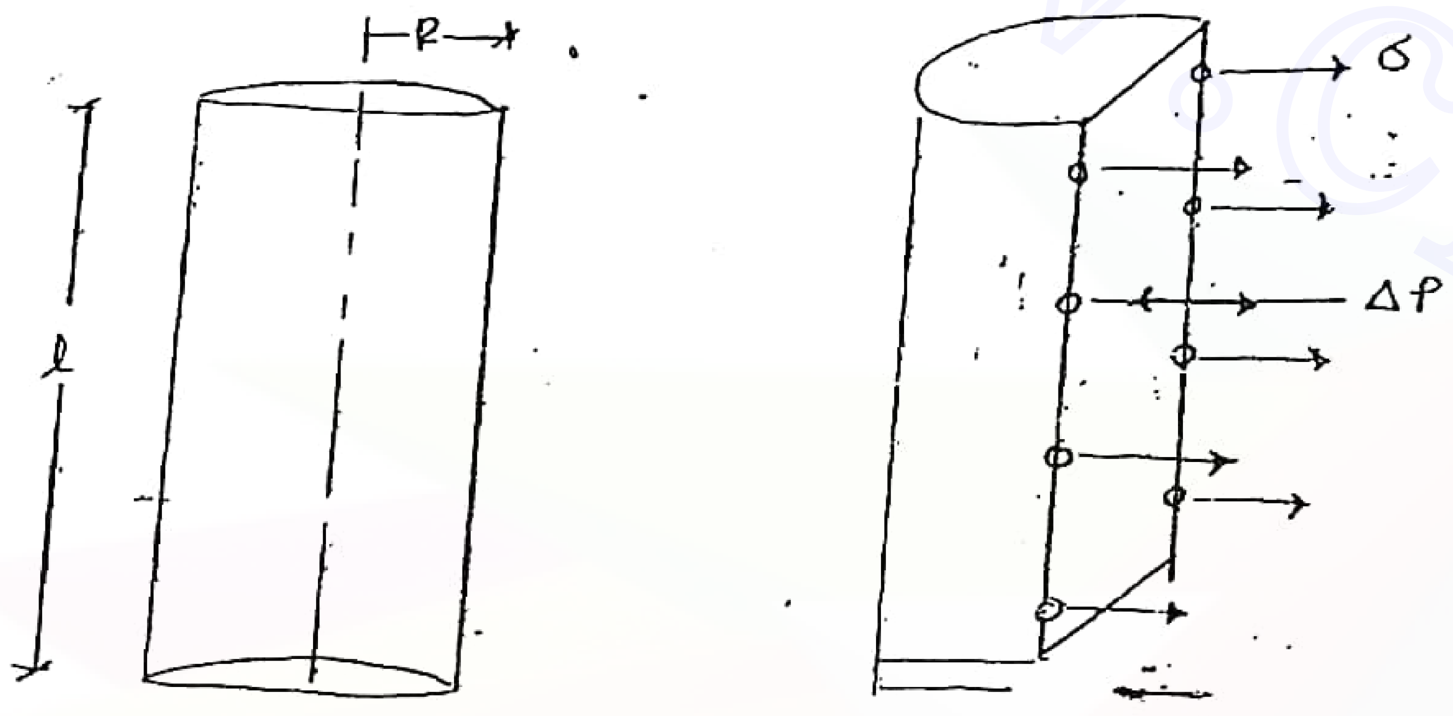
$$\Delta P = \frac{2\sigma}{R}.$$

Bubble formation :

$$\Delta P \cdot \pi R^2 = \sigma \cdot 4\pi R$$

$$\Delta P = \frac{4\sigma}{R}.$$

Jet formation:



$$\Delta P \cdot (2R \cdot l) = \sigma \cdot 2l$$

$$\Delta P = \frac{\sigma}{R}$$

The formation of jet is easy as it requires least excess pressure. So the jet will form first when we apply excess pressure.

The working fluid filled inside — called as drop.

e.g. Water drop has water filled inside.

air bubble has air inside as working fluid, thus air bubble is a Drop.

Wednesday
23rd October 2013

Concept of surface energy:

The work done in creating the surface area of the fluid film is stored in the form of potential energy and it is known as Surface energy.

The higher potential energy state always tries to get converted to lowest potential energy state. i.e. The lowest potential energy state is highly stable state.

The body always tries to acquire low potential

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Work = Force × displacement.

for length l, to strech the material (form film) by x

Work $= (\sigma \cdot l) \cdot x$ $\qquad$ $\sigma \cdot l$ – Force i.e. surface tension.

$= \sigma (l x)$

$E = \sigma \cdot A$

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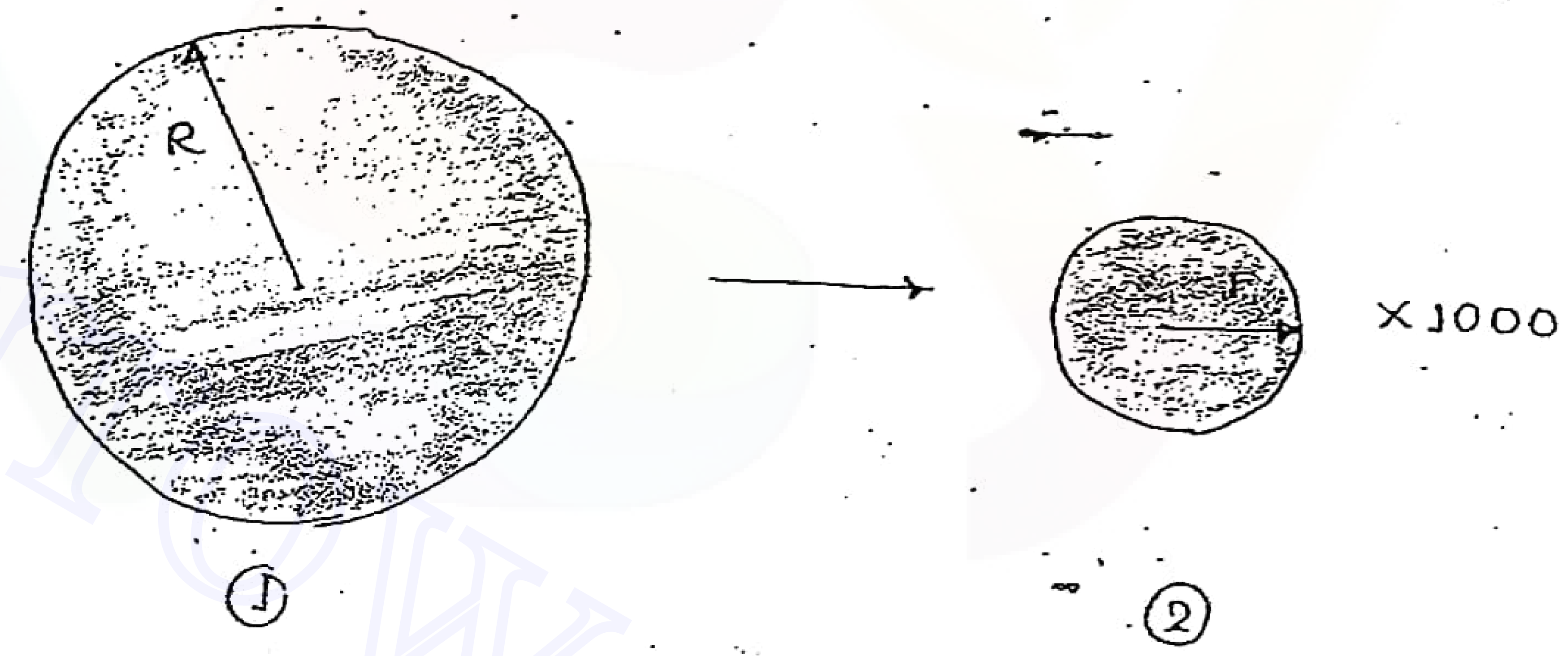
8588058174

e.g.



$$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$$

$$r = \frac{R}{10}$$

For state ①

$E_1 = \sigma_1 (4\pi R^2)$ – lower energy state

for state ②

$E_2 = \sigma (4\pi r^2) \times 1000$

$= \sigma \left(4\pi \times \frac{R^2}{100}\right) \times 1000$

$= 10\,\sigma (4\pi R^2)$ – higher energy sta

$\Delta E = E_2 - E_1$

$= 36\,\sigma \pi R^2$

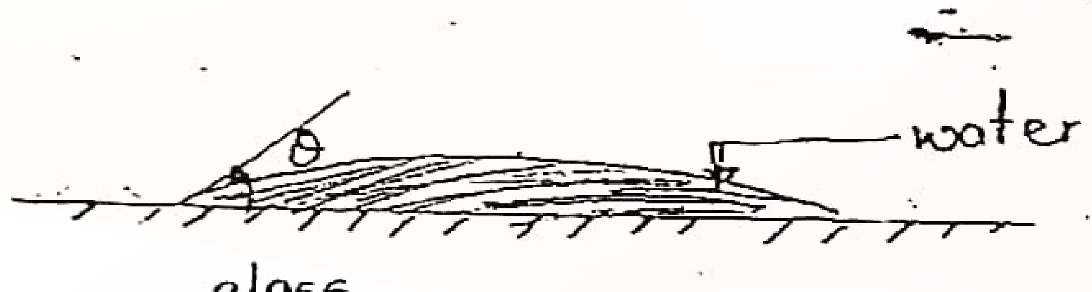
Wetting and non wetting fluids:

It is a mutual property of liquid and surface. It depends on cohesion and adhesion both.

If adhesion >>>> cohesion.

Liquid is wetting the surface.

e.g. Water on the glass (angle of contact $\theta < \pi/2$)

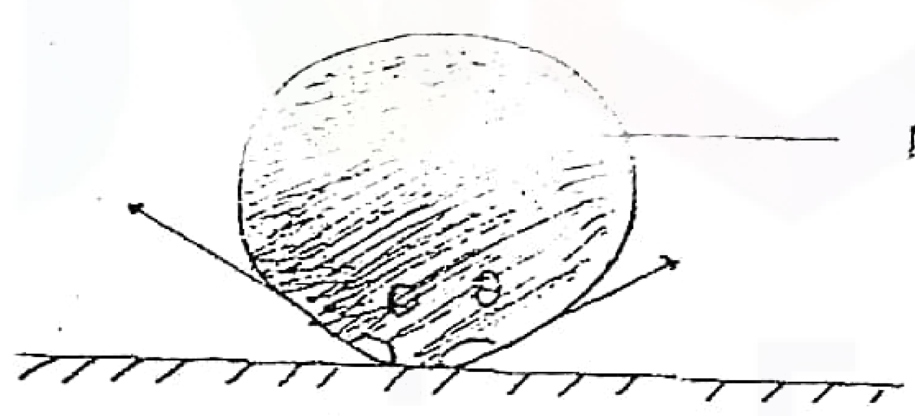


For pure water, angle of contact is 0° (wets surface completely)

If adhesion <<<< cohesion.

Liquid is non wetting the surface

e.g. Mercury on the glass ($\theta > \pi/2$)



Mercury actually touches the glass surface at a point when taken in small quantity, but when large drop is placed on glass surface, contact area is more due to the weight of drop itself.

Angle of contact also depends upon the type of contact surface.

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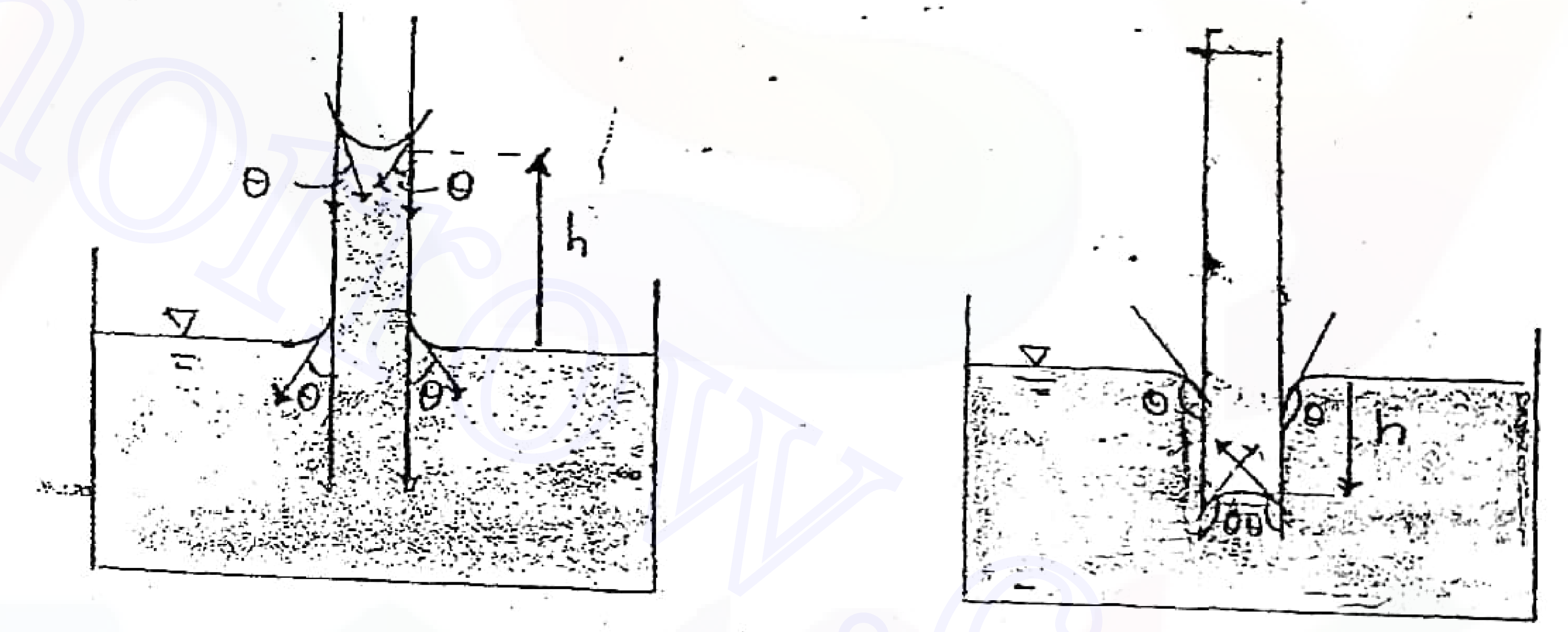
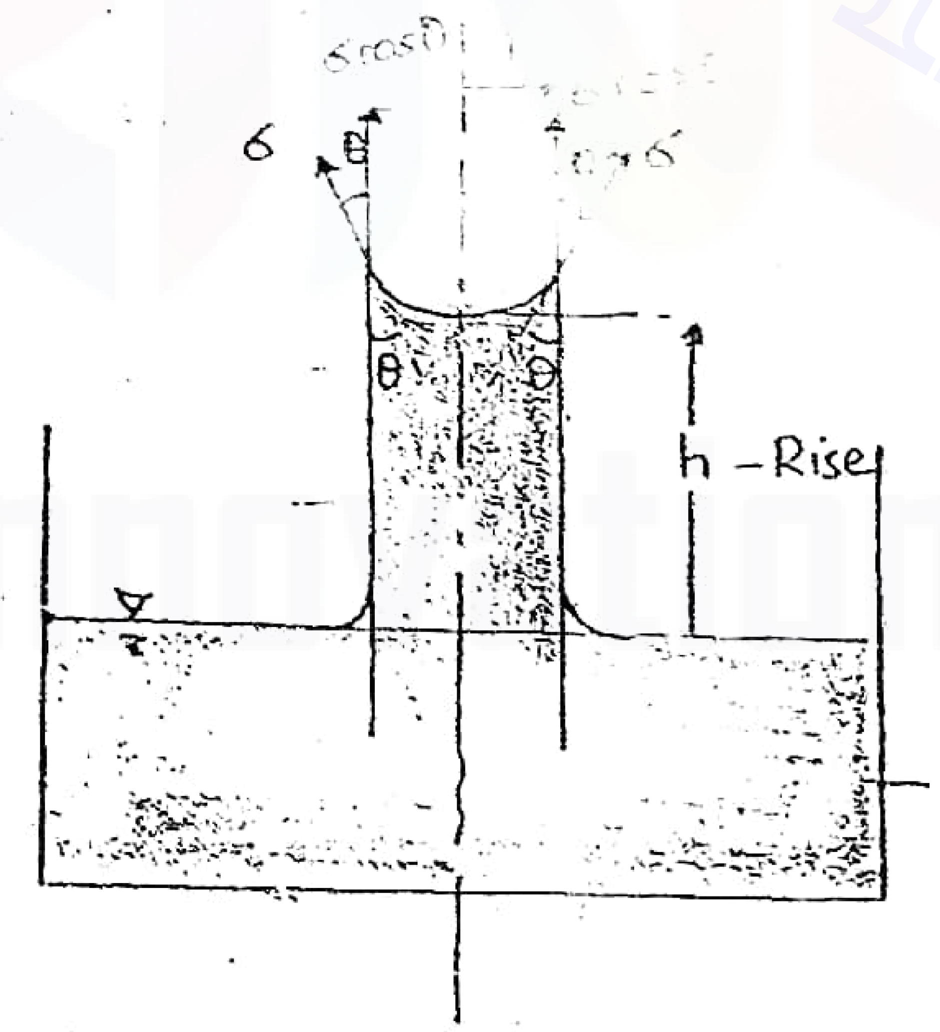
Capillarity:

When a tube of very fine diameter is submerged or emersed inside a fluid then there may be the rise or fall of liquid level inside the tube, depending upon the wetting and non-wetting nature of liquid, with tube surface. This rise or fall of the liquid level in a tube of fine diameter is a phenomenane known as Capillarity and this tube of fine diameter is known as Capillary tube.



Wetting ($\theta < \pi/2$)   Non-wetting ($\theta > \pi/2$)

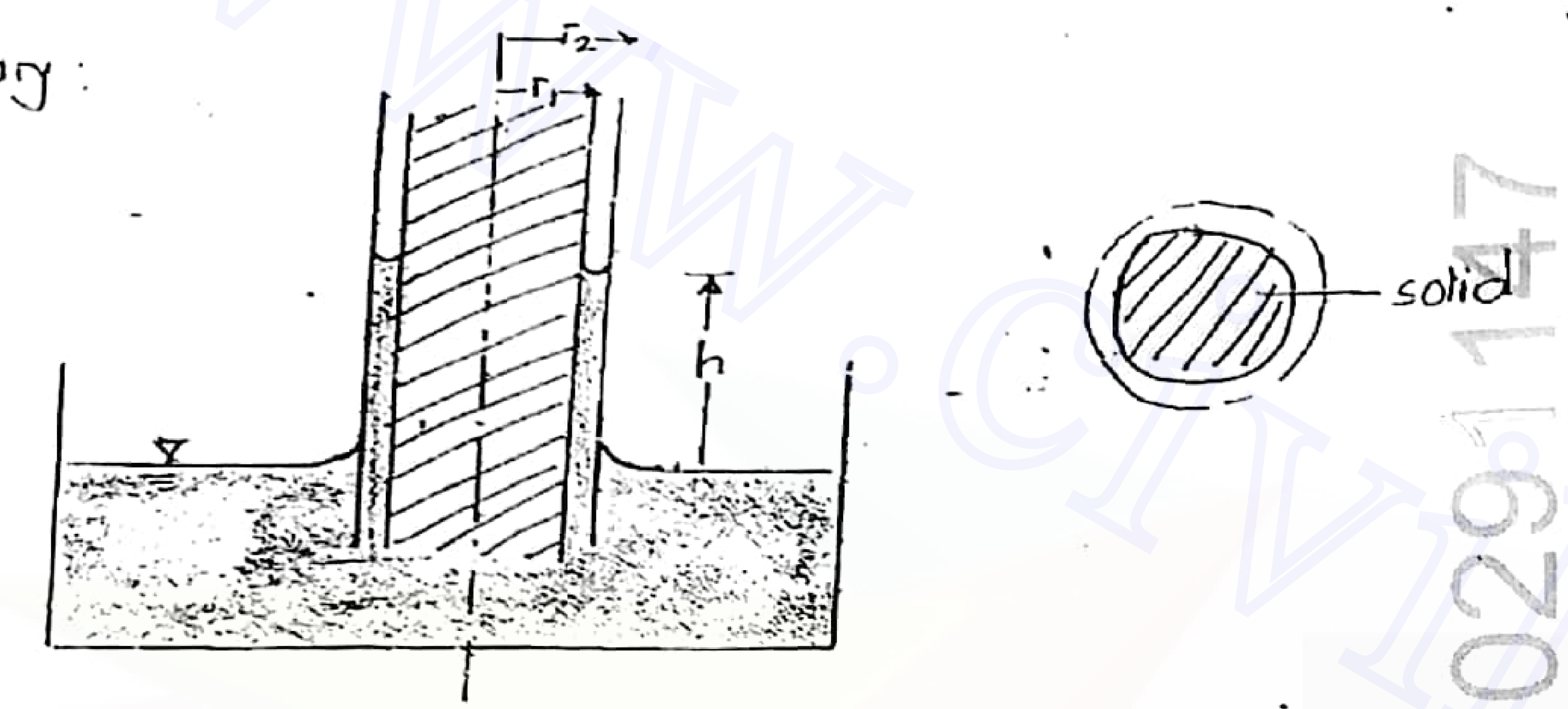
Rise or fall



$$\sigma \cos\theta \cdot 2\pi r = (\pi r^2 \cdot h) \cdot \rho \cdot g$$

$$h = 2\sigma \cos\theta$$

Annular capillary :

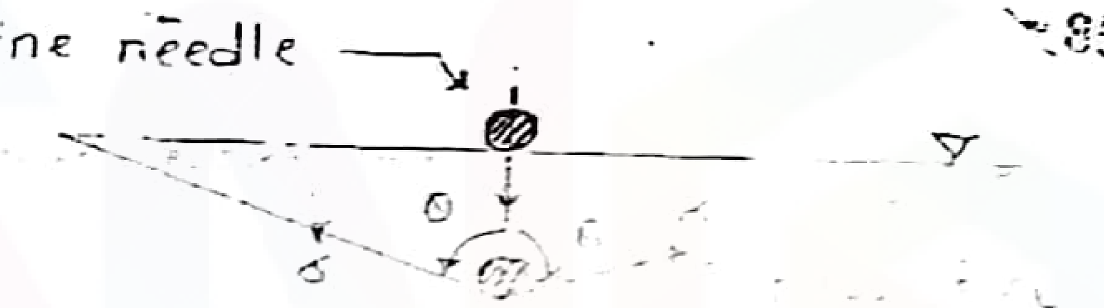


$$\sigma \cos\theta \cdot (2\pi r_1 + 2\pi r_2) = \pi (r_2^2 - r_1^2) \cdot h \cdot \rho \cdot g$$

$$2\pi \cdot \sigma \cos\theta \, (r_1 + r_2) = \pi \, (r_1 + r_2)(r_2 - r_1) \cdot h \cdot \rho \cdot g$$

$$h = \frac{2\sigma\cos\theta}{(r_2 - r_1) \cdot \rho g}$$

Stainless steel needle floating on surface of water.



[illegible] surface.

Needle sinks down due [illegible] weight.

$$(2\sigma\cos\theta) \cdot l = m g$$

l - length of needle

m - mass of needle

$$(2\sigma\cos\theta) \cdot l = (\pi r^2 l) \cdot \rho \cdot g$$

$$2\sigma\cos\theta = \pi r^2 \cdot \rho g$$

r - radius of needle.

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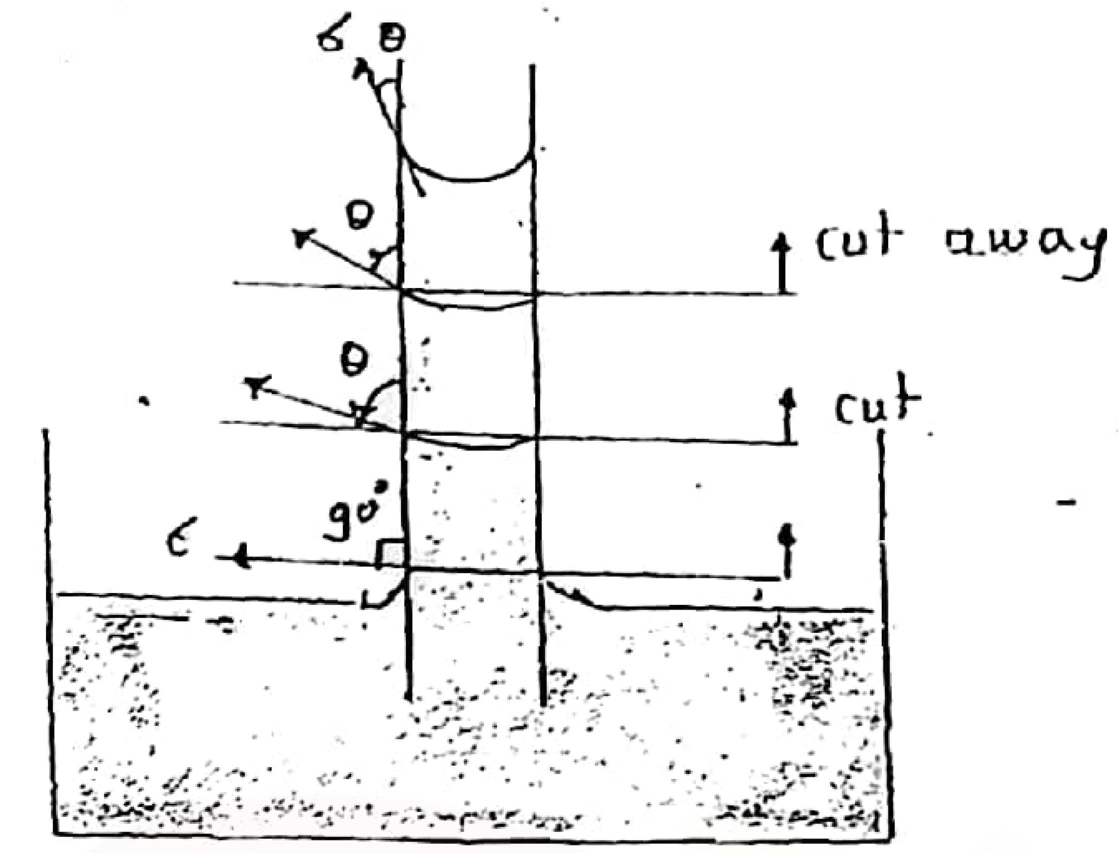


If for certain fluid (angle of contact θ) the rise in capillary tube is h and if the length of tube is less than h then.

(i) Concavity of the free surface of fluid (liquid) inside capillarity will decrease

(ii) Angle of contact (θ) will increase

(iii) But the water will not spill out of capillary.

If the length of capillary is further reduced to the liquid [illegible] container (angle of [illegible]) will become 90°) and [illegible] meniscus will become horizontal.

Pressure and its measurement :

Pressure :-

The existance of molecules in a system is existance of pressure.

Mathematically, intensity of pressure is defined as external normal force per unit area.

$$P = \frac{F}{A} \rightarrow \text{External normal force (thrust)}$$

(gives pressing effect)

Pressure is scalar quantity (direction - intensity is same in all directions) but pressure force is a vector.

Units of pressure :

(i) $1 \ Pa = 1 \ N/m^2$ S.I. unit.

(ii) $1 \ \frac{kg \cdot f}{cm^2} = \frac{9.81 \ N}{10^{-4} \ m^2} = 9.81 \times 10^4 \ N/m^2$

(iii) $1 \ bar = 10^5 \ Pa$

(iv) $1 \ atm = 101.325 \ Pa$

(v) FPS system.

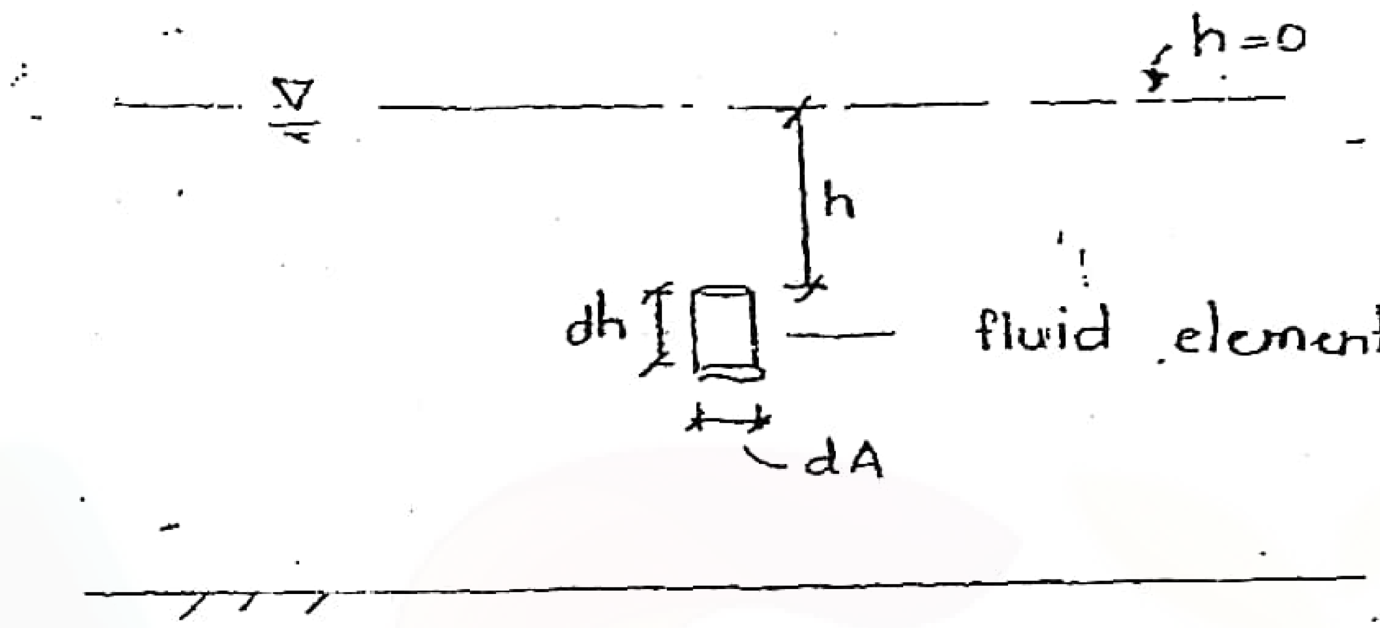
$1 \ \frac{lb \cdot f}{inch^2} = 1 \ P.S.I$ (pound per square inch)

$$= \frac{0.453 \ kg \cdot f}{(2.54 \ cm)^2} = \frac{0.453 \times 9.81}{(2.54)^2 \times (10^{-2})^2}$$

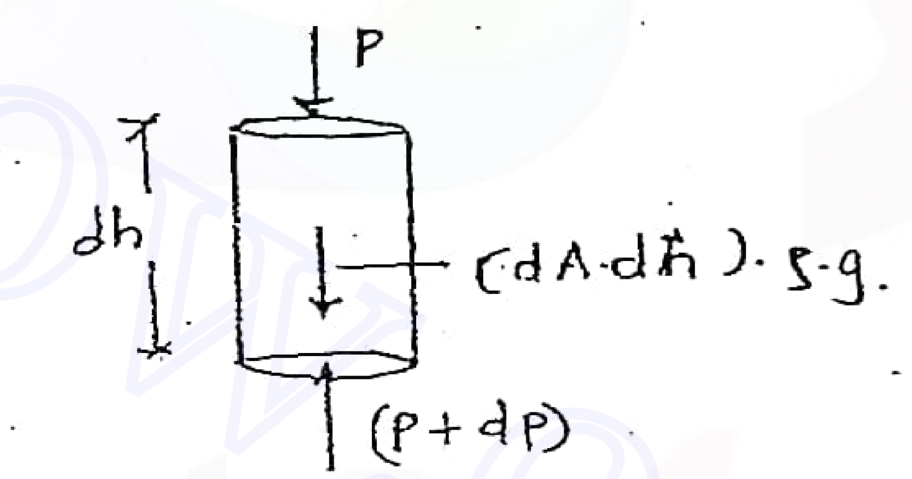
$$= 6888.1 \ Pa.$$

$1 \ atm = 14.7 \ PSI.$

Pressure at a point in static fluid
(Hydrostatic pressure)



Fluid element system :



As it is static fluid.

$$(P+dP)\cdot dA - P\cdot dA - (dA\cdot dh)\cdot \rho g = 0$$

$$\frac{dP}{dh} = \rho g$$

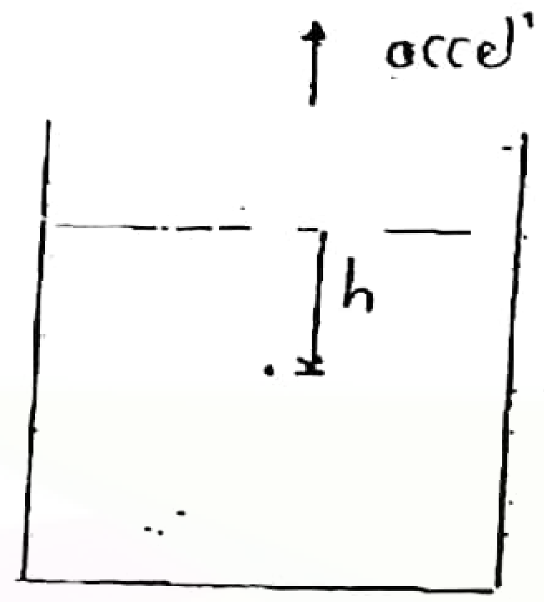
(> 0 means the pressure increasing with height)

$$\int_0^P dP = \int_0^h \rho \cdot g \cdot dh$$

$P = 0 \rightarrow$ at $h = 0$ beco... we are considering only fluid pressure.

$$P = \rho g h.$$

If the container is moving upward with acceleration a_z.



$$(P+dP)\cdot dA - P\cdot dA - (dA\cdot dh)\cdot \rho g = (dA\cdot dh)\cdot \rho\cdot a_z$$

$$\frac{dP}{dh} = \rho(g+a_z)$$

$$P = \rho(g+a_z)\cdot h$$

If container is moving downward with acceleration a_z

$$P = \rho(g-a_z)\cdot h$$

For the free fall of container.

$$a_z = g$$

$$P = \rho(g-g)\cdot h$$

$$= 0 \quad \text{(weighless condition)}$$

6th Unit of pressure:

Pressure can be represented by height of fluid column (as the pressure at bottom of fluid is $\rho g h$)

$$1\ \text{atm} = 101.325\ \text{Pa}$$

In terms of water column.

$$101.325 = 1000\ \text{kg/m}^3 \times 9.8\ \text{m/s}^2 \times h_{water}$$

$$h_{water} = 10.3\ \text{m}$$

In terms of Mercury

$$101325 = 13600 \times g \times h_{Hg}$$

$$h_{Hg} = 760\ \text{mm}$$

$$1\ \text{atm} = 10.3\ \text{m water} \doteq 760\ \text{mm Hg}$$

Different types of pressures:

1. Atmospheric pressure:

The pressure exerted by environmental mass is known as atmospheric pressure.

2. Absolute pressure:

It is the total pressure of the system measured from zero level. This pressure is also known as net pressure.

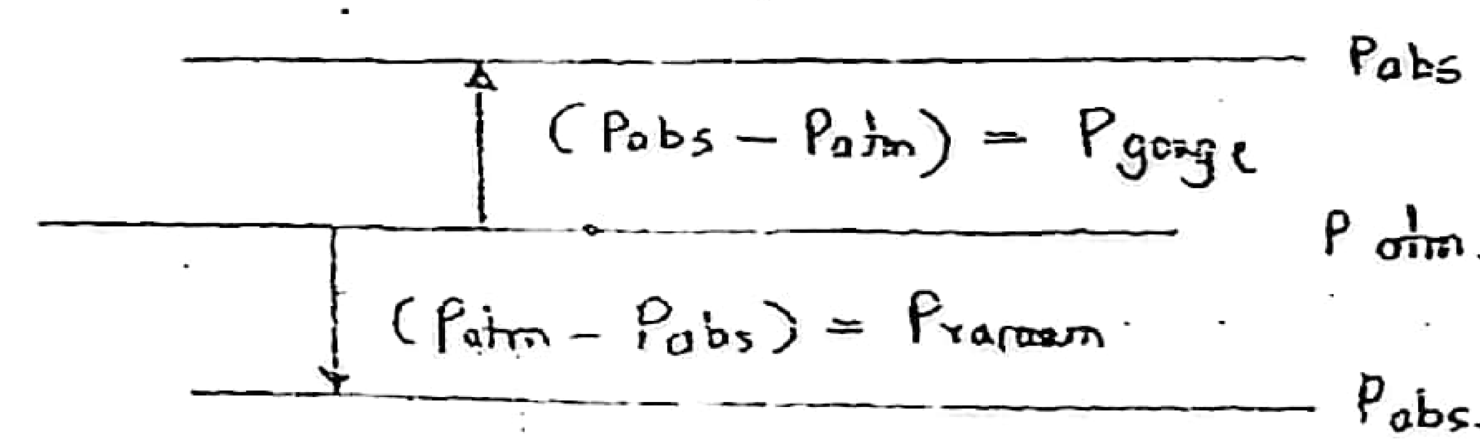
3. Gauge pressure:

It is the pressure of the system measured above atmospheric pressure values.

4. Vacuum pressure:

It is a pressure of the system measured below the atmospheric pressure values.

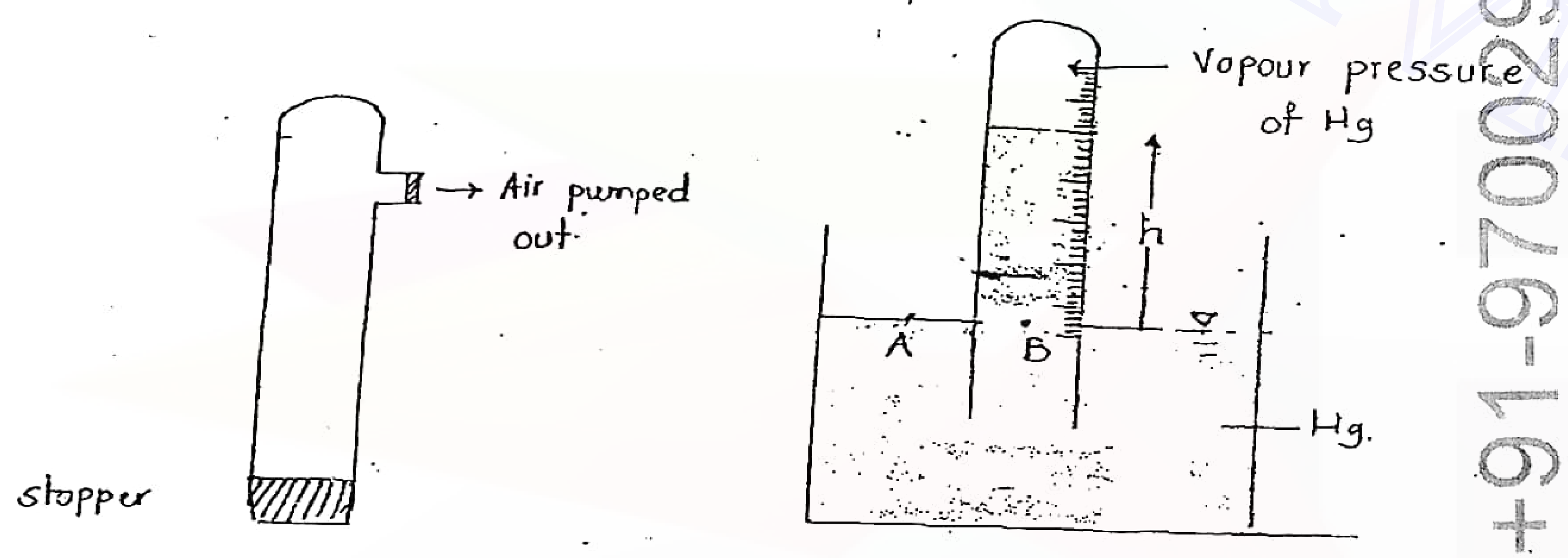
Vaccum pressures are negative gauge pressures and gauge pressures are negative vacuum pressures.



Conventional pressure measurement devices:

(1) Barometer:-

It is a device which is basically used to measure local atmospheric pressure. This device is made by Torri. Cell.



Vapoure pressure is pressure due to the dissolved gases in the liquid.

Here the Mercury rise is not due to capillary action but the pressure difference of the levels of Mercury at A & B when the stopper is just removed.

$P_A = P_B$ (same liquid same height)

$P_{atm} = P_{vapoure (Hg)} + \rho_{Hg} \cdot g \cdot h$

Vapour pressure of Hg is very less, hence can be neglected while vapour pressure of water is $\frac{1}{3}$ rd of atm. pressure which cannot be neglected.

$P_{atm} = \rho_{Hg} \cdot g \cdot h$.

Hg is used in pressure measuring devices because.

(i) It has very high density. (less column height)

(ii) It has very low vapour pressure (can be.

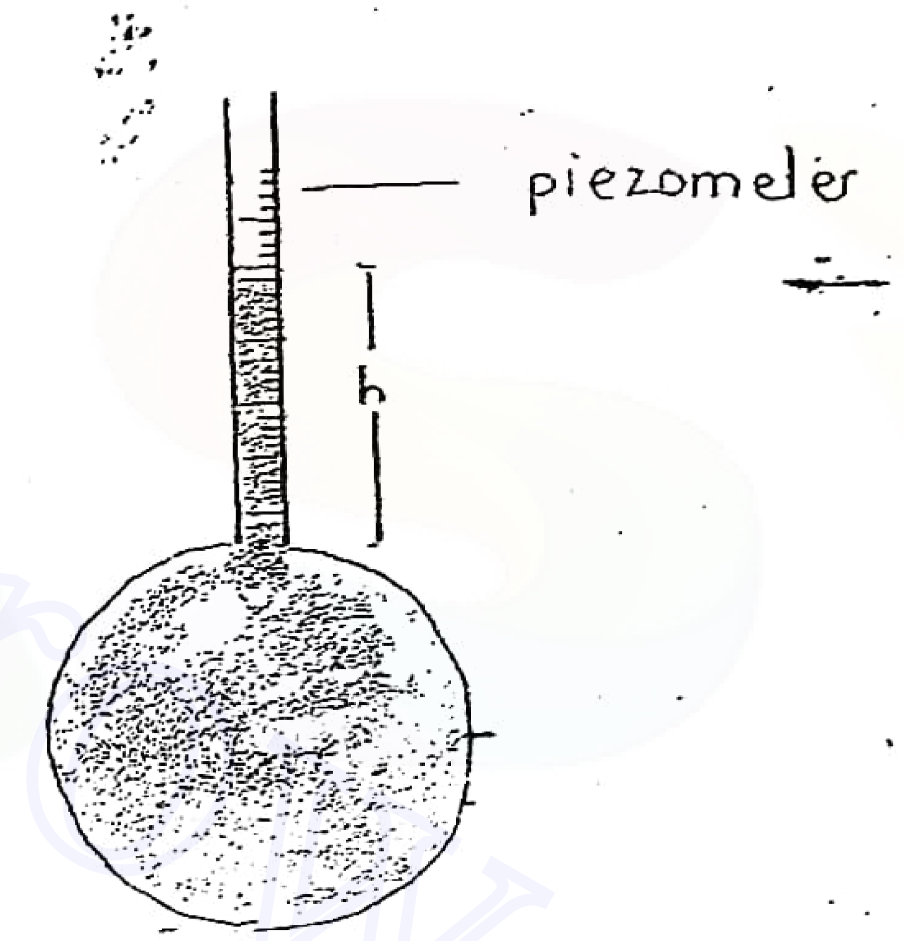
(2) Piezometer.

It is a straight tube used for the pressure measurement

It measures only moderate (low, medium

It measure gauge (+ve) pressures

It measure only liquid pressure



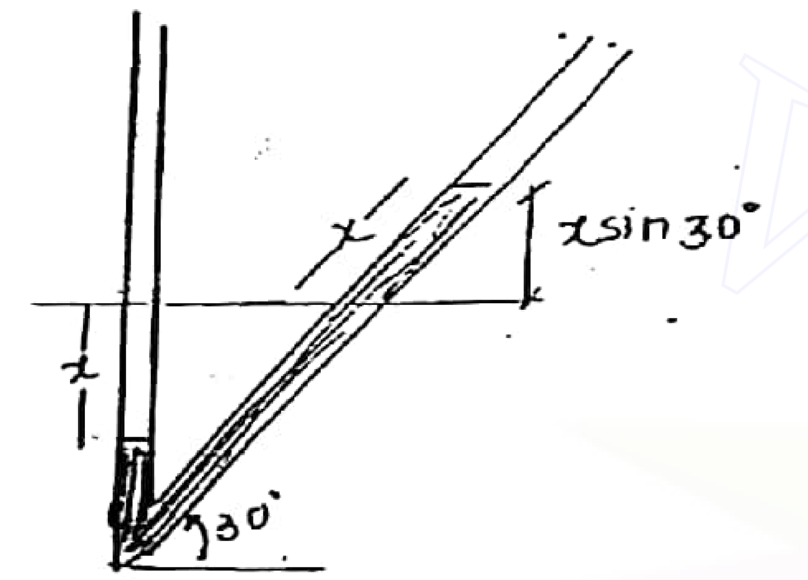
(3) Manometers:

It is a device which is used to measure low, medium and high pressures +ve or -ve gauge pressures of liquids and gases both.

In manometer the additional fluid is used called as Manometeric fluid (density - ρ_m).

7^{th} Unit of pressure

1 tor = 1 mm Hg in Barometer.

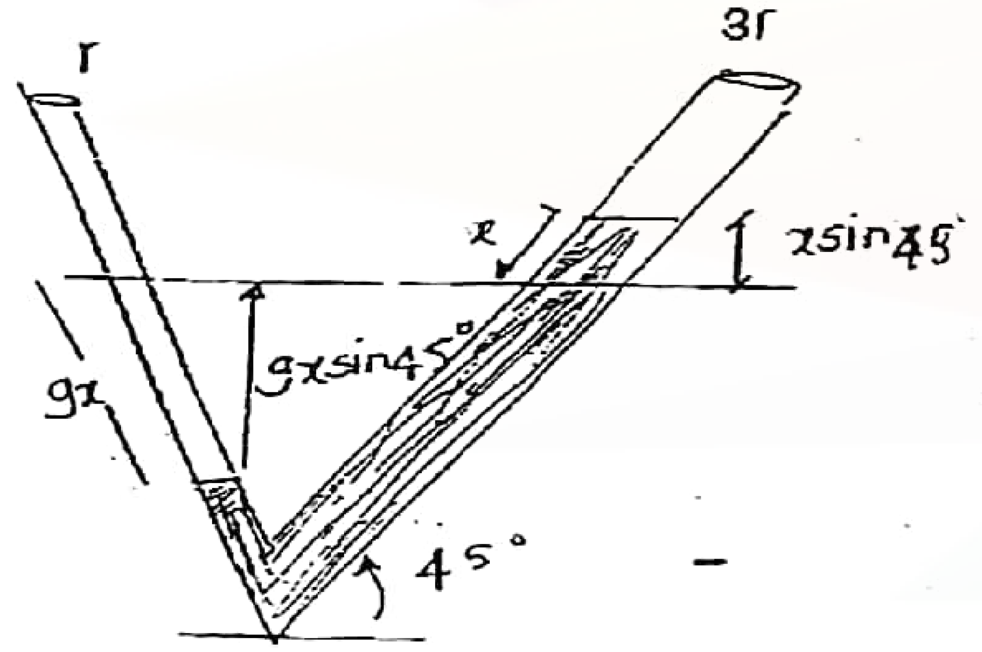


$x \sin 30 + x = 0.2$

$x =$

oil : $(3 - x)$

water : $(1.5 + x \sin 30) =$



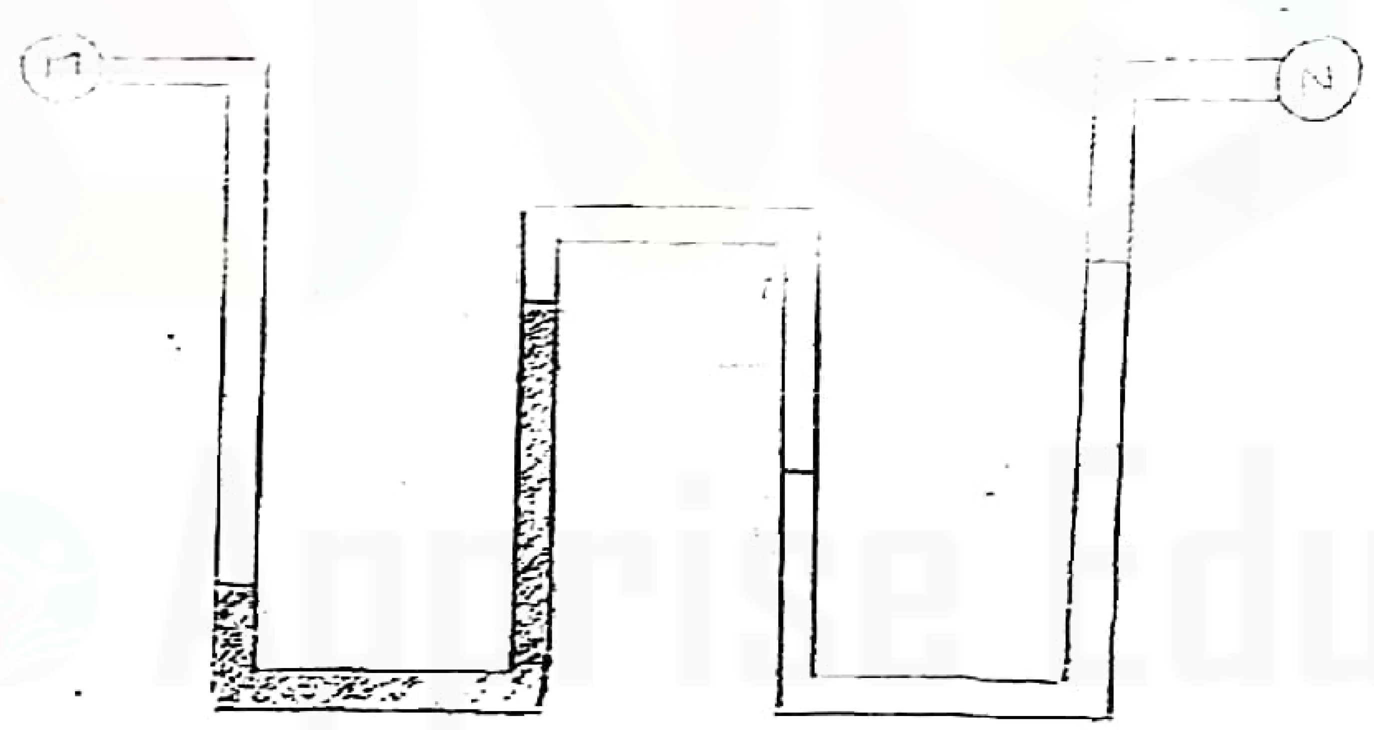
$x \sin 45° + 9x \sin 45° = 0.2$

$x =$

oil : $(3 - 9x \sin 45) =$

water : $(1.5 + x \sin 45) =$

Q. Find difference of pressure $(P_M - P_N)$



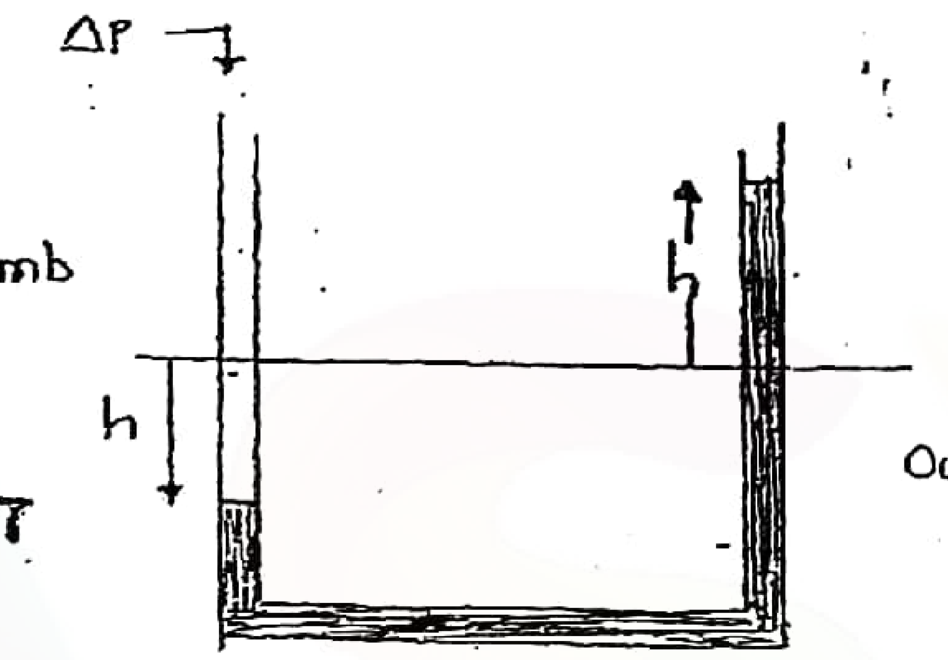
$P_M + (36 \times 800 \times g) - (12 \times 13600 \times g) + (12 \times 800 \times g)$

$- (15 \times 13600 \times g) - (15 \times 800 \times g) = P_N$

$P_M - P_N =$

Sensitivity of Manometer:

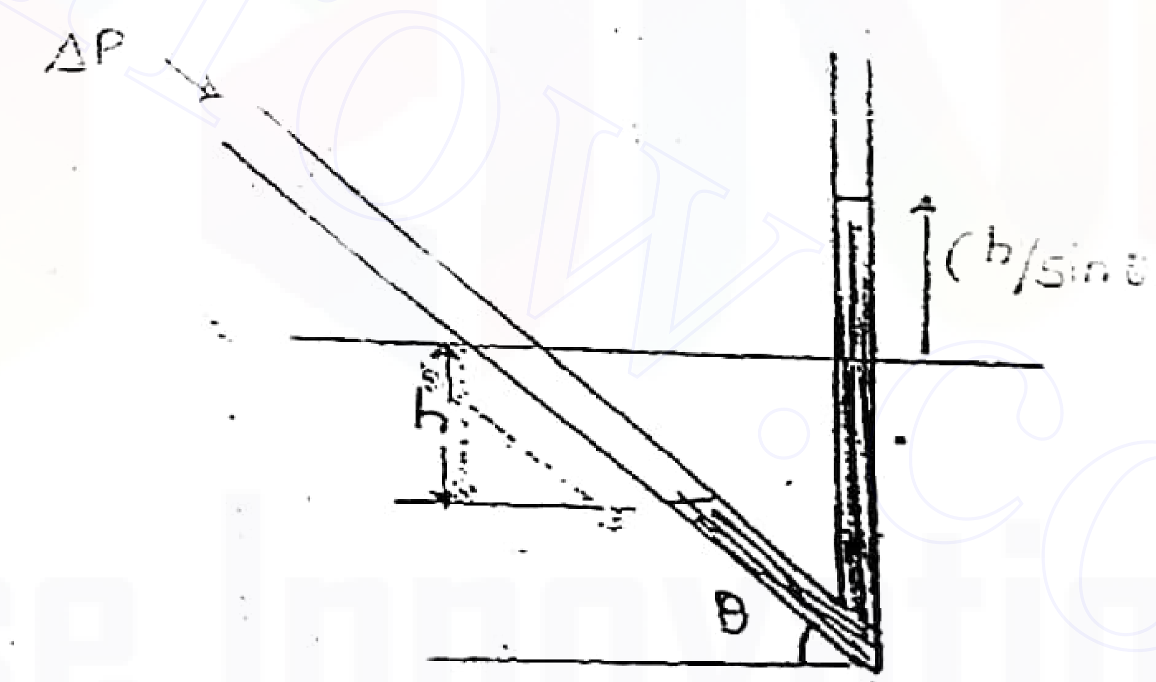
The extent of reading in the output limb of the manometer for a given pressure difference in input limb is known as Sensitivity of Manometer.



'h' is called Sensitivity of Manometer for applied pressure ΔP (excess pressure)

(i) To increase sensitivity:

Input limb is inclined.

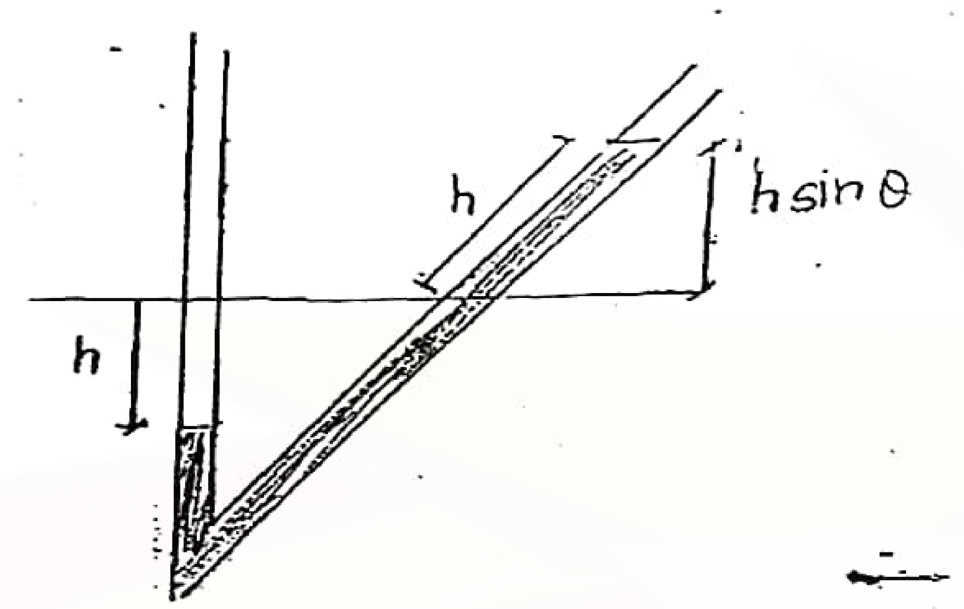


Sensitivity is increased by $\left(\frac{1}{\sin\theta}\right)$ times

When the readings are very small, the sensitivity of the manometer is increased by inclining the i/p limb.

(ii) To decrease sensitivity:

The output limb is inclined.



Sensitivity is decreased by (sin θ) times.

When the pressure difference (readings) is more than the sensitivity of manometer is decreased. (c/s areas can also be increased to reduce sensitivity)

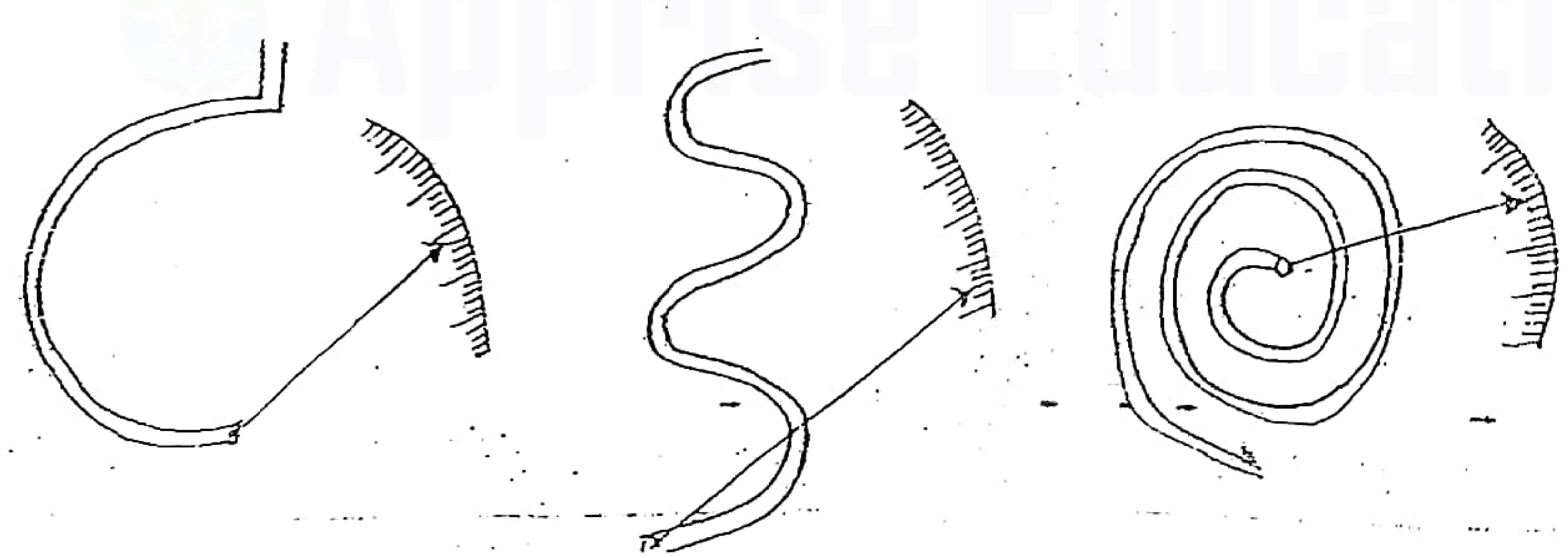
Modern pressure measurement devices:

1. Bourdon gauge:

There is small flexible tube inside this gauge.

It measures the pressure above atmospheric pressures only (gauge pressures)

The property used in pressure measurement is Flexibility.



2. Strain gauge transducer.

It contains very small chips of smart materials (alloys). The small wire coming out of two chips is attached to Data Aquisition System (DAS) to measure the strain developed.

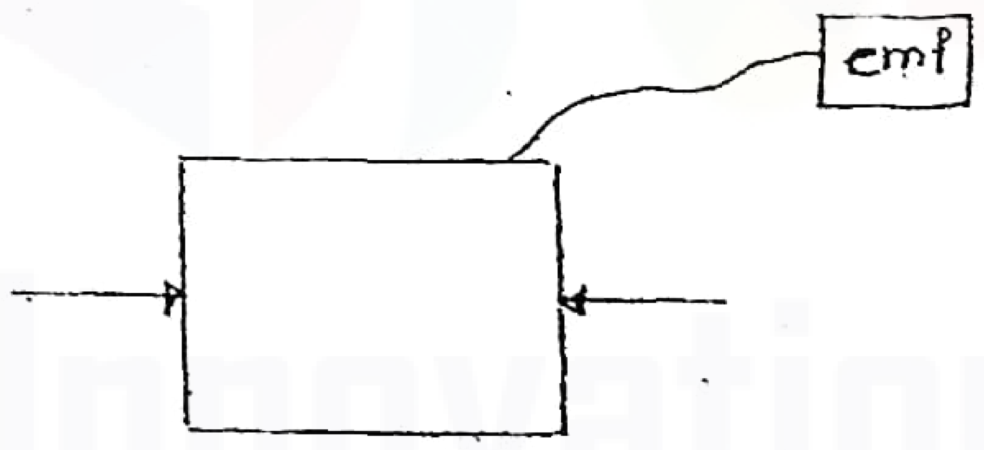
The shock developed are measured in terms of strain. (strain → stress → pressure)

e.g. On road testing of vehicles

3. Piezo-electric transducers:

Piezo - pressure, electric - emf

The emf is developed in the small transducers as the pressure is applied. This property is found only in Quartz and Rochelle salt.



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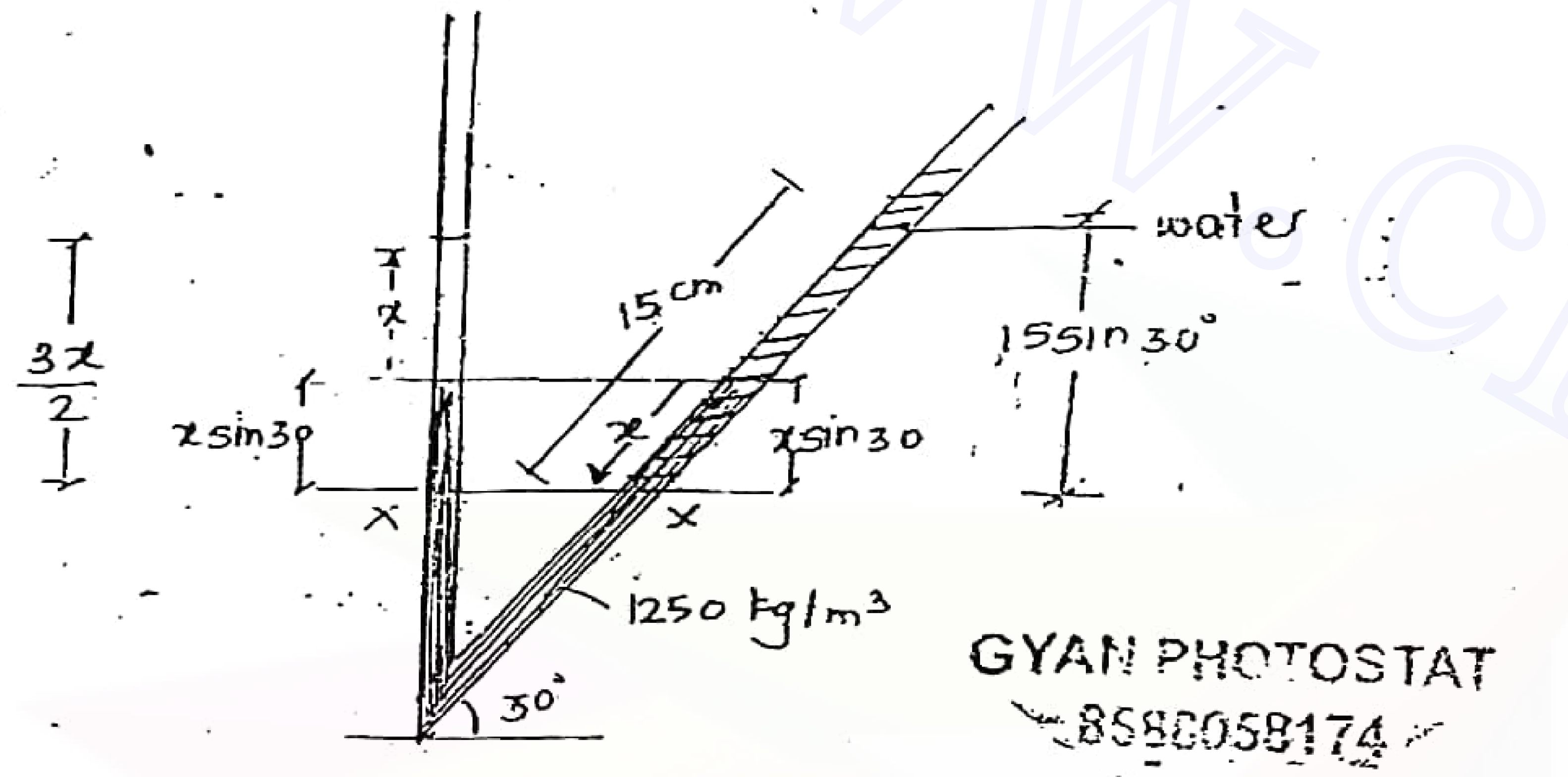

$$\cancel{P_{atm}} + \left(\frac{3x}{2}\right) \times 1250 \times \cancel{g} - 0.075 \times 1000 \times \cancel{g} = \cancel{P_{atm}}$$

$$x = 4\text{ cm}$$

1 atm = 101325 Pa

=

$1 \; \frac{kg.f}{cm^2} = 0.967$ atm

1 Pa = [illegible] $\times 10^{-6}$ atm

1 bar = [illegible] atm

1 tor = 1.31×10^{-3} atm = 1 mm of Hg.

1 PSI = 0.068 atm

1 m water = 0.097 atm

Pa < tor < PSI < water < $\frac{kg.f}{cm^2}$ < bar < atm

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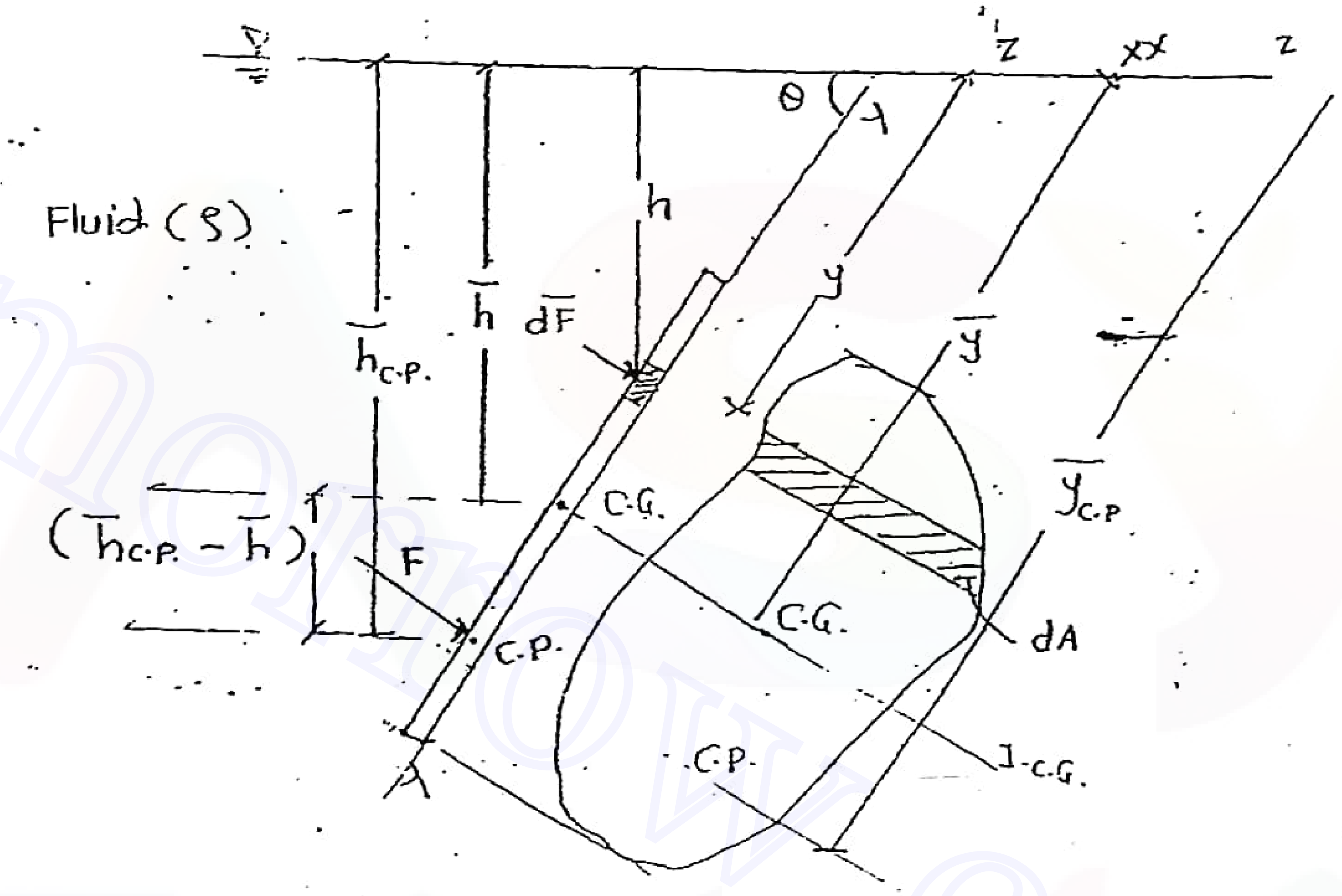
FLUID STATICS

Hydrostatic pressure on the plane surface



$$\sin\theta = \frac{\bar{h}}{\bar{y}} = \frac{h}{y} = \frac{h_{c.p}}{y_{c.p}}$$

Hydrostatic force :

$$F = \int dF$$

$$= \int P \cdot dA$$

$$= \int \rho g h \cdot dA$$

$$= \rho g \int y \sin\theta \cdot dA$$

$$= \rho g \sin\theta \int y \cdot dA$$

$$= \rho g \sin\theta \cdot \bar{y} \cdot A \qquad \because \bar{y} = \frac{\int y \cdot dA}{A}$$

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The resultant force is passing through the point on the body known as centre of pressure.

Location of C.P.

$$\int dF \cdot y = F \cdot \bar{y}_{cp} \quad \text{(varignon's theorem)}$$

$$\rho g \sin\theta \int y^2 \, dA = \rho g \bar{h} A \cdot \frac{\bar{h}_{cp}}{\sin\theta}$$

$$\bar{h}_{cp} = \frac{\sin^2\theta}{\bar{h} A} \cdot I_{xx} \qquad \int y^2 \, dA = I_{xx}$$

$$= \frac{\sin^2\theta}{\bar{h} A} \cdot (I_{cg} + \bar{y}^2 A) \qquad I_{xx} = I_{cg} + \bar{y}^2 A$$

$$= \frac{\sin^2\theta}{\bar{h} A} \cdot \left(\frac{\bar{h}^2}{\sin^2\theta} \cdot A + I_{cg} \right)$$

$$\boxed{\bar{h}_{cp} = \bar{h} + \frac{I_{cg} \sin^2\theta}{\bar{h} \cdot A}}$$

Note:

$$(\bar{h}_{cp} - \bar{h}) = \frac{I_{cg} \sin^2\theta}{\bar{h} \cdot A}$$

I_{cg}, A and θ for given surface are constant.

If surface is taken to more depth, $\bar{h}$ increases so $(\bar{h}_{cp} - \bar{h})$ will decrease. i.e. Centre of pressure (C.P.) shift towards the centre of gravity (C.G.)

If surface is taken to ∞ depth, $\bar{h} \to \infty$.

$$h_{cp} - \bar{h} = 0$$

$$h_{cp} = \bar{h} \qquad \text{or}$$

$$\bar{h}_{cp} = \bar{h} + \frac{I_{cg} \cdot \sin^2\theta}{\bar{h} \cdot A}$$

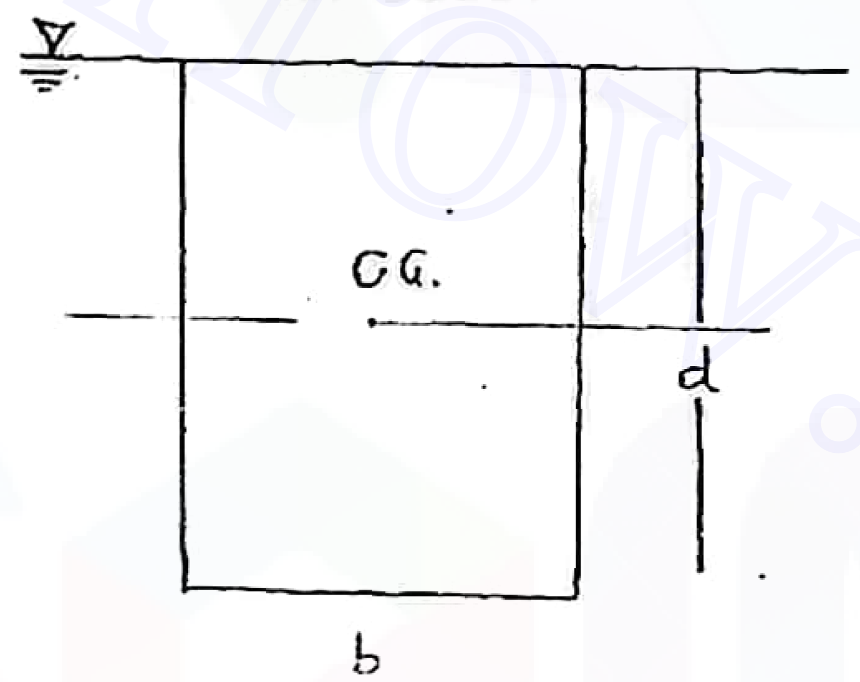
For horizontal surface, $\theta = 0$

$$\bar{h}_{cp} = \bar{h}$$

For vertical surface, $\theta = 90^\circ$

$$\bar{h}_{cp} = \bar{h} + \frac{I_{cg}}{\bar{h} \cdot A}$$

Few vertical surfaces:



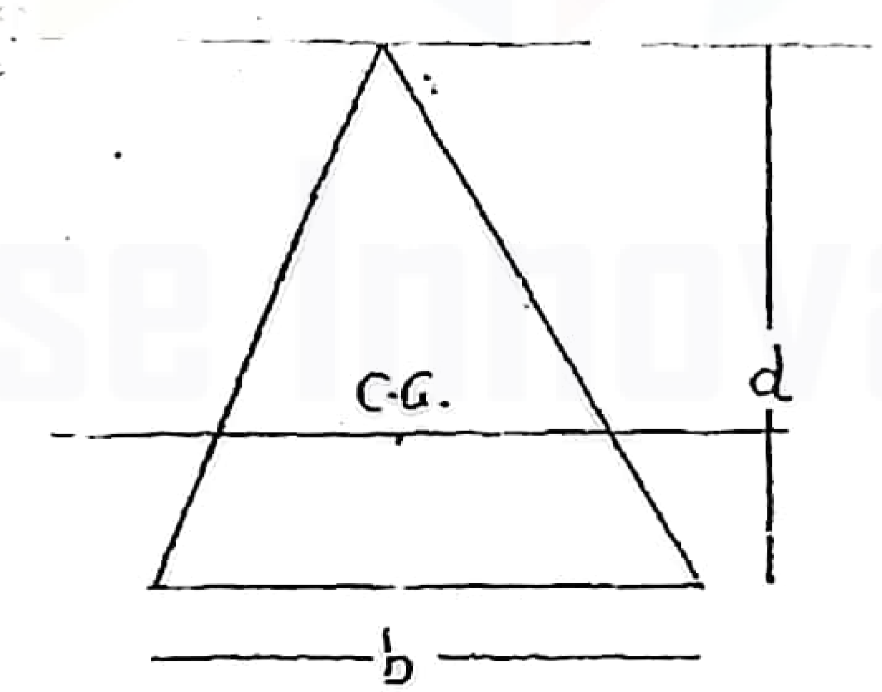
$A = b \cdot d$

$\bar{h} = d/2$

$I_{cg} = \frac{bd^3}{12}$

$\bar{h}_{cp} = \bar{h} + \frac{bd^3/12}{d/2 \cdot bd}$

$\bar{h}_{cp} = \frac{2}{3} d$



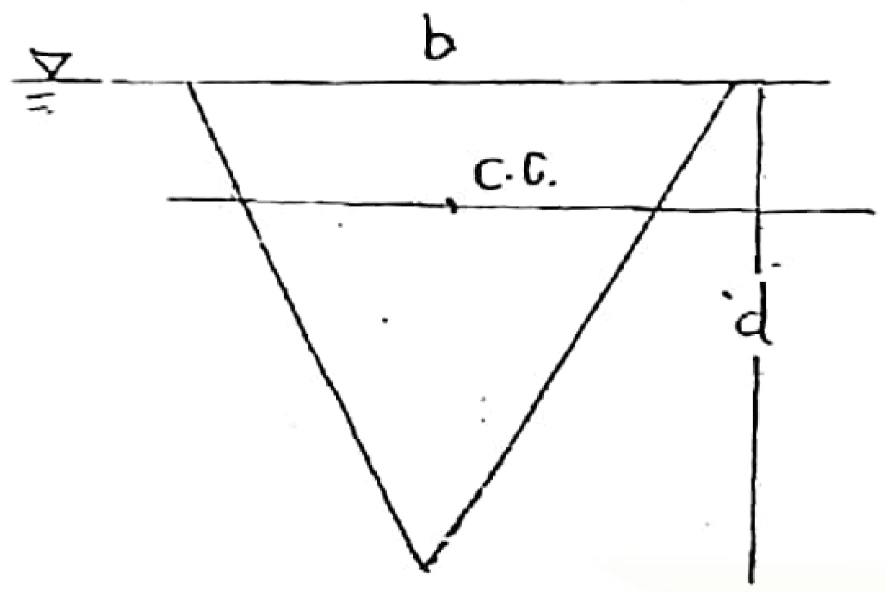
$A = \frac{bd}{2}$

$\bar{h} = \frac{2}{3} \cdot d$

$I_{cg} = \frac{bd^3}{36}$

$\bar{h}_{cp} = \frac{2}{3} d + \frac{bd^3/36}{2/3 \cdot d \times bd/2}$

$\bar{h}_{cp} = \frac{3}{4} d$



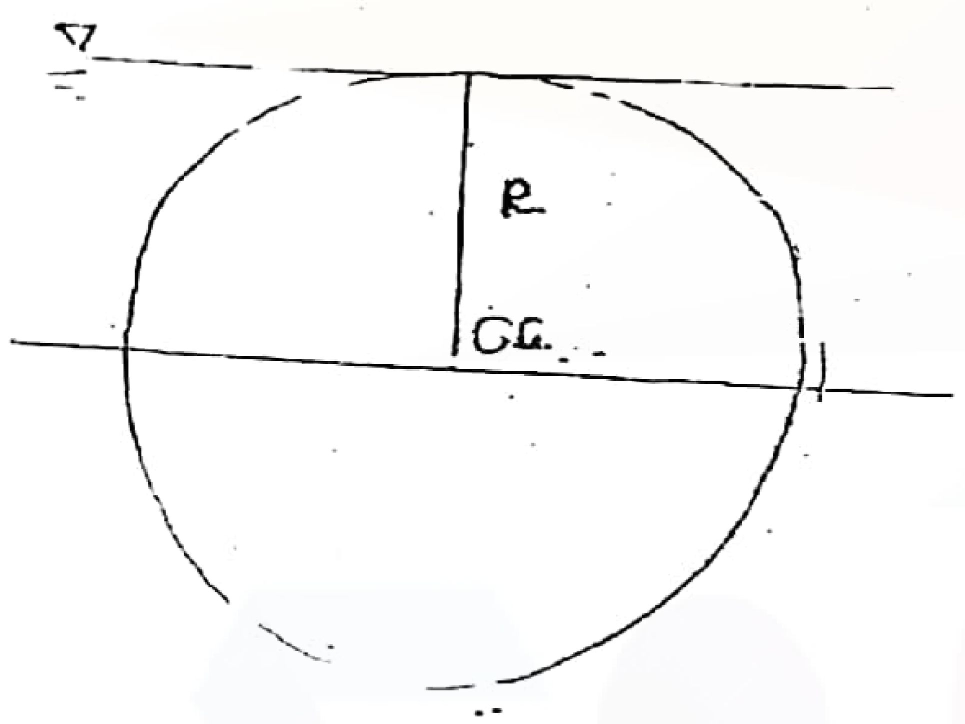
$A = \frac{bd}{2}$

$\bar{h} = \frac{2d}{3}$

$I_{c.G.} = \frac{bd^3}{36}$

$\bar{h}_{c.p.} = \frac{d}{3} + \frac{bd^3/36}{d/3 \times bd/2}$

$= \frac{d}{2}$



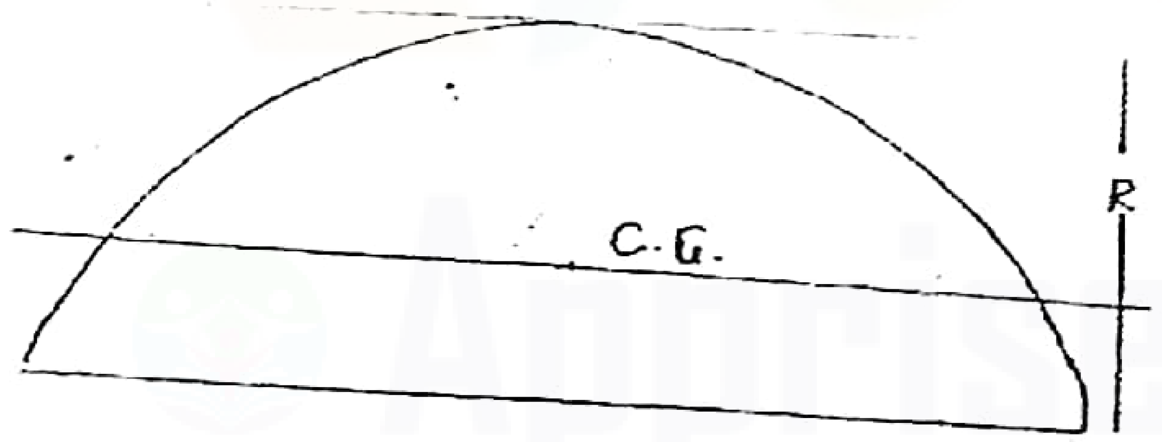
$A = \pi R^2$

$\bar{h} = R$

$I_{.CG} = \frac{\pi R^4}{4}$

$\bar{h}_{c.p.} = R + \frac{\pi R^4/4}{R \cdot \pi R^2}$

$= \frac{5}{4} R$

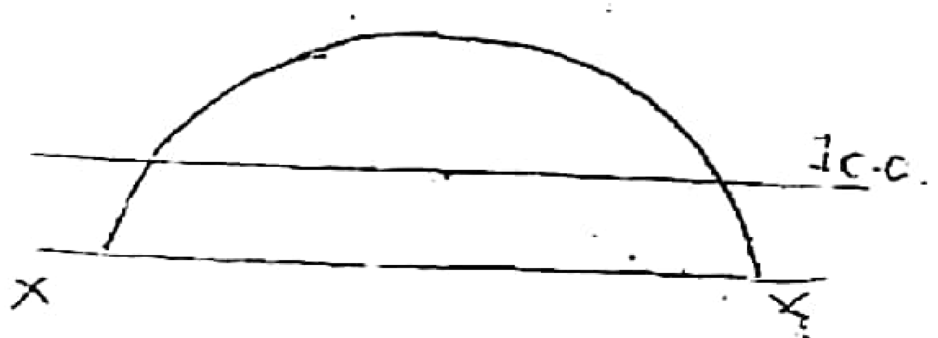


$A = \frac{\pi R^2}{2}$

$\bar{h} = \left(R - \frac{4R}{3\pi}\right)$

$I_{.C.G.} = 0.1097\ R^4$

$I_{c.G} = R^4 \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$



$I_{xx} = I_{c.G} + \left(\frac{4R}{3\pi}\right)^2 \cdot \frac{\pi R^2}{2}$

$\bar{h}_{c.p.} = \left(R - \frac{4R}{3\pi}\right) + \frac{0.1097\ R^4}{\frac{\pi R^2}{2}\left(R - \frac{4R}{3\pi}\right)}$

$= 0.5755 R + 0.1219 R$

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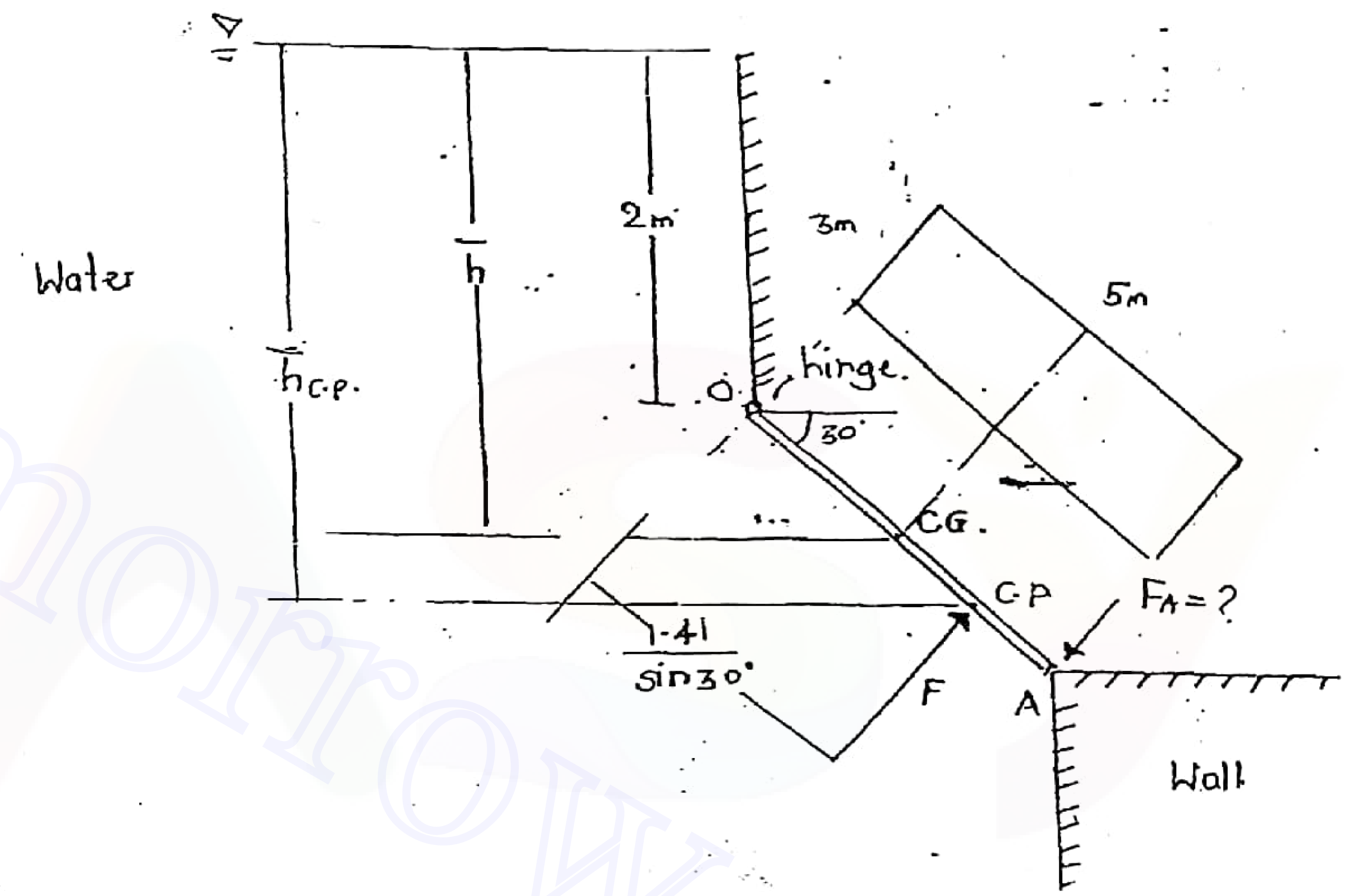
Q. Find the force F_A in order to keep th gate to be closed



Hydrostatic force:

$\bar{h} = 2 + 2.5 \sin 30°$

$\bar{h} = 3.25\ m$

$\theta = 30°$

$A = 5 \times 3 = 15\ m^2$

$I_{.c.G.} = \frac{3(5)^3}{12} = 31.25\ m^4$

$F = \rho g \bar{h} \cdot A$

$= (1000 \times 9.8 \times 3.25 \times 15)$

$= 478.237\ kN$

$\bar{h}_{c.p.} = \bar{h} + \frac{I_{CG} \sin^2\theta}{\bar{h} \cdot A}$

$= 3.25 + \frac{31.25 \times \sin^2(30)}{}$

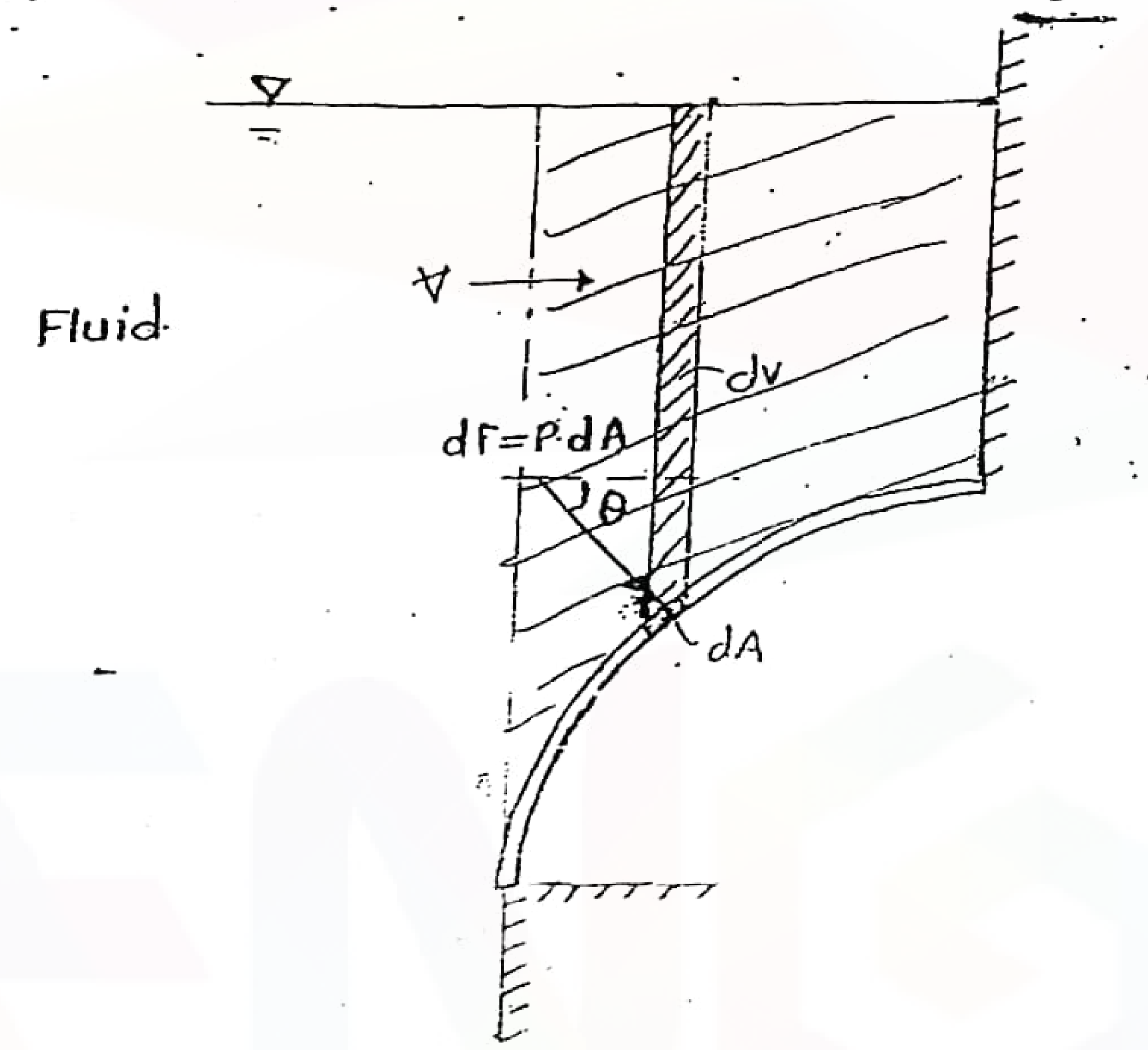
Take $M_O = 0$

$$F \times 2.82 = F_A \times 5$$

$$F_A = \frac{478.237 \times 2.82}{5} \qquad \because \frac{1.41}{\sin 30} = 2.82\,m$$

$$F_A = 269.73\ kN$$

Hydrostatic forces on curved surface:

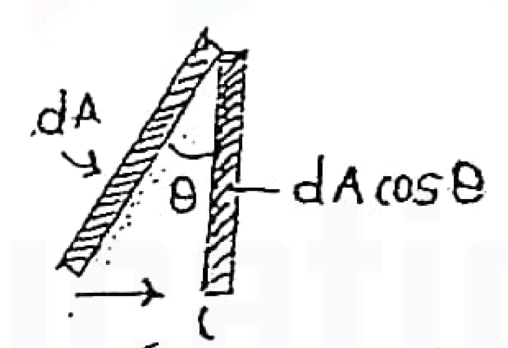


Horizontal analysis.

$$F_H = \int p \cdot dA \cdot \cos\theta$$

$$= \int \rho g h \cdot dA_H$$

$$\because (dA\cos\theta = dA_H)$$



(Area projected in horizontal direction (Area in vertical plane)

$$F_H = \rho g \bar{h} \cdot A_H$$

where A_H - projected area of curved surface in horizontal direction (on vertical plane)

Vertical analysis.

$$F_V = \int P \cdot dA \sin\theta$$

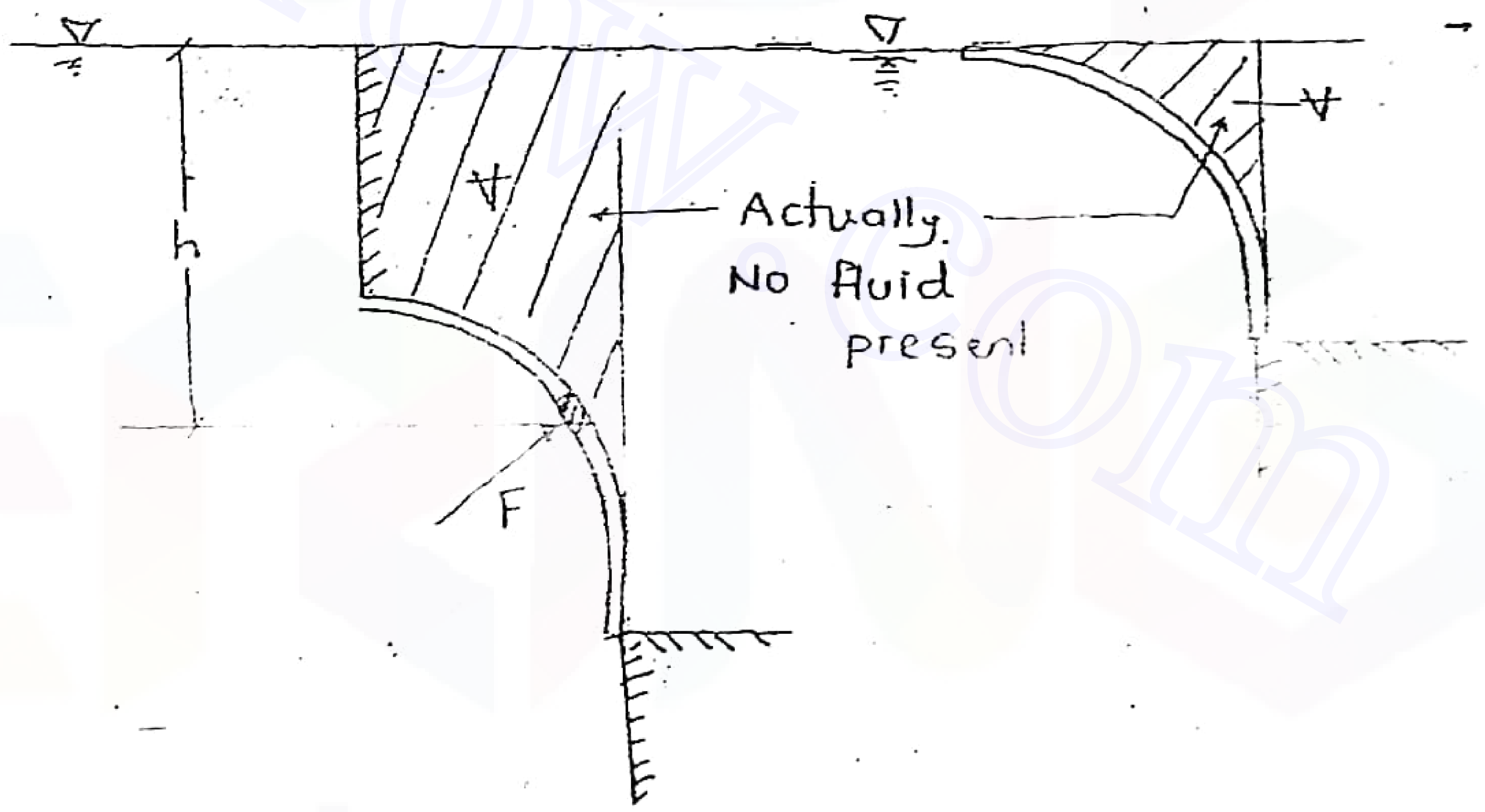
$$= \int \rho g h \cdot dA \cdot \sin\theta$$

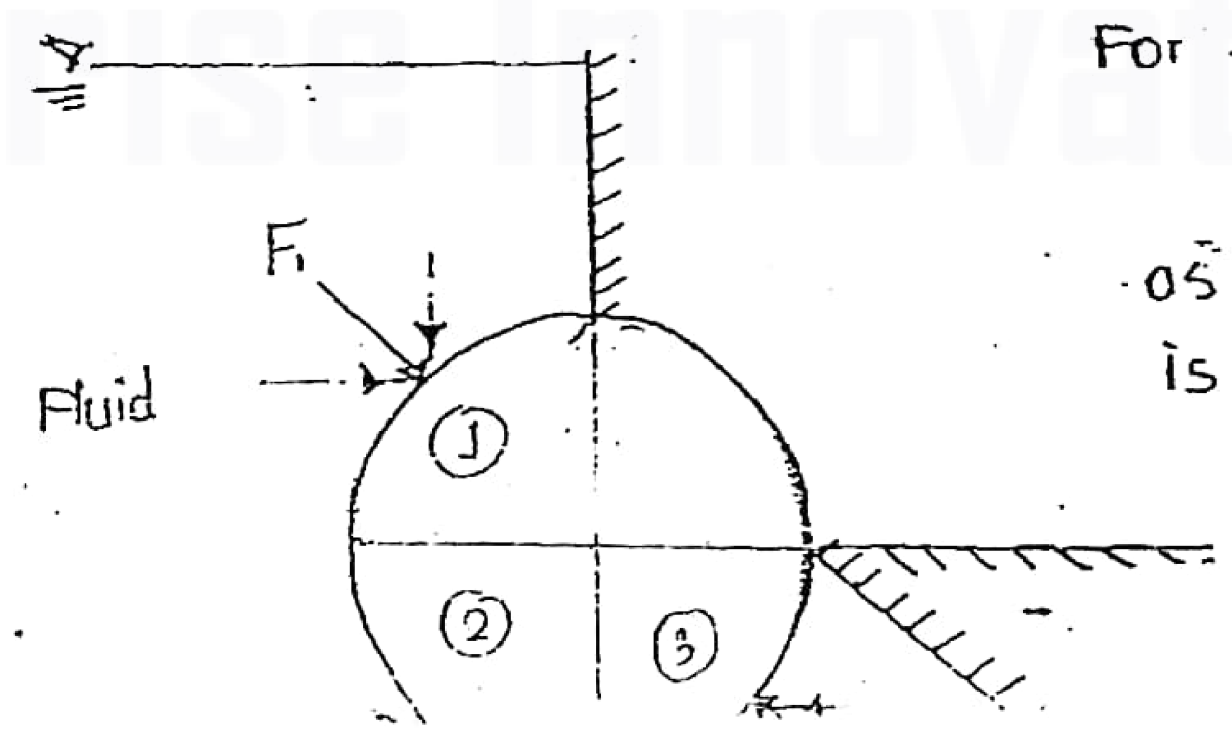
$$= \rho g \int h \cdot dA \cdot \sin\theta$$

$$= \rho g \int d\forall \qquad * \ h \cdot dA \cdot \sin\theta = d\forall$$

$$F_V = \rho g \cdot \forall$$

where $\forall$ - volume above the curved surface upto free surface.





For horizontal analysis

① + ② can be taken together as direction of horizontal force is same. (→)

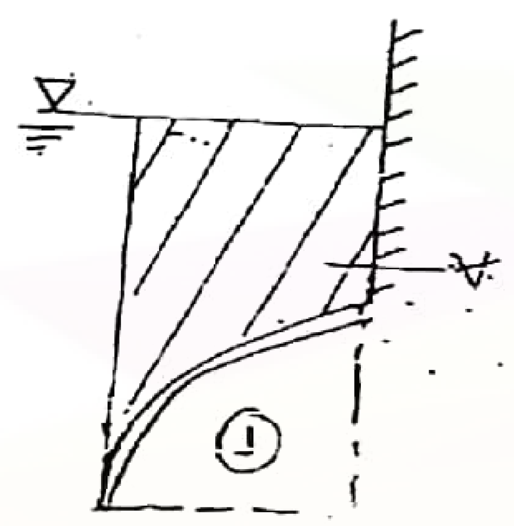
③ can be taken seperately) (←)

For vertical analysis

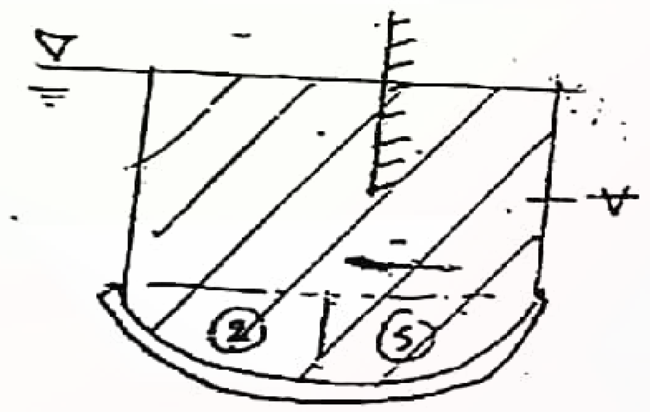
① can be taken alone (↓)

② and ③ can be taken together (↑)

For ① ↓

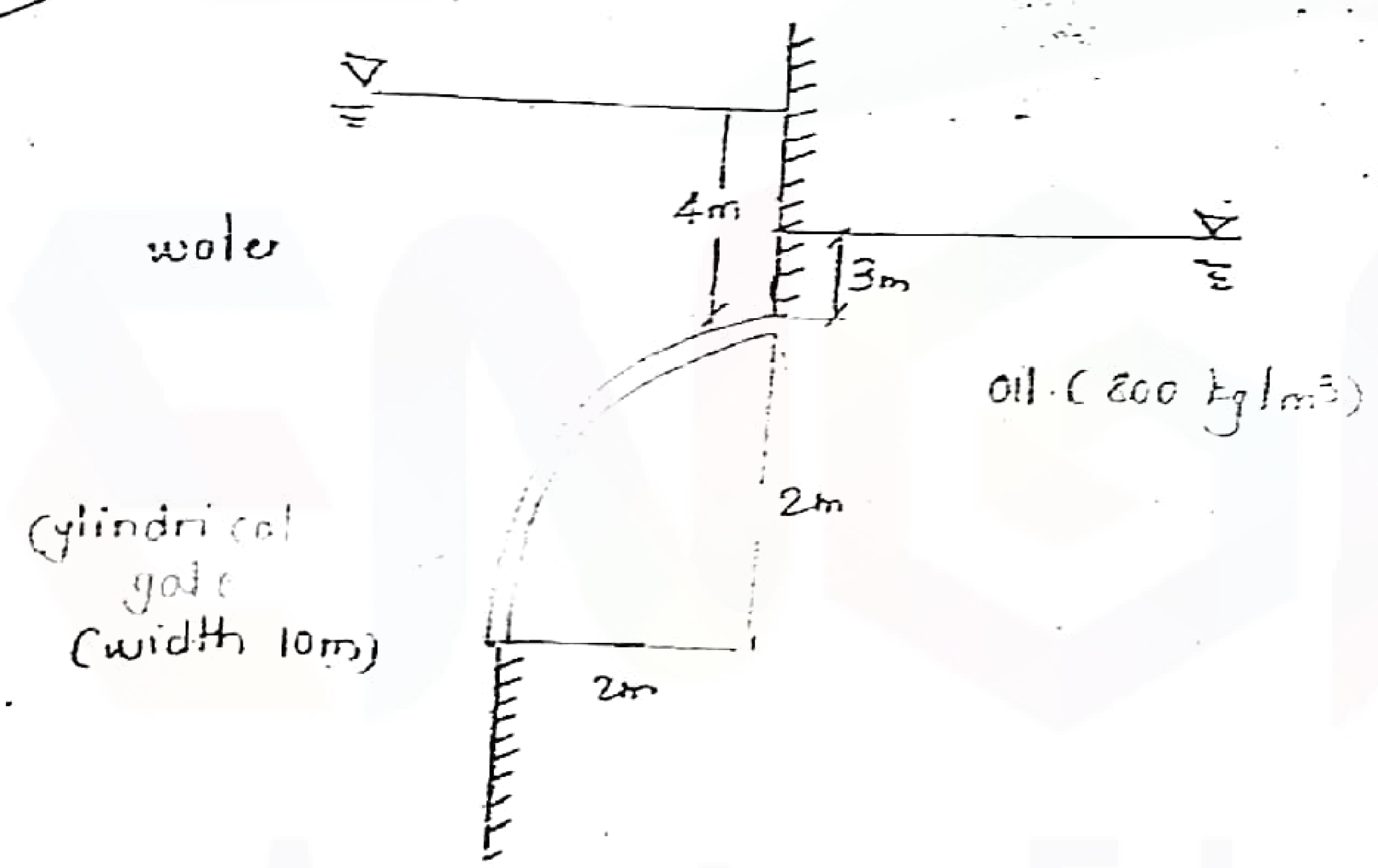


For ② & ③ ↑

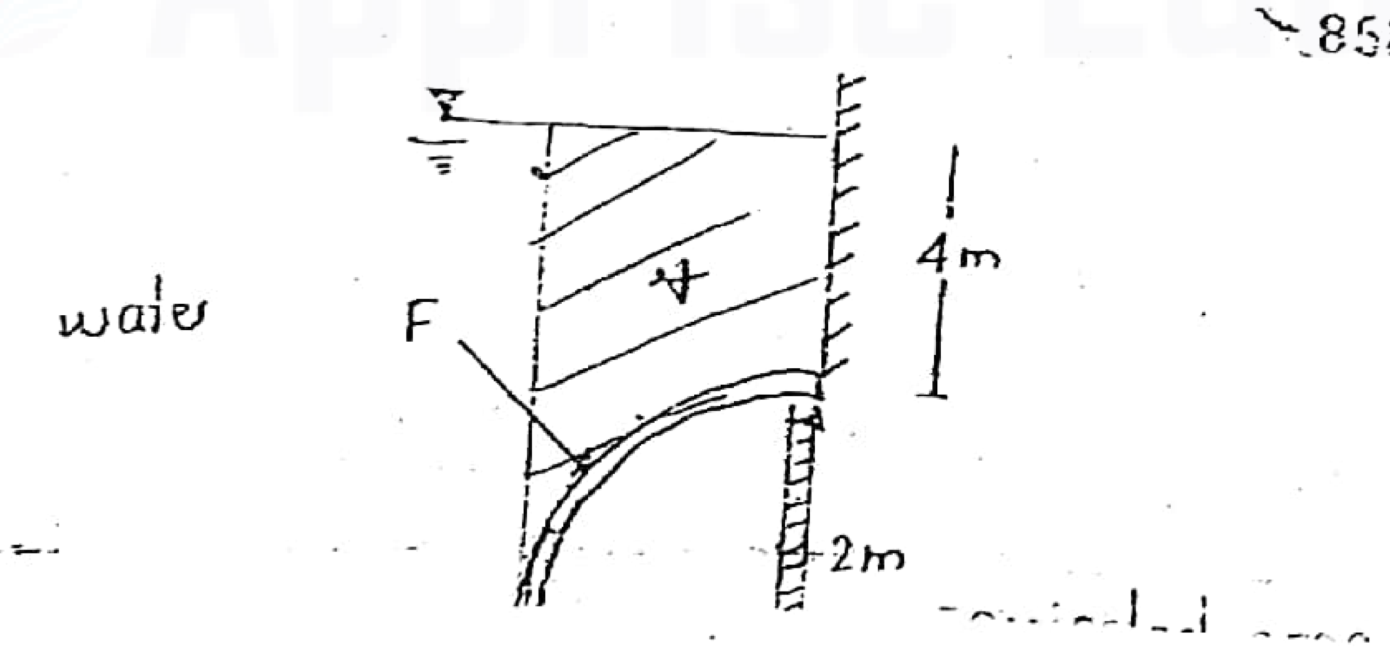


Q. Find the total hydrostatic force on the curved surface.

20 Marks



Force from the water:



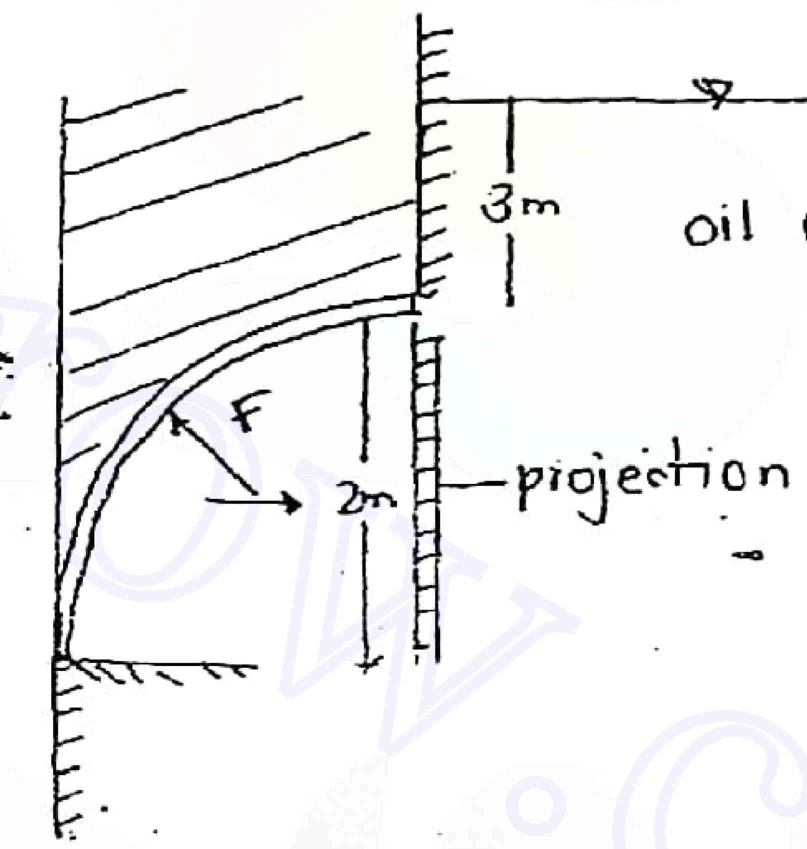
$F_{H_1} = (1000 \times 9.81 \times 5 \underbrace{(2\times10)}_{A_H})\ N \rightarrow$

$= 981\ kN$

$F_{V_1} = 1000 \times 9.81 \times \overbrace{\left[(6\times2) - \frac{\pi (2)^2}{4}\right]}^{\forall} \times 10 \downarrow$

$= 869\ kN$

Force from oil.



$F_{H_2} = (800 \times 9.81 \times 4 \underbrace{(2\times10)}_{A_H}) \leftarrow$

$= 627.84\ kN$

$F_{V_2} = 800 \times 9.81 \times \overbrace{\left[(5\times2) - \frac{\pi (2)^2}{4}\right]}^{\forall} \times 10 \uparrow$

$= 537.69\ kN$

$F_H = |F_{H_1} - F_{H_2}|$

$= 308.16\ kN$

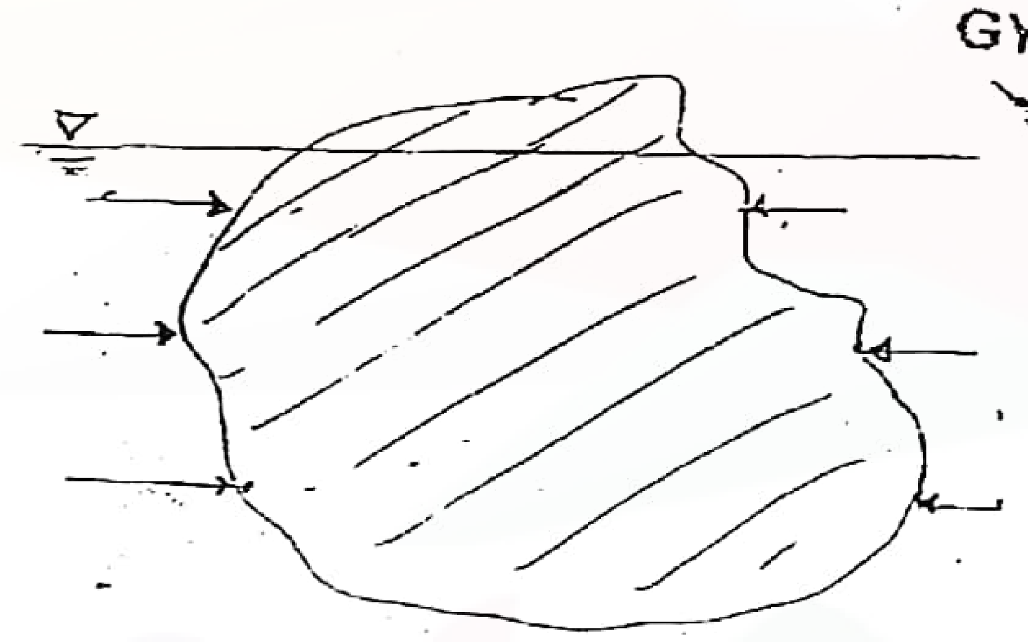
$F_V = |F_{V_1} - F_{V_2}|$

$= 331.31$

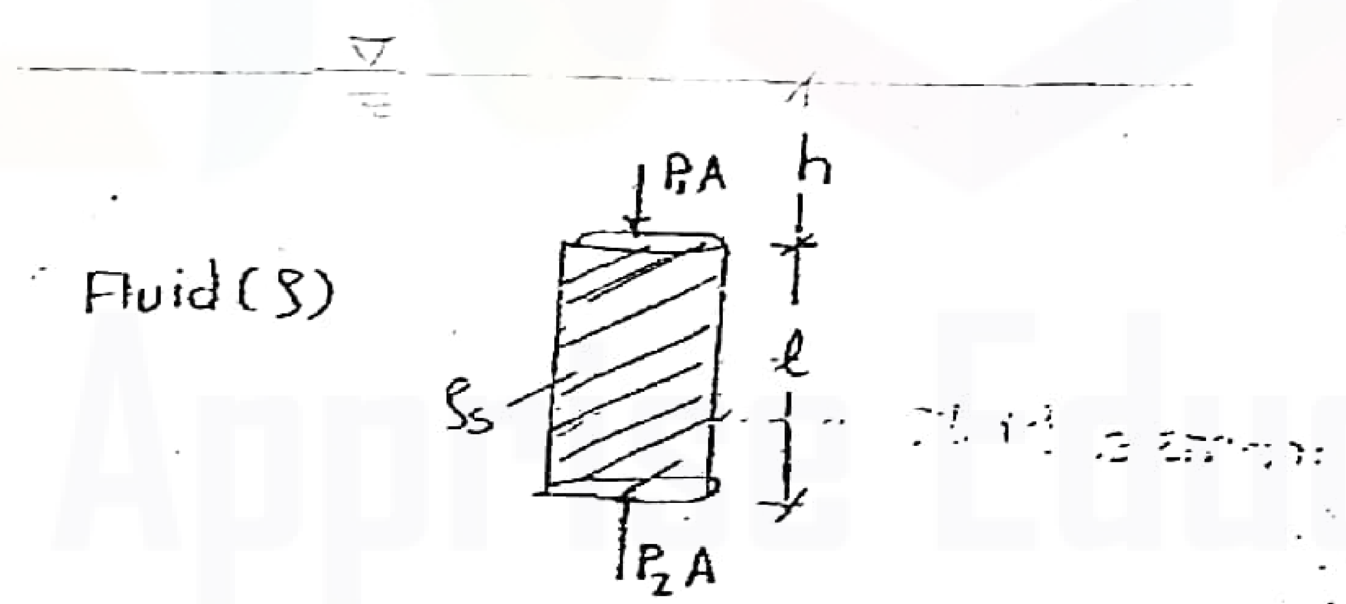
$F_R = \sqrt{F_H^2 + F_V^2}$

Hydrostatic forces on the bodies :
(Archimedes' Principle) - Buoyancy.

"When a body is submerged or emersed fully or partially inside a static fluid, then the resultant hydrostatic force acts on the body in the vertical upward direction. This force is known as Buoyant force, or upthrust and the value of this force is exactly same as the weight of the displaced fluid by the body, when it is submerged."



Net horizontal hydrostatic force on body is zero.
and net vertical hydrostatic force is vertical upward.



Hydrostatic force,

$$F_B = P_2 A - P_1 A$$

$$= [\rho g (h + l) - \rho g h] A$$

$$F_B = \rho g A l$$

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$$\boxed{F_B = \rho g \cdot \forall'_{Body}}$$

m - mass of body
m' - mass of submerged body
$\forall_{Body}$ - volume of body
$\forall'_{Body}$ - volume of submerged body.
$\forall$ - volume of fluid displaced.

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$m' \leq m$ — may be fully or partially
$\forall'_{Body} \leq \forall_{Body}$ — submerged

$\forall = \forall'_{Body}$ - (Archimedes principle)

$$F_B = \rho \cdot g \cdot \forall$$

$$= \rho \cdot g \cdot V_{Body}'$$

$$= \rho \cdot g \left(\frac{m'}{\rho_s} \right)$$

$$= \frac{m' g}{\left(\frac{\rho_s}{\rho} \right)}$$

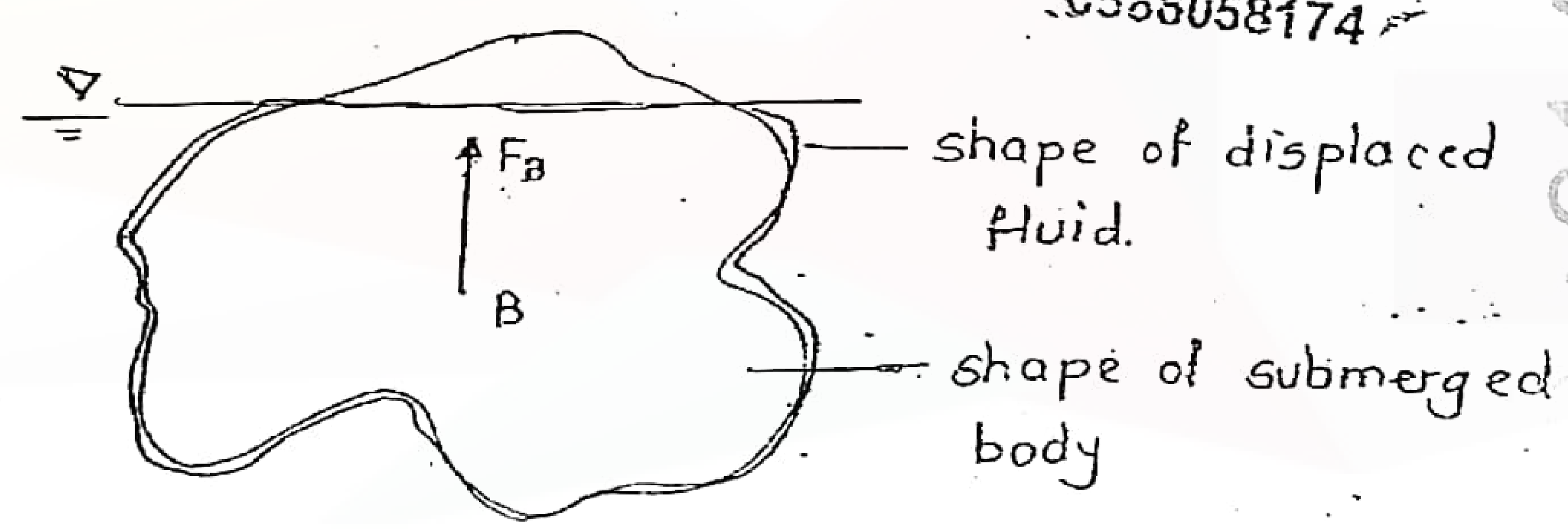
$$\boxed{F_B = \frac{m' g}{(R.D.)}} \qquad \left(R.D. = \frac{\rho_s}{\rho} \right)$$

where R.D. - relative density of body w.r.t. fluid.

Centre of Buoyancy (B):

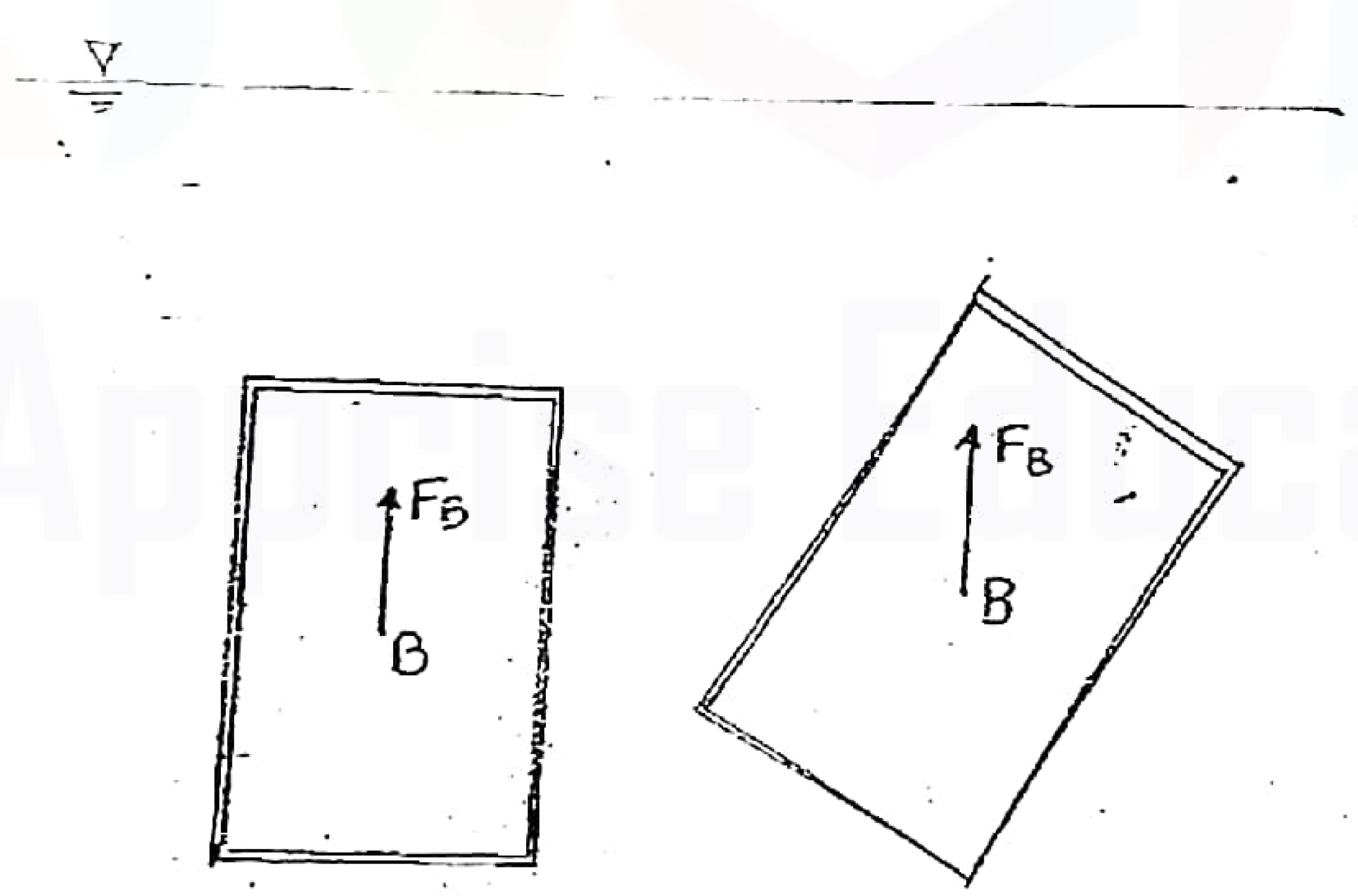
"It is a point in the body from where the buoyant force (upthrust - hydrostatic force) acts."

This point is same as the centre of gravity of displaced fluid. Body can have many surface and thus many centres of pressure. Therefore centre of Buoyancy represents combined effect of all centre of pressures of all surfaces of body.



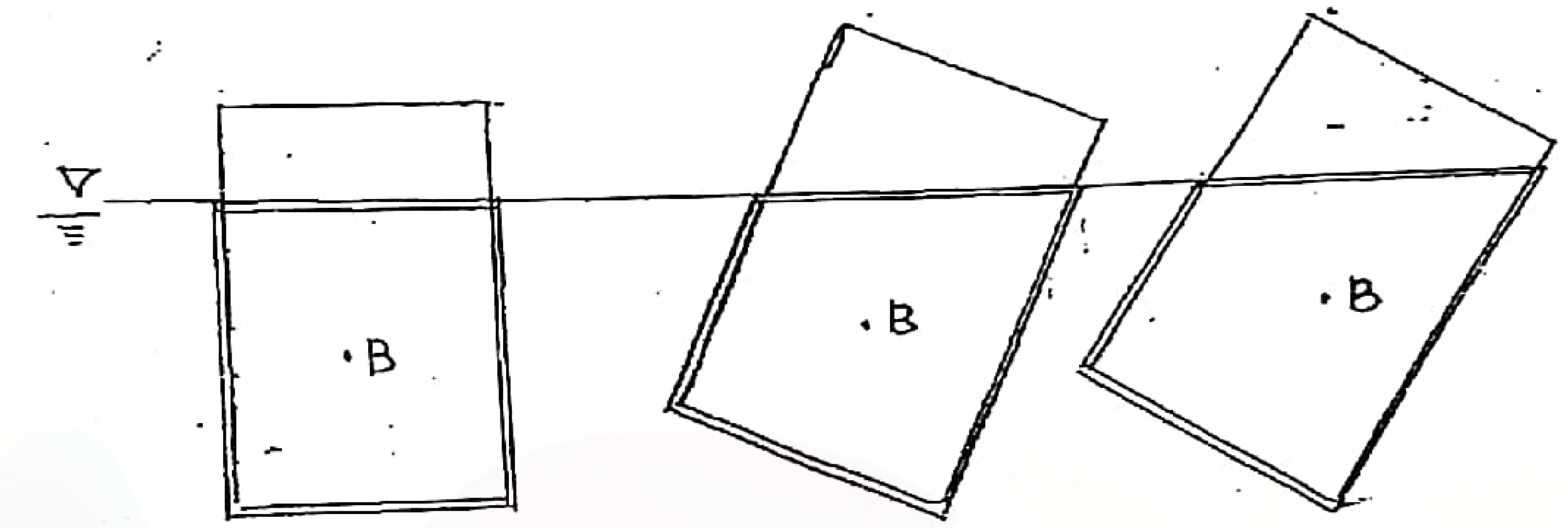
Submerged body:

Under disturbance centre of buoyancy doesn't change.

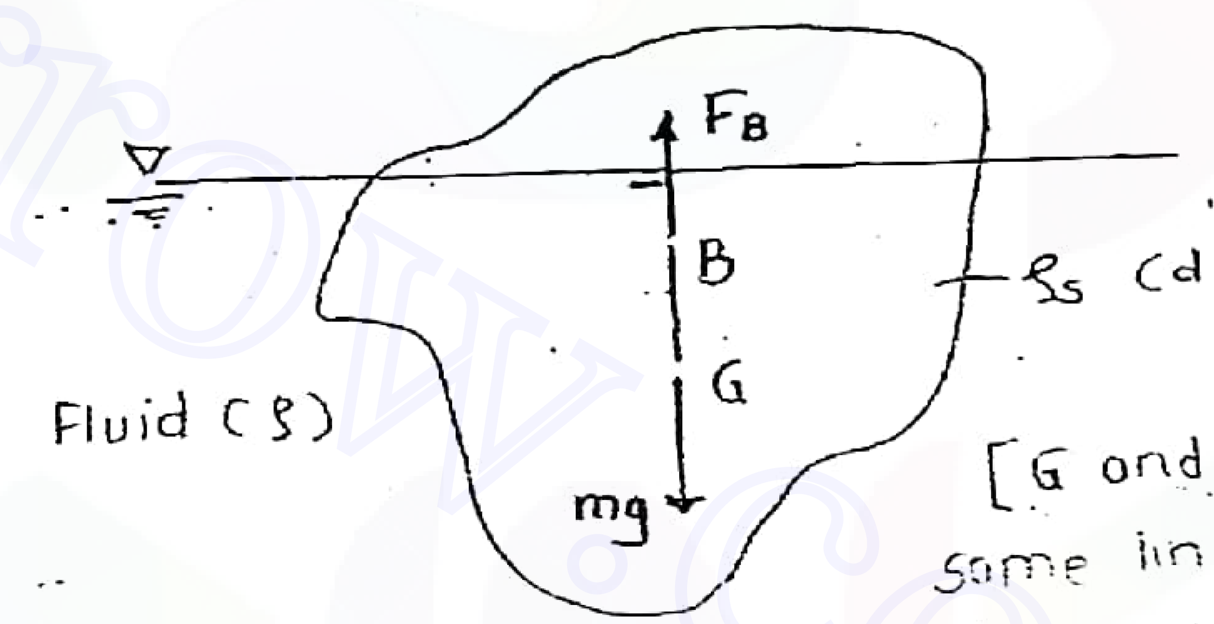


Floating body:

Under disturbance centre of buoyancy changes



Concept of floatation:



[G and B lie along same line for floatation if n... the body will orient itself bring them in line]

For floatation,

$$mg = F_B$$

$$mg = \frac{m'g}{R.D.}$$

$$m' = m \cdot R.D$$

$$m' \leq m$$

$$m \geq m \cdot R.D.$$

$$R.D. \leq 1$$

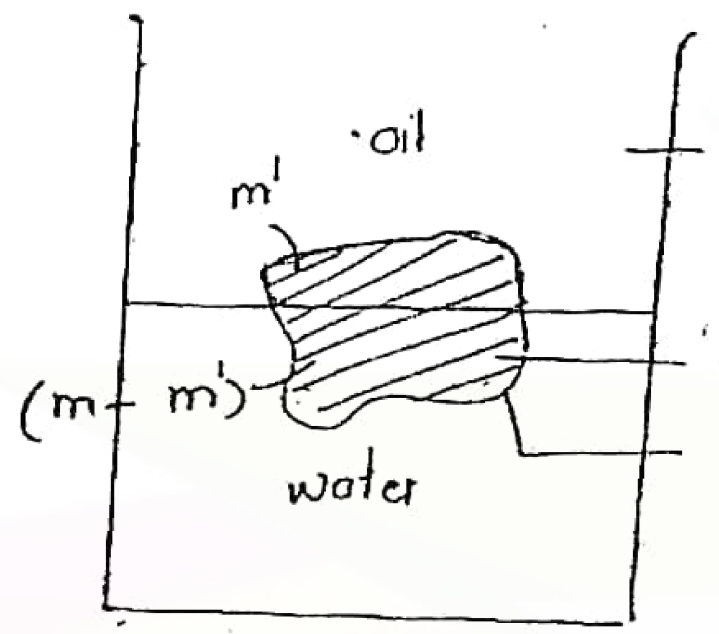
$$\left(\frac{\rho_s}{\rho}\right) \leq 1$$

$$\rho_s \leq \rho$$

m - mass of bo...
m' - submerged ... of body

Q. Find fraction of body in oil and water.

2 Marks



$\rho = 800\ kg/m^3$

$\rho_s = 850\ kg/m^3$

Total mass of body $= m$

For floatation.

$$mg = F_{B\,oil} + F_{B\,water}$$

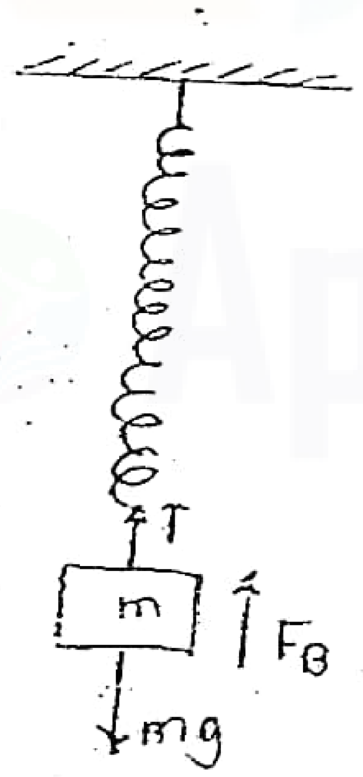
$$mg = \frac{m'g}{\left(\frac{850}{800}\right)} + \frac{(m-m')\cdot g}{\left(\frac{850}{1000}\right)}$$

$$850\ m = 800\ m' + 1000\ m - 1000\ m'$$

$$\frac{m'}{m} = 0.75 \quad \text{i.e. } 75\%\ \text{mass in oil.}$$

Concept of apparent weight :

For real weight (taken in air)



spring balance

T - tension in the spring (reading of spring balance)

F_B - buoyant force by air

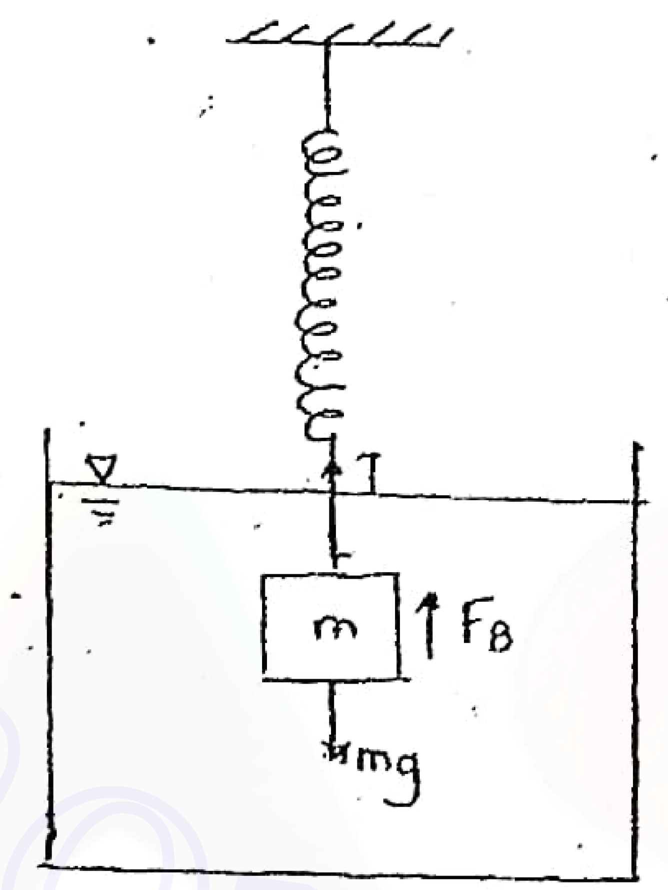
$$T + F_{B\,(by\ air)} = mg$$

$$T = mg - F_{B\,(by\ air)}$$

$$T = mg. \quad \text{- (Real weight)}$$

$F_{B\,(by\ air)} = 0 \quad \because \rho_{air}$ is 0

When weight is taken in liquid :



$\rho_s > \rho$ - to submerge the body in fluid.

$$T + F_B = mg$$

$$T = mg - F_B$$

Apparent weight $(T) = mg - F_B$

$\therefore$ Reduction in weight appeared

$$= mg - (mg - F_B)$$

$$= F_B \quad \text{i.e. Buoyant force.}$$

Q. A body appears to have 1000 N in water and 1200 N in oil of density 800 kg/m³. find

(i) Real wt. of body

(ii) Mass of body

(iii) Volume of body

(iv) Density of body.

For water.

$$1000\ N = mg - \forall_{body} \times 1000 \times g$$

$$1200\ N = mg - \forall_{body} \times 800 \times g$$

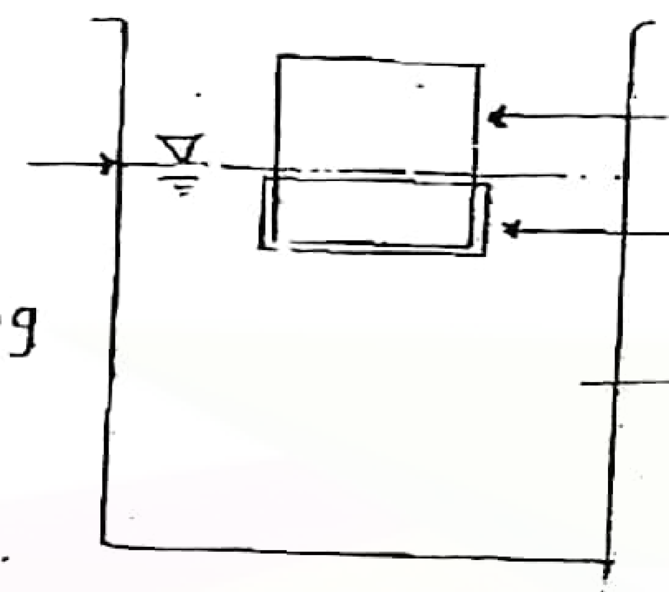
$$1000 + \forall_{body}\ 1000 \times g = 1200 + \forall \times 800 \times g$$

$$1000 + 9000\ \forall_{body} = 1200 + 7200\ \forall_{body}$$

$$1800\ \forall_{body} = 200.$$

$$\forall_{body} = \frac{1}{9}.$$

$$mg = 1000 + 1000 \times 9.8$$



For floatation.

$$mg = F_B$$

$$mg = \forall \cdot \rho g$$

$$\therefore \forall = \frac{m}{\rho}$$

Recovery of water. (conservation of mass)

m kg of ice → m kg of water (after melting)

$$\forall_{water} = \frac{\text{mass of water recovered}}{\text{density of water}}$$

$$\forall_{water} = \frac{m}{\rho}$$

$$\forall = \forall_{water}$$

The water level will remain same after melting of ice.

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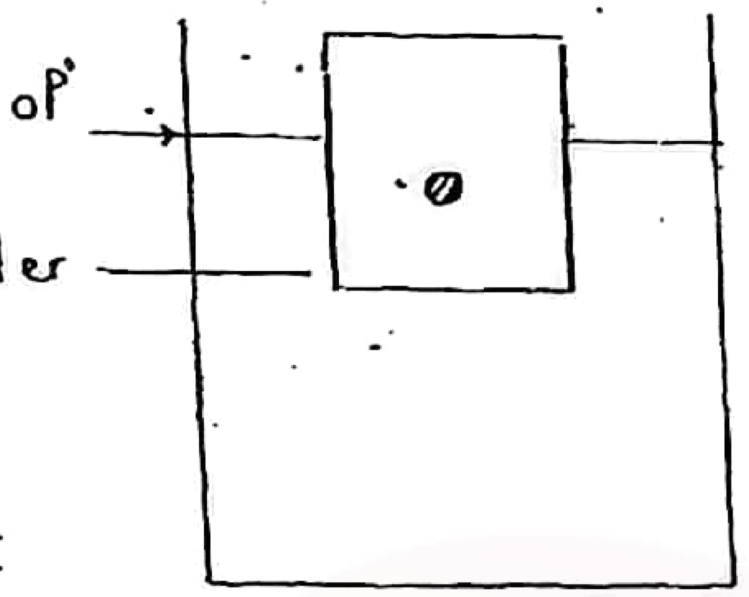

$$mg = F_B$$

$$(m_1+m_2)\,g = \forall \cdot \rho \cdot g$$

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$$\forall = \frac{m_1}{\rho} + \frac{m_2}{\rho}$$

Recovery of water.

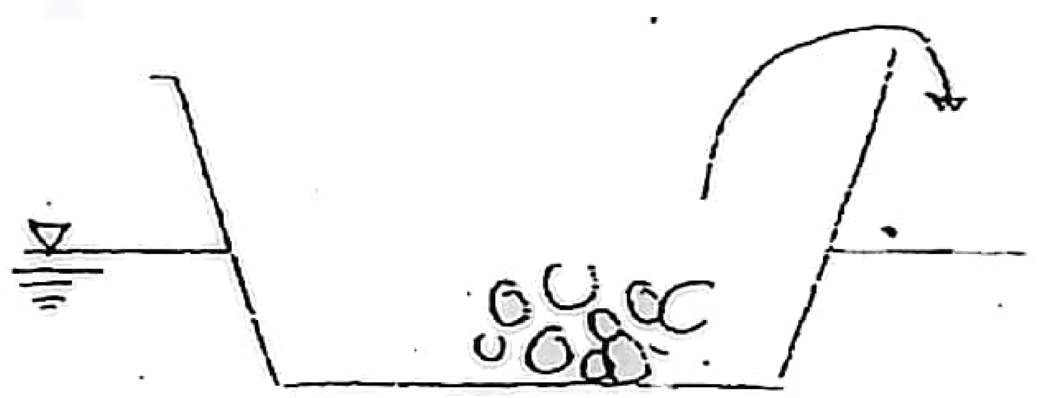
by ice $= \frac{m_1}{\rho}$

by nail $= \frac{m_2}{\rho_{nail}}$

$$\forall_{water} = \frac{m_1}{\rho} + \frac{m_2}{\rho_{nail}}$$

$$\rho < \rho_{nail}$$

The volume of water recovered by nail will be less i.e. water level will go down.



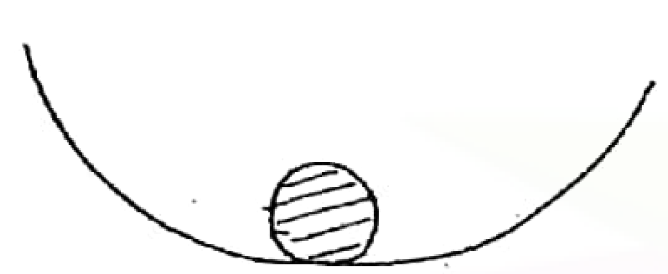
If stone is thrown out of boat then the boat will come up.

If sea water comes from sea water to river water, the

Equilibriums and types of Equilibriums :

$\Sigma \vec{F} = 0$ – translatory equilibrium

$\Sigma \vec{M} = 0$ – rotationary equilibrium.



stable equilibrium

(body will not leave its original equilibrium position after applied force)

unstable equilibrium

(body will never come back to its original equilibrium position)

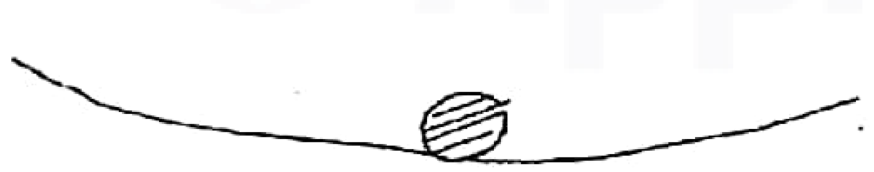
Neutral equilibrium

(body will gain the new equilibrium position)

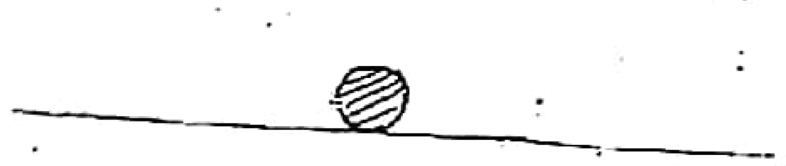
More stable

[The body will oscillate (the no. of oscillation depends on the fluid properties)

More stability – less period of oscillation.]



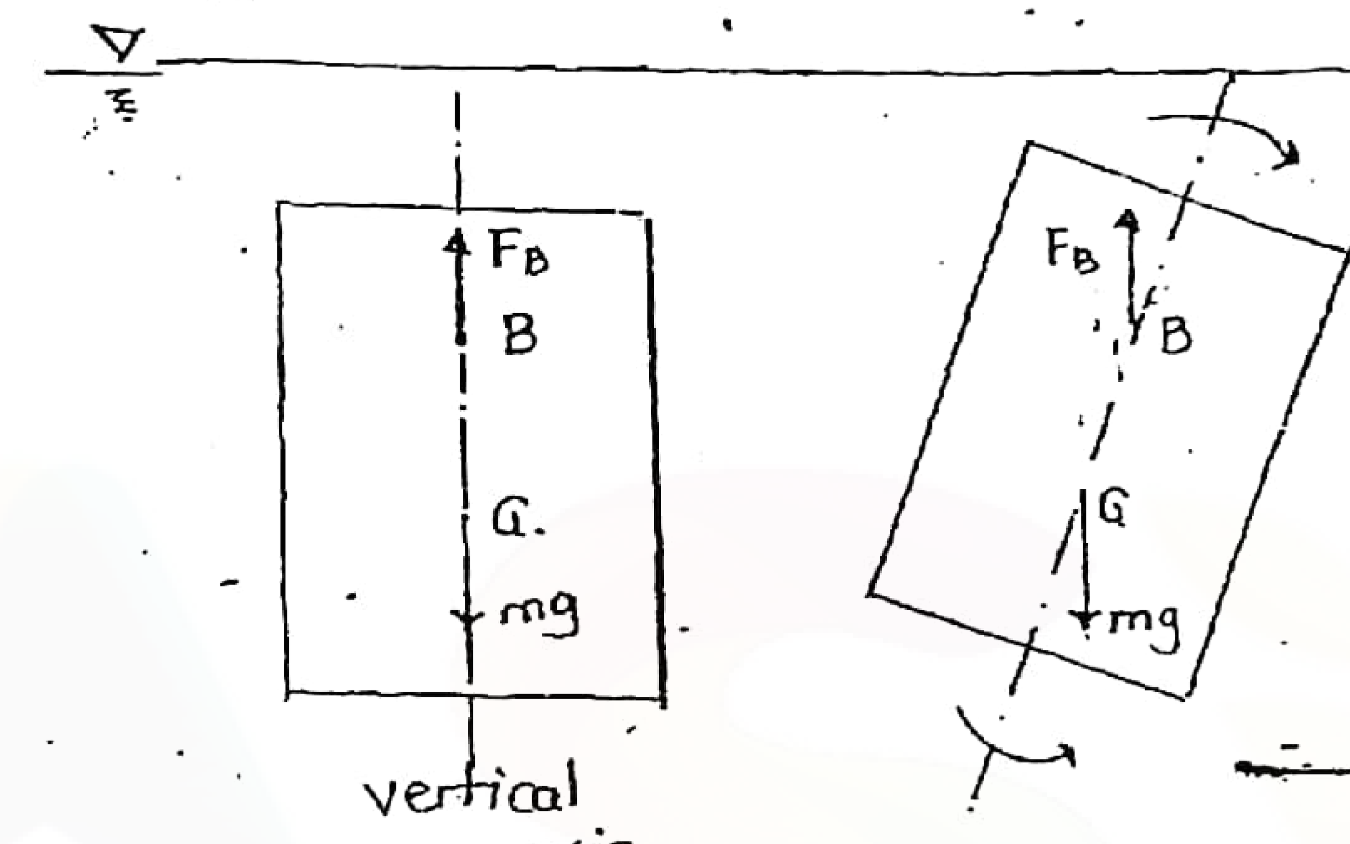
→ less stable



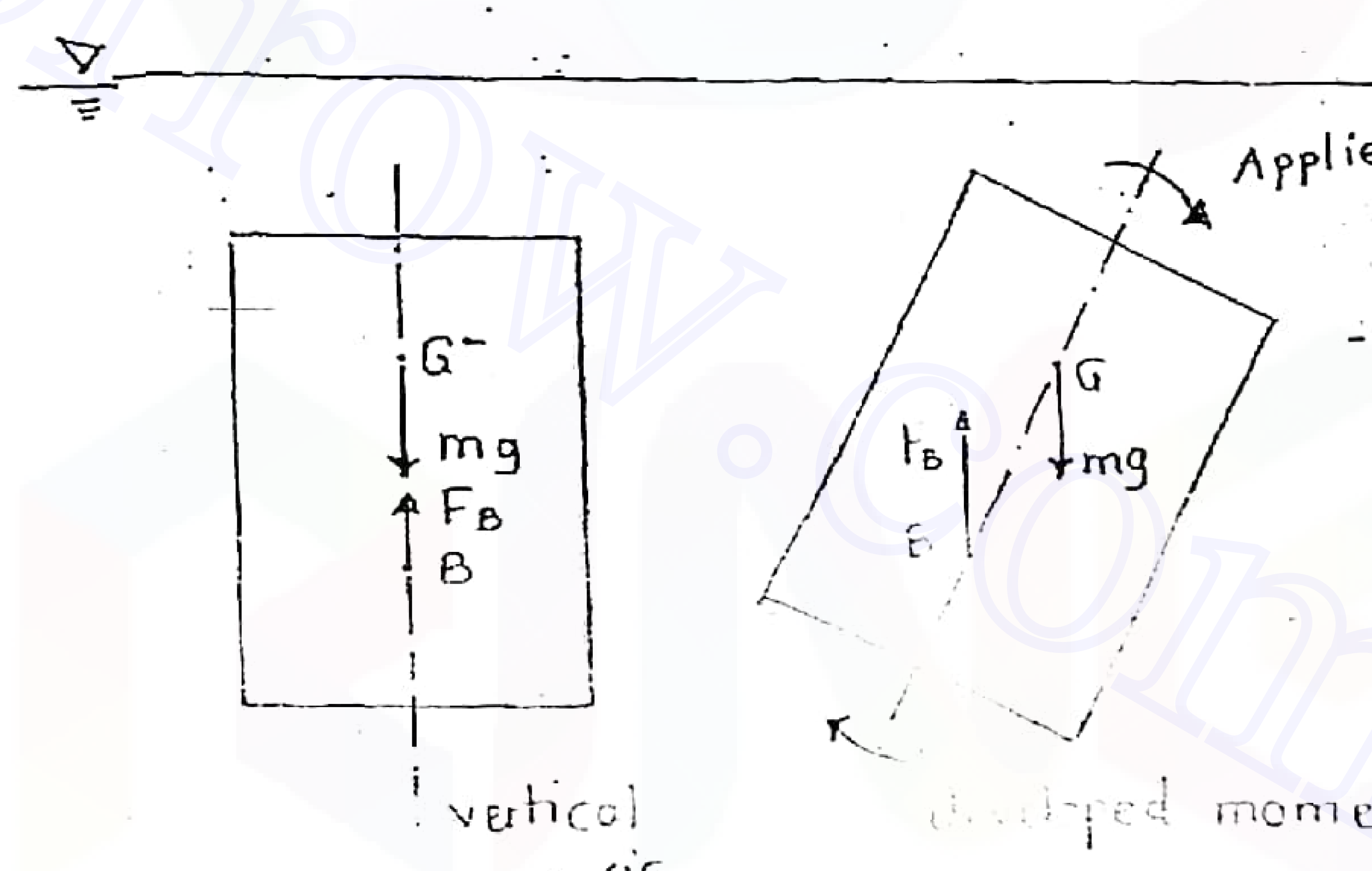
→ stability zero (neutral equilibrium)

The neutral equilibrium is obtained

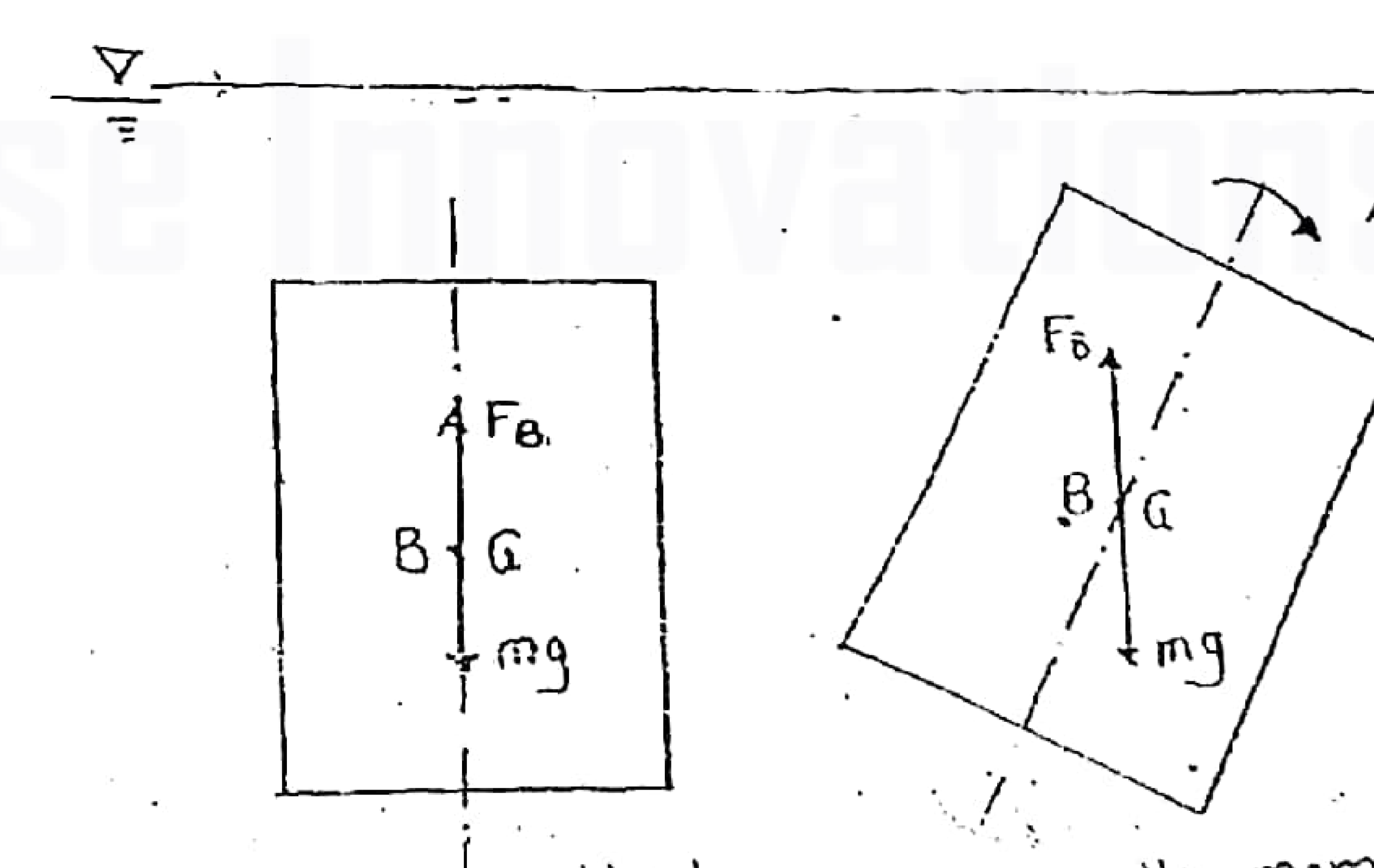
Stability of submerged bodies :



Stable condition



Unstable condition



Stability of floating bodies:

When the floating bodies are disturbed angularly, there centre of buoyancy is continuously changing. Therefore the stabilities of floating bodies are analysed w.r.t. the different point known as Metacentric point which is also known as Metacentre. It is represented by M.

Metacentre (M):

" It is defined as point of ~~intersection~~ of buoyant force line with the vertical axis of the body under very small angular tilt given to the body.

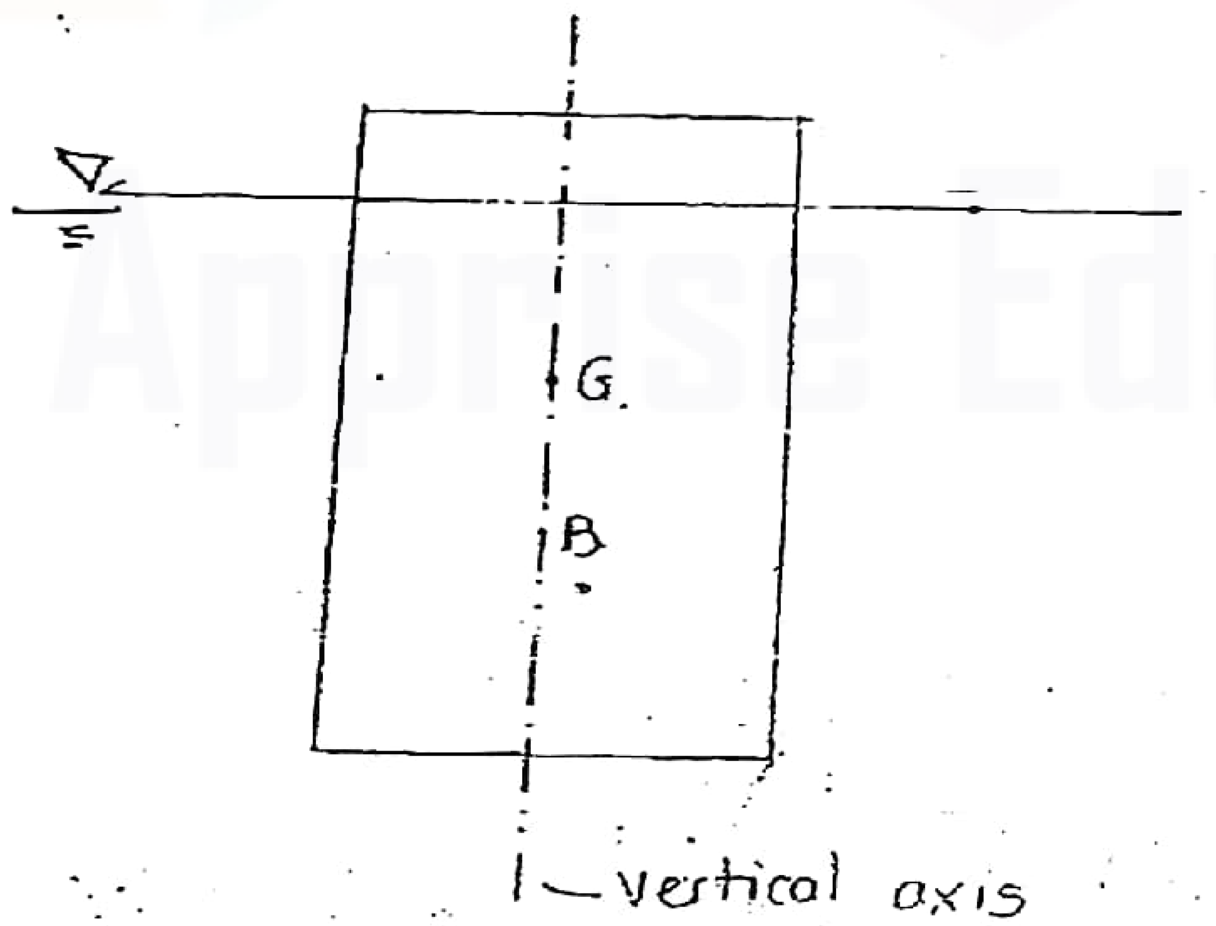
The disturbance should be small so as to not change the position of body.

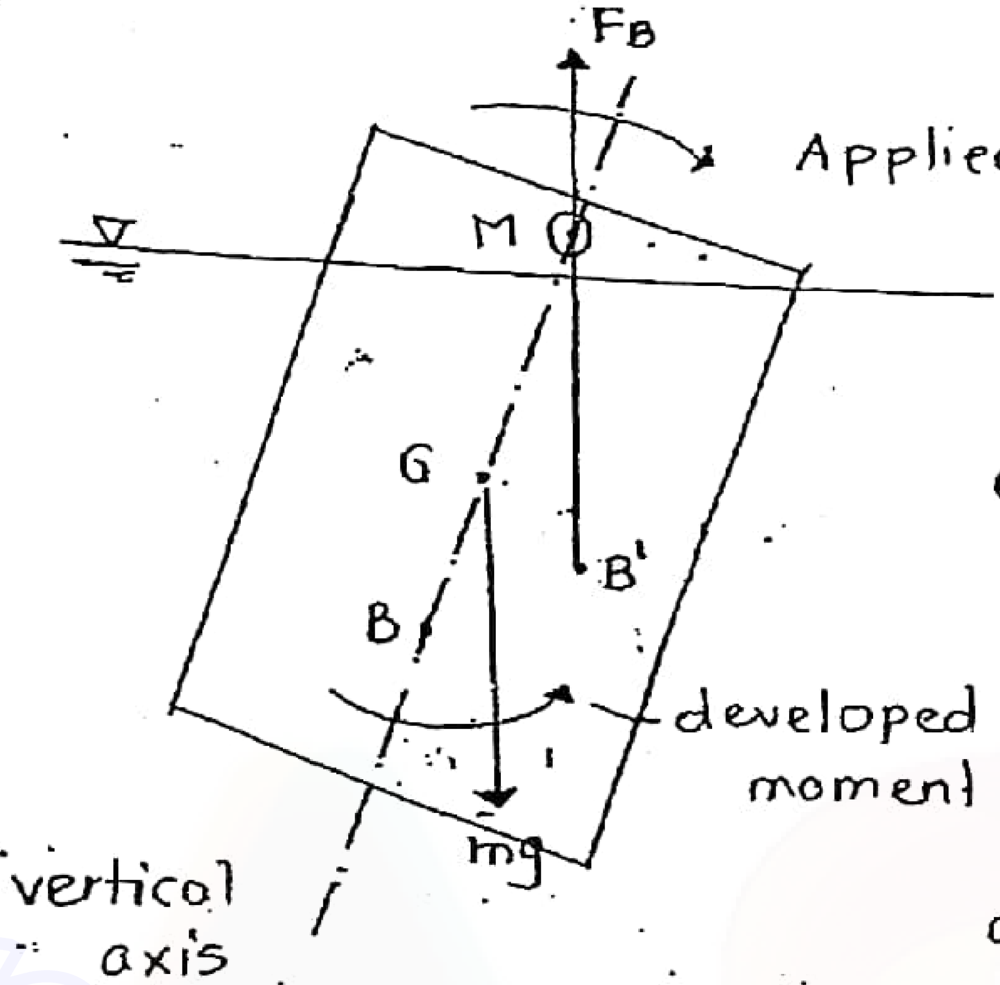
$\sin 2^{\circ} = 0.0348$

$2^{\circ} = 0.349^{c}$ i.e. θ is very small

$\sin\theta \simeq \theta$

This point may be inside or outside the body. It is also defined as a point about which the body is oscillating when they are slightly disturbed and released.

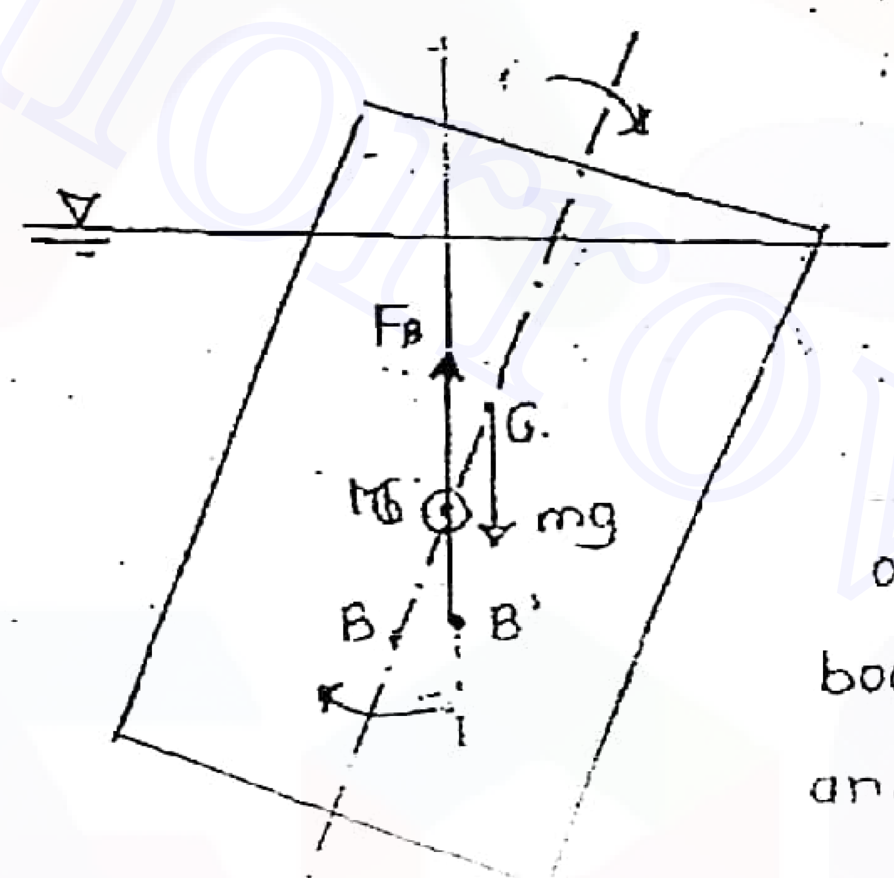




stable equible

(As the applied moment is balanced by developed moment due to mg about M)

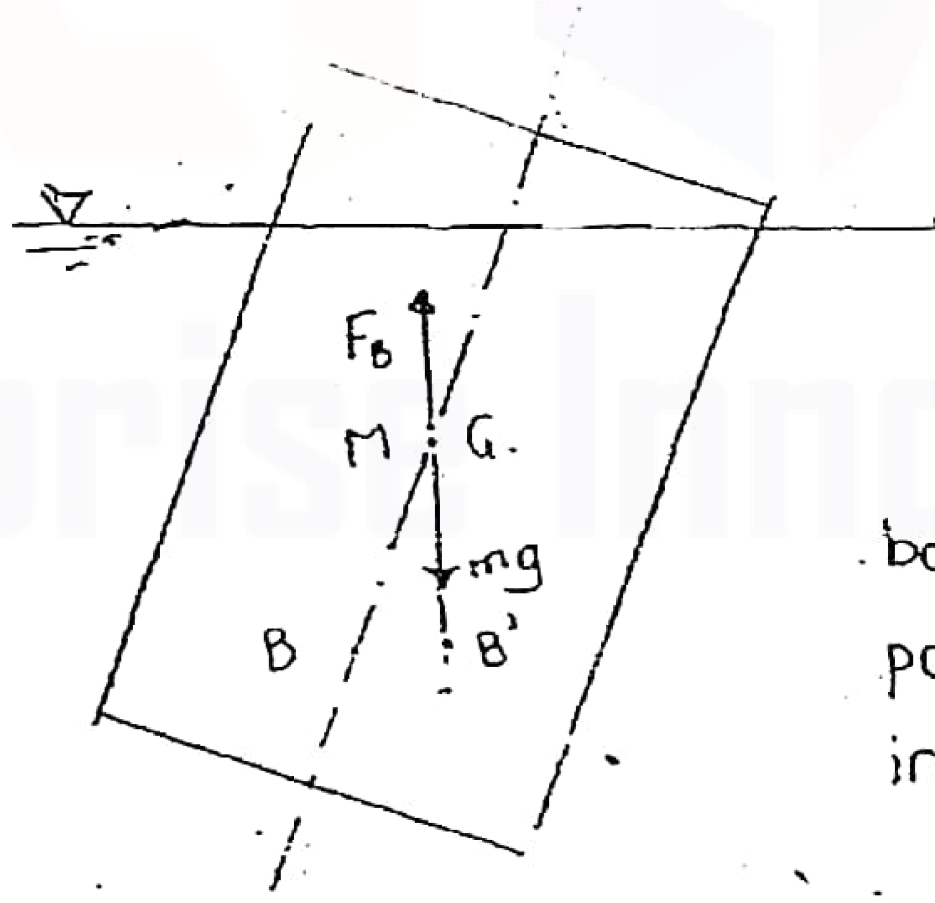
∴ Metacentre (M) should lie above the centre of gravity (C.G.)



Unstable equilibrium

(applied moment causes development of moment in same direction i.e. body will not come to original position or another equilibrium

Metacentre (M) should lie below C.G.

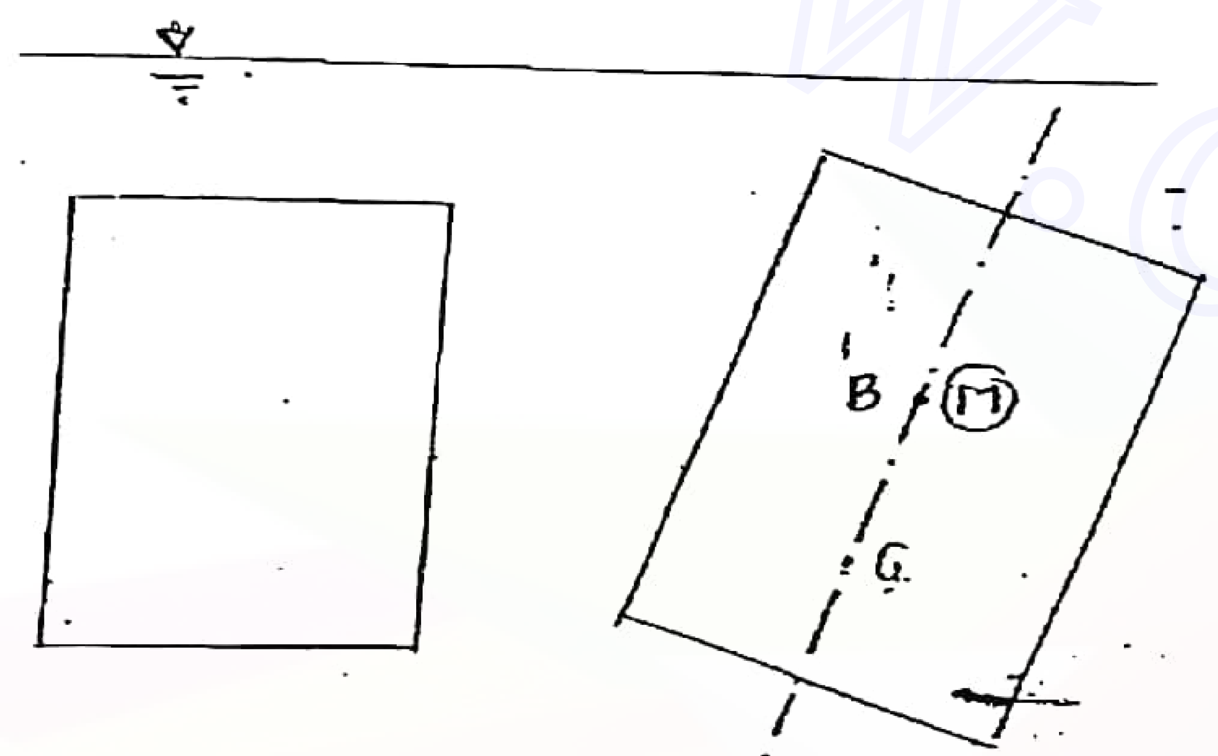


Neutral Equilibrium

(Applied moment rotates the body where it attains new equilibrium position i.e. no moment is developed in the body.

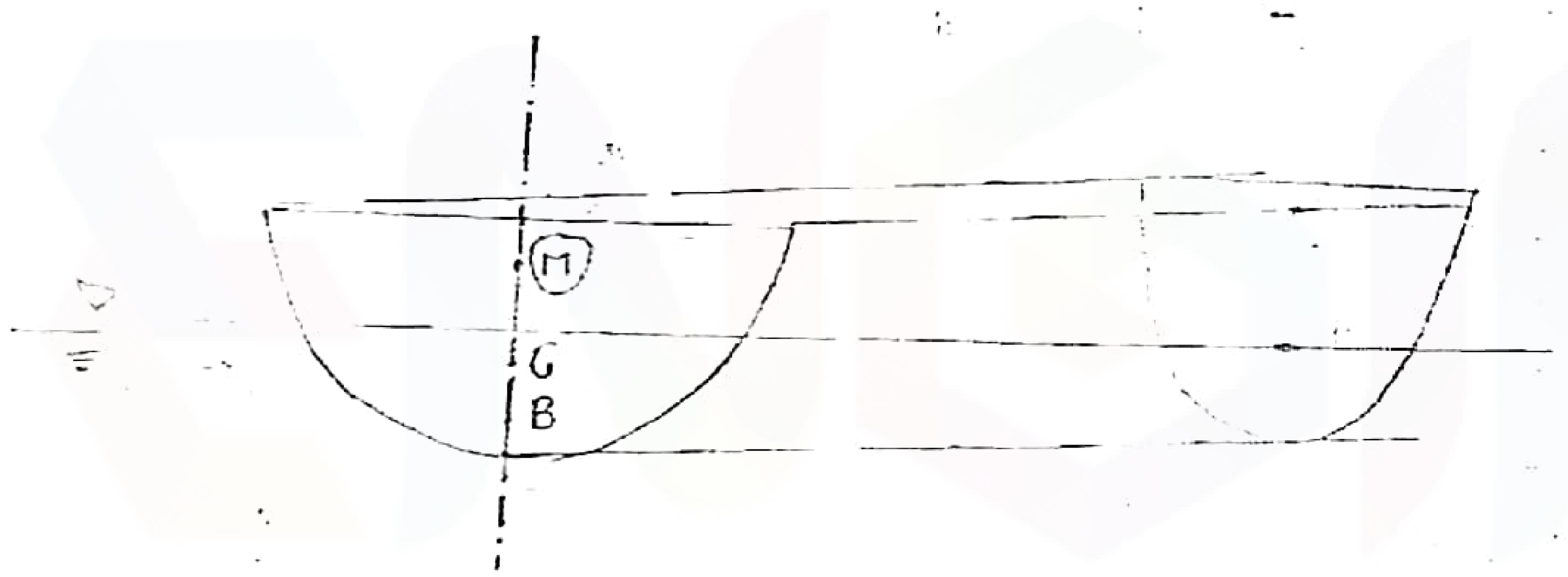
∴ Metacentre coincides with G.

For submerged body.



In a submerged bodies the centre of buoyancy is Mentacentre itself. i.e. the original criteria or the point of reference for checking the stability of a body is Metacentre always.

Metacentric height:



The vertical distance between the M and G is known as Metacentric height. (GM)

$$GM = BM - BG$$

$$GM = \frac{I}{\forall} - BG$$

If $GM > 0$ - stable equilibrium (M above G)

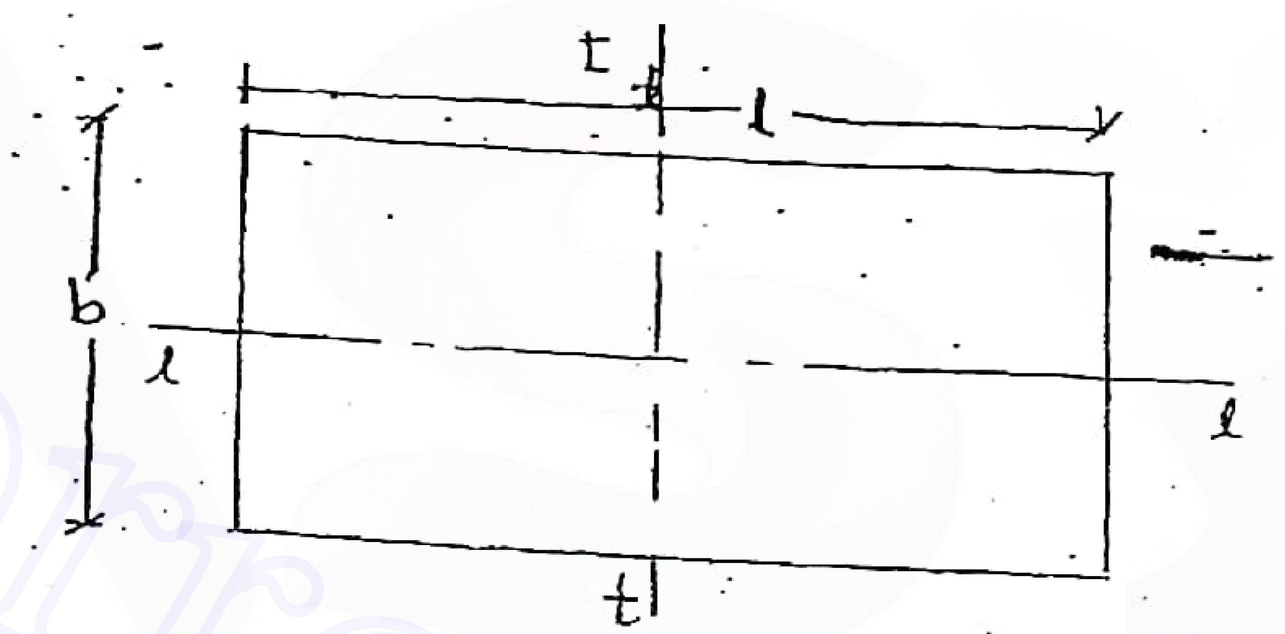
$GM < 0$ - unstable equilibrium (M lies below G)

$GM = 0$ - neutral equilibrium.

$\forall$ - volume of fluid displaced (volume of submerged mass of body)

I - Moment of inertia of surface of body which is intersected by the free water surface.

Face intersected by free surface:



If the body oscillates about the longitudinal axis, the phenomenon is called Rolling phenomenon.

$$I_{ll} = \frac{lb^3}{12} \quad (\text{@ longitudinal axis})$$

If the body oscillates about the transverse axis the phenomenon is called Pitching phenomenon.

$$I_{tt} = \frac{bl^3}{12} \quad (\text{@ transverse axis})$$

As $I_{ll} <<< I_{tt}$

The body oscillations will be more about the longitudinal axis. (less stable about longitudinal axis)

$$(GM)_{Rolling} <<< (GM)_{pitching}$$

the body will be sensitive for rolling action i.e. the rolling is more dangerous

The stabilities of floating bodies are always designed under the rolling

$$GM = \frac{I}{V} - BG.$$

The time period of oscillation is

$$T = 2\pi\sqrt{\frac{k^2}{(GM)\cdot g}}$$

k - radius of gyration.

$I = A \cdot k^2$

$$T \propto \frac{1}{\sqrt{GM}}$$

i.e. More is the metacentric height less is the period of oscillation.

Passenger ships (Cargo ships) have metacentric height 0.5 - 1 m for the purpose of more comfort to the passengers. The less metacentric height indicates less stable and high period of oscillation i.e. deflected ship takes more time to reach it's original position so smoothly that passengers inside will barely notice.

For War ships metacentric height is more (1 - 1.8 m)

War ships are meant to work in battle situations where planes, helicopters may land on it. The landing of aircraft is critical phenomenon which needs to be taken care. Thus the War ship should stable so that period of oscillation is very less when striked by the water waves. Thus Metacentric height is large.

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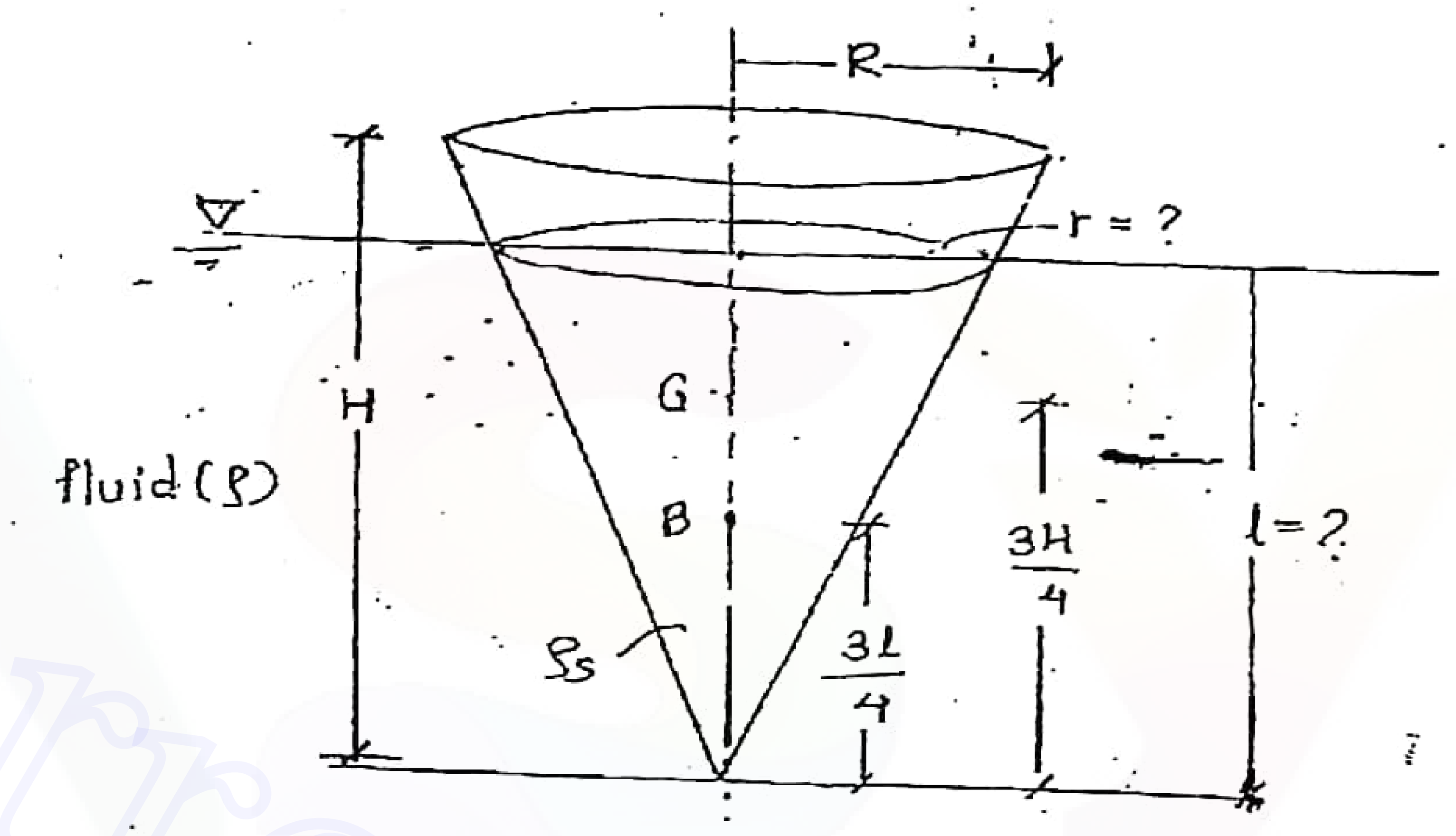
Q. A cone having a max. radius R, height H and density ρ_s is floating in fluid of density ρ with its axis vertical and apex down. Find the condition for the stability of cone.

0 Marks



Relative density (R.D.) $= \frac{\rho_s}{\rho}$

For floatation.

$$mg = F_B$$

$$g\left(\frac{1}{3}\pi R^2 H\right)\cdot \rho_s = \forall \cdot \rho g$$

$$\frac{1}{3}\pi R^2 H \cdot \rho_s = \frac{1}{3}\pi r^2 l \cdot \rho$$

$$r^2 \cdot l = R^2 H \cdot \frac{\rho_s}{\rho}$$

$$r^2 \cdot l = R^2 \cdot H \; (R.D.)$$

$$\frac{R}{H} = \frac{r}{l}$$

$$r = \left(\frac{R}{H}\right)\cdot l \qquad \text{from similar triangle}$$

$$\left(\frac{R^2}{H^2}\right) l^2 \cdot l = R^2 \cdot H \; (R.D.)$$

$$l^3 = H^3 \cdot (R.D.)$$

$$l = H\,(R.D.)^{1/3}$$

$$r = \frac{R}{H} \cdot H \cdot (R.D.)^{1/3}$$

$$= R \cdot (R.D.)^{1/3}$$

To find BG.

$$BG = \frac{3H}{4} - \frac{3l}{4}$$

$$= \frac{3}{4}(H - l)$$

$$= \frac{3}{4}\left(H - H \cdot R.D.^{1/3}\right)$$

$$BG = \frac{3}{4} H \left(1 - R.D.^{1/3}\right)$$

To find BM.

$$BM = \frac{I}{\forall}$$

$$= \frac{\frac{\pi r^4}{4}}{\frac{1}{3}\pi r^2 l}$$

$$= \frac{3}{4}\,\frac{r^2}{l}$$

$$= \frac{3}{4}\,\frac{R^2 (R.D.)^{2/3}}{H\,(R.D.)^{1/3}}$$

$$B.M. = \frac{3}{4}\,\frac{R^2}{H}\,(R.D.)^{1/3}$$

$$GM = B.M - B.G.$$

For stability.

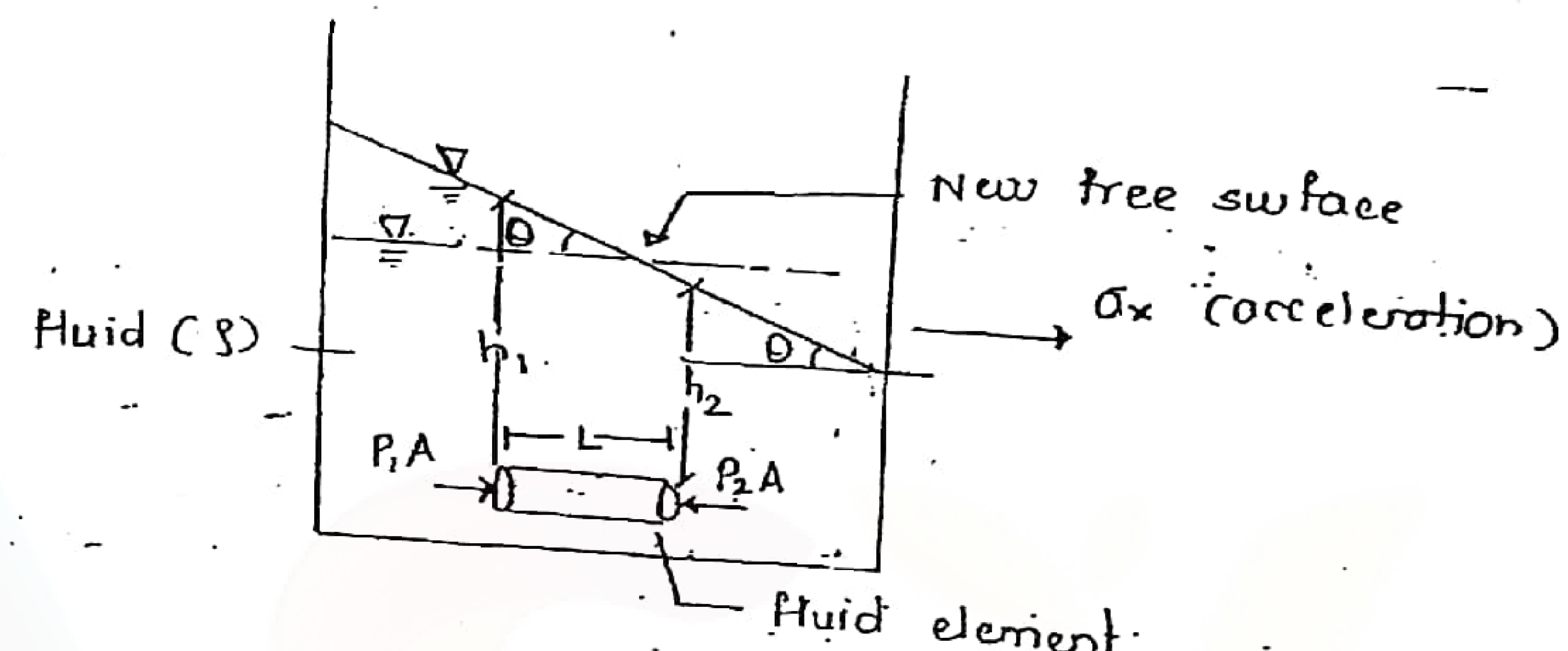
$$GM > 0$$

$$BM > B.G.$$

$$\frac{3}{4}\,\frac{R^2}{H}\,(R.D.)^{1/3} > \frac{3}{4}\,H\left(1 - R.D.^{1/3}\right)$$

$$\frac{R^2}{H^2} > \frac{1 - R.D.^{1/3}}{R.D.^{1/3}}$$

Concept of linearly accelerated vessels containing liquid:



By Newton's 2nd law of motion.

$$P_1 A - P_2 \cdot A = m \cdot a_x$$

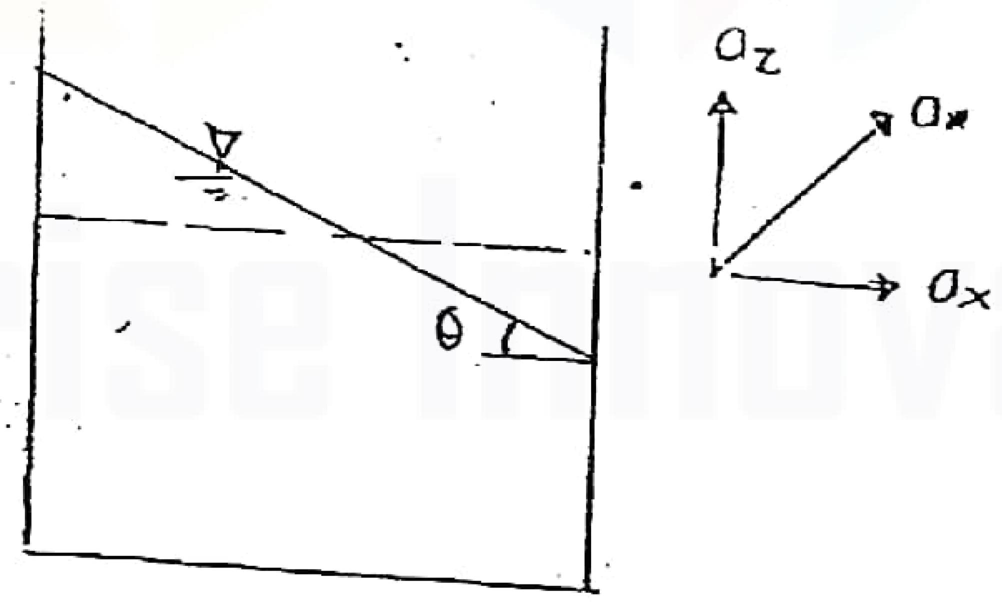
$$(P_1 - P_2) \cdot A = (A \cdot L \cdot \rho) \cdot a_x$$

$$(P_1 - P_2) = L \cdot \rho \cdot a_x$$

$$\rho g (h_1 - h_2) = L \rho \cdot a_x$$

$$\left(\frac{h_1 - h_2}{L}\right) = \frac{a_x}{g}$$

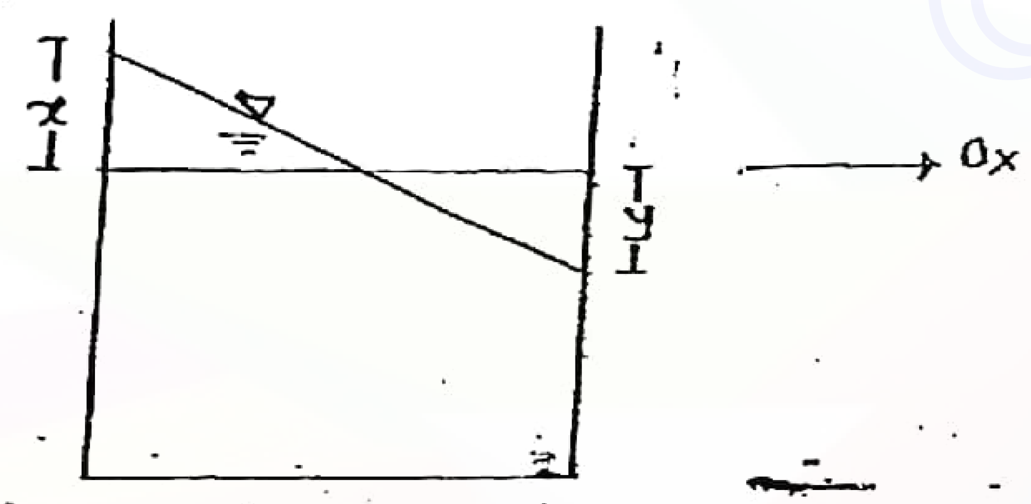
$$\tan\theta = \frac{a_x}{g}$$



$$\tan\theta = \frac{a_x}{g + a_z}$$

Conservation of volume :

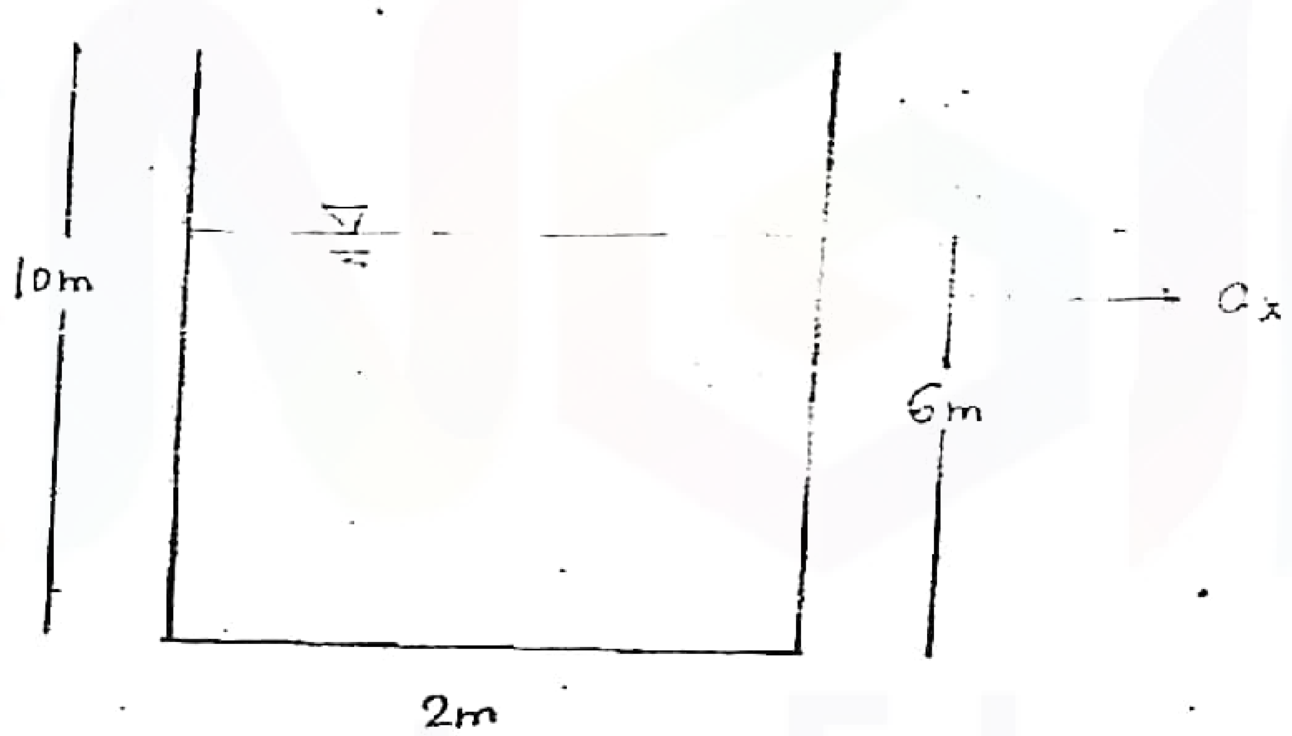
It is applicable if not even a single drop of liquid is spilled out.



If a container filled with liquid is accelerated in horizontal direction then rise of liquid level on one side is equal to fall of liquid level on other side.

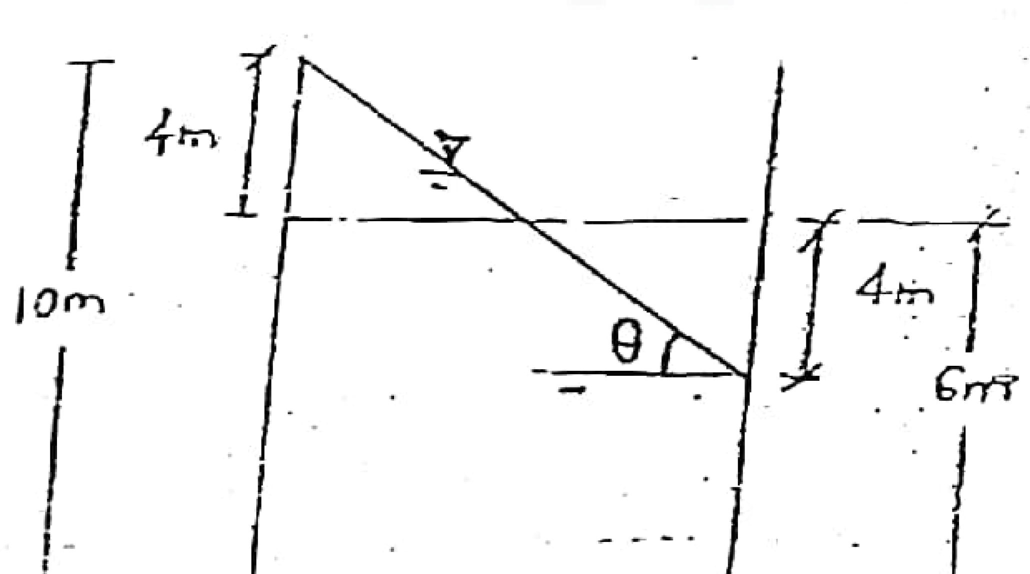
$x = y$.

Q. Find the max. acceleration so that water doesn't spill out.



Width $= 3m$

As water is not spilling out, law of conservation of volume is applicable.



$$\tan\theta = \frac{8}{2} = 4.$$

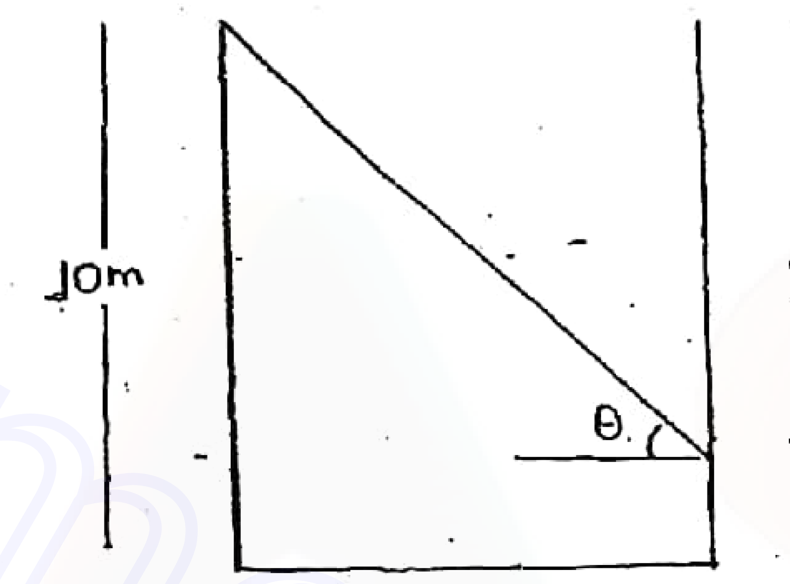
$$\tan\theta = \frac{(a_x)_{max}}{g}$$

$$(a_x)_{max} = 4.9$$

Q. Find the volume of water spilled if $a_x = 45\ m/s^2$

$$\tan\theta = \frac{a_x}{g} = \frac{45}{9.81}$$

$$\tan\theta = 4.587 \quad < 5 \quad \text{for } \left(\frac{10}{2} = \tan\theta\right)$$



$$\tan\theta = \frac{y}{2}$$

$$y = 2 \times 4.587$$

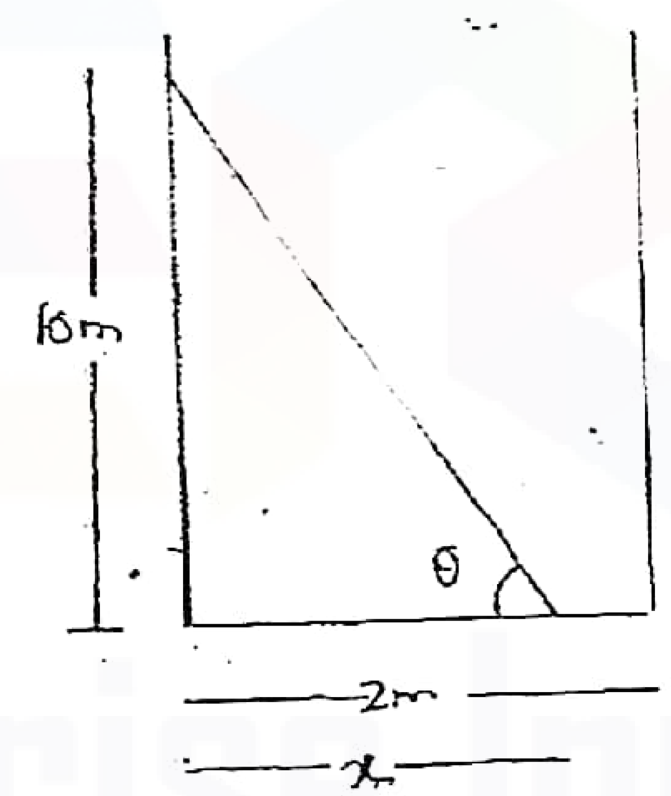
$$= 9.174\ m$$

$$V_{final} = \left[\frac{9.174 \times 2}{2} + 0.826 \times 2\right] \times 3$$

$$V_{initial} = 2 \times 6 \times 3$$

$$V_{spilled} = 3.522\ m^3$$

Q. Find spilled volume if $a_x = 60\ m/s^2$



$$\tan\theta = \frac{60}{9.81} = 6.116$$

$$\tan\theta = \frac{10}{x}$$

$$x = 1.635\ m$$

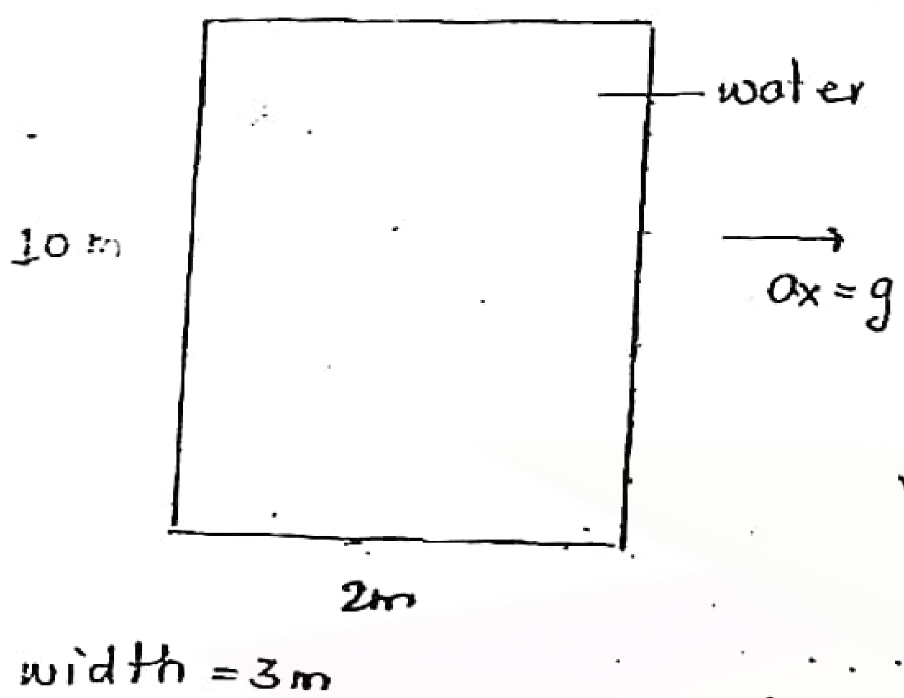
$$V_{final} = \left(\frac{10 \times 1.635}{2}\right) \times 3$$

$$=$$

$$V_{initial} = (2 \times 6 \times 3)$$

$$V_{spilled} = 11.475\ m^3$$

Q. If the closed container moved with an acceleration g m/s². what is force at the bottom?



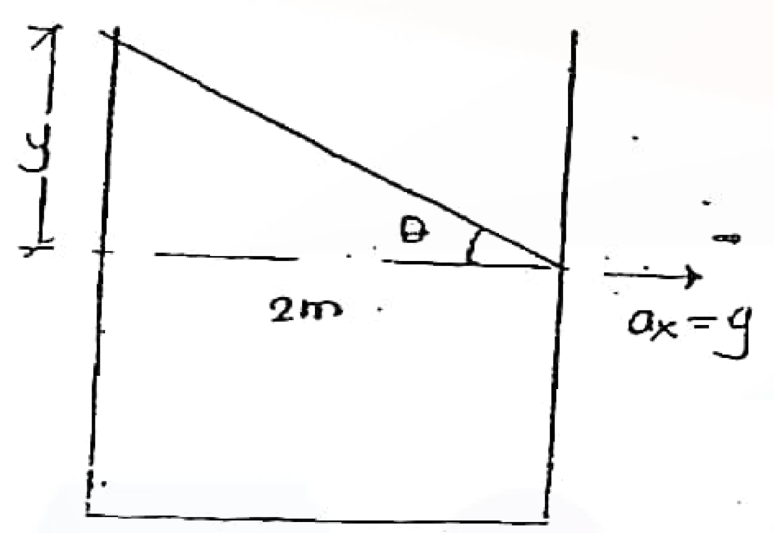
The hydrostatic force will always there in the fluid.

∴ Hydrostatic force at the bottom of vessel

$F_1 = (\rho g h) \cdot A$

$= (1000 \times 9.8 \times 10) \times (2 \times 3)$

$=$



If the container was open. the volume of water spilled $\forall$.

As the container is closed. $\forall$ water will apply force on top surface of container as it is trying to spill.

$\tan\theta = \frac{a_x}{g}$

$\frac{y}{2} = \frac{g}{g}$

$y = 2$

$\forall_{spilled} = (\frac{1}{2} \times 2 \times 2) \times 3$

$= 6\,m^3$

Force applied on the top (or the force applied on bottom due to $\forall_{spilled}$)

$F_2 = \rho g \forall$

$= 1000 \times 9.8 \times 6$

$=$

Total force on bottom $= F_1 + F_2$

KINEMATICS OF FLUID FLOW

most difficult

P1V

It is the study of fluid particle motion without the help of basic cause of motion i.e. force.

1. Lagrangian method: (used to Ph.D)

This approach is particle concentration approach i.e. the entire concentration goes to a particular fluid particle and its motion is analysed. After completing the study of motion of one fluid particle. the concentration goes to another fluid particle. Since the number of fluid particles in a system are very large. therefore this approach is highly time consuming but very accurate approach. Just because of time consumption to be high. this approach is not used in Classical Fluid Mechanics.

computational fluid dynamics + finite volume / element Technique (5-6 yrs Re-search

2. Eulerian method:

(1st to read/observe vibrations of cantilever beam)

This method is space concentration approach i.e. the entire concentration goes to a particular space or zone and all the fluid particles passing through that space or zone are analysed simultaneously. Therefore, the results obtained with the method are the average results which are not correct particle by particle but on an average these results are 100 % correct for overall bulk motion of fluid particles. Therefore this approach is very time saving approach. Hence we prefer this approach in classical fluid Mechanics.

e.g. If one travells 20 km distance in 15 min ($\frac{1}{4}$ hr) with differen velocities.

Avg. velocity $= \frac{20}{(1/4)} = 80$ km/hr.

i.e. when one travells with constant velocity of 80 km/hr

Different types of flows in the fluid flow system:

1. Steady and unsteady flows:

If the properties in flow are not changing w.r.t time, such a flow is known as steady flow. and if the properties are changing w.r.t. time it is unsteady flow.

$$\left.\frac{\partial R}{\partial t}\right|_{space\text{-}fixed} = 0 \qquad R\text{- fluid properties.}$$

2. Uniform flow and non-uniform flow:

If the properties in the flow are not changing w.r.t. space then such a flow is known as uniform flow

$$\left.\frac{\partial R}{\partial space}\right|_{time\text{-}fix} = 0 \qquad \text{uniform flow.}$$

3. Incompressible flow and compressible flow:

If the density is not changing w.r.t. pressure then such a flow is known as Incompressible flow.

$$\frac{\partial R}{\partial P} = 0 \qquad \text{Incompressible flow.}$$

4. Irrotational flows and rotational flows:

In a flow if the fluid particles are also rotating about there own centre of masses then such a flow is known as rotational flow and if the particles are not rotating about their own centre of masses then such a flow is known as irrotational flow.

e.g. The swirling water in a bucket is rotational flow.

5. Laminar flow and turbulent flow:

"If all the fluid particles lying in a layer are having there velocities in the direction of flow of layers then there will not be any kind of intermixing of the fluid particles between the adjacent layers. Such a well organised flow of fluid particles in the laminated form of layers is known as Laminar flow. This flow is also known as Streamline flow." (Name given by Raynold)

If all the fluid particles lying in a layer are having their velocity component in different directions, then there will be huge intermixing of fluid particles between the adjacent layers. Such a most chaotic flow is known as Turbulent flow. Shear force between particles also contribute

Some mathematical tools:

1: Taylor series:

$$f(x+h) = f(x) + f'(x)\,h + f''(x)\frac{h^2}{2!} + f'''(x)\frac{h^3}{3!} + \dots$$

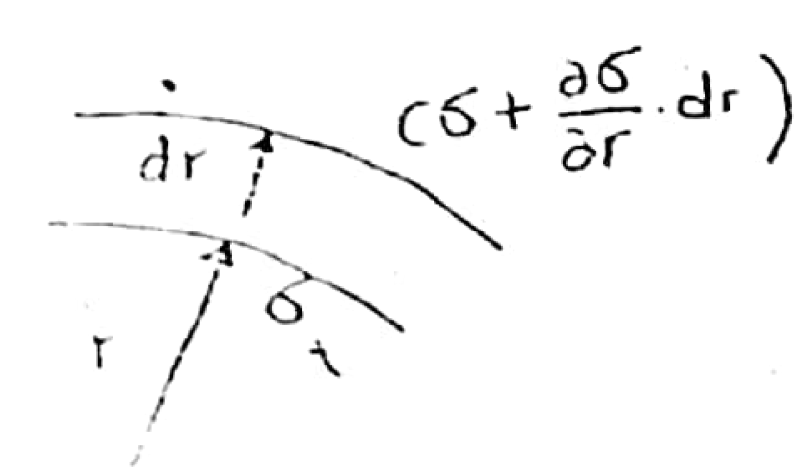
If h is very small i.e. of the order of dx.

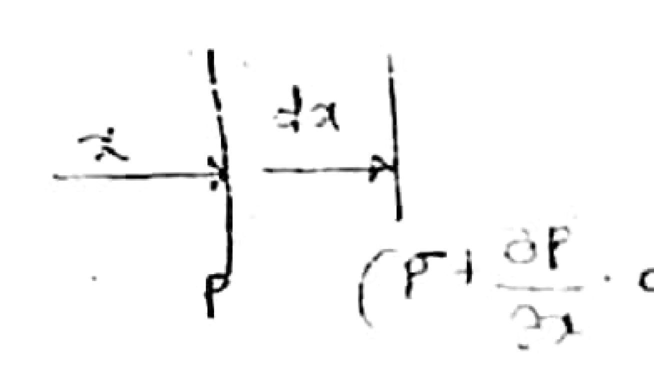
$$f(x+dx) = f(x) + f'(x)\cdot dx$$

– (Taylor series approximation) upto 1st order

Application:

σ-stress at r





2. If F is function of many variables

$$F = f(x, y, z)$$

Total change = sum of partial changes in x, y and z.

$$dF = \frac{\partial F}{\partial x}\cdot dx + \frac{\partial F}{\partial y}\cdot dy + \frac{\partial F}{\partial z}\cdot dz$$

(gradient) (length in X-direction)

(Partial change in X-direction)

Continuity equation:
(conservation of mass)

In general, for 3-D flows, the velocity is given by

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

where,
u, v, w are velocity components in X, Y and Z directions.

The magnitudes of u, v, w are the functions of (x, y, z, t)

$$\vec{V} = f(x, y, z, t)$$

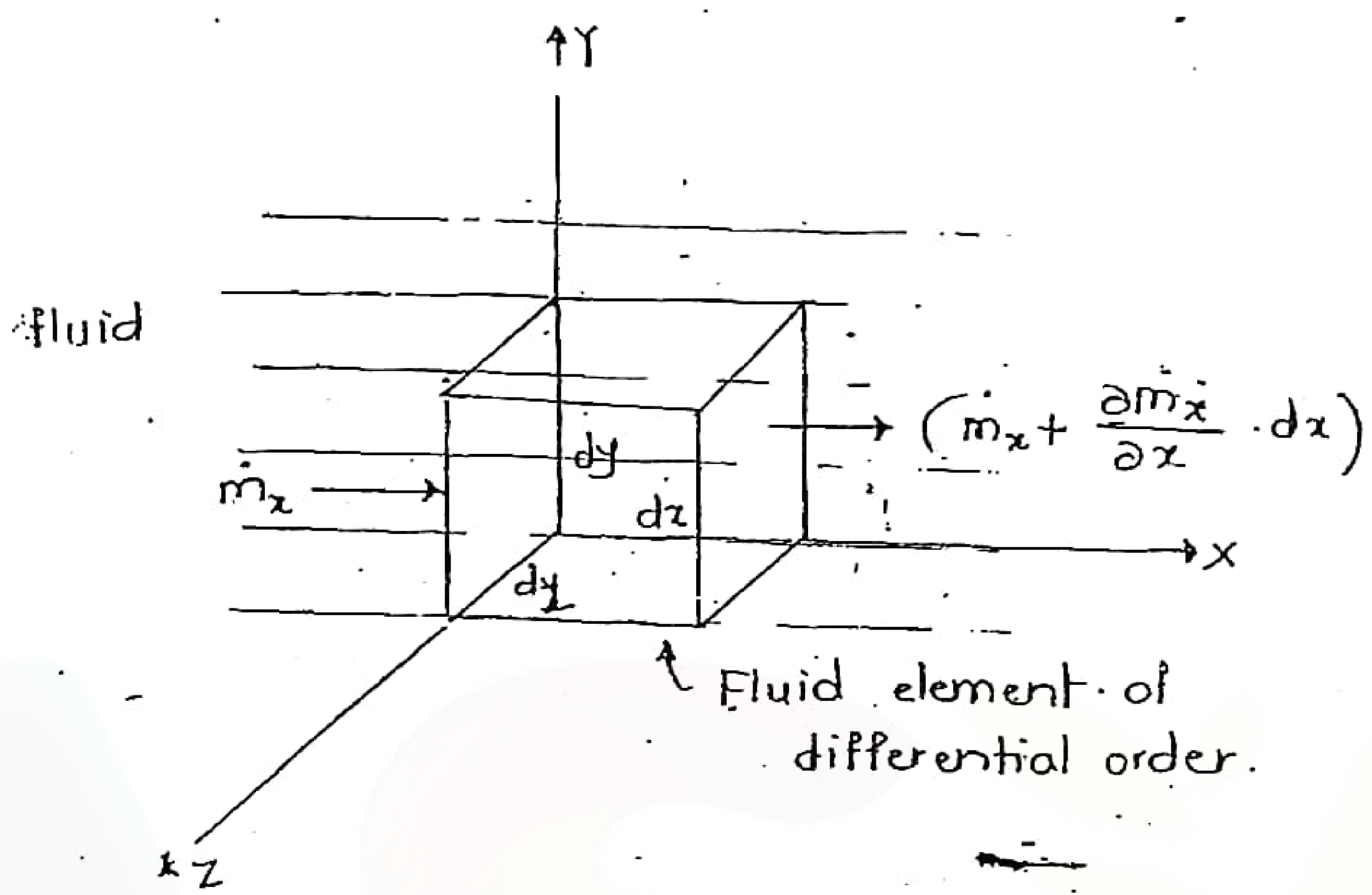
conservation of mass

$$\dot{m}_{in} - \dot{m}_{out} = \dot{m}_{stored}$$

where,
$\dot{m}_{in}$ - mass entering the system per sec.
$\dot{m}_{out}$ - mass leaving the system per sec
$\dot{m}_{stored}$ - mass stored per sec.

$\dot{m}_{in}$ and $\dot{m}_{out}$ are surface phenomenon (as the mass will enter or leave the system through a section)

$\dot{m}_{stored}$ is bulk / volumetric phenomenon as mass is



In X-direction,

$$(\dot{m}_{in} - \dot{m}_{out})_x = \dot{m}_x - \left(\frac{\partial \dot{m}_x}{\partial x}\cdot dx + \dot{m}_x\right)$$

$$= -\frac{\partial \dot{m}_x}{\partial x}\cdot dx$$

$$= -\frac{\partial}{\partial x}\underbrace{(\rho\cdot dy\cdot dz\cdot u)}_{\text{rate of mass entering per sec}}\cdot dx$$

$$= -\frac{\partial}{\partial x}(\rho u)\cdot d\forall \qquad dx\cdot dy\cdot dz = d\forall$$

similarly,

$$(\dot{m}_{in} - \dot{m}_{out})_y = -\frac{\partial}{\partial y}(\rho v)\cdot d\forall$$

$$(\dot{m}_{in} - \dot{m}_{out})_z = -\frac{\partial}{\partial z}(\rho w)\cdot d\forall$$

$$(\dot{m}_{in} - \dot{m}_{out}) = -d\forall\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right]$$

$$\dot{m}_{st} = \frac{\partial}{\partial t}(m_{st}) \quad \text{- stored mass per sec}$$

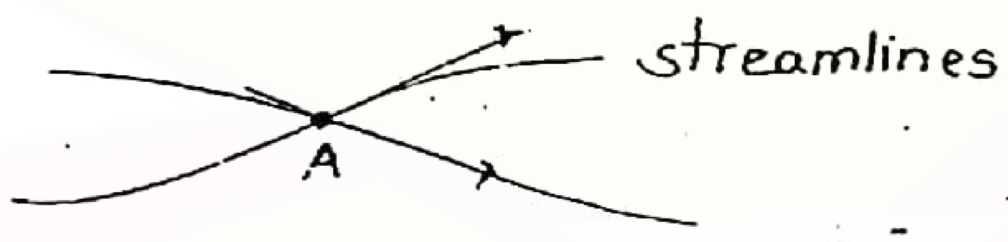
$$= \frac{\partial}{\partial t}(\rho\cdot d\forall)$$

$$= d\forall\cdot\frac{\partial \rho}{\partial t}$$

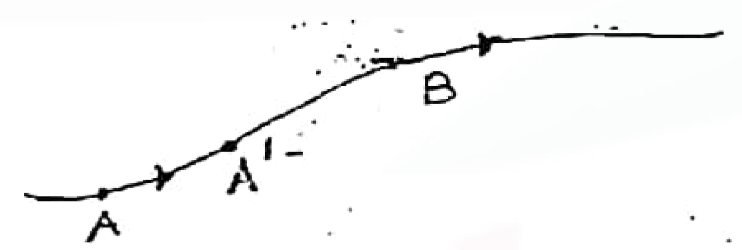
Streamlines :

It is an imaginary line drawn in the flow field in such a way such that tangent drawn at any point on this line directly represents direction of velocity vector of fluid particle at that point.

Two different streamlines can never intersect each other at any one moment.

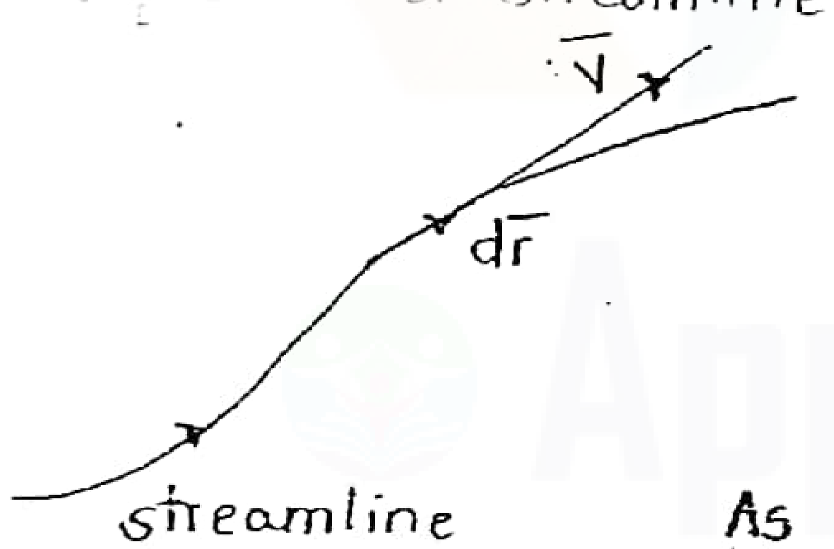


If two streamlines cross each other, particle A will have two different directions at the same point which is not possible. (Particle can have different directions at different points but not at single point)



Streamline give the velocity direction of a particle at that point, but it doesn't give velocity direction of particle when it changes its position (A')

Equation of streamline:



Take a differential position vector along streamline.

$$d\vec{r} = (dx)\hat{i} + (dy)\hat{j} + (dz)\hat{k}$$

As velocity vector ($\vec{V}$) and position vector ($d\vec{r}$) are along same direction (angle 0)

$$d\vec{r} \times \vec{V} = 0$$

$$\begin{vmatrix} i & j & k \\ dx & dy & dz \end{vmatrix} = 0$$

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$$\hat{i}(w\,dy - v\,dz) - \hat{j}(w\,dx - u\,dz) + \hat{k}(v\,dx - u\,dy) = 0$$

$$\hat{i}(w\,dy - v\,dz) = 0$$

$$\frac{dy}{v} = \frac{dz}{w}$$

$$\hat{j}(w\,dx - u\,dz) = 0$$

$$\frac{dx}{u} = \frac{dz}{w}$$

$$\hat{k}(v\,dx - u\,dy) = 0$$

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Q. A 2-D steady incompressible flow is given by velocity $\vec{V} = (3x\hat{i} - 3y\hat{j})$. Find equ^n of streamline passing through (1,1).

$u = 3x$

$v = -3y$

for 2-D steady incompressible flow,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 3 + (-3)$$

$$= 0 \quad \text{(flow is possible)}$$

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\int \frac{dx}{3x} = \int \frac{dy}{-3y}$$

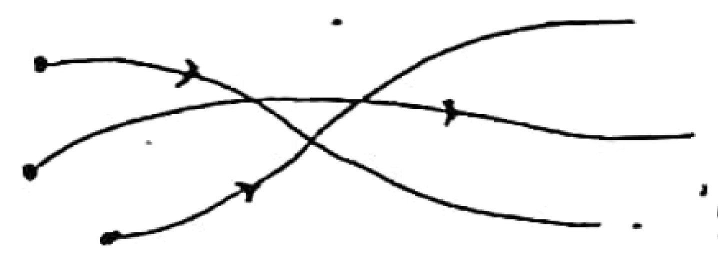
$$\ln x = -\ln y + \ln c$$

$$xy = c$$

$$C = 1 \times 1 = 1$$

Pathlines:

It is an actual path traced by an individual fluid particle.



Pathlines can intersect each other.

Streaklines:

It is a locus of fluid particle at a moment which have been crossed from the same point.

Streamline - direction of velocity vector.

Path lines - Individual motion of fluid particle.

Streak line - Identification of location of fluid particles.

Note: If flow is steady and uniform all three lines will look mathematically similar.

Acceleration of fluid particle:

$$\bar{V} = (u\hat{i} + v\hat{j} + w\hat{k}) \text{ — function of } (x, y, z, t)$$

$$d\bar{V} = \frac{\partial \bar{V}}{\partial x} \cdot dx + \frac{\partial \bar{V}}{\partial y} \cdot dy + \frac{\partial \bar{V}}{\partial z} \cdot dz + \frac{\partial \bar{V}}{\partial t} \cdot dt$$

$$\vec{a} = \frac{d\bar{V}}{dt} = \frac{\partial \bar{V}}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial \bar{V}}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial \bar{V}}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial \bar{V}}{\partial t} \cdot \frac{dt}{dt}$$

$$\bar{a} = \underbrace{u \cdot \frac{\partial \bar{V}}{\partial x} + v \cdot \frac{\partial \bar{V}}{\partial y} + w \cdot \frac{\partial \bar{V}}{\partial z}}_{\text{Convective acceleration}} + \underbrace{\frac{\partial \bar{V}}{\partial t}}_{\text{Local, temporal}}$$

$$a_x = u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial v}{\partial y} + w \cdot \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u \cdot \frac{\partial w}{\partial x} + v \cdot \frac{\partial w}{\partial y} + w \cdot \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} \text{ units.}$$

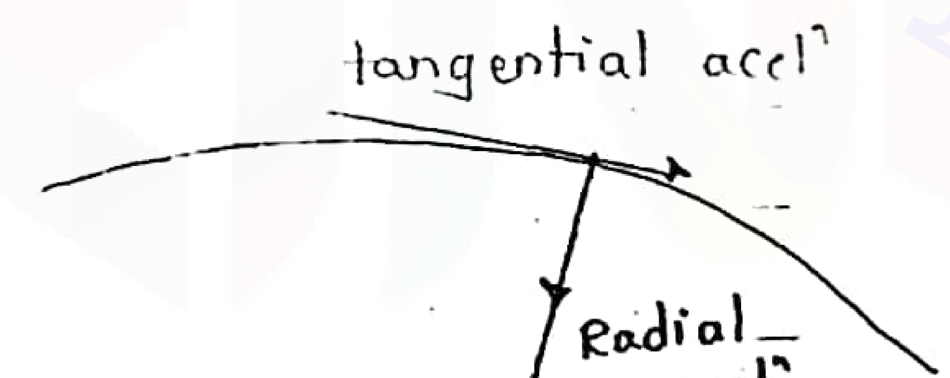
If flow is uniform,
convective acceleration $= 0$

If flow is steady,
Local acceleration $= 0$

Linear motion:

acceleration because of change in magnitude of velocity

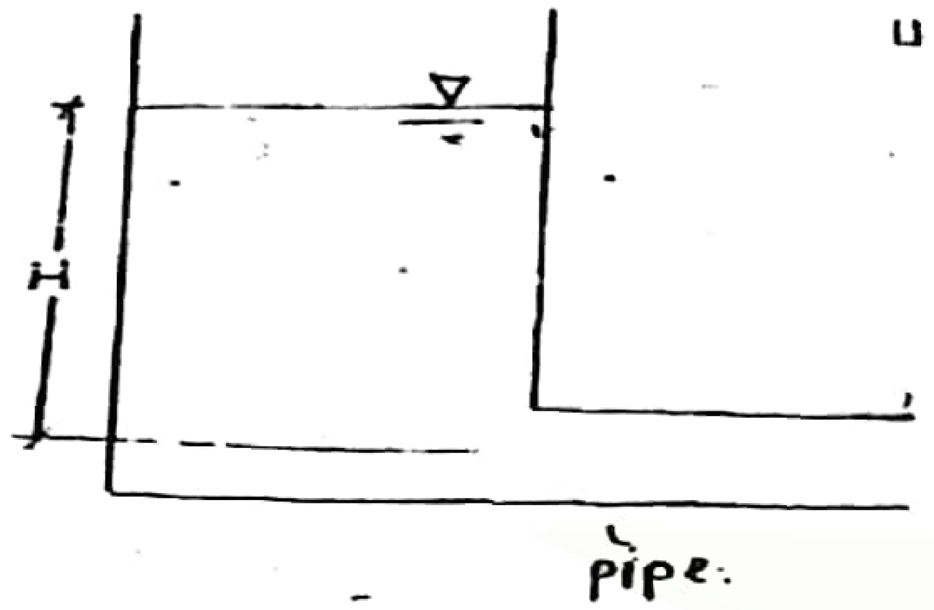
Curvilinear motion



Because of change in direction of velocity - normal accelⁿ (Centripetal acceleration, radial acceleration - towards the centre.

Because of change in magnitude of velocity - tangential acceleration.

(i) Flow through straight pipe of uniform diameter.



Convective Accel^n = 0 (c/s = const)

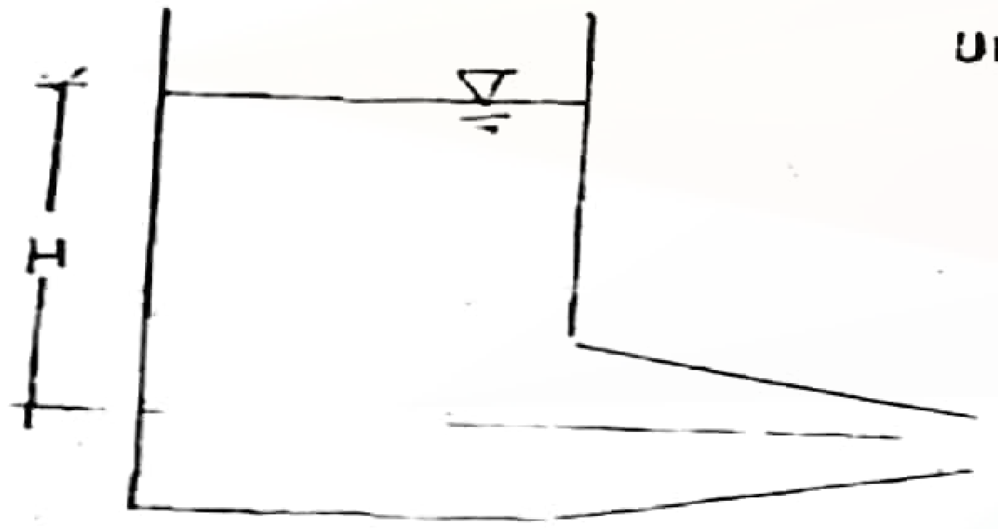
If H = constant (steady flow)

Local accl^n = 0

If H = variable (unsteady flow)

Local accl^n $\neq 0$

(ii) Flow through straight pipe of non-uniform diameter.



Convective accl^n $\neq 0$ (c/s changing)

If H = constant (steady flow)

local accl^n = 0

If H = variable (unsteady flow).

Local accel^n $\neq 0$

(iii) Flow through curved pipe of uniform diameter.

Convective tangential accl^n = 0

convective radial accl^n $\neq 0$

If H = constant (steady flow)

Local tangential accl^n = 0

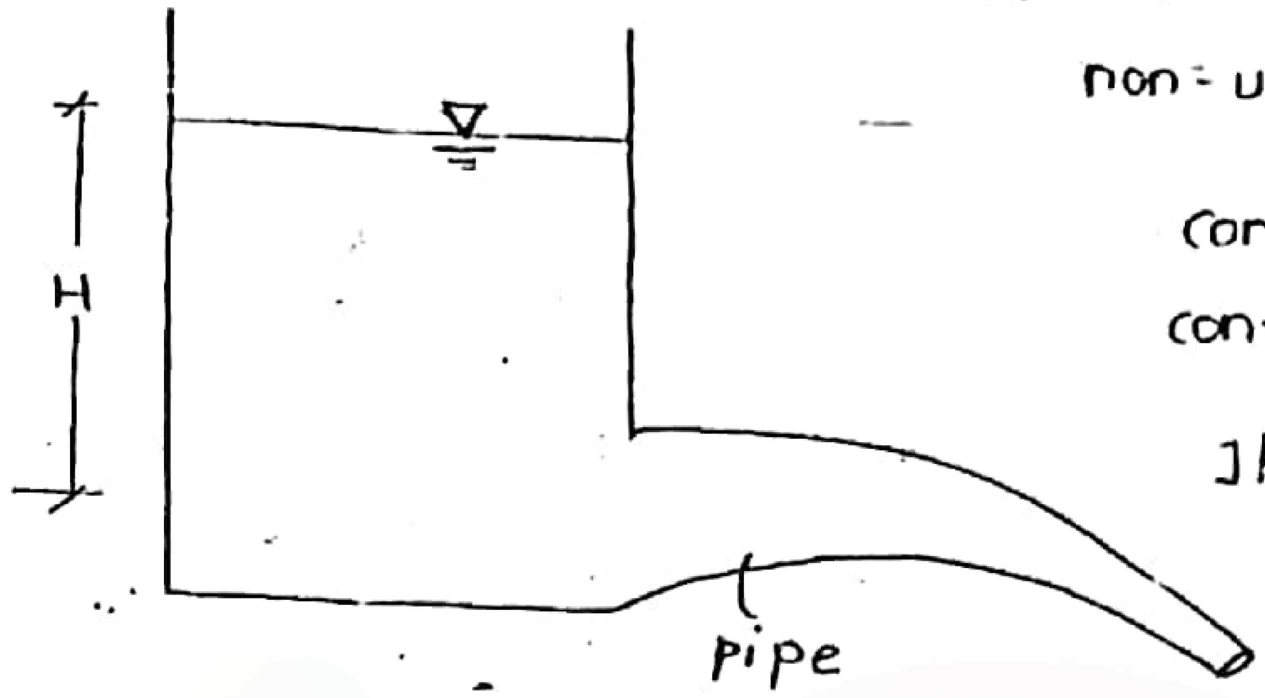
Local radial accl^n = 0

If H = variable (unsteady flow)

Local tangential accl^n $\neq 0$

Local radial accl^n = 0

(iv) Flow through curved pipe of non-uniform diameter



conventional tangential accl^n $\neq 0$

conventional normal accl^n $\neq 0$

If H = constant (steady)

Local tangential accl^n = 0

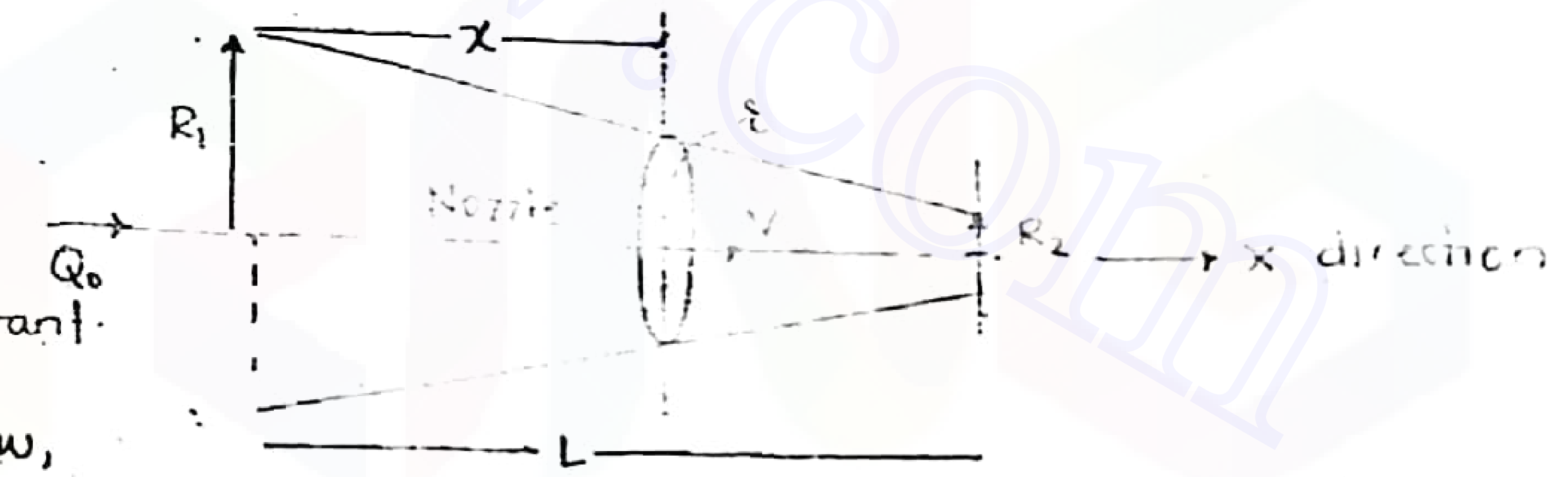
Local radial accl^n = 0

If H = variable (unsteady)

Local tangential accl^n $\neq 0$

Local radial accl^n = 0

Q. Find the acceleration of fluid at exit section.



Q_0 constant ↙ steady flow, constant head.

At the section x-x.

$a = 0$ convective

$= V \cdot \frac{\partial V}{\partial x}$ (unidirectional flow)

$= \frac{Q_0}{A} \cdot \frac{\partial}{\partial x}\left(\frac{Q_0}{A}\right)$

$\frac{R_1 - R_2}{L} = \frac{R_1 - r}{x}$

$$z = R_1 - Bx \qquad B = \left(\frac{R_1 - R_2}{L}\right)$$

$$a = \frac{Q_0}{\pi (R_1 - Bx)^2} \cdot \frac{\partial}{\partial x}\left(\frac{Q_0}{\pi (R_1 - Bx)^2}\right)$$

$$= \frac{Q_0^2}{\pi^2 (R_1 - Bx)^2} \cdot \frac{\partial}{\partial x}\left(\frac{1}{(R_1 - Bx)^2}\right)$$

$$= \frac{2B \cdot Q_0^2}{\pi^2 (R_1 - Bx)^5}$$

At exit section,

$$a = \frac{2B \cdot Q_0^2}{\pi^2 (R_1 - BL)^5}$$

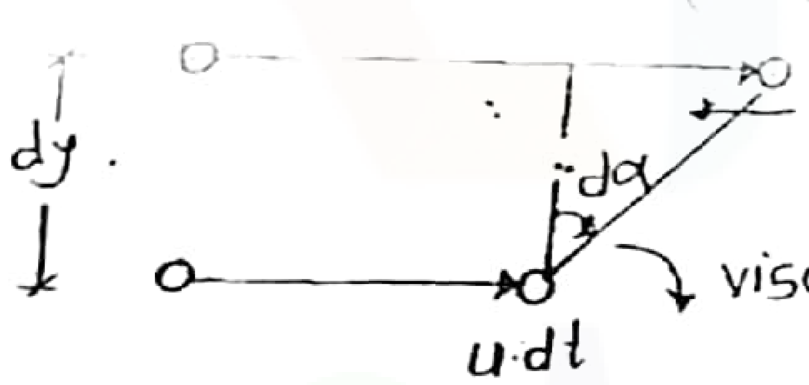
Rotational components of fluid :

$\circlearrowright$ − ve $\qquad \circlearrowleft$ + ve

In XY plane,

$$\vec{V} = (u\hat{i} + v\hat{j})$$

(i) u share of rotation

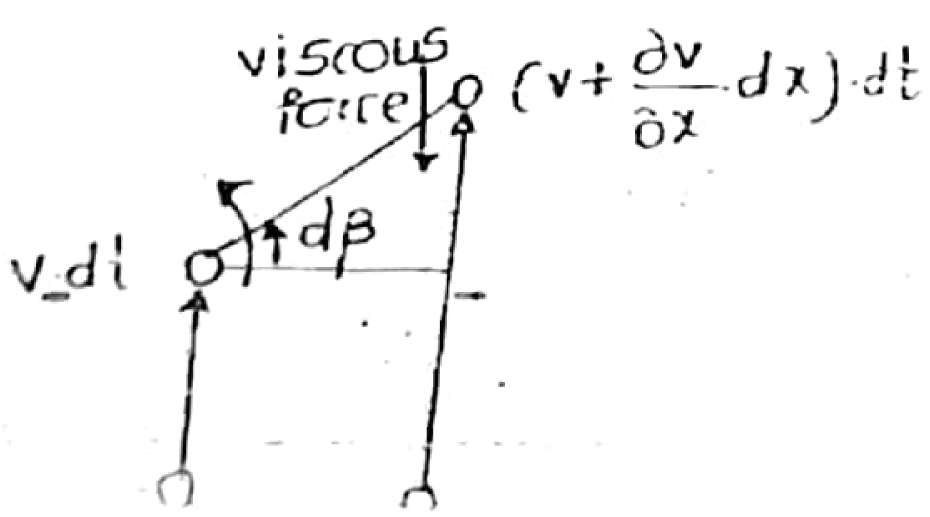


$$\tan d\alpha = d\alpha = \frac{-\frac{\partial u}{\partial y} dy \cdot dt}{dy}$$

$$d\alpha = -\frac{\partial u}{\partial y} \cdot dt$$

$$\frac{d\alpha}{dt} = -\frac{\partial u}{\partial y}$$

(ii) v-share of rotation :



$$\tan d\beta = d\beta = \frac{\frac{\partial v}{\partial x} \cdot dx \cdot dt}{dx}$$

$$= \frac{\partial v}{\partial x} \cdot dt$$

∴ Angular velocity of fluid particle about z-axis

$$\omega_z = \frac{d\beta}{dt} + \frac{d\alpha}{dt}$$

$$= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

— since the obtained rotation is effect of 2-particles (divide by 2)

In general in 3-D flows,

$$\vec{\omega} = \frac{1}{2}(\nabla \times \vec{V})$$

$$\vec{\omega} = \frac{1}{2}\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ u & v & w \end{vmatrix}$$

$$= \frac{1}{2}\left[i\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\right] - j\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right) + k\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial y}\right)\Big]$$

$$= \frac{1}{2}\left[\omega_x - \omega_y + \omega_z\right]$$

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

If $\omega_x = \omega_y = \omega_z = 0$

$\vec{\omega} = 0$. irrotational flow.

If any of ω_x or ω_y or $\omega_z \neq 0$, —

$\vec{\omega} \neq$. — rotational flow.

For 2-D flow

If $\omega_z = 0$ — irrotational flow.

$\omega_z \neq 0$ — rotational flow.

Vorticity :

It is rotational strength in the pair of fluid particles. (angular velocity between two particles)

$$2\vec{w} = (\nabla \times \vec{V})$$

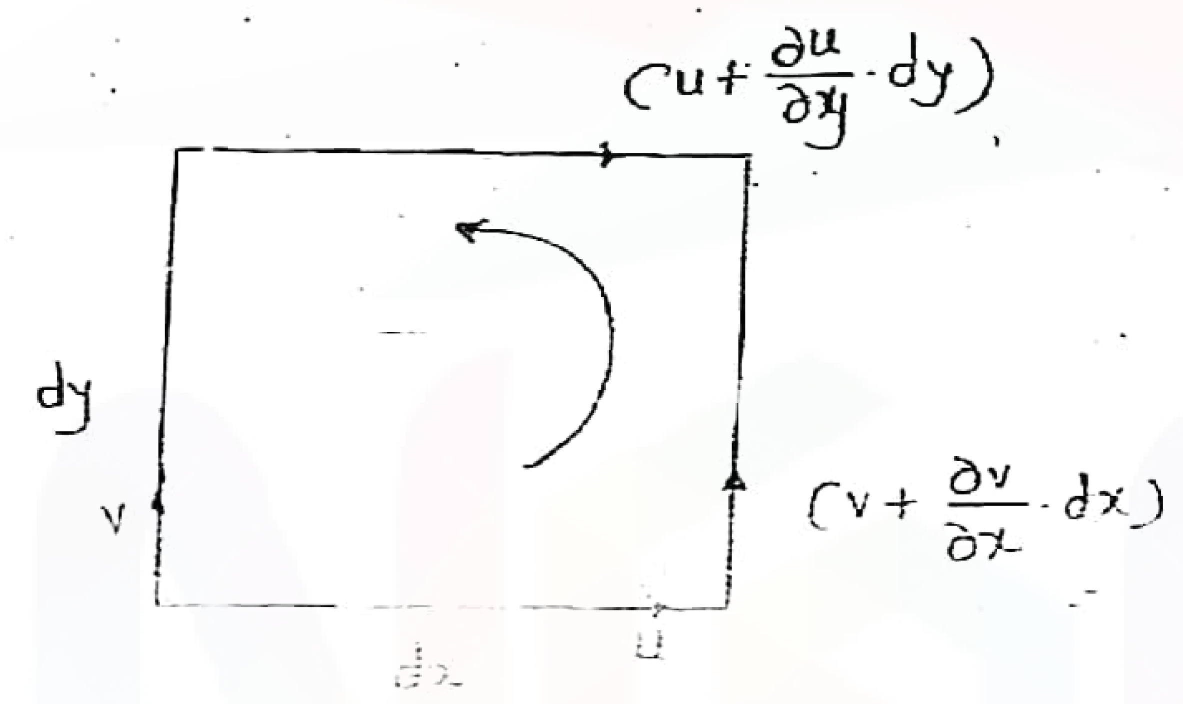
Circulation (Γ)

It is strength of rotation in fluid zone.

It is defined as the line integral of velocity vector taken along a closed loop.

$$\Gamma = \oint \vec{V} \cdot \vec{dr} \qquad \text{line - dot product}$$

For a differential (very small) system



$$\Gamma = \oint \vec{V} \cdot \vec{dr} = u \cdot dx + \left(v + \frac{\partial v}{\partial x} \cdot dx\right) \cdot dy - \left(u + \frac{\partial u}{\partial y} \cdot dy\right) \cdot dx$$

$$- v \cdot dy.$$

$$= \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \cdot dx \cdot dy$$

$$= 2 w_z \cdot A$$

A - area of differential zone. — valid for diff. system

$$\Gamma = \text{vorticity} \times \text{area of zone}$$

It is applicable only when vorticity is constant. If vorticity

Q. (Page 26, Q.38)

$r = 2$ units.

$u = 2x + 3y$

$v = -2y$.

$$\text{vorticity} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

$$= 0 - 3$$

$$= -3 \quad \text{— constant}$$

$$\Gamma = -3 \times \pi r^2$$

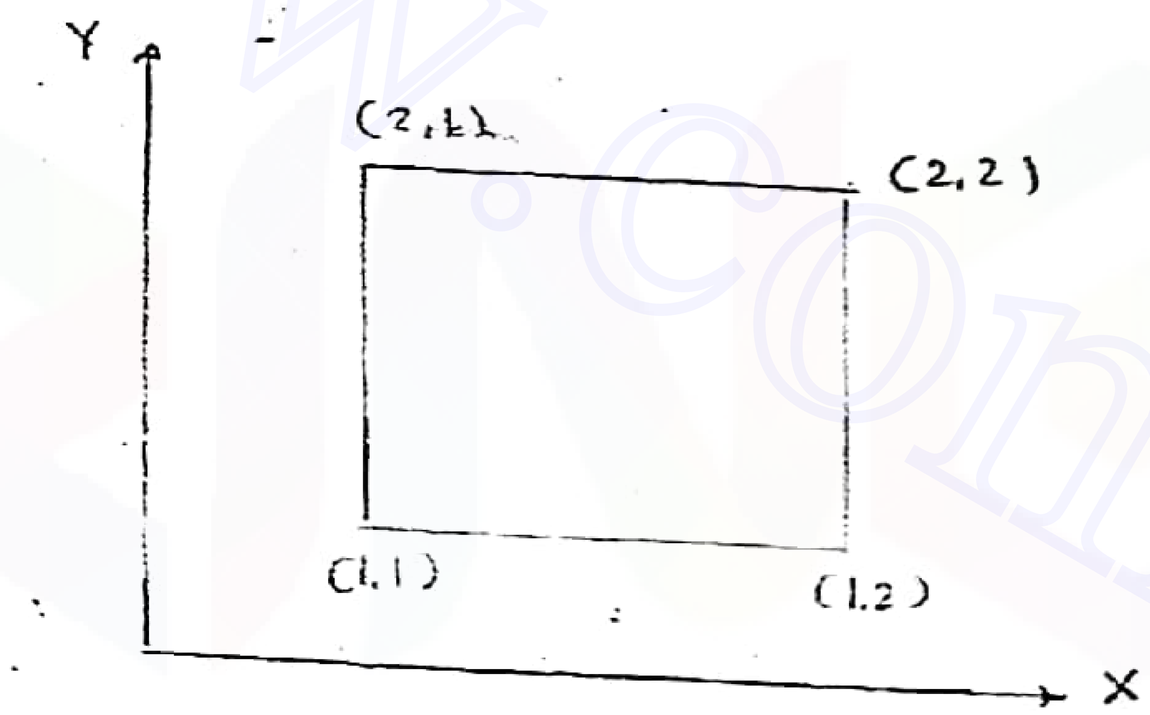
$$= -3 \times \pi \times (2)^2$$

$$= -12\pi \quad \text{i.e. clockwise}$$

Q. (Page 29, Q 9)

$u = x^2$

$v = 2xy$.



$$\Gamma = \oint \vec{V} \cdot \vec{dr} = \int_1^2 x^2 \cdot dx + \int_1^2 2(2y) \cdot dy + \int$$

$$= 3 \text{ units.}$$

Velocity potential function (ϕ)

It is defined as function of space and time in such a way that negative derivative of this function w.r.t. space directly gives velocity in that direction.

By definition

$$-\frac{\partial \phi}{\partial x} = u$$

$$-\frac{\partial \phi}{\partial y} = v$$

$$-\frac{\partial \phi}{\partial z} = w$$

Boundation of ϕ.

For 2-D steady, incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$ — for flow to occur. – (continuity equ^n)

$$\frac{\partial}{\partial x}\left(\frac{-\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{-\partial \phi}{\partial y}\right) = 0$$

$$-\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

i.e. $\nabla^2 \phi = 0$

Laplacian operator

ϕ must satisfy the Laplace's equation

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Physical significance: of ϕ:

Angular velocity of particle,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{-\partial \phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{-\partial \phi}{\partial x}\right)\right]$$

GYAN PHOTOSTAT
8588058174

$$= \frac{1}{2}\left[\frac{-\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial y \partial x}\right]$$

$$\omega_z = 0$$

i.e. flow is irrotational.

Velocity potential function (ϕ) only exists for the irrotational flow.

Equipotential lines:

(ϕ - constant)

It is line joining the points of equal potential function values

for steady, incompressible flow

$$\phi = f(x, y)$$

$$d\phi = \frac{\partial \phi}{\partial x} \cdot dx + \frac{\partial \phi}{\partial y} \cdot dy$$

$$d\phi = -u \cdot dx - v \cdot dy$$

for equipotential lines,

$$\phi = \text{constant}.$$

$$d\phi = 0$$

$$-u \cdot dx - v \cdot dy = 0$$

$$\frac{dy}{dx} = \frac{-u}{v}$$ — slope of equipotential line.

Stream function (ψ)

In general, this function is defined as function of space and time in such a way such that continuity equation is satisfied.

By definition,

$$u = -\frac{\partial \psi}{\partial y}$$

$$v = +\frac{\partial \psi}{\partial x}$$

By continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial}{\partial x}\left(\frac{-\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right)$$

$$= \frac{-\partial \psi}{\partial x \cdot \partial y} + \frac{\partial \psi}{\partial x \cdot \partial y}$$

$$= 0 \quad \text{– (satisfied automatically)}$$

ψ exists in rotational as well as irrotational flow.

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial y}\right)\right]$$

$$= \frac{1}{2}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right]$$

$$= \frac{1}{2}\nabla^2 \psi$$

If ψ satisfies Laplace's equ^n

i.e. $\nabla^2 \psi = 0$ – irrotational flow $(\omega = 0)$.

and if $\nabla^2 \psi \neq 0$

Equi-stream function lines.

(ψ -constant line).

It is the line joining the points having same stream function value.

For 2-D steady, incompressible flow.

$$\psi = f(x, y)$$

$$d\psi = \frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} \cdot dy$$

$$= (v \cdot dx - u\, dy)$$

For equi-stream function lines, ψ = constant

$$d\psi = 0$$

$$v \cdot dx - u\, dy = 0$$

$$\frac{dy}{dx} = \frac{v}{u} \quad \text{— slope of equi-stream function line}$$

$$\frac{dy}{v} = \frac{dx}{u} \quad \text{is equation of streamline.}$$

∴ Stream lines are the only equi-stream function lines.

$$\therefore \left(\frac{dy}{dx}\right)_{\phi\text{-const line}} \times \left(\frac{dy}{dx}\right)_{\psi\text{ const line}} = \frac{-u}{v} \times \frac{v}{u}$$

$$= -1.$$

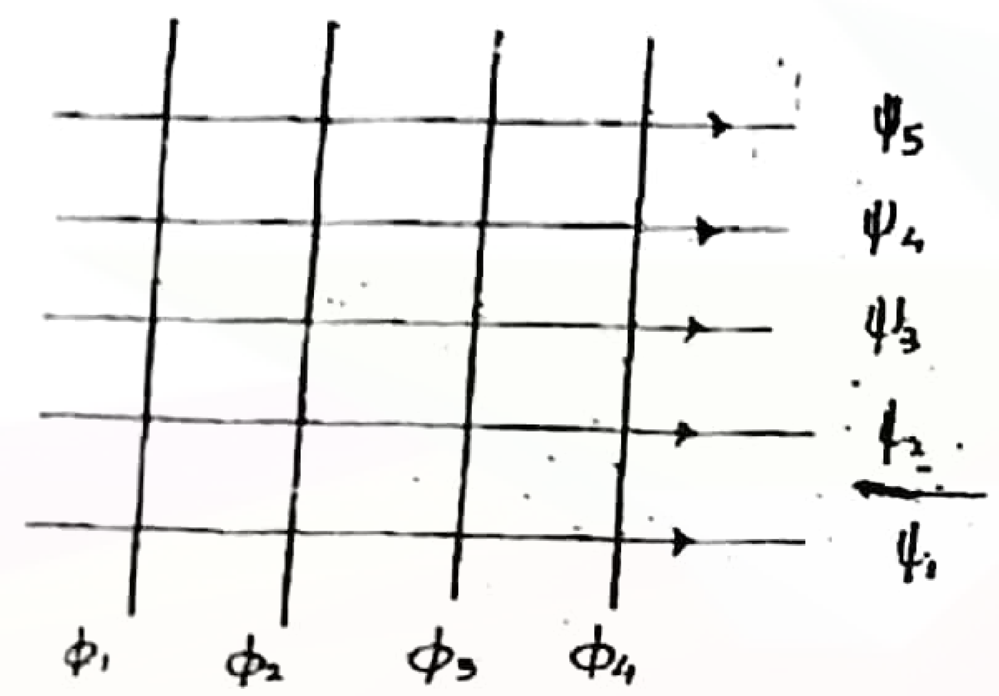
Equipotential lines and streamlines in irrotation flow are always orthogonal to each other.

A graphical diagram representing equipotential lines and streamlines in an irrotational flow is known as flow net.

Thursday
21st Nov' 2013

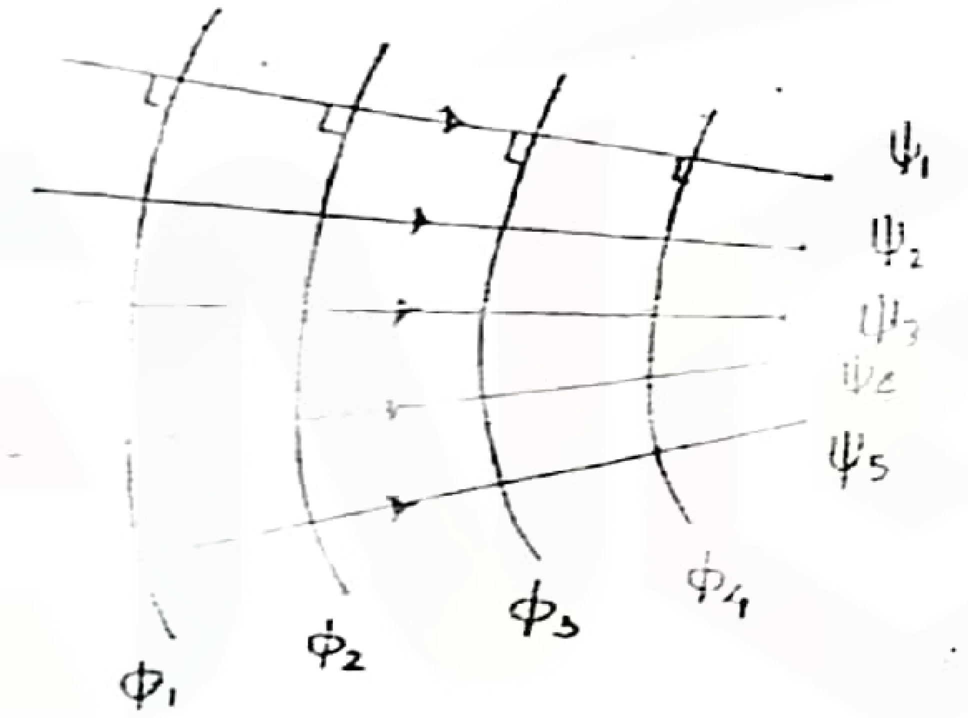
Flow nets in different Flows :

(i) Uniform Flows :



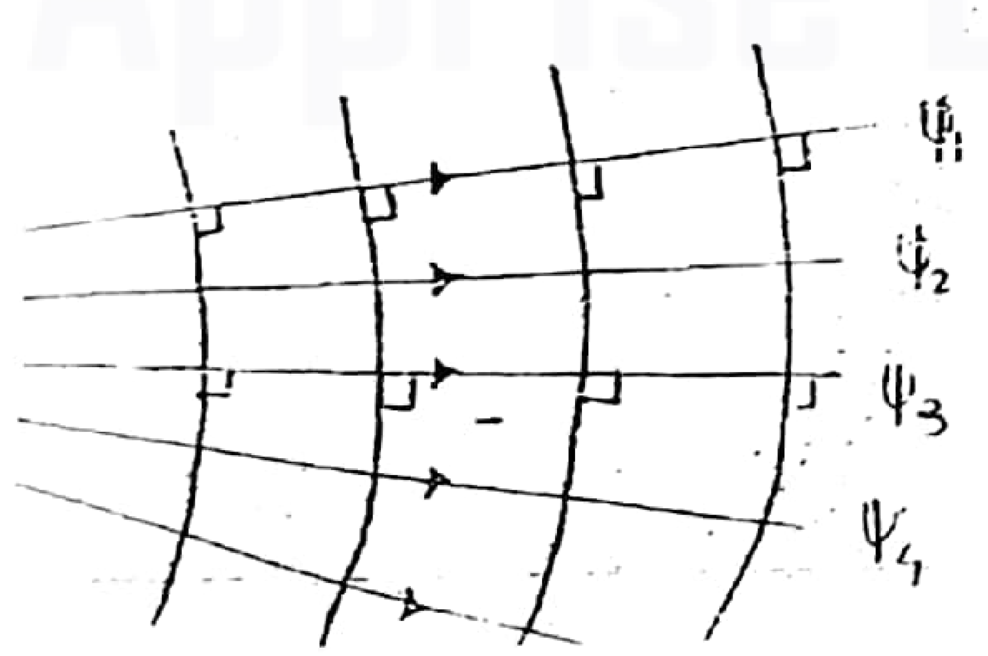
(ii) Accelerated flow :

(streamlines are converging)



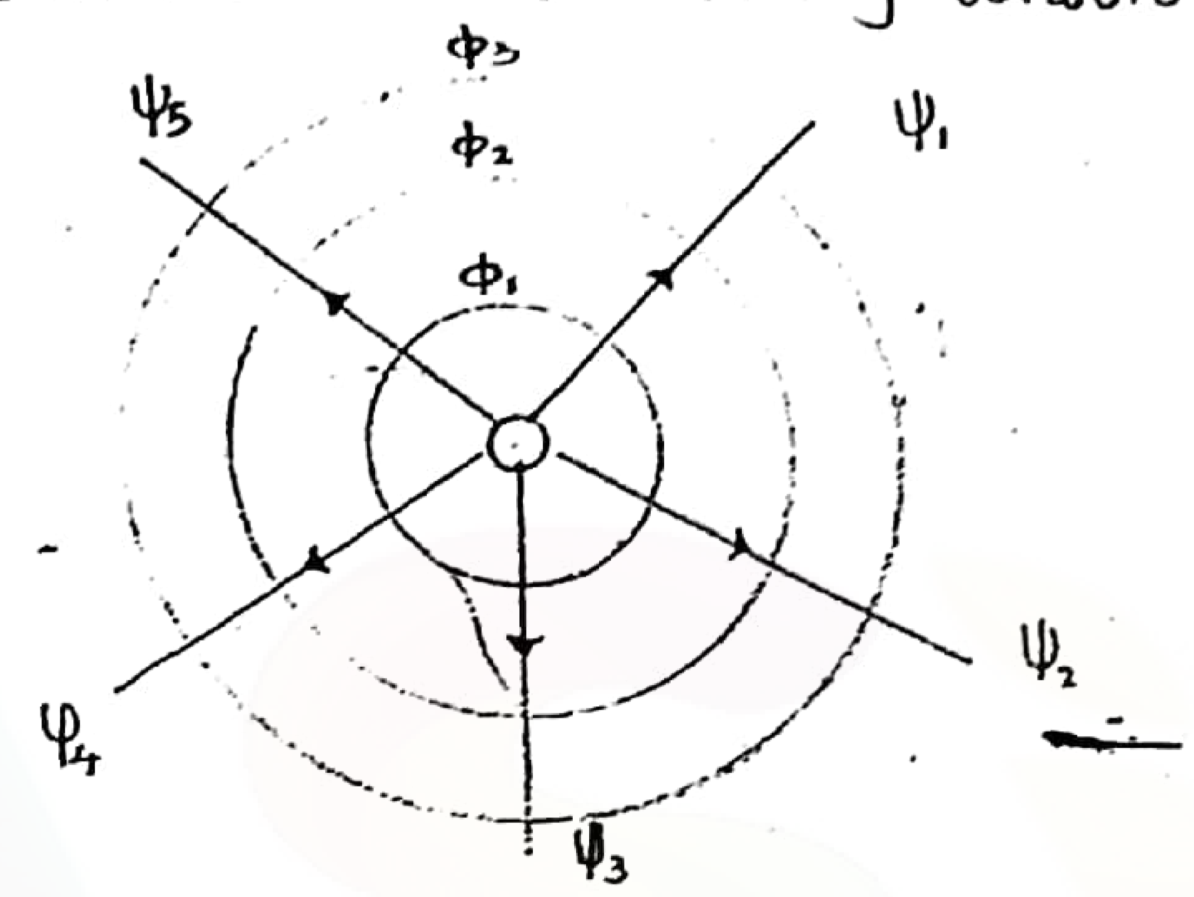
(iii) Retarding flow :

(streamlines are diverging)



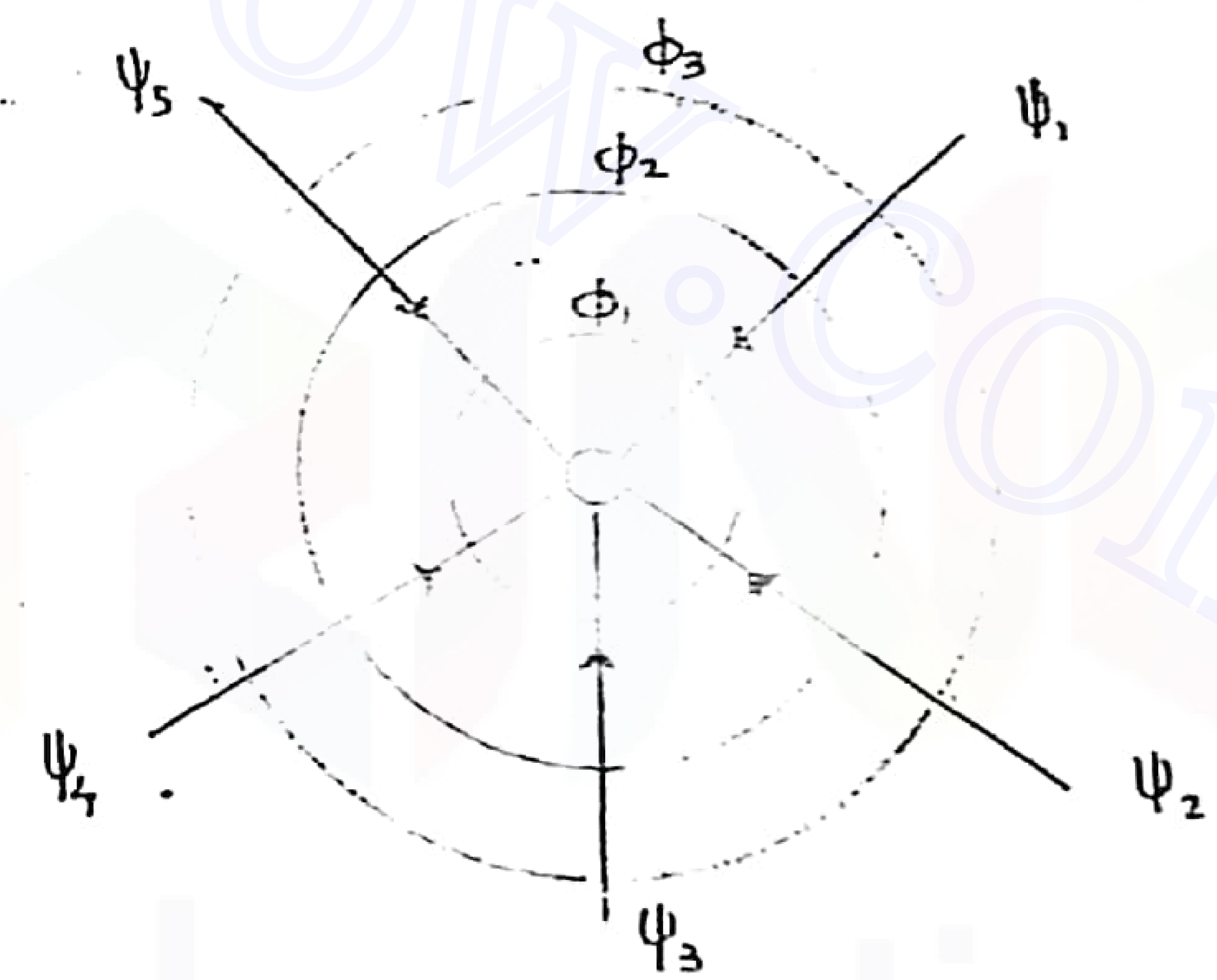
(iv) Source flow :

(streamlines move radially outward) —

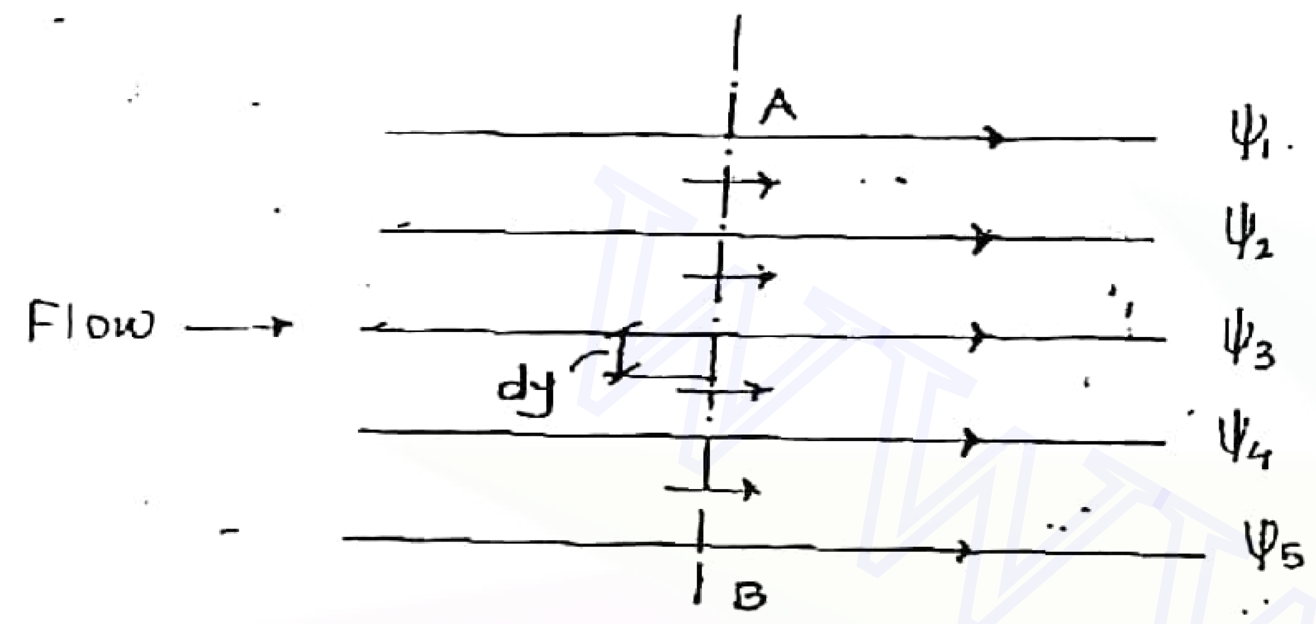


(v) Sink :

(streamlines move radially inward.)



Physical significance of stream function (ψ):



In 2-D steady incompressible flow,

$$\psi = f(x,y)$$

$$d\psi = \cancel{\frac{\partial \psi}{\partial x}}^{0} dx + \frac{\partial \psi}{\partial y} \cdot dy$$

(no distance in x-direction)

$$d\psi = \frac{\partial \psi}{\partial y} \cdot dy$$

$$= -u \cdot dy$$

$$-d\psi = u \cdot dy$$

$$= u \cdot (dy \times 1) \quad \text{- unit width (depth) of flow}$$

$$\int_A^B -d\psi = \int dQ \; (\text{per unit width of flow})$$

$$\Delta \psi = Q \text{ per unit width of flow.}$$

i.e.

the difference of stream function value between any two points in flow gives the discharge per unit width of flow between these two points.

Q.

Page 24, Q-18

Q. A 2-D steady, incompressible flow is given by

$$\vec{V} = (3xy)\,i + \left(\frac{3}{2}x^2 \pm \frac{3}{2}y^2\right) j$$

Find the relevant potential and stream function.

$$u = 3xy$$

$$v = \left(\frac{3}{2}x^2 \mp \frac{3}{2}y^2\right)$$

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 3y + (0 \cancel{-3y})$$

$$= 0 \quad \text{- Flow is possible.}$$

For potential function (ϕ)

.... Angular velocity (ω_z) -

$$= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= \frac{1}{2}\left(\frac{3x}{2} - 3x\right)$$

$$= 0 \quad \text{- Irrotational flow}$$

ϕ will exist for given flow

$$u = -\frac{\partial \phi}{\partial x}$$

$$\frac{\partial \phi}{\partial x} = -u = -3xy$$

$$\partial \phi = -3xy \; \partial x$$

$$\int \partial \phi = -y \int 3x \, \partial x$$

$$\phi = -\frac{3x^2}{2}y + f(y) + c$$

There can be pure

similarly, $v = \frac{-\partial\phi}{\partial y}$

$$\frac{\partial\phi}{\partial y} = -v = \frac{3}{2}y^2 - \frac{3}{2}x^2$$

$$\int\partial\phi = \int\left(\frac{3}{2}y^2 - \frac{3}{2}x^2\right)\partial y$$

$$\phi = \frac{-3}{2}\cdot\frac{y^3}{3}x^2 + \frac{y^3}{2} + f(x) + c \quad \text{—— (ii)}$$

There can be pure function of x.

from —(i) and —(ii)

$$\phi = \frac{-3}{2}x^2y + \frac{y^3}{2} + c.$$

For stream function

$$u = -\frac{\partial\psi}{\partial y}$$

$$\int\partial\psi = \int -3xy\cdot\partial y$$

$$\psi = -\frac{3x}{2}y^2 + f(x) + c \quad \text{—— (iii)}$$

There can be pure function of x.

and $v = \frac{\partial\psi}{\partial x}$

$$\int\partial\psi = \int\left(\frac{3}{2}x^2 - \frac{3}{2}y^2\right)\cdot\partial x$$

$$\psi = \frac{x^3}{2} - \frac{3}{2}y^2\cdot x + f(y) + c \quad \text{—— (iv)}$$

from —(iii) and —(iv)

$$\psi = \frac{-3xy^2}{2} + \frac{x^3}{2} + c$$

Couchy - Riemann equation
(Irrotational flow)

$$u = -\frac{\partial\phi}{\partial x} = -\frac{\partial\psi}{\partial y} \quad \text{—— (i)}$$

&

$$v = -\frac{\partial\phi}{\partial y} = +\frac{\partial\psi}{\partial x} \quad \text{—— (ii)}$$

from —(i) & —(ii)

$$\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y}$$

and

$$-\frac{\partial\phi}{\partial y} = \frac{\partial\psi}{\partial x}$$

Note:

(i) Angular velocity $(\omega_z) = \frac{\left(\frac{d\beta}{dt} + \frac{d\alpha}{dt}\right)}{2} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$
(vector quantity)

(ii) Rate of shear deformation
(scalar quantity)

$$= \frac{\left|\frac{d\beta}{dt}\right| + \left|\frac{d\alpha}{dt}\right|}{}$$

$$= \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

DYNAMICS OF FLUID FLOW

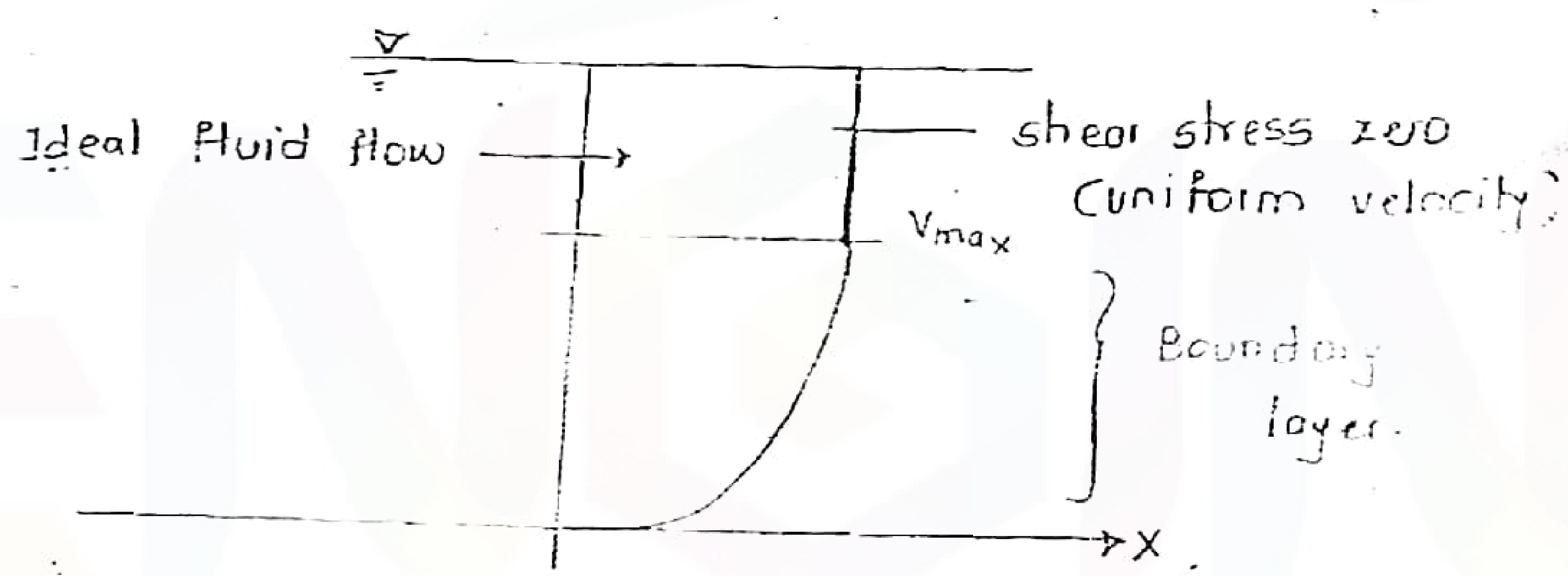
"Study of fluid particle motion with the consideration of basic cause of motion i.e. Force.

It is studyied taking mass of fluid as system. The Newton's second law of motion is applied to get the Momentum equation.

Assumptions :

(i) Flow is Laminar
(ii) Flow is irrotational
— Navier-stoke's equation

(iii) Flow is inviscid (ideal fluid flow)
— Euler's equation of motion



(iv) Steady flow
(v) Incompressible flow
— Bernoulli's equ^n (Energy equation)*

Bernoulli further integrated the equation

$$\int m.v = \frac{m v^2}{2} - \text{Energy}$$

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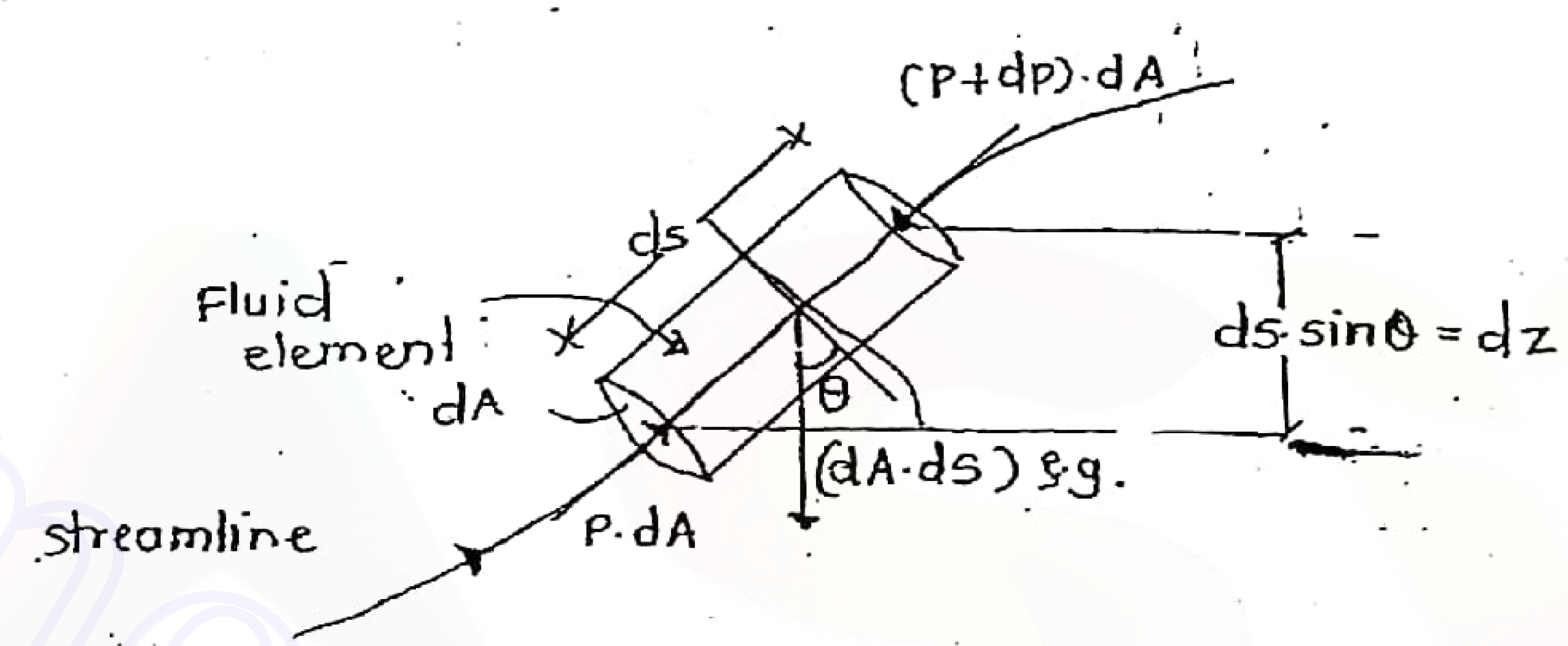
Euler's equation of motion along streamline :

Assumptions :

(i) Flow Laminar.
(ii) Flow is irrotational
(iii) Flow is inviscid.



Mass of an element,

$$dm = (dA \cdot ds) \cdot \rho$$

velocity of element,

$$v = f(s, t)$$

Acceleration,

$$a = v \cdot \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}$$

Applying Newton's second law of motion in direction of flow.

$$\underbrace{p\,dA}_{F_1} - \underbrace{(p+dp).dA}_{F_2} - \underbrace{(dA \cdot ds \cdot \rho g \sin\theta)}_{F_3} = \underbrace{(dA \cdot ds \cdot \rho)}_{m}\underbrace{\left[v \cdot \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}\right]}_{a}$$

$$\frac{dp}{\rho} + ds\left(v \cdot \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}\right) + g \cdot dz = 0$$

Euler's equation

If flow is steady
(Bernoulli's 1st assumption)

$$\frac{\partial v}{\partial t} = 0 \quad \therefore \quad v = f(s) \text{ only}$$

$$\frac{\partial v}{\partial s} = \frac{dv}{ds}$$ only one function deper

$$\frac{dP}{\rho} + v \cdot dv + g \cdot dz = 0$$

Euler's equation for steady flow

Integrating further
(Bernoullis work)

$$\int \frac{dP}{\rho} + \int v \cdot dv + \int g \cdot dz = \text{constant}$$

Here P is function of ρ wherein in actual ρ is function of P for compressible flow. Thus Bernoulli assumed ρ constant.

$\therefore$ If flow is incompressible (ρ = constant)
(Bernoullis 2nd assumption)

$$\frac{1}{\rho} \int dP + \int v \cdot dv + g \int dz = \text{const.}$$

$$\frac{P}{\rho} + \frac{v^2}{2g} + gz = \text{const.}$$

$$P + \frac{1}{2} \rho v^2 + \rho g z = \text{const.}$$ – Energy equatic

K.E. per unit volume

P.E. per unit volume

Pressure (P) ⟶ Pressure force (P·A) ⟶ work done (P·A·x)

volume.

$$P = \frac{P \cdot A \cdot x}{(A x)}$$ – pressure energy per unit volume.

$\therefore$ Pressure is form of energy. (work)

Under the above five assumptions inside the flow if they are valid, the summation of all the energies (pressure energy, Kinetic energy and potential energy) per unit volume will be constant throughout the flow.

This concept is explained by Bernoulli, known as Bernoullis theorem but in general it is known as Energy equation.

Head form of energy equation:

When energy of fluid is represented as a height of a fluid column then that height of fluid column is known as Head.

(i) Pressure head:

pressure energy per unit volume = P

unit volume = $\rho g \cdot h$

$$P = \rho g \cdot h$$

$$h = \frac{P}{\rho g}$$

h - pressure head.

(ii) Kinetic head (velocity head or dynamic head)

$$K.E.\ \text{per unit volume} = \frac{1}{2}\rho v^2$$

$$\text{[illegible]} = \rho g h$$

$$\frac{1}{2}\rho v^2 = \rho g h$$

$$h = \frac{v^2}{2g}$$

(iii) Potential head :

$$\rho g z = \rho g h$$

$$h = z$$

It is also called datum head or reference head.

$$P + \frac{1}{2}\rho v^2 + \rho g z = \text{constant}$$

$$\frac{P}{\rho g} + \frac{v^2}{2g} + z = \text{const} \quad - \text{total head}$$

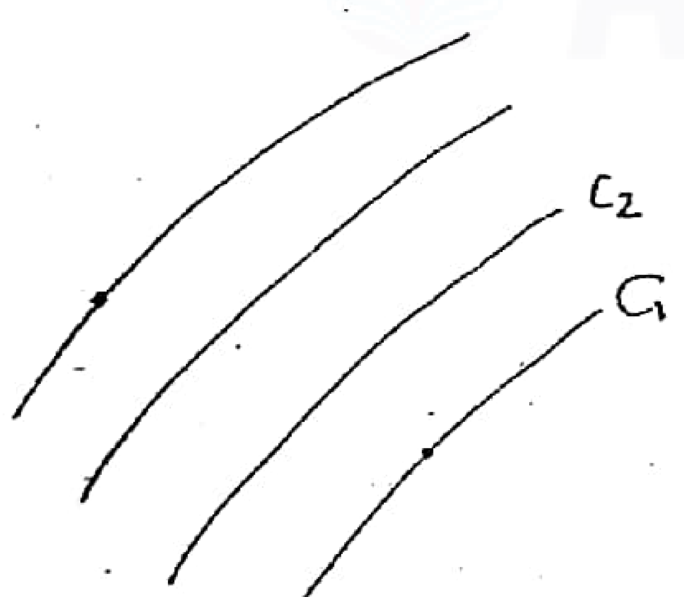
- Head form of energy equation

$$\left(\frac{P}{\rho g} + z\right) - \text{piezometric head - static head}$$

$$\frac{P}{\rho g} + \frac{v^2}{2g} + z = C - \text{total head}$$

If flow is irrotational

$$C_1 = C_2 = C_3 = \ldots$$



Energy equ^n can be applied between points on different streamlines

If flow is rotational

$$C_1 \neq C_2 \neq C_3$$

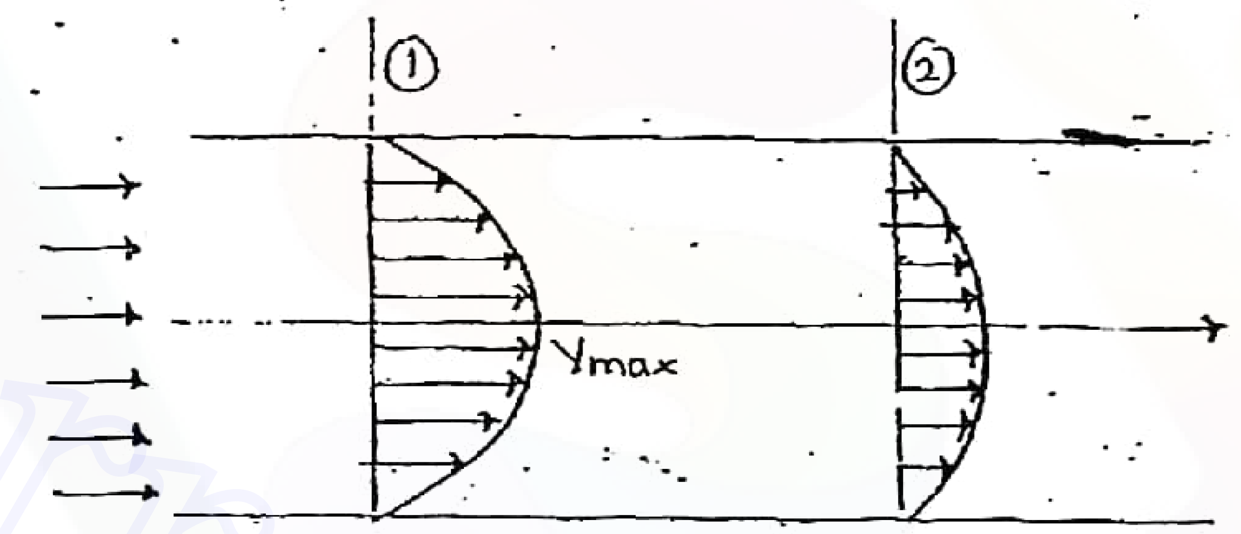
Note :

General equation can also be applied in Rotational fluid flows between the points which lie on same streamline

Friday
22nd Nov 2013

Energy equation applied in viscous fluid flow :

(pipe flows, boundary layer flow, viscous flow)
(Internal flow)



velocity distribution in pipe flow.

The velocity distribution in pipe flow is as shown in fig. The velocity will be max. at the centre and zero at the ends.

As pipe flow is viscous flow, the velocity at ② will be less than at ①.

But since,

$$Q_1 V_1 = Q_2 V_2 \quad - \text{for pipe flow}$$

Due to $V_2 < V_1$ continuity equ^n will not be satisfied. Thus the flow will not be possible.

To make The flow will be open channel flow if the velocity goes on decreasing ($V_2 < V_1$)

To make flow possible, the energy must be supplied at ② to satisfy continuity equation.

This energy is provided by the total head only on the section ① so that it would compensate the energy loss due to friction

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

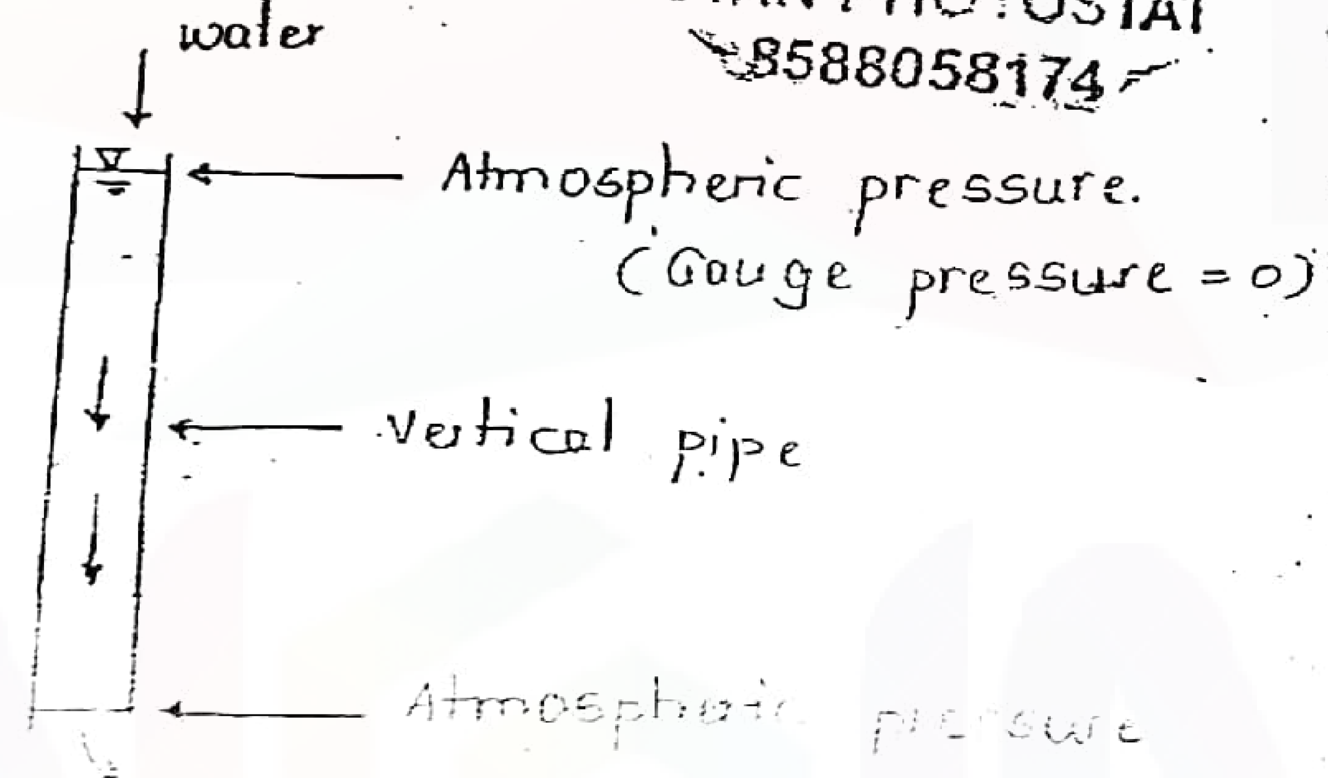
$Q = A_1 V_1 = A_2 V_2$ is the continuity equation for the pipe flows only. (for 1D flow)

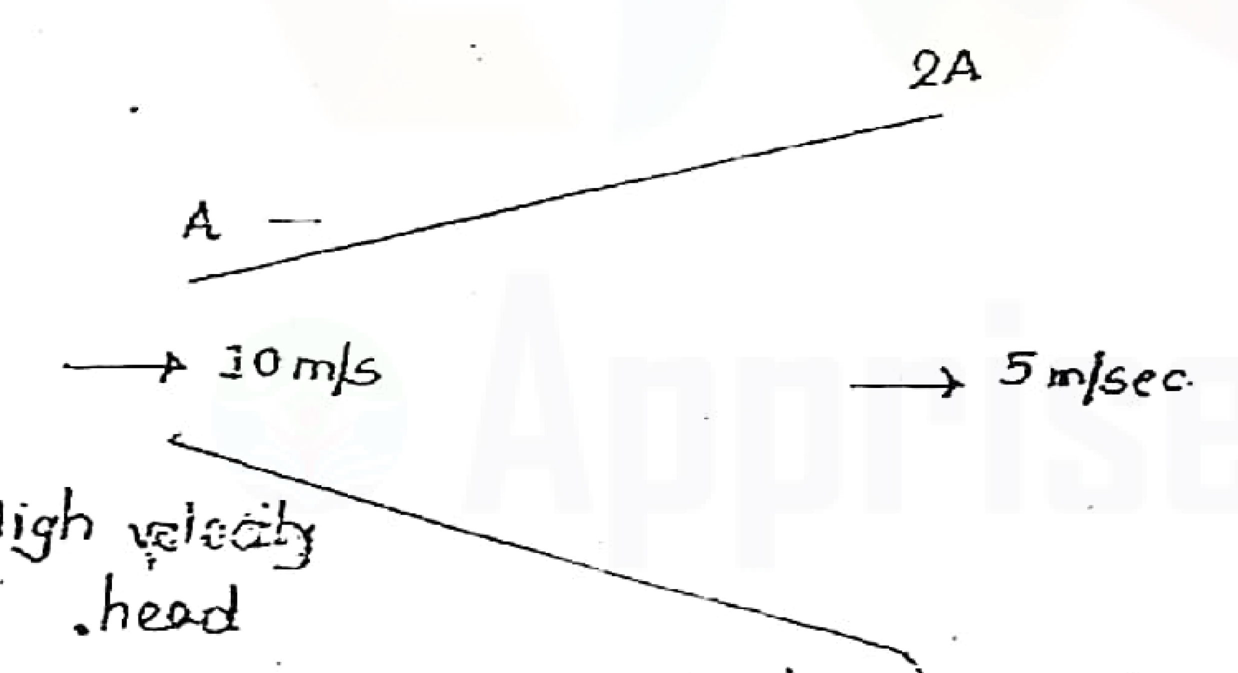
Shear resistance reduces the velocity of flow. Thus the mean velocity of the flow goes on decreasing.

The fluid flow is function of total head and not the pressure head. Thus difference ~~in~~ total head between two points of fluid cause flow from higher total head to lower total head.

Example:

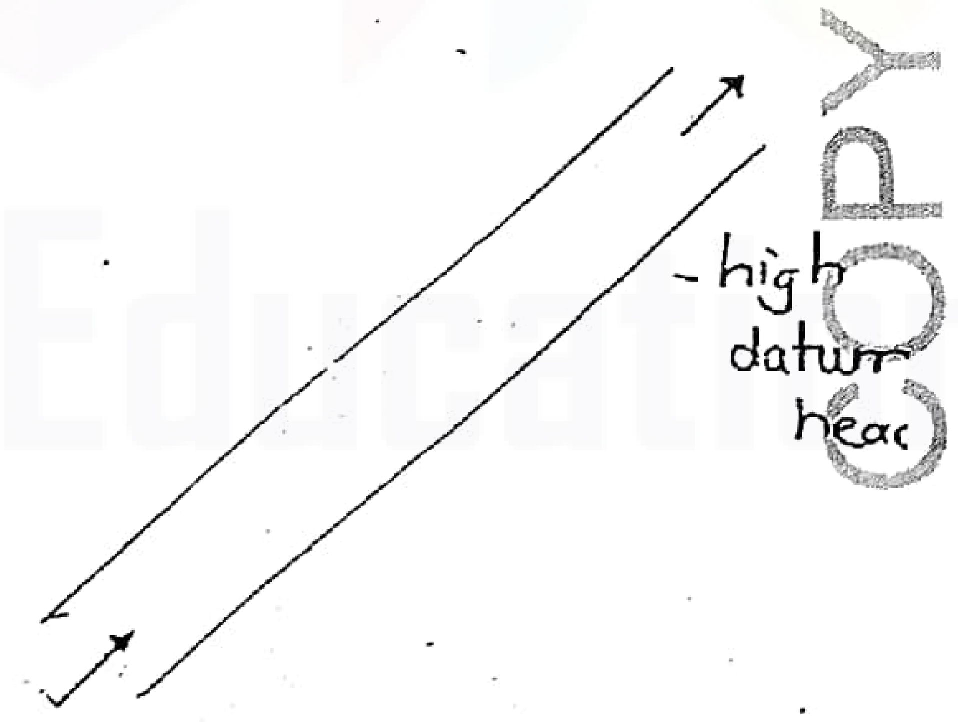
Flow is possible without a pressure difference. (atmospheric pressure at both ends.)





- High velocity head
- low pressure
- High total head

- High pressure head
- low velocity head
- Low Total head



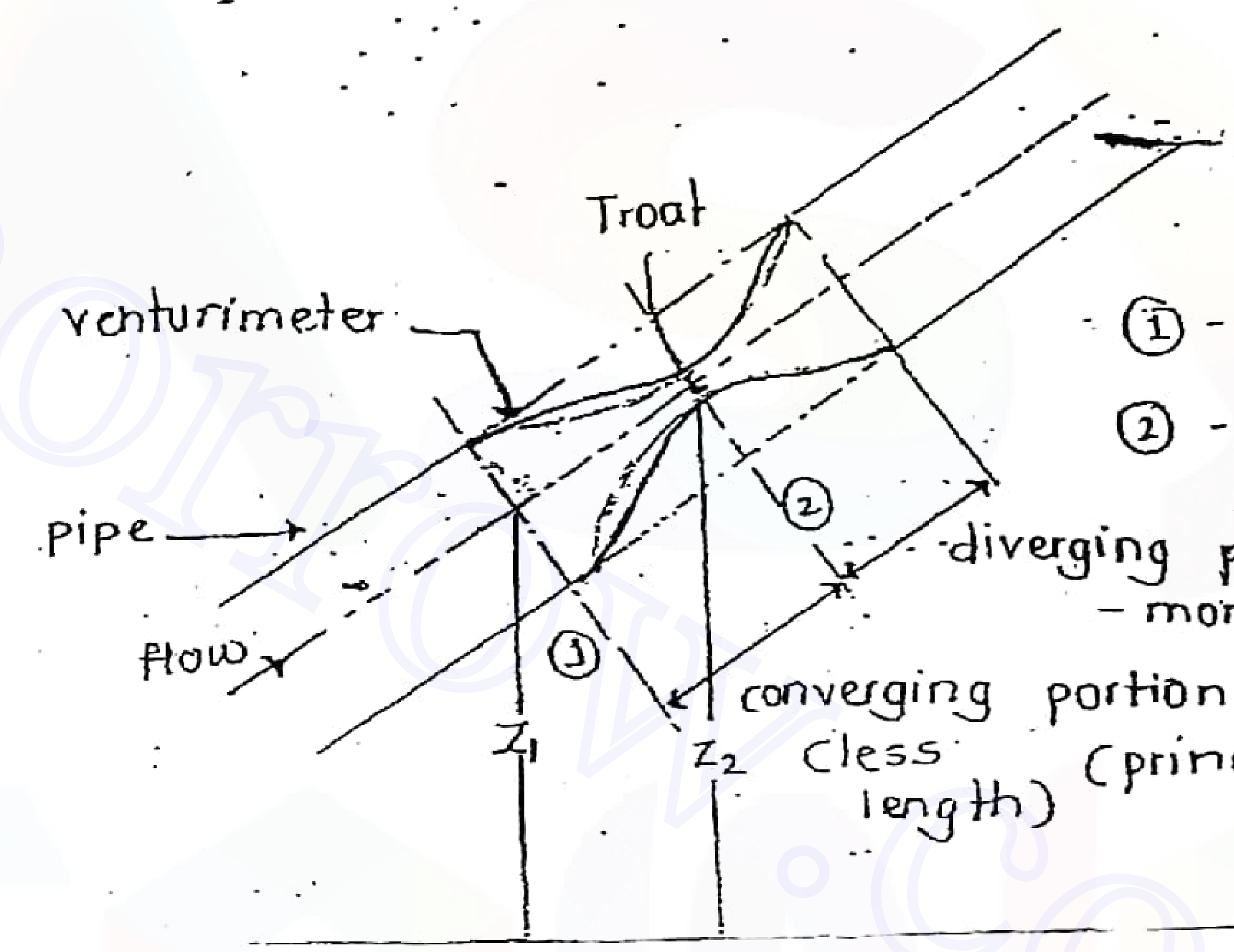
- low datum head
- velocity head cannot be increase (Area constant)

Flow measurement devices:-

(i) Venturimeter:

It is a device which is basically used for the measurement of flow rate of fluids flowing through pipes.

It is a converging-diverging section, installed in the pipe having the minimum c/s area which is known as throat.



① - Normal section

② - Throat section

diverging portion (for flow of more length, water to normal c/s - only)

converging portion (less length) (principle, function of venturimeter)

Theoretical analysis (head loss not considered)

for sections ① and ②, energy equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{V_1^2}{2g} - \frac{V_2^2}{2g} = \underbrace{\left(\frac{P_2}{\rho g} + Z_2\right) - \left(\frac{P_1}{\rho g} + Z_1\right)}_{=h}$$

difference of piezometric head between normal and throat section

$$V_2^2 - V_1^2 = 2gh$$

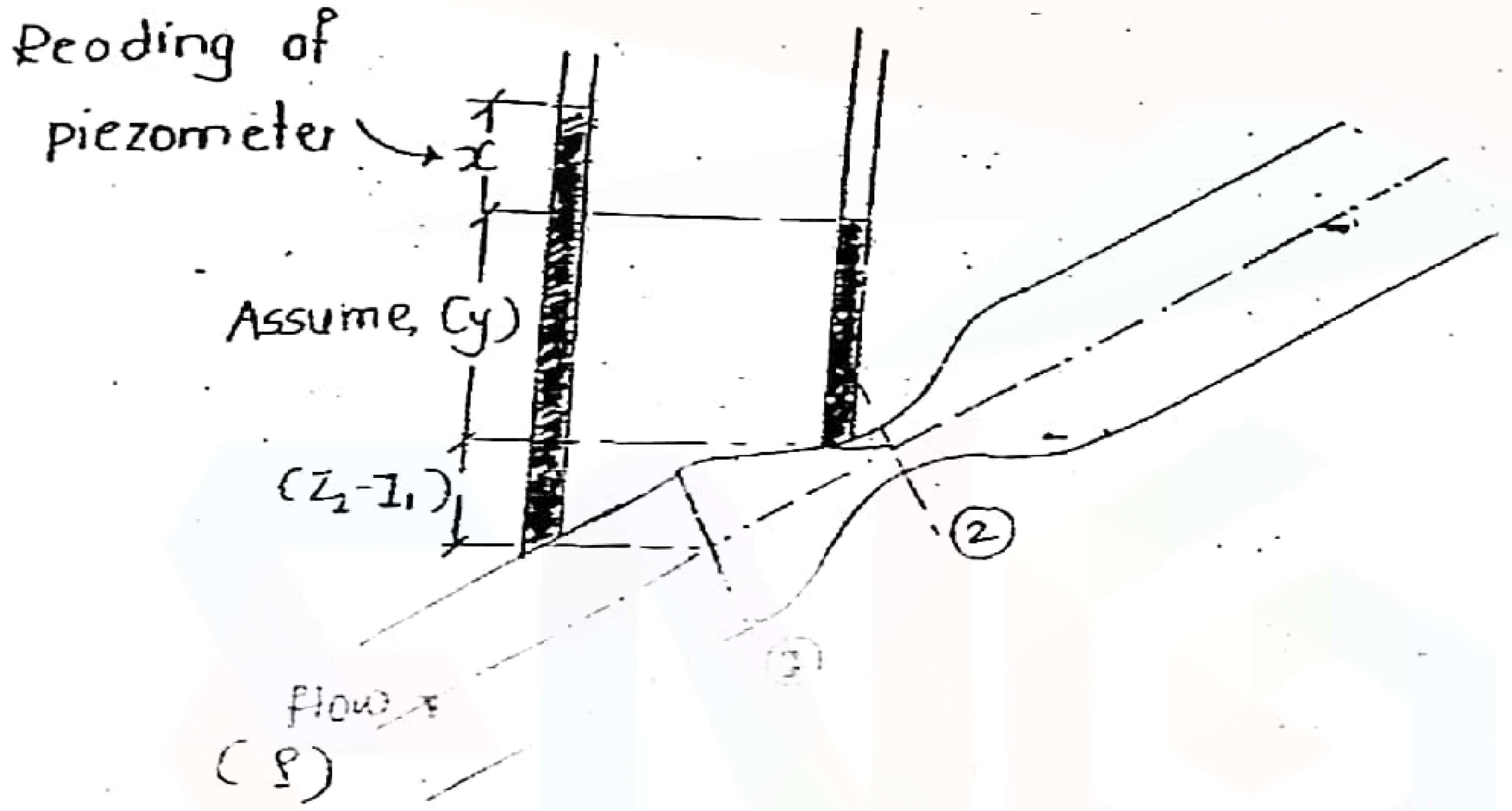
$$\frac{Q_{th}^2}{A_2^2} - \frac{Q_{th}^2}{A_1^2} = 2gh$$

$Q_{th} = A_1V_1 = A_2V_2$ - continuity equatio

$$Q_{th} = A_1 \cdot A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

Calculation of differential head of Venturimeter:

1. Venturimeter with piezometers:



$$P_1 = [x + y + (z_2 - z_1)]\,\rho g$$

$$P_2 = y \cdot \rho g$$

$$\frac{P_1}{\rho g} = x + y + (z_2 - z_1)$$

$$\frac{P_2}{\rho g} = y$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = x + (z_2 - z_1)$$

$$\left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = x = h$$

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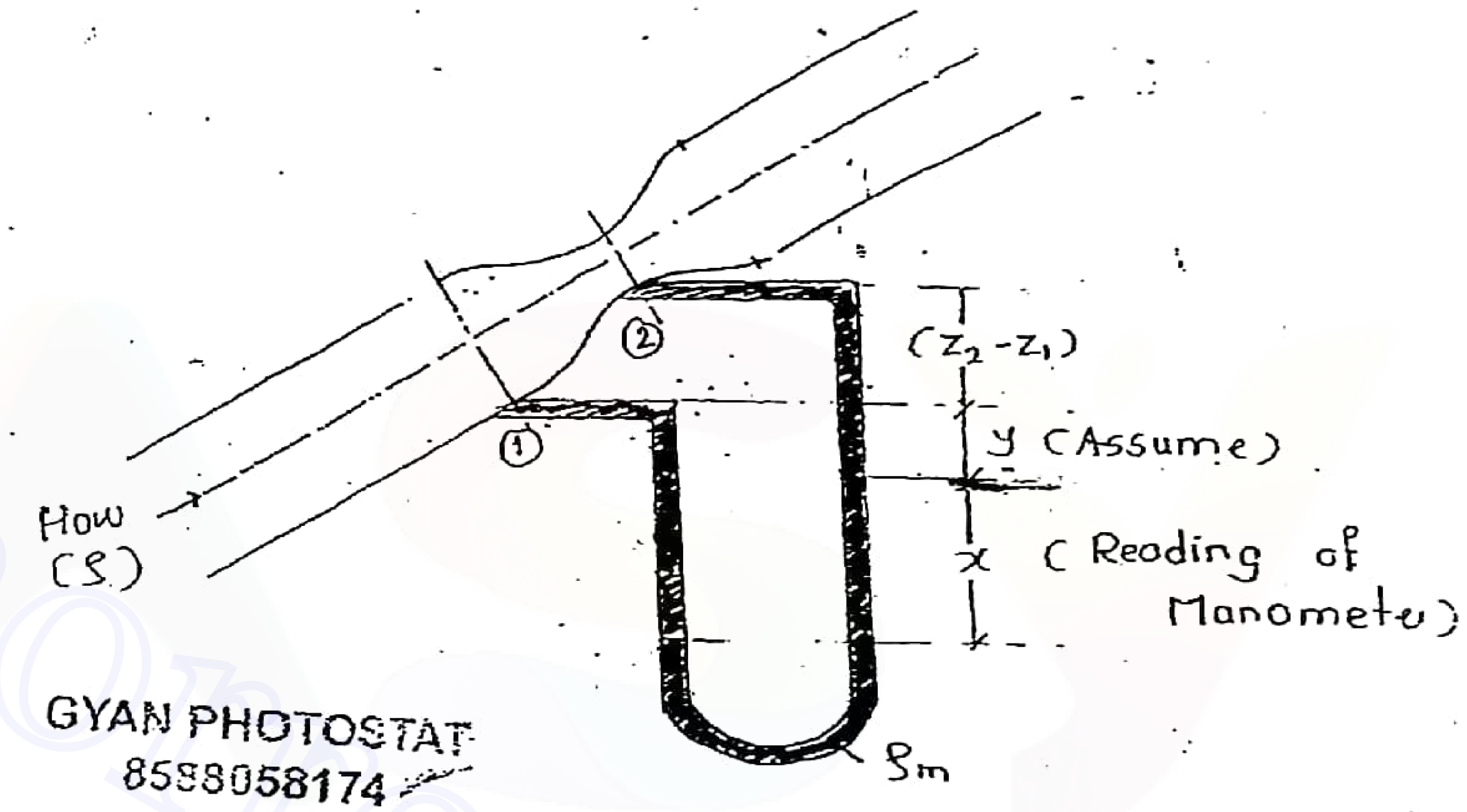
2. Venturimeter with Manometer:

(i) $\rho_m > \rho$



$$P_1 + (y + x)\rho g - x \cdot \rho_m \cdot g - [y + (z_2 - z_1)]\rho g = P_2$$

$$\frac{P_1}{\rho g} + x - x \cdot \frac{\rho_m}{\rho} - z_2 + z_1 - \frac{P_2}{\rho g} = 0$$

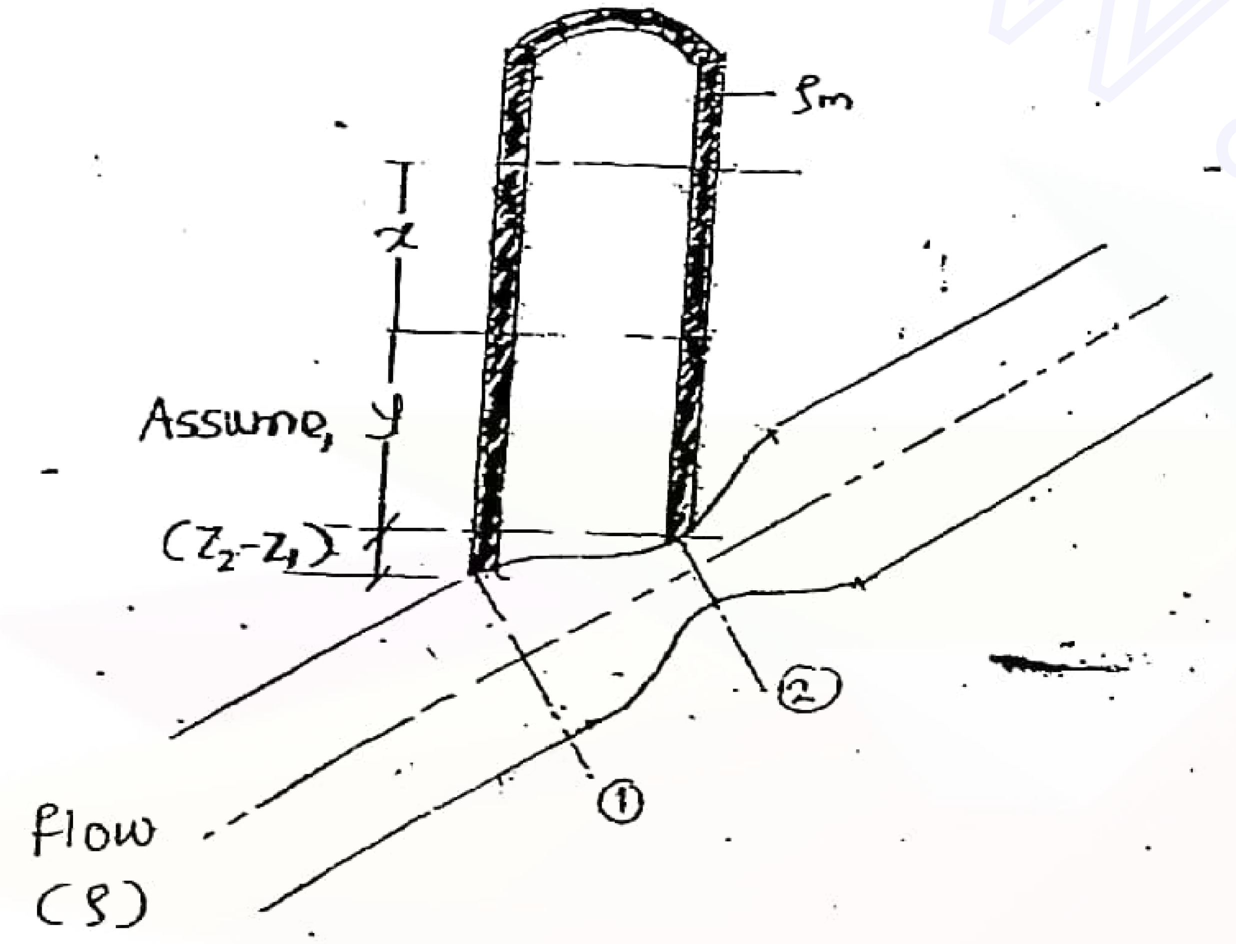
$$\left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = x\left(\frac{\rho_m}{\rho} - 1\right)$$

$$h = \left(\frac{\rho_m}{\rho} - 1\right)$$

If the Manometer is attalch above the pipe, the fluid inside pipe will rise due to less density and manometric fluid will drop in the pipe due high density.

Similarly for fluids with $\rho > \rho_m$, if manometer is attached below the pipe then Manometric fluid will flow into the pipe and will get carried away with flow in pipe.

(ii) $\rho_m < \rho$.



No turbulence inducing - gradual convergence

$$P_1 - [(Z_2 - Z_1) + y + x]\rho g + x \cdot \rho_m g + y \cdot \rho g = P_2$$

$$\frac{P_1}{\rho g} - Z_2 + Z_1 - x + \frac{x \rho_m}{\rho} - \frac{P_2}{\rho g} = 0$$

$$\left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right) = x\left(1 - \frac{\rho_m}{\rho}\right)$$

$$h = x\left(1 - \frac{\rho_m}{\rho}\right)$$

Actual analysis:

for normal section and throat section of venturimeter.

Energy equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) = \left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right) - h_L$$

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = (h - h_L)$$

$$\therefore V_2^2 - V_1^2 = 2g(h - h_L)$$

$$\frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2} = 2g(h - h_L)$$

$$Q = A_1 A_2 \sqrt{\frac{2g(h - h_L)}{A_1^2 - A_2^2}}$$

$$Q = \underbrace{\sqrt{\frac{h - h_L}{h}}}_{\text{coefficient of discharge of Venturimeter}} \cdot \underbrace{A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}}_{\text{Theoretical discharge}}$$

$$Q = C_D \cdot A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

where,

$h = x$, if piezometer is used.

$= x\left(\frac{\rho_m}{\rho} - 1\right)$ if monometer $(\rho_m > \rho)$

$= x\left(1 - \frac{\rho_m}{\rho}\right)$ if monometer used $(\rho_m < \rho)$

C_d for venturimeter $= 0.97 - 0.99$

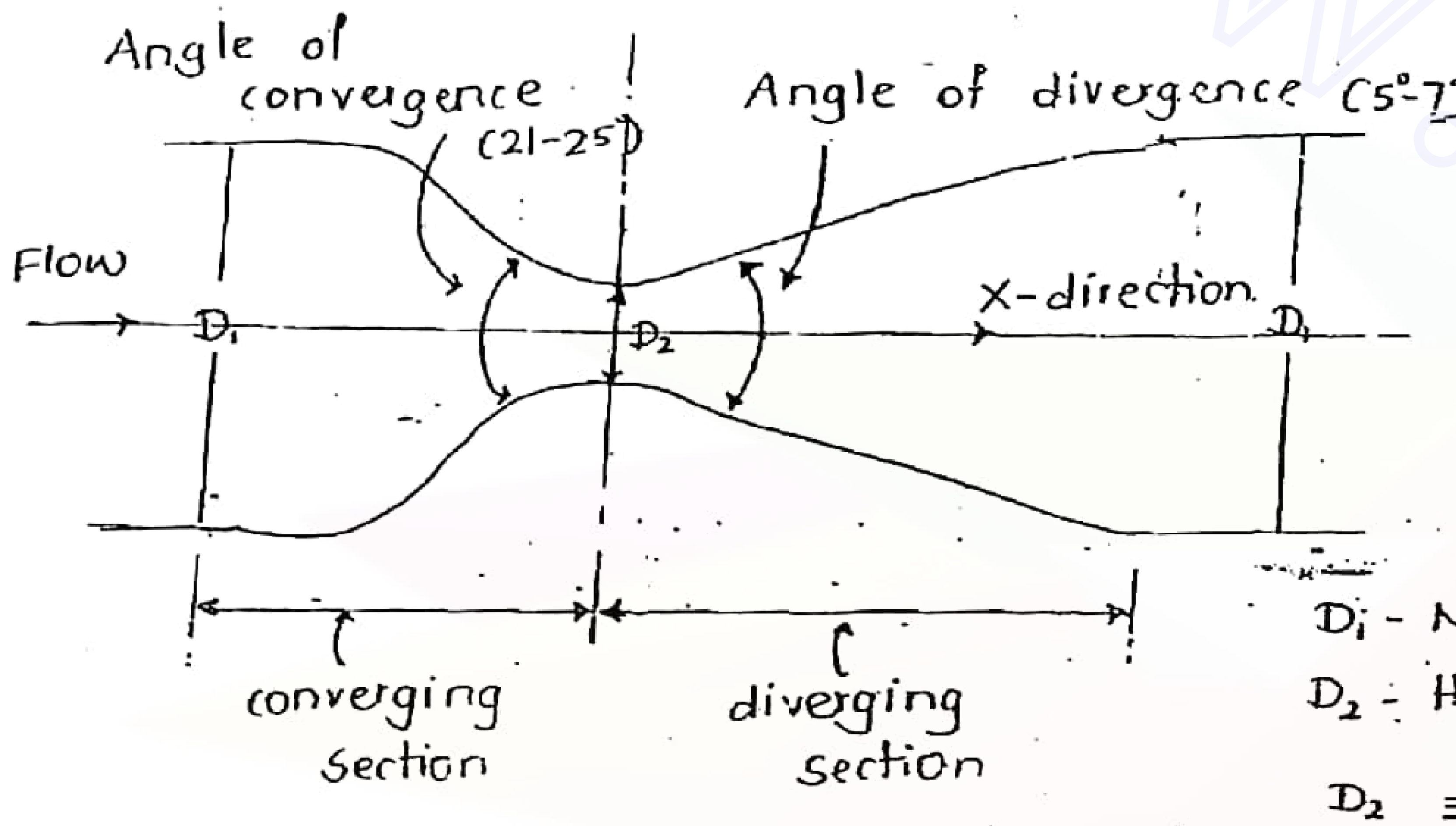
Note:

In the pipe flows whenever the pressures are given at any section, it is assumed that this pressure is Gauge pressure.

The Cd for the venturimeter is 0.97-0.99 i.e. theoretical discharge is nearly same as that of actual discharge. This is because of Gradual convergence of venturimeter over a very less length.

If the sudden convergence is provided high turbulence will be created causing energy dissipation.

Design parameters of venturimeter :



D_1 - Nominal diameter

D_2 - throat diameter

$$D_2 = \left(\frac{D_1}{3}, \frac{D_1}{2}\right)$$

i.e. $0.33\,D_1 < D_2 < 0.5\,D_1$

In converging section.

Area of flow is decreasing in flow-direction. Thus velocity increases. Therefore pressure is decreasing in flow direction.

$$\frac{\partial P}{\partial x} < 0$$ decreasing pressure

This is called Favourable pressure gradient. (i.e. accelerated flow) As velocity is increasing, making flow easy

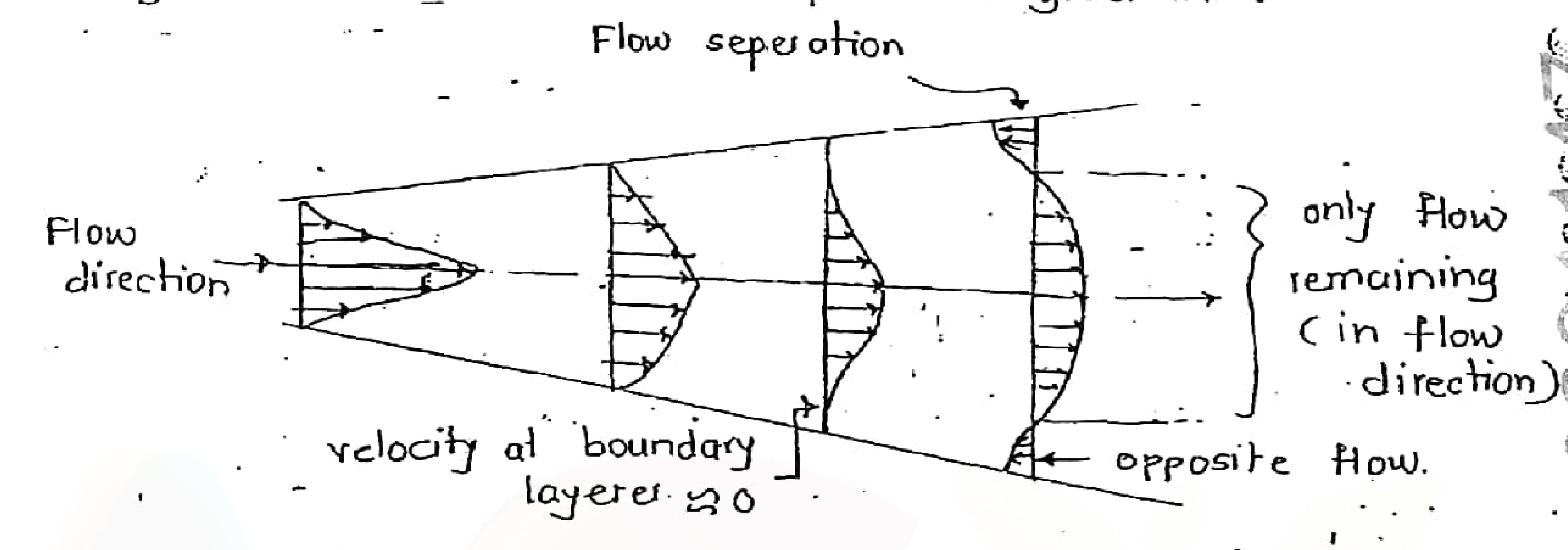
In diverging section,

In direction of flow area is increasing thus velocity is decreasing and pressure is increasing thereby

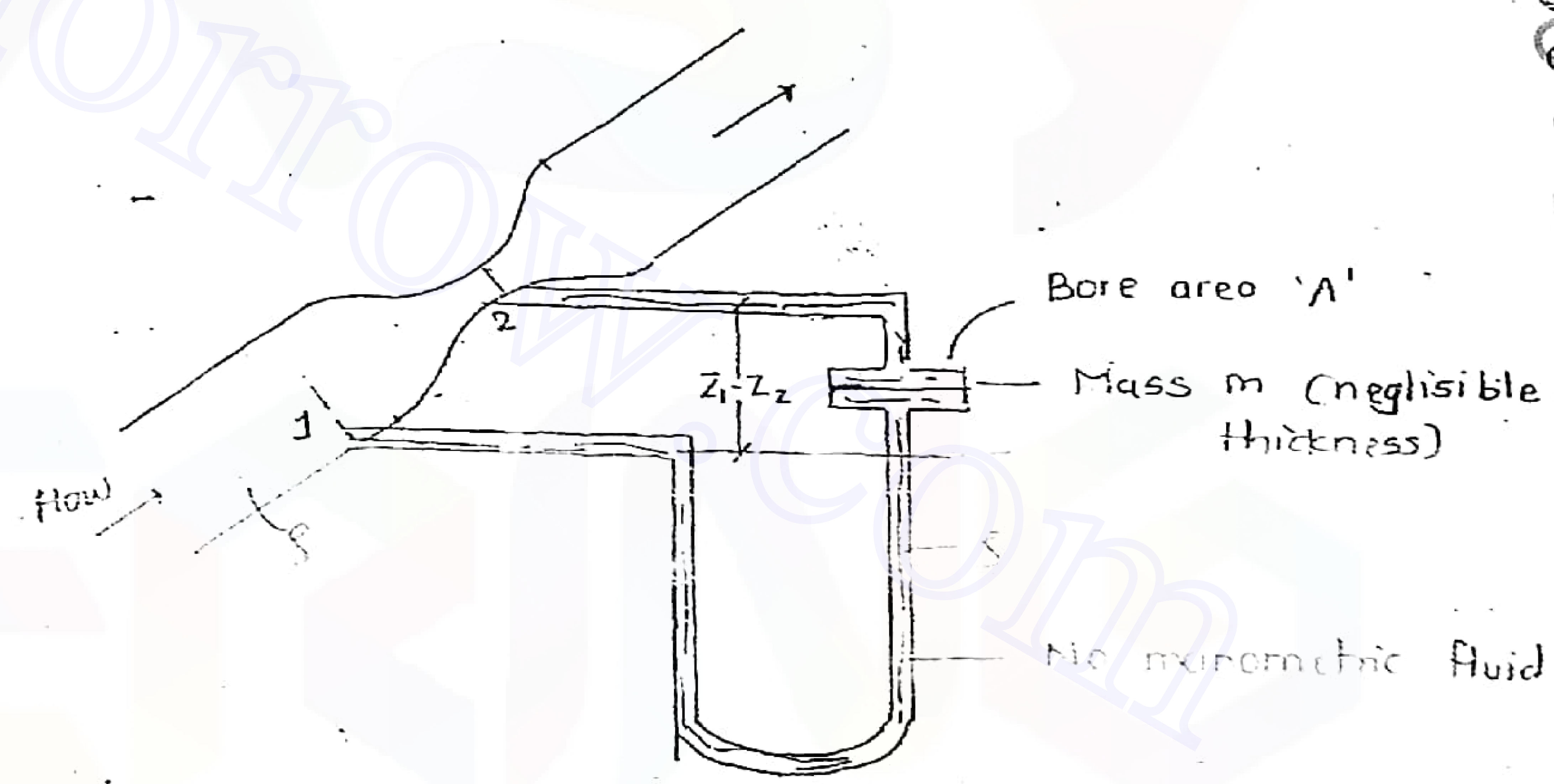
$$\frac{\partial P}{\partial x} > 0$$

- It is called Adverse pressure gradient (retarding flow) because in this region velocity is decreasing making flow difficult.

Velocity distribution in adverse pressure gradient :



Q. Find discharge through venturimeter shown:- 10 Marks



$$P_1 - (Z_1 - Z_2)\rho g - \left(\frac{mg}{A}\right) = P_2$$

$$\frac{P_1}{\rho g} - Z_2 + Z_1 - \frac{mg}{\rho g A} = \frac{P_2}{\rho g}$$

$$\left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right) = \frac{mg}{\rho g A}$$

$$h = \frac{m}{\rho A}$$

Area ratio of venturimeter :

The ratio of area's of normal section to throat section is known as area ratio of venturimeter.

Q. Two venturimeters one with coefficient of discharge 0.98 and area ratio 2 and the other having coefficient of discharge 0.97 are installed in same pipe. If differential head of 2^{nd} venturimeter is 5 times of that of 1^{st} venturimeter calculate the area ratio of 2^{nd} ratio venturimeter. 2 Marks

$$Q = C_d \cdot A_1 \cdot A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

$$= C_d \cdot A_1 \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

∴ Pipe is same, discharge will be same and A_1 will be same for both venturimeters.

$$0.98 \cdot A_1 \sqrt{\frac{2gh}{(2)^2 - 1}} = 0.97 \cdot A_1 \sqrt{\frac{2g \cdot 5h}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

$$\left(\frac{A_1}{A_2}\right) = 3.96.$$

Q. Water is flowing through horizontal pipe of dia. 30 cm and its flow rate is measured with the help of venturimeter installed in the pipe, and its throat dia is 15 cm. The pressures at the normal section and throat sections are recorded as 200 kPa and 40 cm of Mercury vacuum respectively. If 4% of differential head is lost in converging section of venturimeter, find the rate of flow of water through pipe. Marks 15

Working fluid is water ($\rho = 1000$ kg/m³)

Pipe is horizontal, $z_1 = z_2$

∴ Normal area, $A_1 = \frac{\pi}{4} (0.30)^2 = 0.07$ m²

Throat area, $A_2 = \frac{\pi}{4} (0.15)^2 = 0.017$ m²

$P_1 = 200$ kPa $= 2\times10^5$ Pa.

$P_2 = 40$ cm of Hg Vaccum.

$= -40$ cm of Hg

$= -0.40 \times 13{,}600 \times 9.81$ Pa

Gauge pressures

∴ Differential head.

$$h = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right)$$

$$h = \frac{P_1 - P_2}{\rho g}$$

$$= \frac{2\times10^5 - (-0.40\times13600\times9.81)}{1000 \times 9.81}$$

$$= 25.821 \text{ m}$$

loss of head. $h_L = 4\%$ of h

$= 0.04$ h

$$C_d = \sqrt{\frac{h - h_L}{h}} \quad \text{for venturimeter}$$

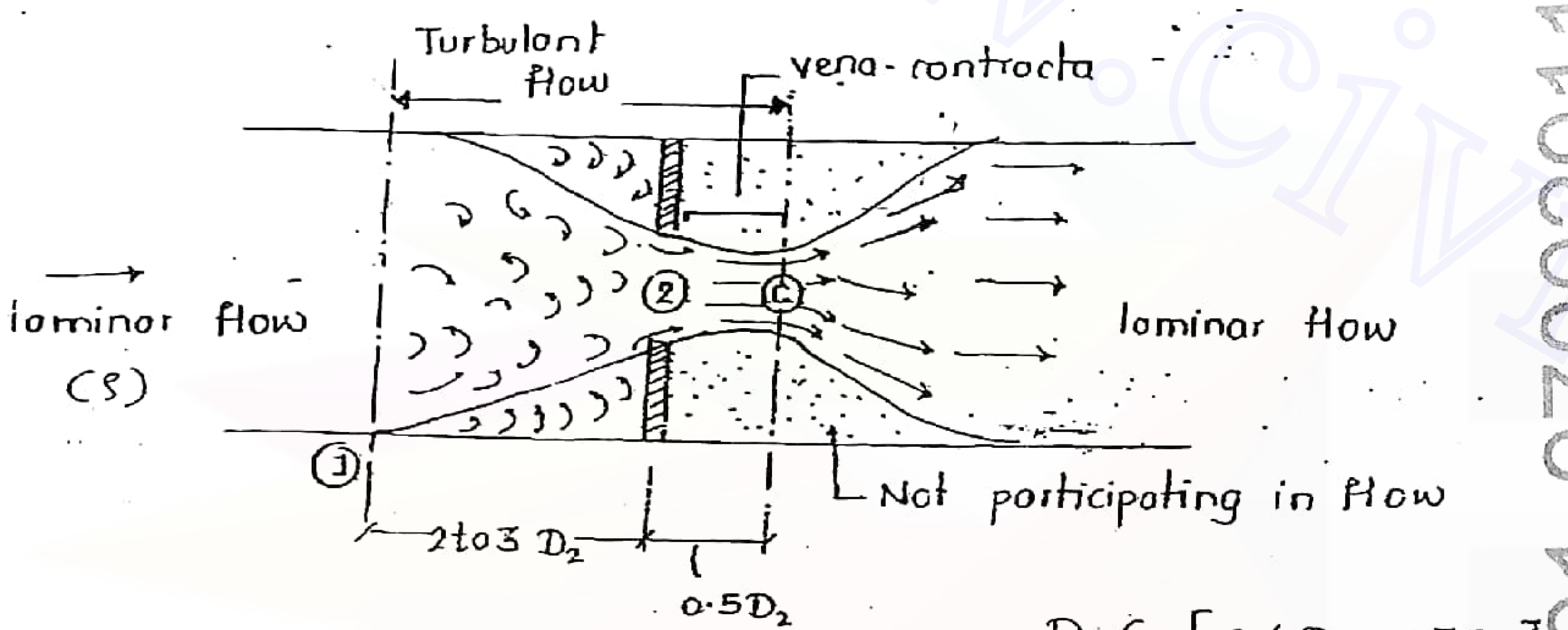
$$= \sqrt{\frac{h - 0.04h}{h}}$$

$$= 0.979$$

$$Q = C_d \, A_1 A_2 \sqrt{\frac{h_L \cdot 2g}{A_1^2 - A_2^2}}$$

(ii) Orifice meter:

It is an instrument which is basically used for the flow measurements inside the pipes. (opaque medium)



$D_2 \in [0.4 D_2, 0.7 D_1]$

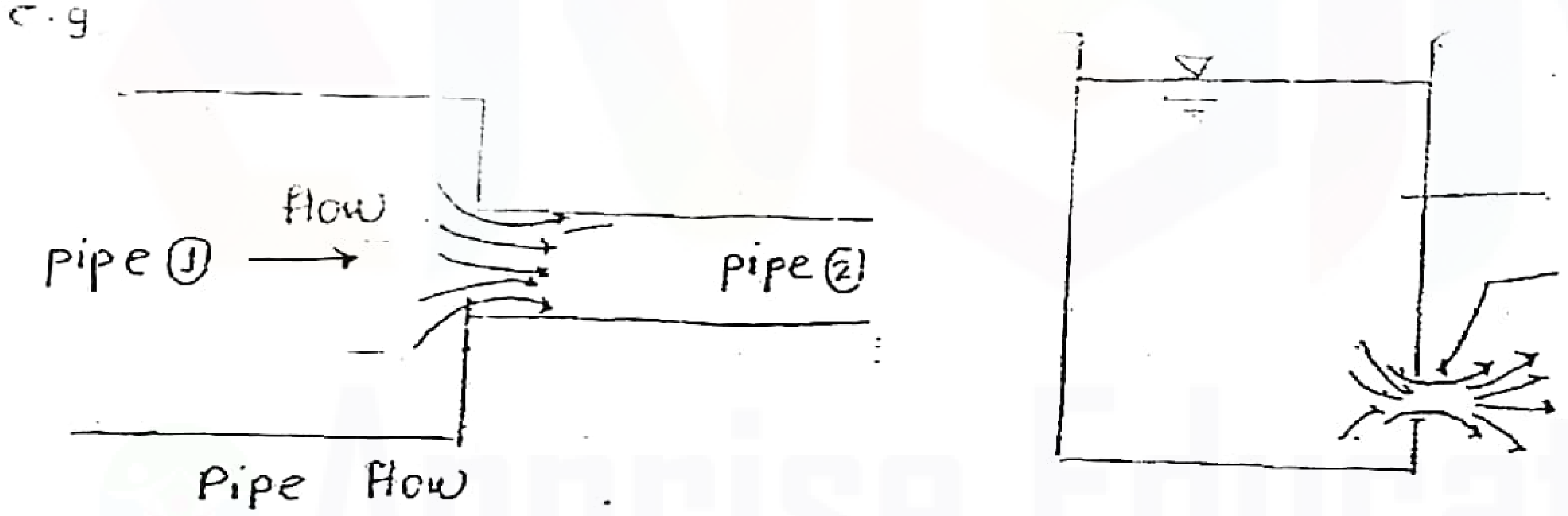
D_1 - diameter of pipe

D_2 - diameter of orifice

Vena contracta is reagion formed just after the sudden contraction after which flow again becomes laminar.

Sudden contraction is change in diameter of pipe at some section.

e.g.



$\frac{A_c}{A_2} = C_c$ - coefficient of contraction for orifice (< 1 always)

A_c - area of flow at 'C'

A_2 - area of flow at orifice.

C_c value depends upon the head loss in the orifice

Differential head of orifice meter:

$$h = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_c}{\rho g} + z_2\right)$$

$= x$ - for piezometers

$= x \left|\frac{S_m}{S} - 1\right|$ - for Manometer.

Energy equation cannot be applied to turbulant flow but applied to the sections of laminar flow only.

Pressure measuring devices (piezometer, venturimeter) are attached at sections where the continuity equations can be applied only) so that piequired data can be collected from these points. (Here at ① and ©)

Applying energy equation at section ① and ©

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_c}{\rho g} + \frac{V_c^2}{2g} + z_c + h_L$$

(Actual analysis)

$$\frac{V_c^2 - V_1^2}{2g} = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) - h_L$$

$$\frac{V_c^2 - V_1^2}{2g} = h - h_L$$

$$V_c^2 - V_1^2 = 2g(h - h_L)$$

∵ Applying continuity equation at ① and ©

$$Q = A_2 \cdot V_2 = A_c \cdot V_c$$

$$V_c = \left(\frac{A_2}{A_c}\right) \cdot V_2$$

$$V_c = \quad V_2$$

$$\frac{C_c \cdot V_2^2 - V_1^2}{C_c} = 2g\,(h - h_L)$$

$$\frac{Q^2}{A_2^2 \cdot C_c} - \frac{Q^2}{A_1^2} = 2g\,(h - h_L)$$

$$Q = C_c \cdot A_1 \cdot A_2 \sqrt{\frac{2g\,(h-h_L)}{A_1^2 - A_2^2 \cdot C_c^2}}$$

$$= C_c \sqrt{\frac{A_1^2 - A_2^2}{A_1^2 - A_2^2 \cdot C_c^2}} \sqrt{\frac{h-h_L}{h}}\; A_1 \cdot A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

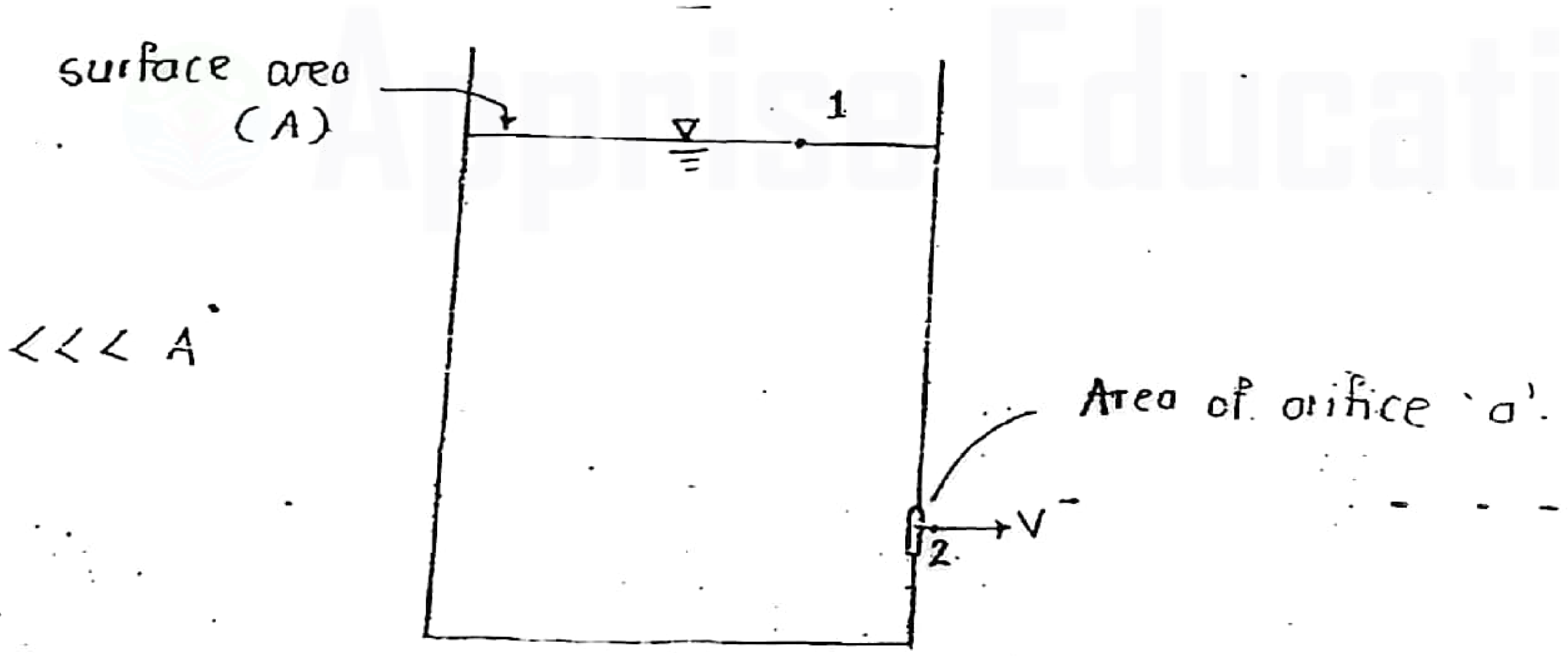
$$= C_d \cdot A_1 \cdot A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

C_d for orificemeter $= 0.62$ to 0.67

Thus venturimeter gives nearly same theorotical discharge as that of actual discharge ($C_d = 0.97$ to 0.99) but for orificemeter we need to know C_d first to find exact value of the discharge.

Tuesday
26th November '13

Flow through Orifices :



$a <<< A$

Consider point 1 - at the free surface and point 2 - just outside orifice.

Applying energy equation at ① and ②.
(done by Torri-Celli)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

(We neglected head loss - as there is no flow between 1 to 2)

Pressure at points 1 and 2 is atmospheric.
Velocity of the falling water surface level is nearly zero

Proof : Applying continuity equation at ① and ②

$$A_1V_1 = A_2V_2$$

$$A \cdot V_1 = a \cdot V_{th}$$

$$V_1 = \frac{a}{A} \cdot V_{th} \simeq 0 \quad \text{as } \frac{a}{A} \text{ is s}$$

$$\frac{P_{atm}}{\rho g} + 0 + h = \frac{P_{atm}}{\rho g} + \frac{V_{th}^2}{2g} + 0$$

considering orifice at datum

$$V_{th}^2 = \sqrt{2gh}$$

This was not actual velocity

Actual velocity from orifice.

$$V = C_v \sqrt{2gh}$$

where. C_v = coefficient of velocity for orifice.

$$C_v = \frac{V_{act}}{V_{th}}$$

Discharge through orifice :

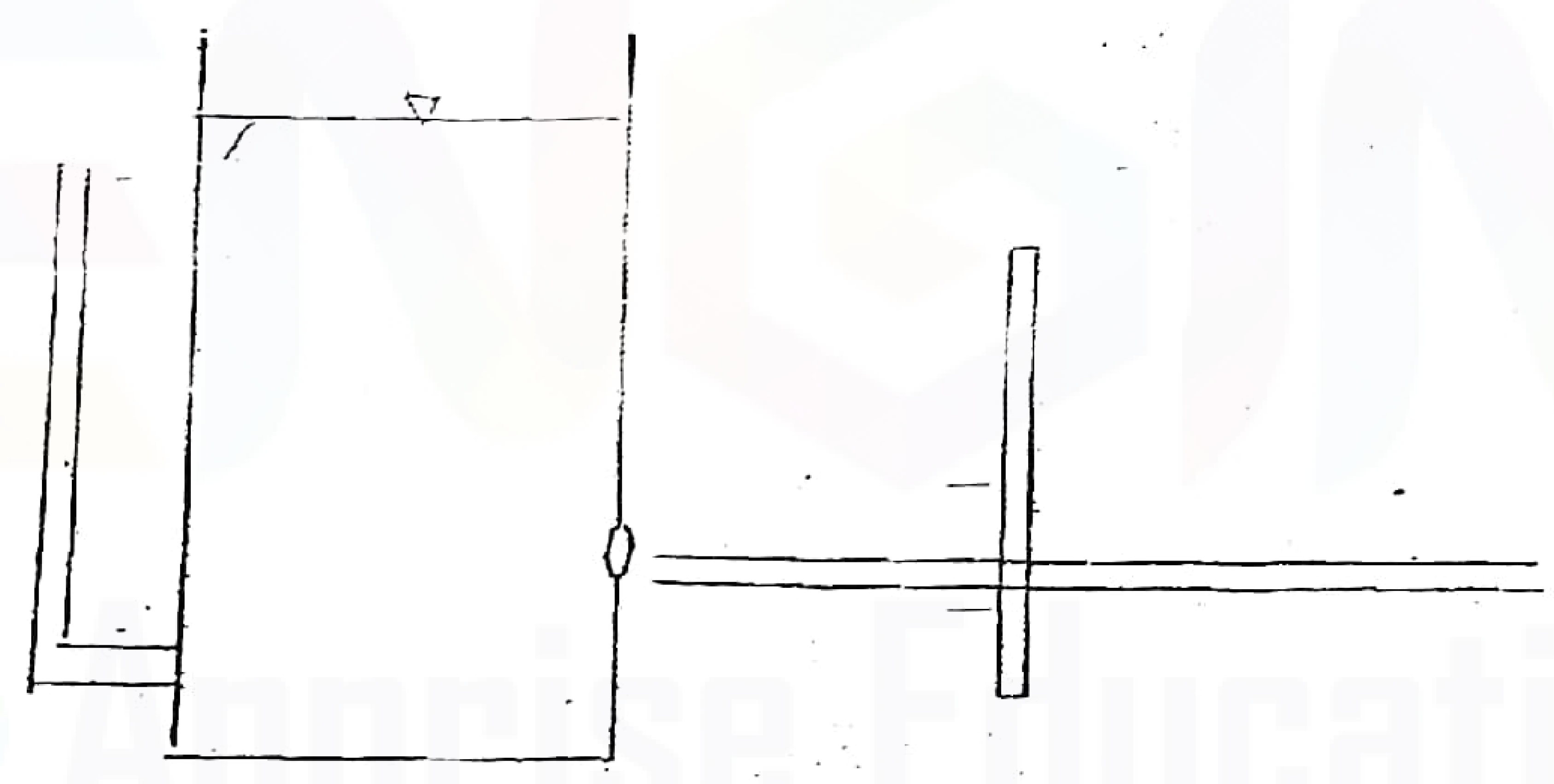
$$Q_{th} = a \cdot \sqrt{2gh}$$

$$Q = C_d \cdot a \sqrt{2gh}$$

where C_d - coefficient of discharge for orifice.

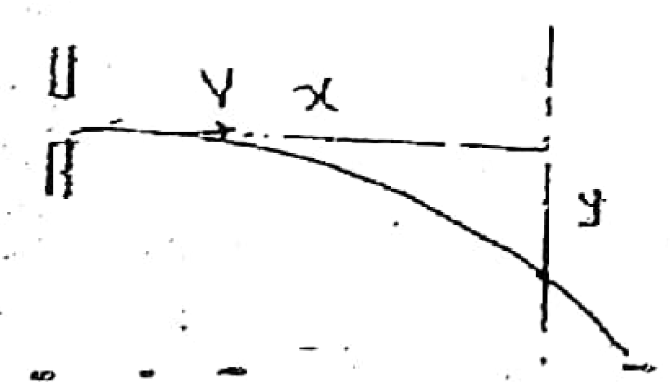
$$C_d = C_c \times C_v$$

Experimental determination of C_v for orifice.



$$x = v \times t$$

$$t = \frac{x}{v}$$



$$y = ut + \frac{at^2}{2}$$

$$= 0 + \frac{gt^2}{2} \qquad u = 0 \text{ at initial condition}$$

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$$y = \frac{1}{2} g \left(\frac{x^2}{v^2}\right)$$

$$v^2 = \frac{gx^2}{2y}$$

$$v = x\sqrt{\frac{g}{2y}}$$

$$C_v = \frac{V}{V_{th}} = \frac{x\sqrt{g/2y}}{\sqrt{2gh}}$$

$$C_v = \frac{x}{2\sqrt{gh}}$$

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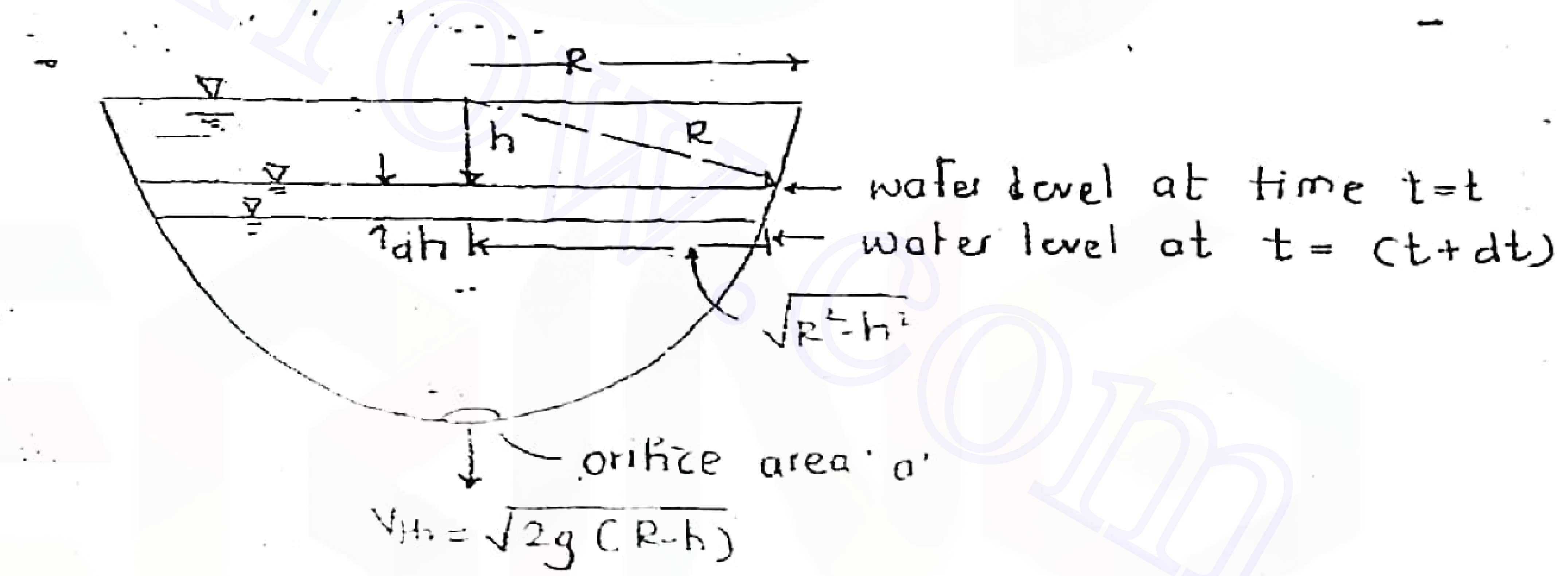
8588058174

Q. A hemispherical vessel is attached small orifice of area a and coefficient of discharge C_d. find the time required to empty the vessel.



Discharge through orifice

$$Q = C_d \cdot a\sqrt{2g(R-h)} \qquad \text{at time } t = t$$

volume of water released in time. dt

$$V = C_d \cdot a\sqrt{2g(R-h)} \cdot dt$$

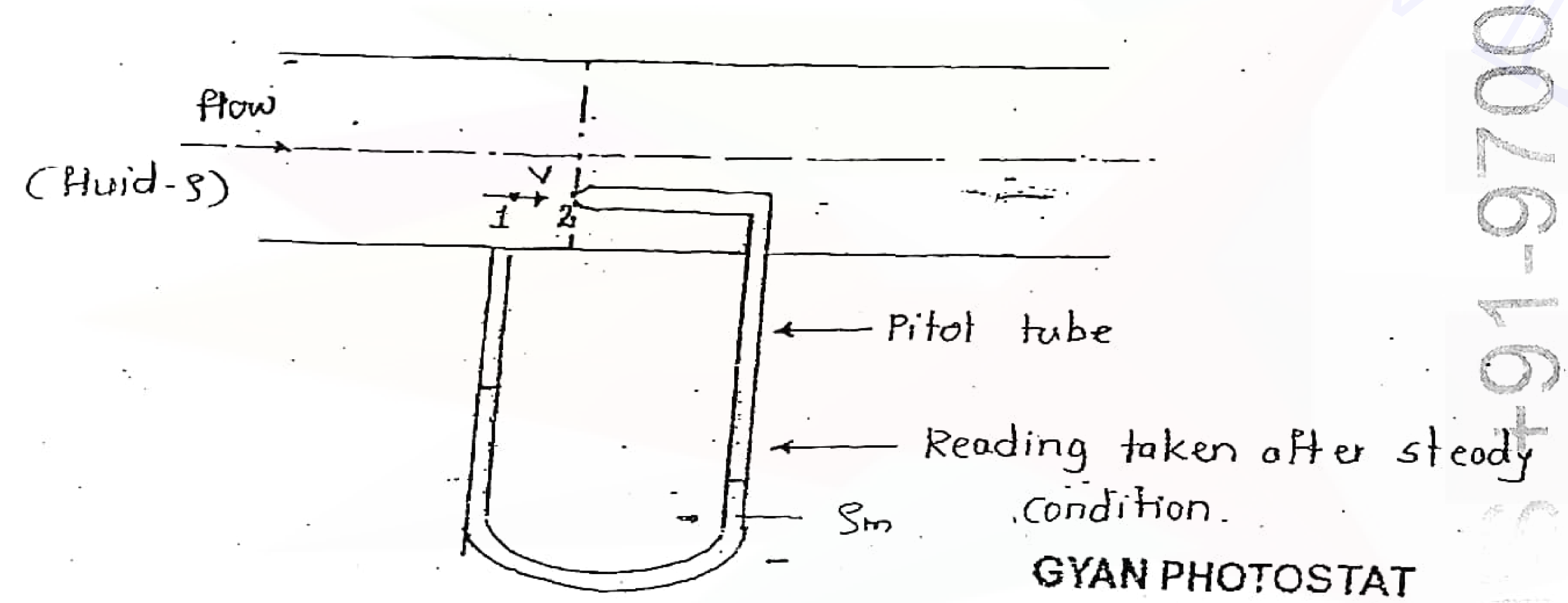
$$= \pi(R^2-h^2) \cdot dh$$

$$\int^{R} \frac{(R^2-h^2)}{\sqrt{R-h}} dh = \frac{C_d \cdot a\sqrt{2g}}{\pi} \int^{t} dt$$

(iii) Pitot tube :

It is a device which is basically used for the measurement of local velocities inside the pipes.

It connot measure discharge as average velocity is unknown. It is 90° bended tube having a very small opening at its mouth.



Consider point 1 - just before striking the tube
point 2 - just after striking the tube.

$V_1 = V_{th}$ - local velocity to be found

$V_2 = 0$

Applying energy equation at 1 and 2.

$$\frac{P_1}{Sg} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{Sg} + \frac{V_2^2}{2g} + Z_2$$

(No head loss as 1 and 2 are very close points)

$$\frac{V_1^2 - V_2^2}{2g} = \underbrace{\left(\frac{P_2}{Sg} + Z_2\right) - \left(\frac{P_1}{Sg} + Z_1\right)}_{h \text{ - differential head of pitot tube}}$$

only for pitot tube -

h_2 = diff. of pizometer levels of 2 and 1

$$\frac{V_{th}^2 - 0}{2g} = h$$

$$V_{th} = \sqrt{2gh}$$

where

$h = x$ - piezometer

$= x\left[\frac{S_m}{S} - 1\right]$ - manometers.

$V_{th} = \sqrt{2gh}$ - local velocity.

Actual velocity (local)

$$V = C_v\sqrt{2gh}$$

where

C_v - coefficient of velocity of pitot tube.

$C_v \in [0.97, 0.99]$

Note:

In any fluid flow system if any point, velocity of flow becomes zero then that point is called stagnation point.

In pitot tube.

point 2 - stagnation point (velocity zero)

point 1 - static point (will follow flow parameters)

$\therefore$ Stagnation piezometeric head - static piezometric head = (K.E. head (Th.))

$$\left(\frac{P_2}{Sg} + Z_2\right) - \left(\frac{P_1}{Sg} + Z_1\right) = \frac{V_{th}^2}{2g} = (h)_{K.E.\,Th}$$

Q. Water is flowing through horizontal pipe of dia 30 cm. Stagnation and static pressures recorded by pitot tube at the centreline of pipe are 100 kPa and 37 cm of mercury. If the mean velocity at a section is half of the maximum velocity, find rate of flow of water through the pipe. Take coefficient of velocity for pitot tube 0.97.

water. $\rho = 1000\ kg/m^3$

Horizontal pipe. $Z_1 = Z_2$

$P_2 = 100 \times 10^3 = 10^5\ Pa$

$P_1 = 37$ cm of Hg

$= (0.37 \times 13{,}600 \times 9.8)$ Pa.

Theoretical kinetic head,

$$\frac{V^2_{th\ Centreline}}{2g} = \left(\frac{P_2}{\rho g} + Z_2\right) - \left(\frac{P_1}{\rho g} + Z_1\right)$$

$$= \left(\frac{10^5}{1000 \times 9.8}\right) - \left(\frac{0.37 \times 13{,}600 \times 9.8}{1000 \times 9.8}\right)$$

$V_{th\ (centreline)} = 10.06$ m/sec

$$\frac{V_{th\ centreline}}{V_{centreline}} = C_v$$

$V_{centreline} = C_v \times 10.06$

$= 0.97 \times 10.06$

$= 9.76$ m/sec

$V_{mean} = \frac{1}{2} V_{centreline}$ (given)

$= \frac{1}{2} \times 9.76$

$= 4.88$ m/sec

$Q = A \cdot V_{mean}$

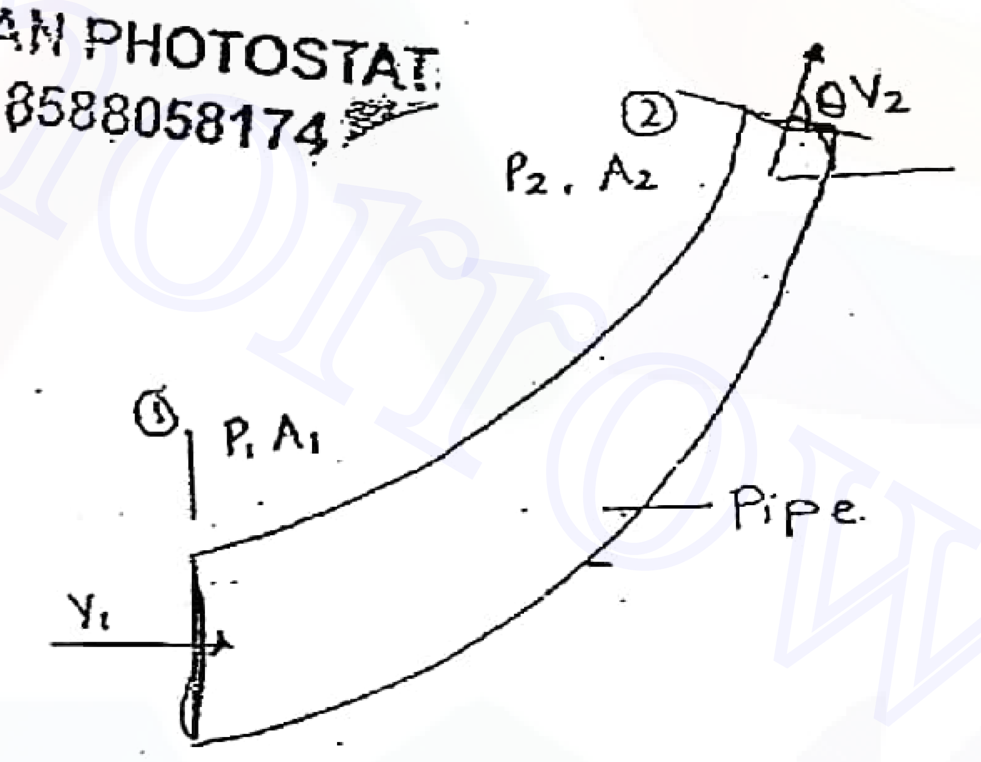
$= \frac{\pi}{4}(0.30)^2 \times 4.88$

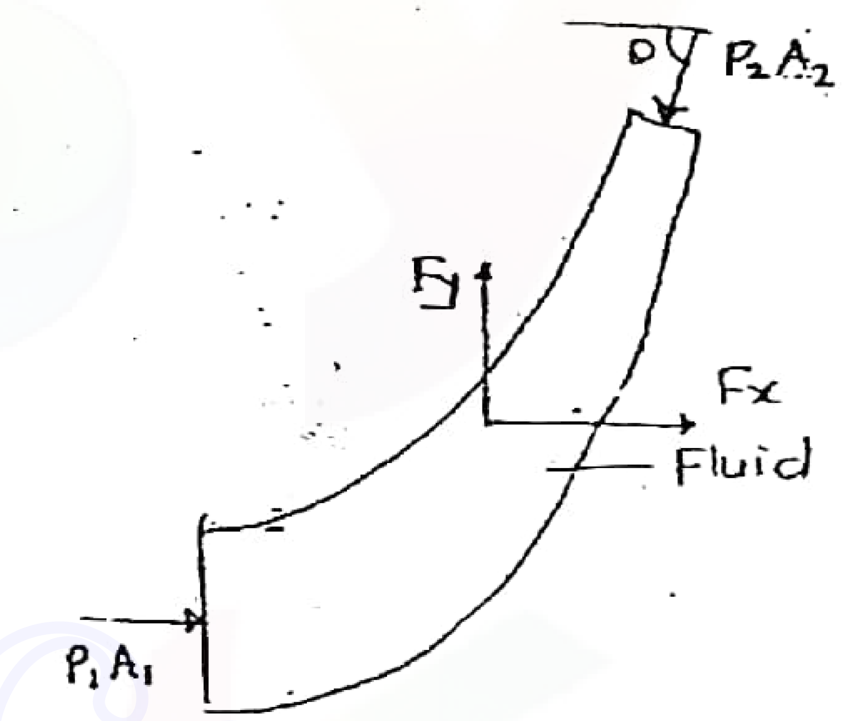
$= 0.344\ m^3/sec$

Forces on the point bends :

(bends in horizontal plane)

Calculations for force exerted by bend on the fluid.





(FBD of fluid)

Consid fluid as system

Applying Newton's 2nd law

In X- direction. rate of change $\left(\frac{d}{dt}\right)$

$P_1 A_1 + F_x - P_2 \cdot A_2 \cdot \cos\theta = \dot{m} V_2 \cos\theta - \dot{m} V_1$

$F_x =$

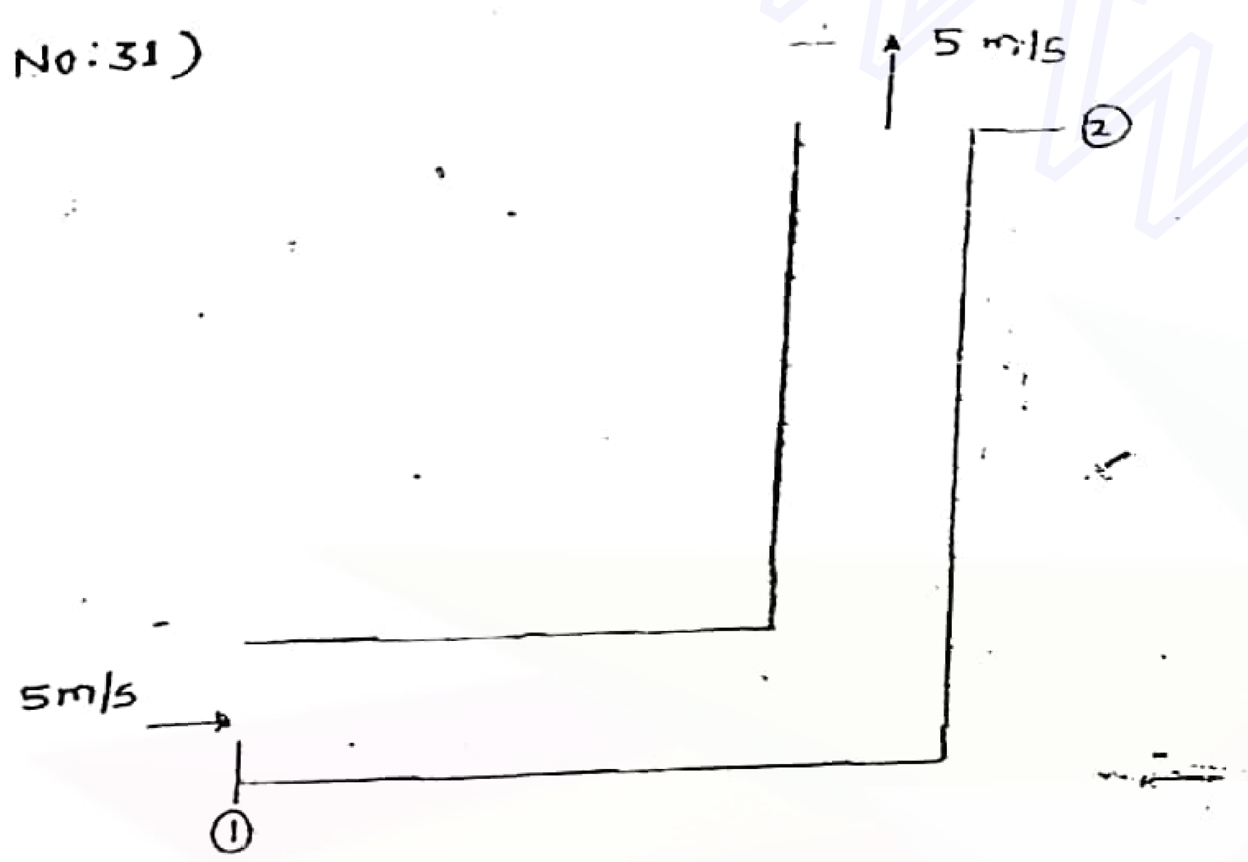
In Y- direction

$F_y - P_2 \cdot A_2 \cdot \sin\theta = \dot{m} V_2 \sin\theta - 0$

$F_y =$

Similar forces will be there on bend by fluid in -

Q. 3
(Page No: 31)



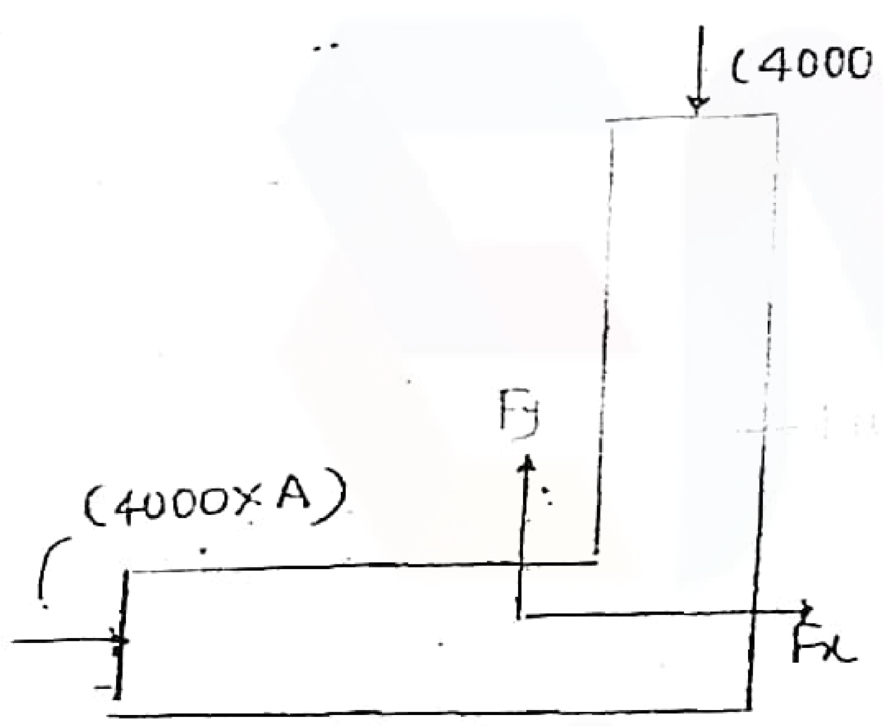
$D = 0.30\ m$

$A = \frac{\pi}{4} \times (0.3)^2 =$

$\dot{m} = (1000\ kg/m^3 \times \frac{\pi}{4} \times (0.3)^2 \times 5)$

$= 353.25\ kg/sec$

$P_1 = P_2 = 4000\ Pa$



$F_y - 4000\,A = \dot{m} \times 5 - 0$

$F_y = 353.25 \times 5 + 4000\,A$

$= 2.05\ kN$

This is force on fluid

∴ Force on bend will be $-2.05kN$.

∴ Force required to keep bend in equilibrium $= 2.05\ kN$

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Vortex flows :

When certain amount of fluid is rotating w.r.t. some axis then such a flow is known as Vortex flow.

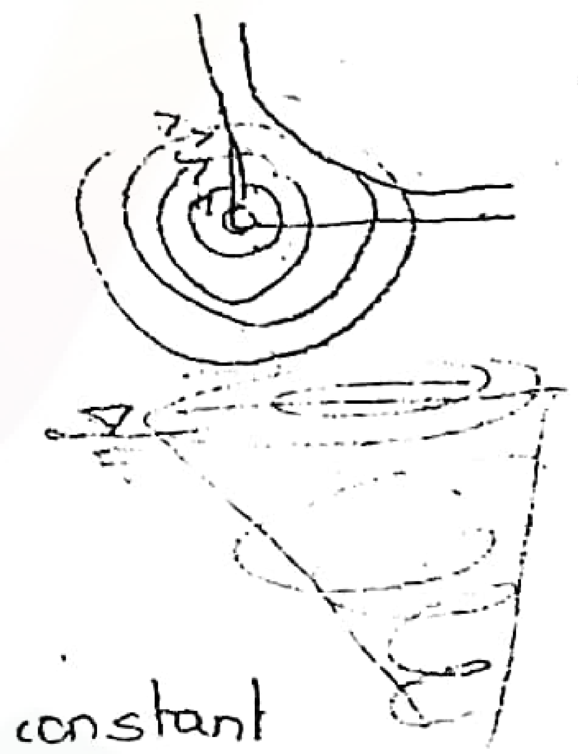
There are two types of vortex flows.

1. Free Vortex flow :

(Natural vortex)

In these vortex flows there will not be any requirement of external torque.

e.g. flow of water in wash basins, eddies and whirlpools in rivers and canals.



$\vec{T}_{ext} = 0$

$\frac{d}{dt}(\text{angular momentum}) = 0$

i.e. Angular momentum = constant

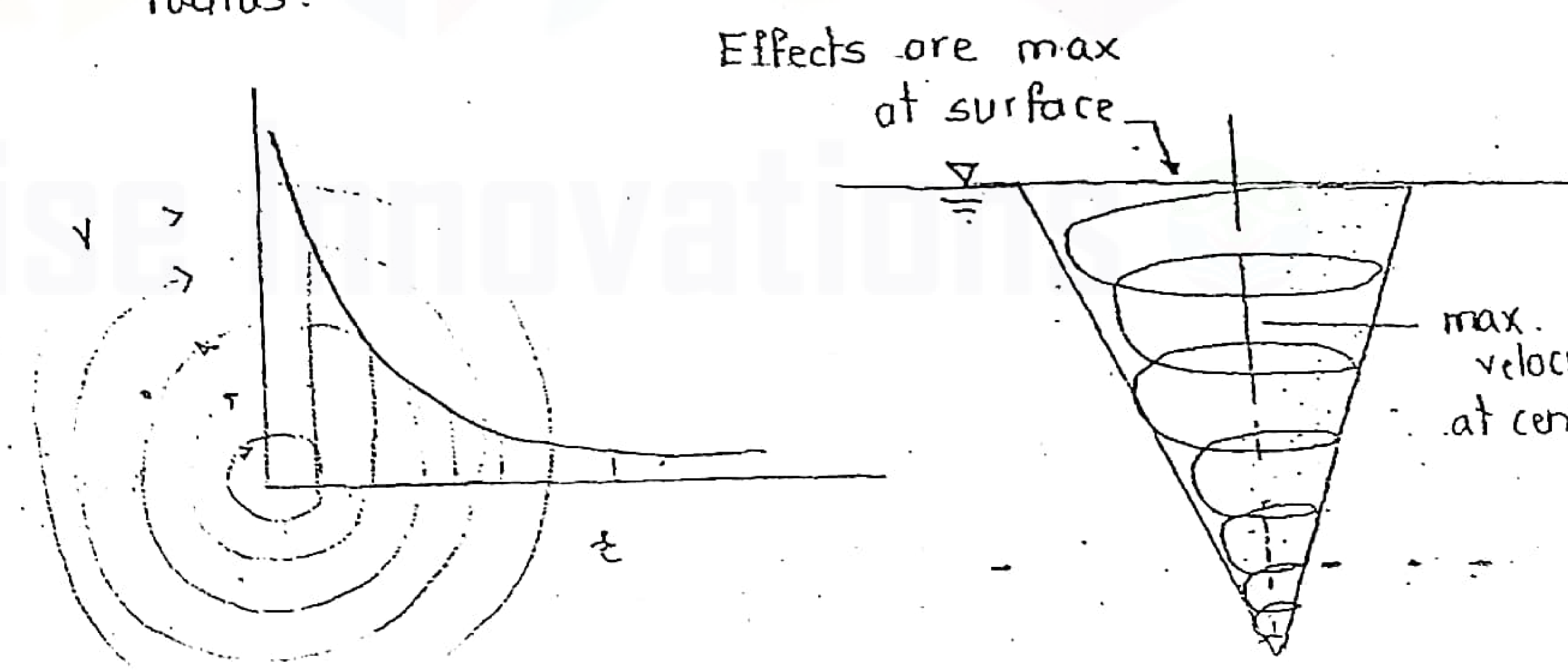
i.e. rate of chang

Moment of linear momentum = constant

$(mv)\cdot r = \text{constant}$

$v \propto \frac{1}{r}$ m - constant

i.e. the velocity in vortex will decrease with increasing radius.



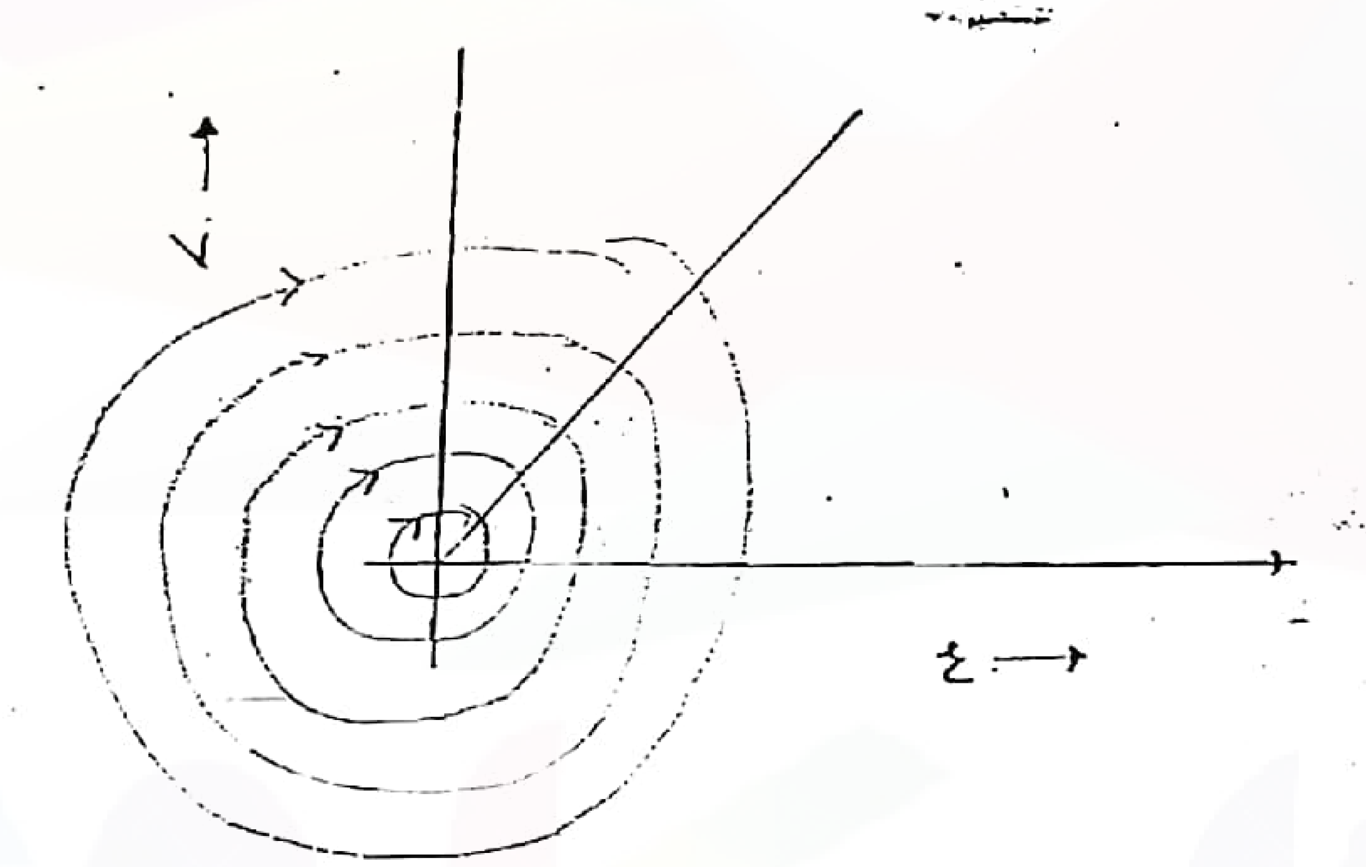
2. Forced Vortex Flow :

In these vortex flows, a continuous external torque is required to maintain the angular velocity of the flow to constant.

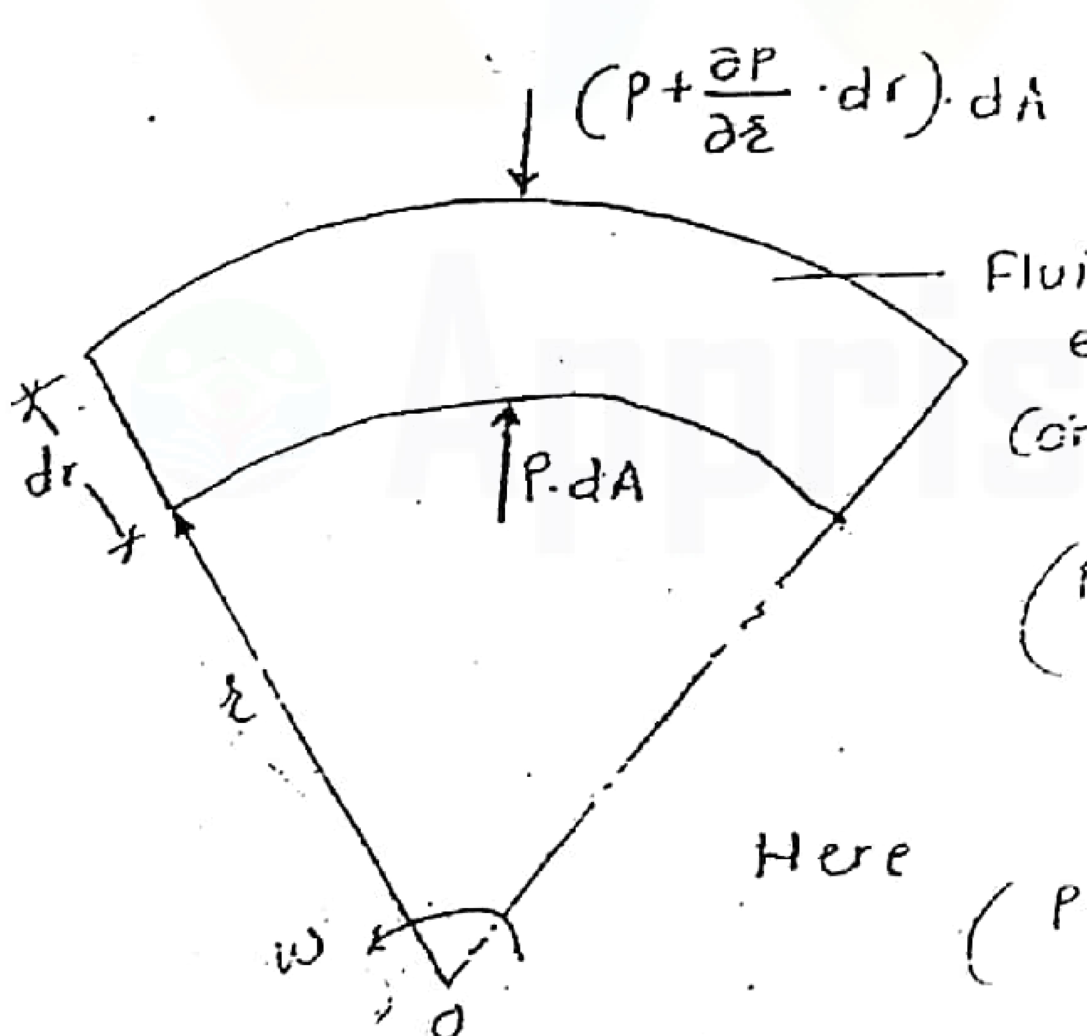
Angular velocity = constant

$$\frac{V}{r} = \text{constant}$$

$$V \propto r$$



Fundamental equation of vortex flows :



Applying Newton's 2nd law of motion, (towards the centre in normal direct

$$\left(P + \frac{\partial P}{\partial r}\cdot dr\right)\cdot dA - P\cdot dA = \underbrace{dm \cdot \frac{V^2}{r}}_{\text{centripetal force}}$$

Here $\left(P + \frac{\partial P}{\partial r} dr\right) dA > P \cdot dA$

Because normal accl^n is towards centre.

$$\frac{\partial P}{\partial r}\cdot dr \cdot dA = (dA \cdot dr)\cdot \rho \cdot \frac{V^2}{r}$$

$$\frac{\partial P}{\partial r} = \frac{\rho V^2}{r} \qquad (+ve)$$

i.e. Pressure is increasing in direction of radius (outward)

In general in Vortex flow :

$$P = f(r, z)$$

where z - datum head (from bottom in the upward direction)

h is in downward direction.

$$dP = \frac{\partial P}{\partial r}\cdot dr + \frac{\partial P}{\partial z}\cdot dz$$

$$= \frac{\rho V^2}{r}\cdot dr - \left(\frac{\partial P}{\partial z}\right)\cdot dz$$

$$dP = \frac{\rho V^2}{r}\cdot dr - (\rho g)\cdot dz \qquad \text{As } \frac{\partial P}{\partial z} \text{ is -ve (datum)}$$

– Fundamental equation of vortex flow

This equation is applied at any two points on the same plane of the vortex.

For free vortex flow :

$$V \propto \frac{1}{r}$$

$$V = \frac{C}{r} \qquad C\text{- constant.}$$

$$dP = \frac{\partial P}{\partial r}\frac{\rho V^2}{r} dr - \left(\frac{\partial P}{\partial z}\right)\cdot dz$$

$$dP = \frac{\rho C^2}{r^3}\cdot dr - \rho g \cdot dz$$

Integrating between any two points in vortex

$$\int_1^2 dP = \rho C^2 \int_1^2 \frac{dr}{r^3} - \rho g \int_1^2 dz$$

$$P_2 - P_1 = \frac{\rho c^2}{2}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] - \rho g (z_2 - z_1)$$

$$P_2 - P_1 = \frac{1}{2}\rho V_1^2 - \frac{1}{2}\rho V_2^2 - \rho g z_2 + \rho g z_1$$

$$P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2 = P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1$$

Energy equation.

i.e. Energy conservation is valid between any two points in Free vortex flow:

Thus flow is irrotational flow. i.e. particles in free vortex flow don't rotate about their own centre of masses.

For forced vortex flow:

$$V \propto r$$

$$V = \omega \cdot r \qquad \omega - \text{constant}$$

$$dP = \left(\frac{\rho V^2}{r} \cdot dr - \rho g\, dz\right)$$

$$dP = \rho r \omega^2 \cdot dr - \rho g\, dz$$

Integrating between any two points in vortex.

$$\int_1^2 dP = \rho \omega^2 \int_1^2 r\, dr - \rho g \int_1^2 dz$$

$$P_2 - P_1 = \frac{\rho \omega^2}{2}(r_2^2 - r_1^2) - \rho g (z_2 - z_1)$$

$$P_2 - \frac{1}{2}\rho V_2^2 + \rho g z_2 = P_1 - \frac{1}{2}\rho V_1^2 + \rho g z_1$$

$$P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2 \neq P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1$$

Energy equation is not valid in case of forced vortex.

Thus flow is rotational flow in forced vortex

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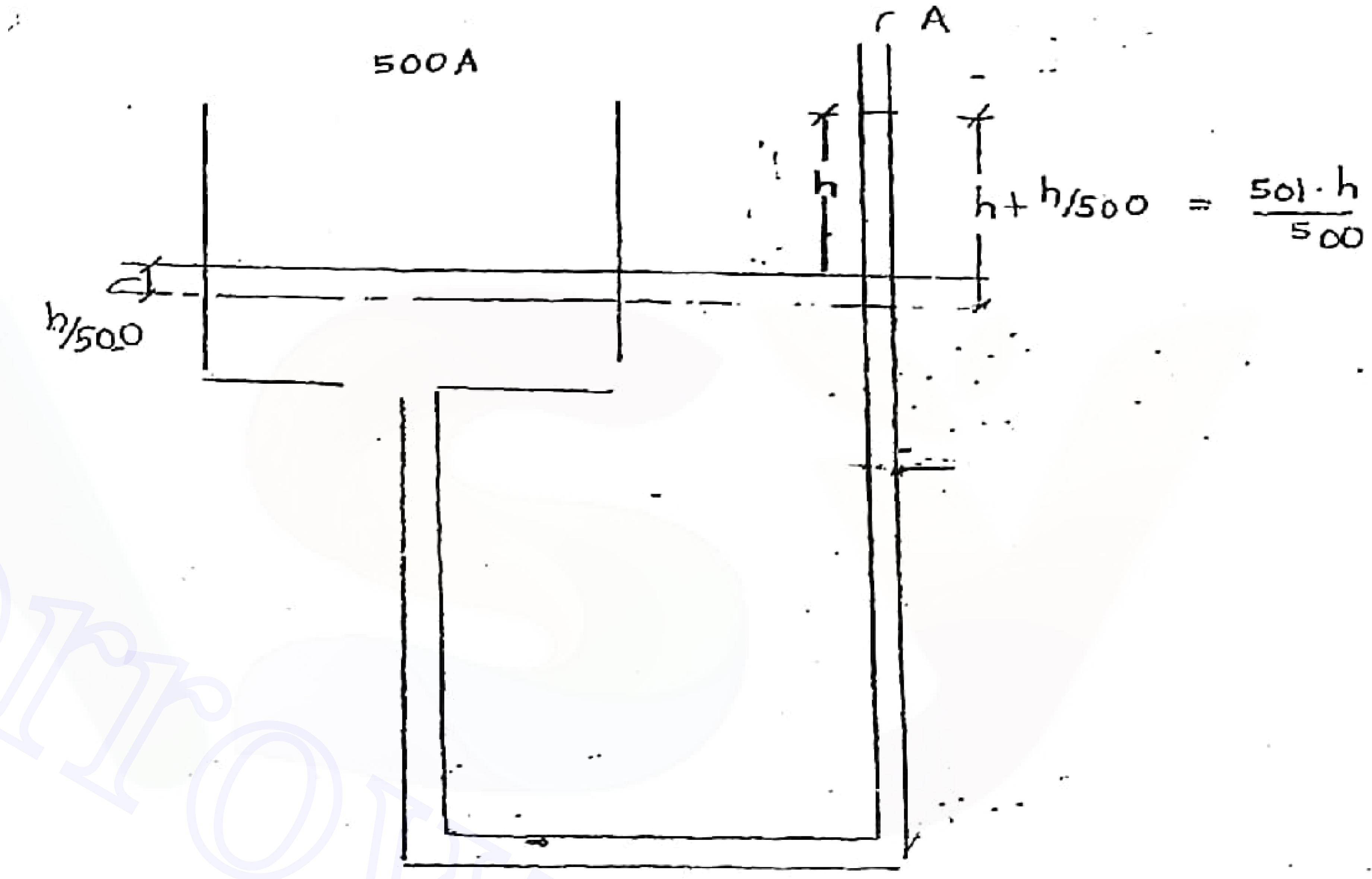
Workbook:

Fluid statics

Q. 14.



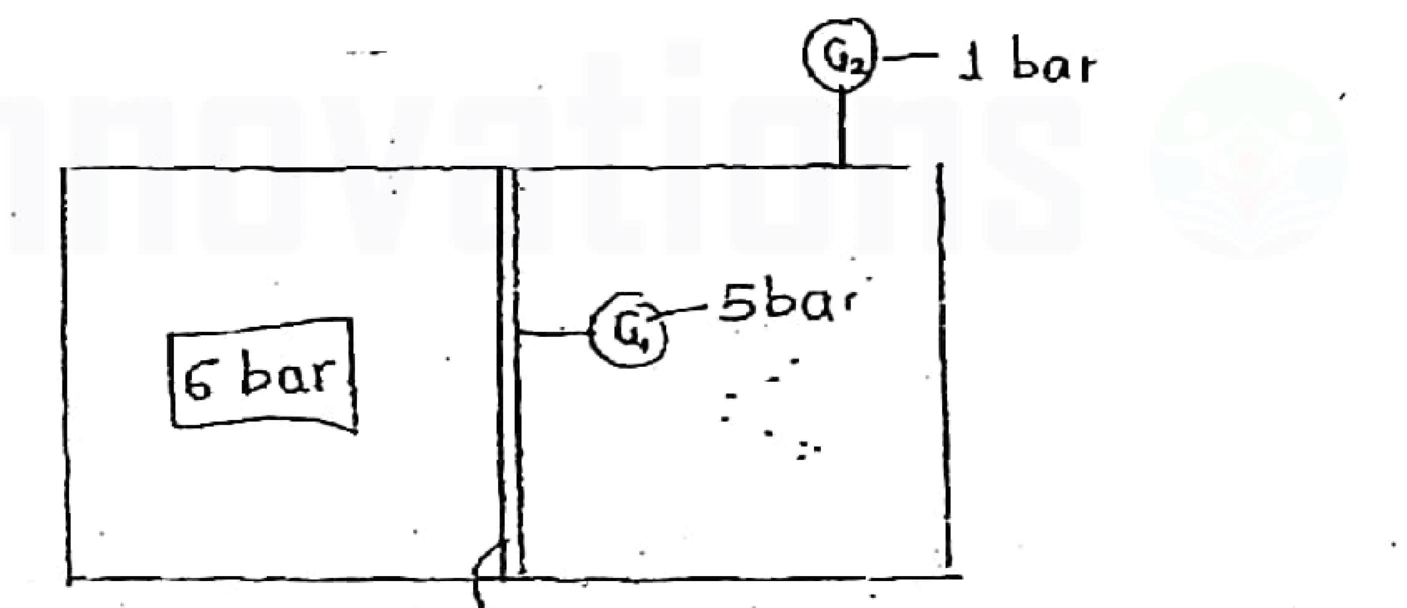
Reading taken = h

Actual reading = $\frac{501 \cdot h}{500}$

$$\text{Error} = \frac{\left(\frac{501 \cdot h}{500} - h\right)}{\left(\frac{501 \cdot h}{500}\right)} \times 100$$

$$= 0.199\ \%$$

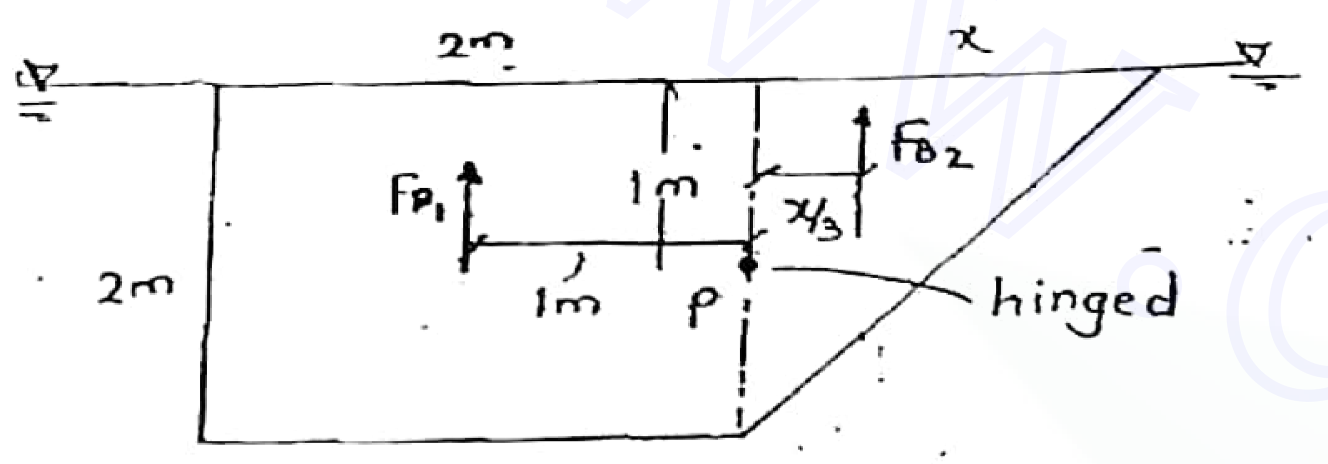
Q. 11.



on piston − (5+1) = 6 bar

P = atm pressure + 6

Q. 35



$M_o\ P = 0$

$$F_{B_1} \times 1 = F_{B_2} \times \frac{x}{3}$$

$$(\rho \times 2 \times 2 \times w \times g) \times 1 = \rho \times \frac{x \times x}{2} \times w \times g \times \frac{x}{3}$$

$$x = 2\sqrt{3}\ m$$

Equation of free surface in forced vortex flow :

$$dP = \frac{\rho v^2}{r} \cdot dr - \rho g \cdot dz$$

Fundamental equation.

In forced vortex

$$v \propto r$$
$$= \omega r$$

where, ω - constant

$$dP = (\rho r \omega^2) \cdot dr - \rho g \cdot dz$$

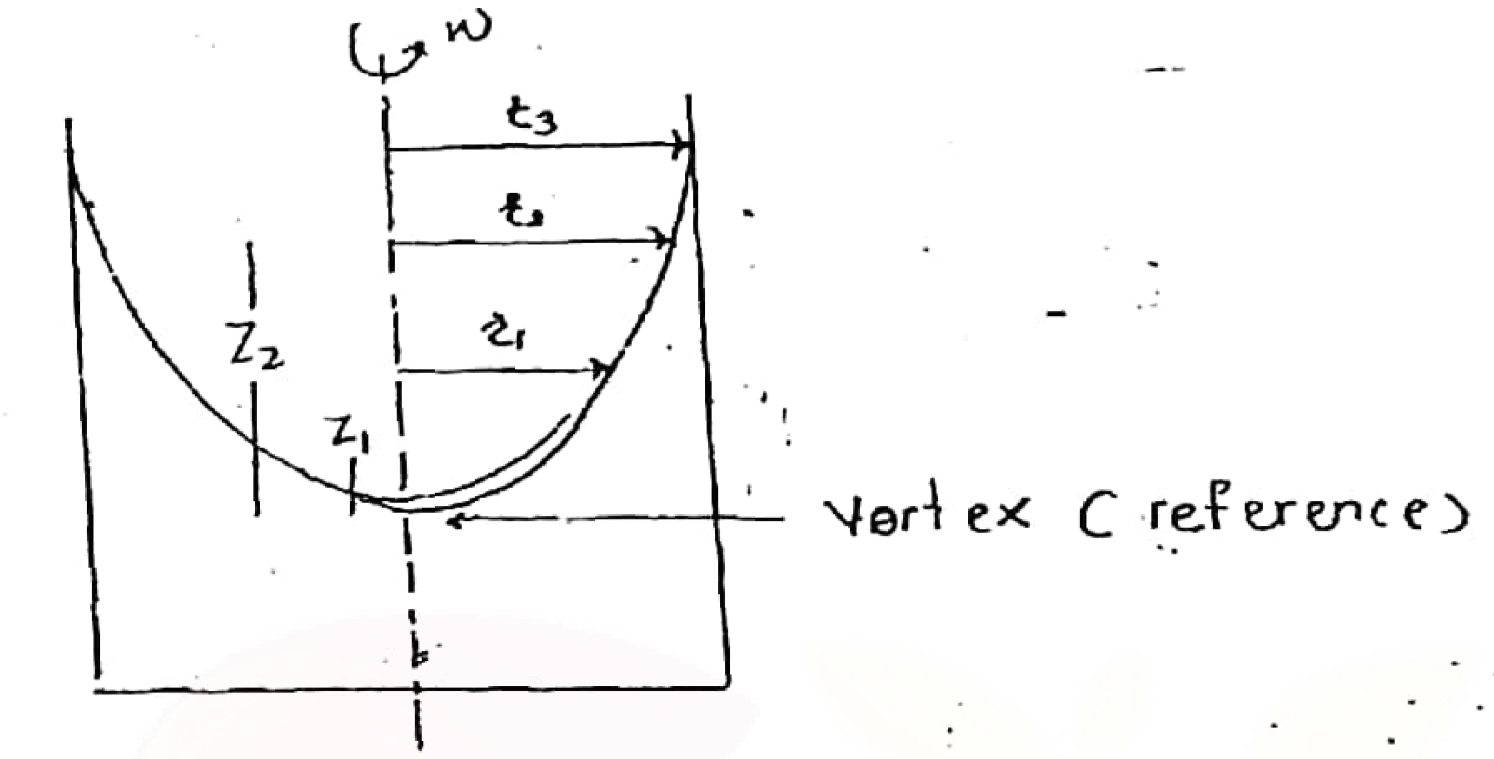
At the free surface

$dP = 0$ ∴ P - constant

$$0 = \rho r \omega^2\, dr - \rho g\, dz$$

$$\int dz = \int r \frac{\omega^2}{g} \cdot dr$$

$$z = \frac{r^2 \omega^2}{2g} + C \quad \text{- equation of}$$



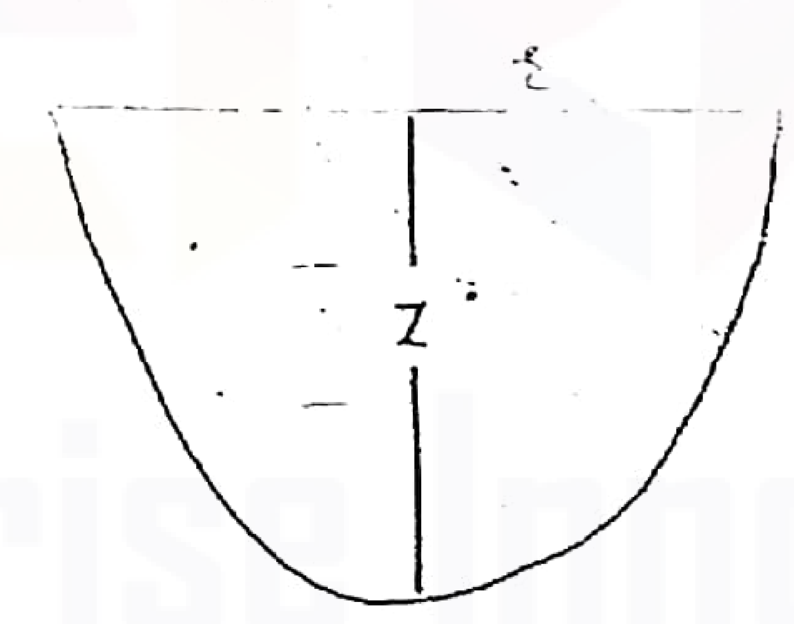
To determine indefinite constant C, assume vertex of parabola as reference

∴ At $r = 0$, $z = 0$

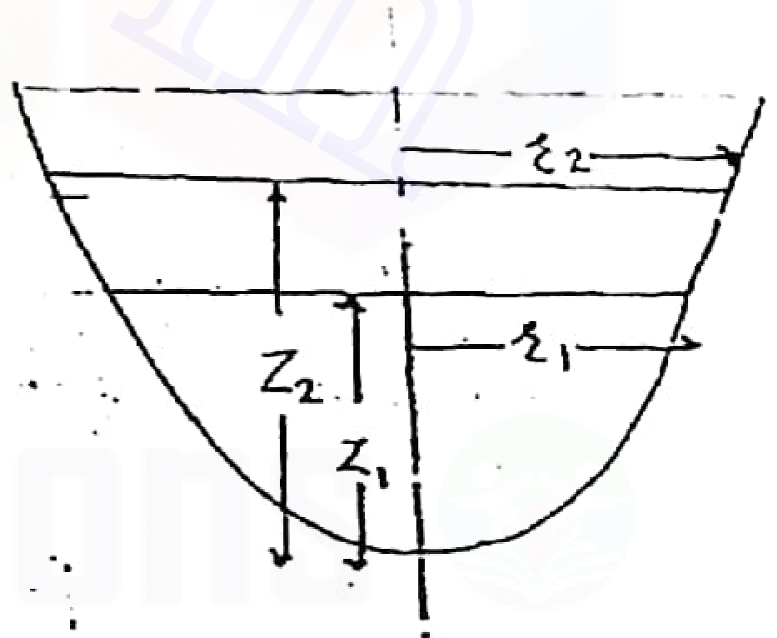
$$C = 0$$

$$z = \frac{r^2 \omega^2}{2g}$$

Volume of paraboloid

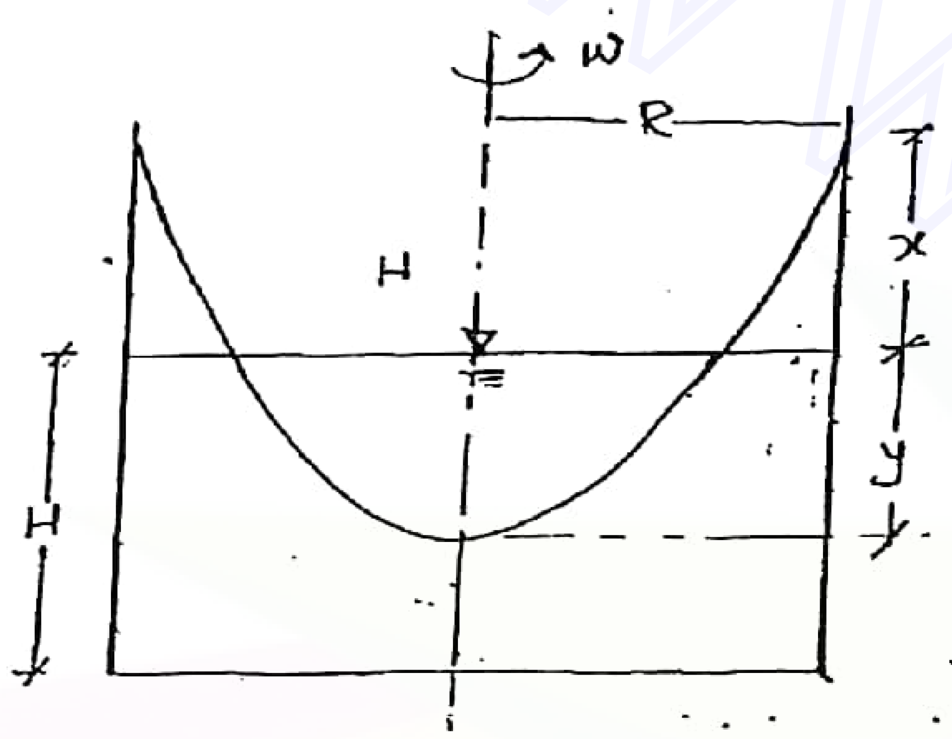


$$V = \frac{1}{2}(\pi r^2) \cdot Z$$



$$V = \left(\frac{1}{2}\pi r_2^2 \cdot Z_2\right) - \left(\frac{1}{2}\pi r_1^2 \cdot Z_1\right)$$

Conservation of volume :



Initial volume of water (liquid)

$$V_i = (\pi R^2) \cdot H$$

Final volume,

$$V_f = \pi R^2 (H \pm x) - \frac{1}{2} \pi R^2 (x + y)$$

$$= \pi R^2 \left[H + x - \frac{x}{2} - \frac{y}{2} \right]$$

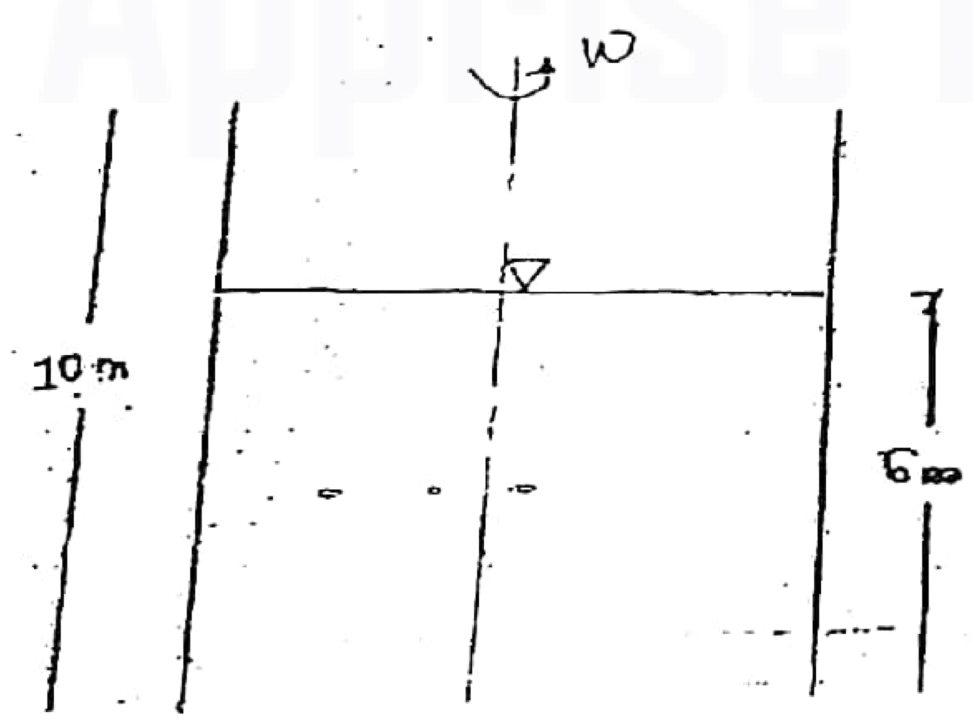
If no water / liquid drop is spilled.

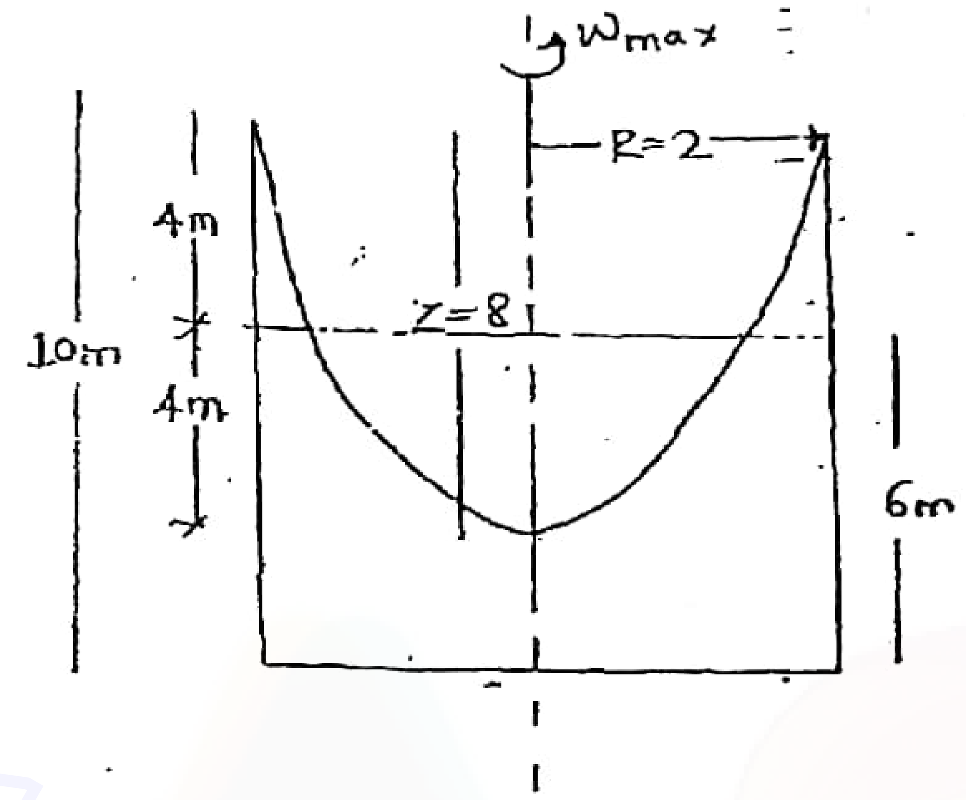
$$V_i = V_f$$

$$\pi R^2 H = \pi R^2 \left[H + \frac{x}{2} - \frac{y}{2} \right]$$

$$x = y$$

Q. Find the max. value of angular velocity so that water does not spill out of container.



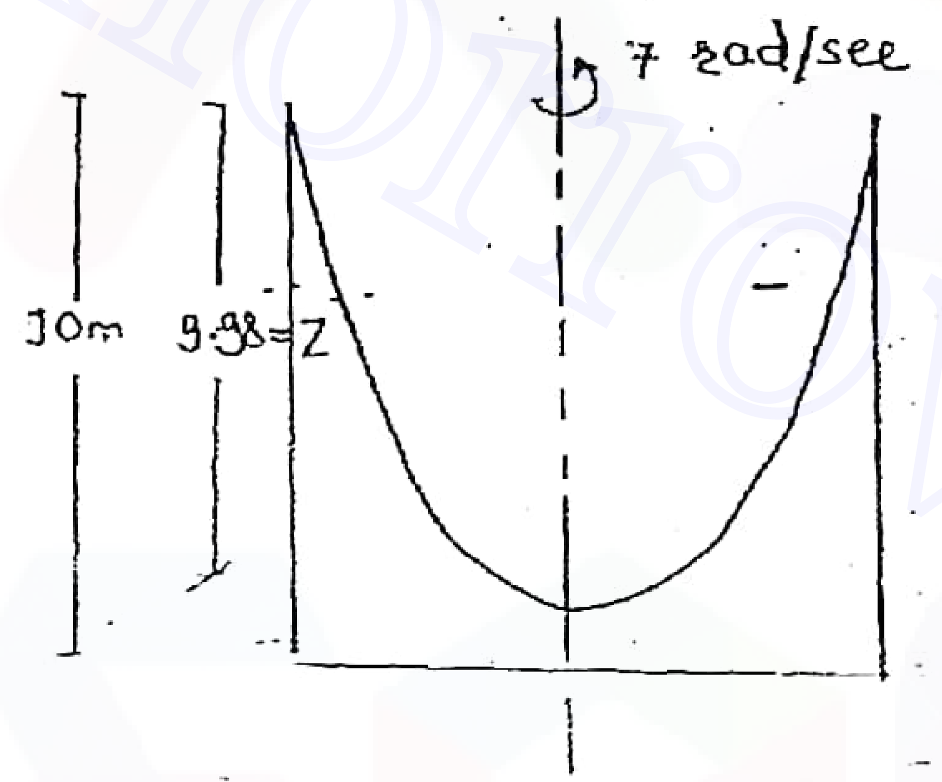


$$Z = \frac{r^2 \omega^2}{2g}$$

$$8 = \frac{(2)^2 \omega_{max}^2}{2 \times 9.8}$$

$$\omega_{max} = 6.26 \text{ rad/sec.}$$

Q. If angular velocity is 7 rad/sec find volume of water spilled.



$$Z = \frac{(2)^2 \times (7)^2}{2 \times 9.8}$$

$$= 9.98 \text{ m} \quad \text{(from vertex)}$$

$$V_f = \left[\pi \times 2^2 \times 10 - \frac{1}{2} (\pi \times 2^2 \times 9.98) \right]$$

$$= 62.95 \text{ m}^3$$

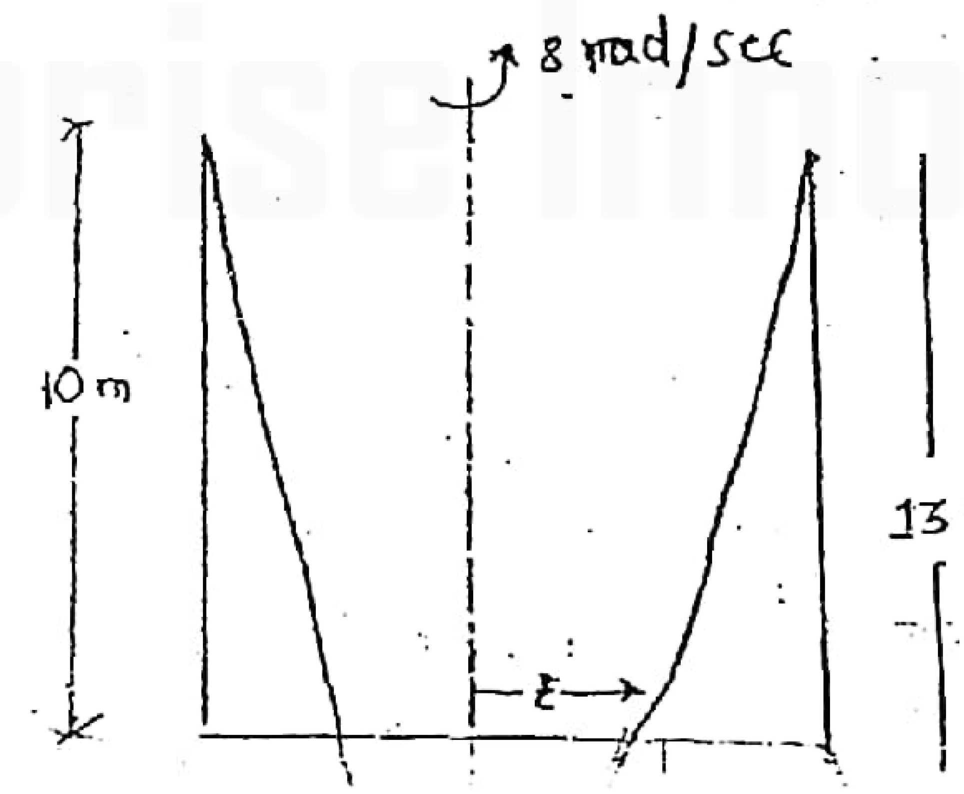
$$V_i = \pi (2)^2 \times 6$$

$$= 75.39 \text{ m}^3$$

Volume of water spilled = $V_i - V_f$

$$= 12.44 \text{ m}^3$$

Q. Find volume of water spilled if angular velocity is 8 rad / sec



$$Z = \frac{(2)^2 \times (8)^2}{2 \times 9.8}$$

$$= 13.05 \text{ m}$$

$$\therefore 3.05 = \frac{(r)^2 \times (8)^2}{2 \times 9.8}$$

$$r = 0.966 \text{ m}$$

$V_P = \pi(2)^2 \times 10 - \left[\frac{1}{2}\pi(2)^2 \times 13.05 - \frac{1}{2}\pi(0.96)^2 \times 3.05\right]$

$= 48.08\ m^3$

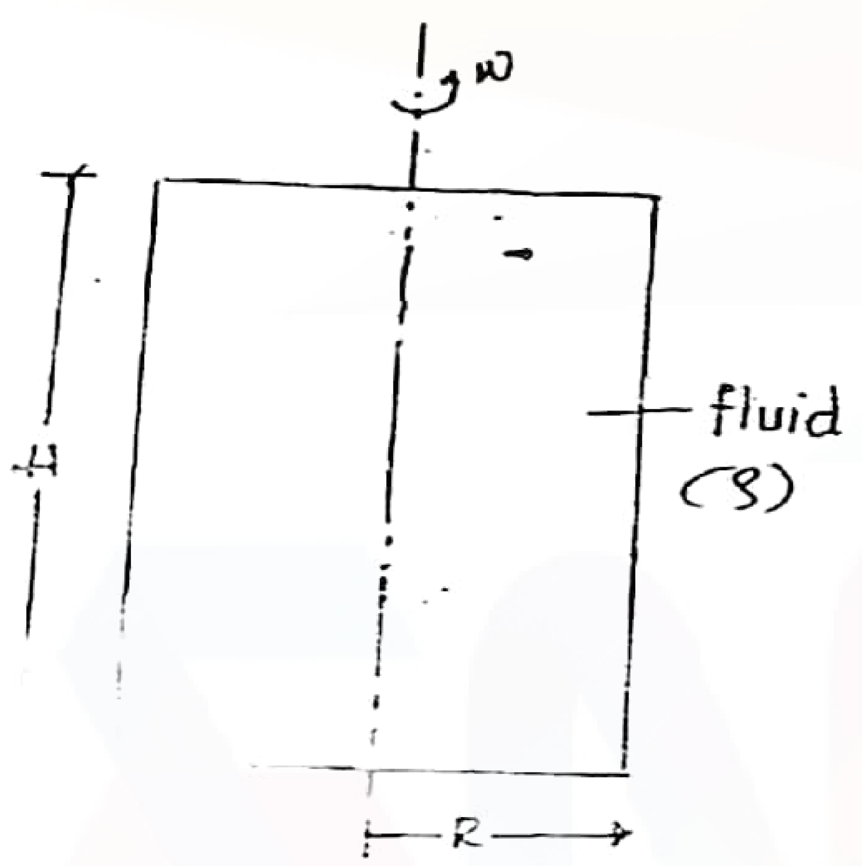
$V_i = \pi(2)^2 \times 6$

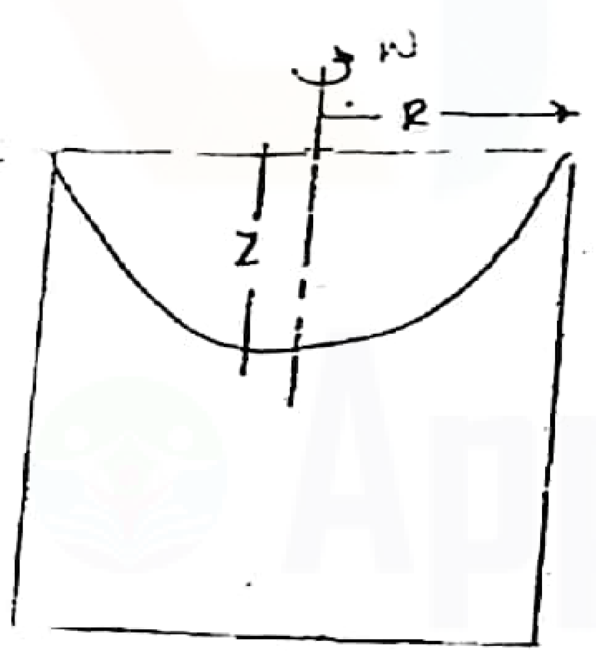
$= 75.39\ m^3$

Volume of water spilled $= V_i - V_P$

$= 27.26\ m^3$

Q. Find the total force at the bottom of vessel shown.





Hydrostatic force at bottom.

$\therefore F_1 = (\rho g H) \cdot \pi R^2$

Force due to volume of water which would have spilled if vessel was open.

$F_2 = \left(\frac{1}{2}\pi R^2 \times z\right) \cdot \rho g$

$= \frac{1}{2}\pi R^2 \cdot \frac{\omega^2 R^2}{2g} \cdot \rho g$

$= \frac{\rho \pi \omega^2 R^4}{4g} \times g$

$= \frac{\rho \pi \omega^2 R^4}{4}$

Total force on bottom

$F = F_1 + F_2$

$= (\rho g H)\pi R^2 + \frac{\rho \pi \omega^2 R^4}{4}$

$= \pi R^2\left[\rho g H + \frac{\rho \omega^2}{4}\right]$

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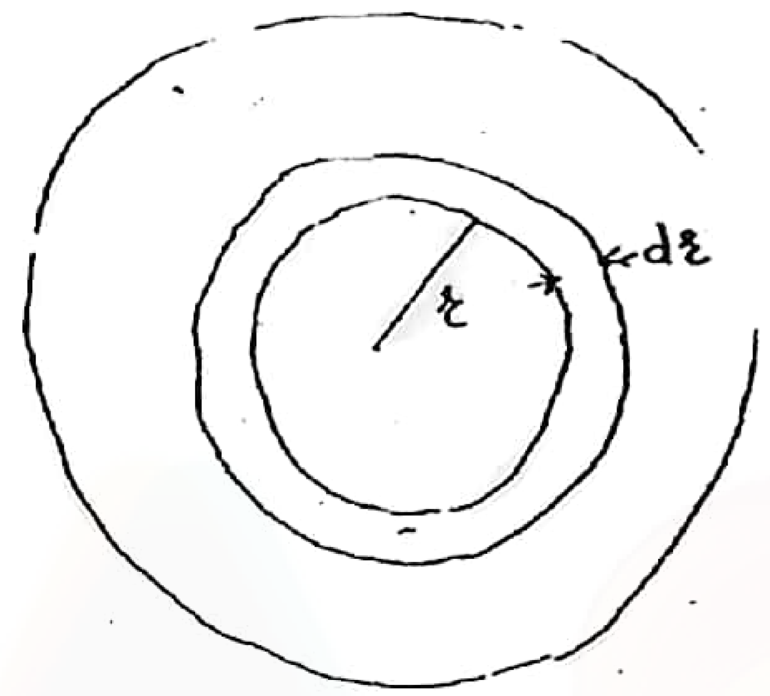
Alternate method.

consider bottom of vessel.

$\therefore$ z - constant (same plane)

$\therefore \frac{dP}{dr} = \frac{\rho v^2}{r}$



– P is function of r only

$\int_{\rho g H}^{P} dP = \int_0^r \rho r \omega^2 \cdot dr$

$P - \rho g H = \frac{\rho r^2 \omega^2}{2}$

$P =$

$\therefore$ Force at bottom of small element :

$dF = P \cdot 2\pi r \cdot dr$

Total force

$\int_0^R dF = \int_0^R P \cdot 2\pi r \cdot dr$

$F = \int_0 \left(\rho g H + \frac{\rho r^2 \omega^2}{2}\right) 2\pi r \cdot dr$

$= \pi r^2\left[\rho g H + \frac{\rho \omega^2}{4}\right]$

LAMINAR FLOW THROUG PIPES.

(Incompressible viscous fluid flows)

Charact eristic dimension is the dimension of the section (pipe or channel) which plays important role in the fluid flow through that section.

For pipes, Characteristic dimension is Hydraulic mean diameter.

It is the diameter of circular section which carries same discharge as that of the given c/s.

$$\text{Hydraulic mean diameter} = \frac{4A}{P} \quad (\text{for pipes})$$

$$= \frac{4\left(\frac{\pi D^2}{4}\right)}{\pi D \times 4}$$

$= D$ – for circular pipe.

Reynold's number is important in case of pipe flows

$$Re = \frac{\text{inertial force}}{\text{viscous force}}$$

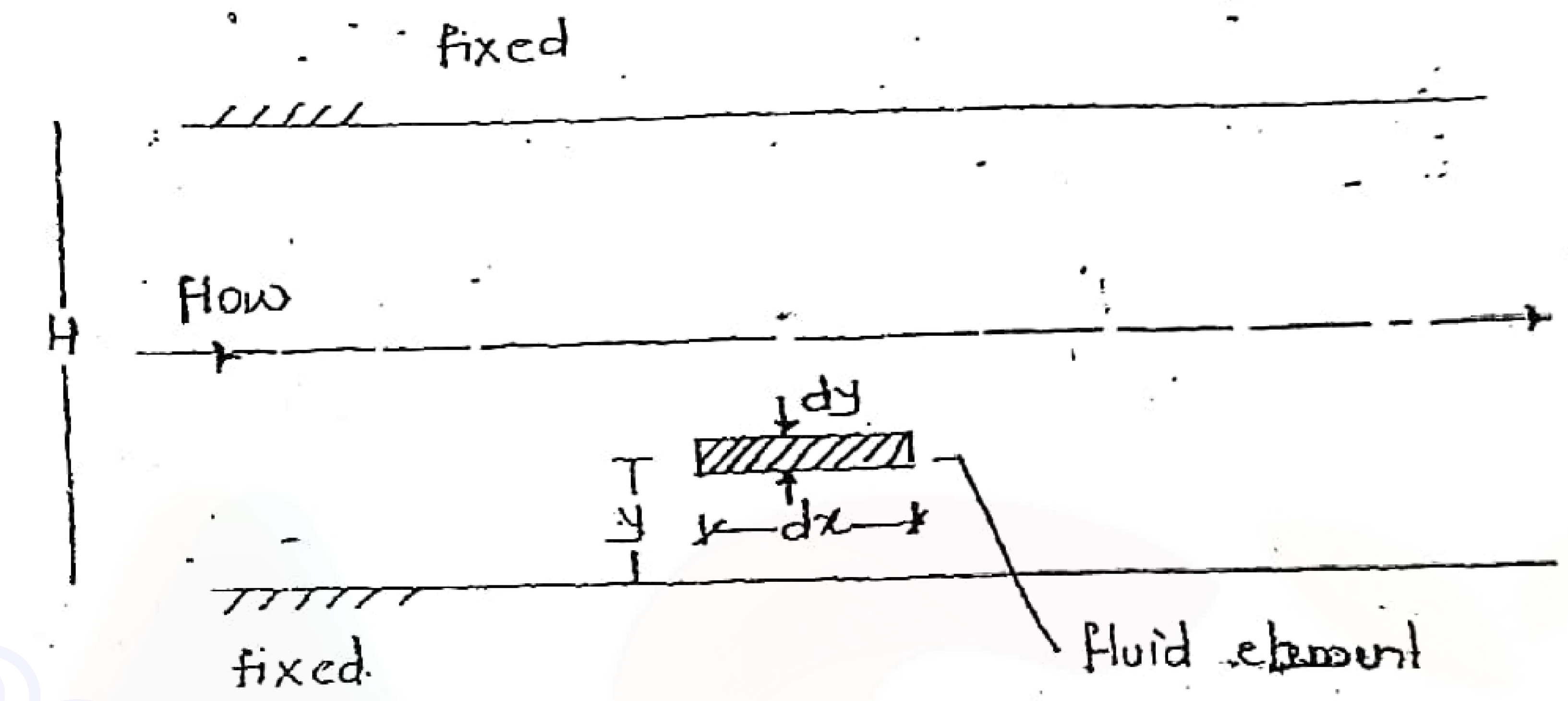
$$= \frac{\rho V D}{\mu} = \frac{VD}{\nu} \qquad V - \text{avg. velocity}$$

If $Re_D < 2000$ – Laminar] only for Pipes.
$Re_D > 4000$ – Turbulent]

In between 2000 and 4000 flow is transitional for which no equation are available. Thus, in problem we consider lower critical value of Re to be 2000.

If $Re > 2000$ – we assume turbulent flow

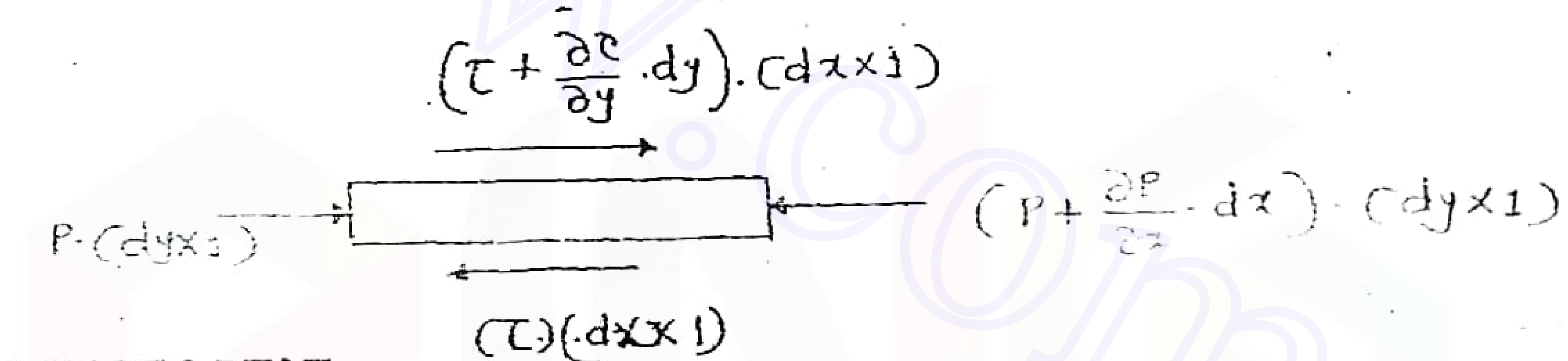
Laminar flow between two fixed parallel plates:



Consider the width of flow to be unity

Here, the weight component of fluid element in the direction flow is zero (thus not considered)

Consider fluid element.



F.B.D. of element of fluid.

In the direction of flow.

Velocity = constant

∴ convective accel^n = 0 – steady flow – Thus Local accel^n is already zero

∴ Total force in X- direction = 0

$$P \cdot dy - \left(P + \frac{\partial P}{\partial x} dx\right) \cdot dy + \left(\tau + \frac{\partial \tau}{\partial y} dy\right) dx - \tau \cdot dx = 0$$

$\partial\tau$ ∂P

Flow is Laminar, Newton's law of viscosity is applicable.

$$\tau = \mu \frac{\partial u}{\partial y}$$

$$\frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) = \frac{\partial P}{\partial x}$$

$$\mu \frac{\partial^2 u}{\partial y^2} = \frac{\partial P}{\partial x}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial P}{\partial x}$$

Integrating

$$\frac{\partial u}{\partial y} = \frac{y}{\mu}\frac{\partial P}{\partial x} + C_1$$

Again

$$u = \frac{y^2}{2\mu}\frac{\partial P}{\partial x} + C_1 \cdot y + C_2$$

Applying boundary conditions

at $y = 0$ $u = 0$

$C_2 = 0$

at $y = H$. $u = 0$

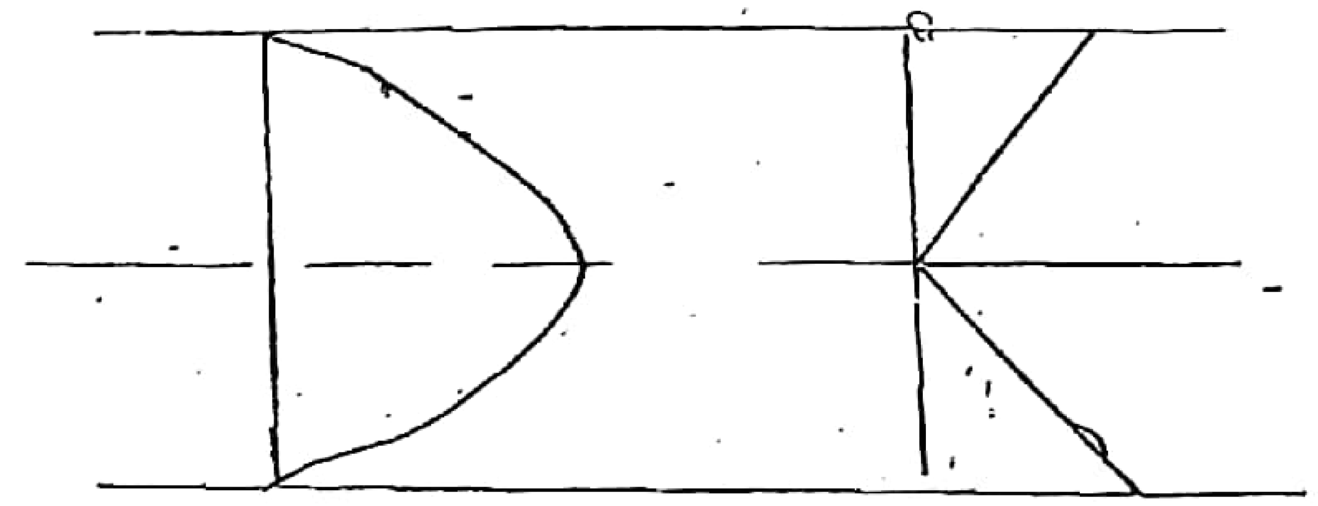
$$0 = \frac{H^2}{2\mu}\frac{\partial P}{\partial x} + C_1 \cdot H$$

$$C_1 = \frac{-H}{2\mu}\frac{\partial P}{\partial x}$$

$$u = \frac{y^2}{2\mu}\frac{\partial P}{\partial x} - \frac{Hy}{2\mu}\frac{\partial P}{\partial x}$$

$$u = \frac{-1}{2\mu}\frac{\partial P}{\partial x}\left(Hy - y^2\right)$$ - parabolic

(u as function



velocity distribution   shear stress distribution

For U_{max},

$y = H/2$

$$U_{max} = \frac{-H^2}{8\mu}\frac{\partial P}{\partial x}.$$

For shear stress,

$$\tau = \mu \frac{\partial u}{\partial y}$$

$$= \mu \frac{\partial}{\partial y}\left[\frac{-1}{2\mu}\frac{\partial P}{\partial x}\left(Hy - y^2\right)\right]$$

$$\tau = -\left(\frac{\partial P}{\partial x}\right)\left(\frac{H}{2} - y\right)$$ - linear

Since $\left(\frac{\partial P}{\partial x}\right)$ is negative here. Because pressure is decreasing in the direction of flow. Thus shear stress is positive.

y is always measured from the fixed end

Local velocity profile is,

$$u = \frac{-1}{2\mu}\frac{\partial P}{\partial x}(Hy - y^2)$$

Mean velocity : (V. or $\bar{u}$)

It is that constant velocity of a section which is crossing same mass flow rate ($\dot{m}$) as it is crossed by actual velocity.

As mean velocity is defined using $\dot{m}$, it is used directly in the continuity equation (which is conservation of mass)

$$\dot{m} = \int_0^H \rho u \, dy \quad - \text{ for actual velocity}$$

$$= \rho H \bar{u} \quad - \text{ for mean velocity}$$

$$\int_0^H \rho u \, dy = \rho H \bar{u}$$

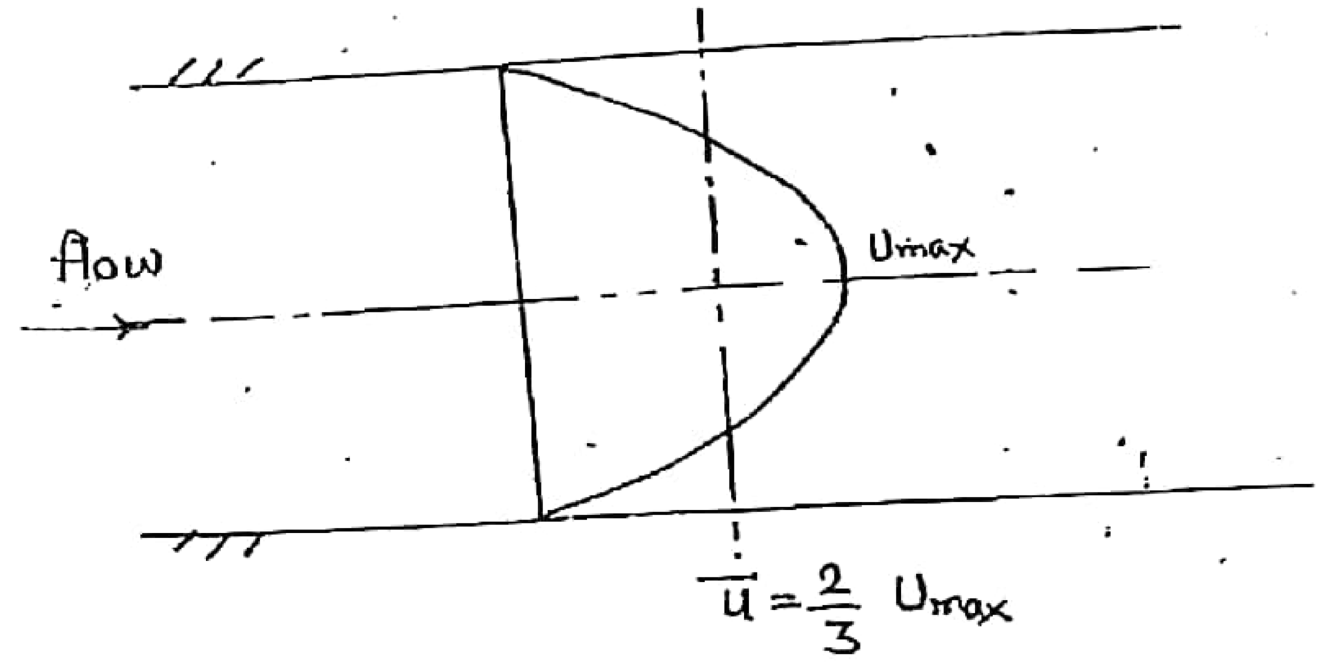
$$\bar{u} = \frac{1}{H}\int_0^H u \, dy$$

$$= \frac{1}{H}\int_0^H \left(\frac{-1}{2\mu}\right)\frac{\partial P}{\partial x}(Hy - y^2)\, dy$$

$$\bar{u} = \frac{-H^2}{12\mu}\left(\frac{\partial P}{\partial x}\right)$$

$$\frac{\bar{u}}{U_{max}} = \frac{2}{3}$$

$$\bar{U} = \frac{2}{3} U_{max}$$



Head loss between two sections in direction of flow.

The mean velocity is used in the energy equations and all other purposes, consider mean velocity

$$\bar{u} = \frac{-H^2}{12\mu}\left(\frac{\partial P}{\partial x}\right)$$

$$\int_1^2 -\partial P = \frac{12\mu\bar{u}}{H^2}\int_1^2 \partial x$$

$$P_1 - P_2 = \frac{12\mu\bar{u}L}{H^2}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{12\mu\bar{U}L}{\rho g H^2}$$

This is total head loss since

$$\left(\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1\right) - \left(\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2\right) = \frac{12\mu\bar{U}L}{\rho g H^2}$$

Since the two plates are horizontal and H is same (same velocity). Therefore, velocity head and datum head difference will be zero.

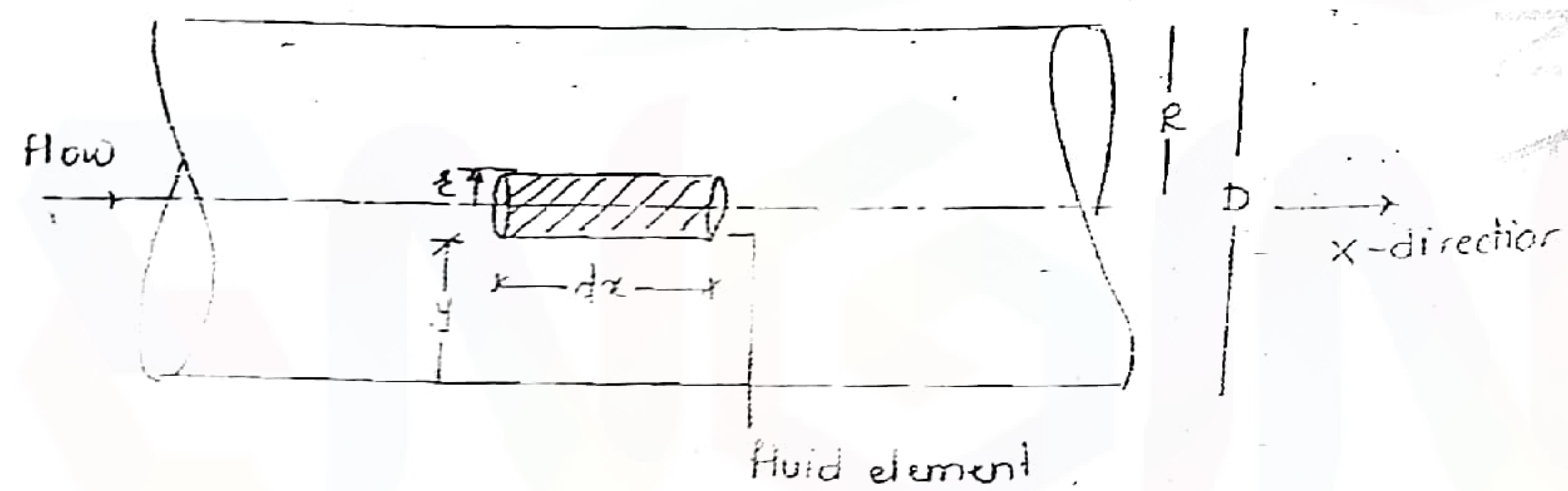
$$\text{Total head loss} = \frac{12\mu\bar{u}L}{\rho g\cdot H^2}$$

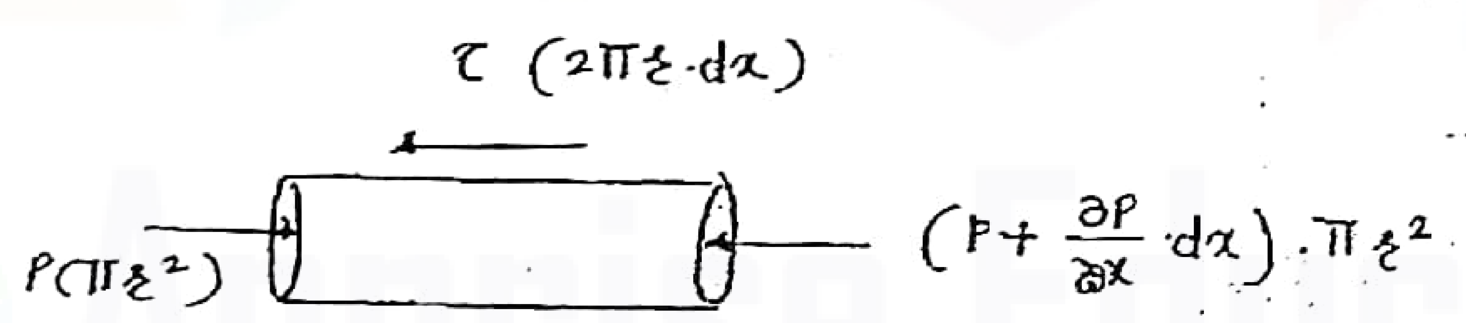
For Laminar flow through two fixed plates

(i) $u = \frac{-1}{2\mu}\frac{\partial P}{\partial x}(Hy - y^2)$

(ii) $\bar{U} = V = \frac{2}{3}U_{max}$.

Laminar flow through pipes:





In the direction of flow,

velocity = constant — (c/s not changing)

$a = 0$

$F_x = 0$

$$P\cdot\pi r^2 - \left(P + \frac{\partial P}{\partial x}\cdot dx\right)\pi r^2 - \tau\cdot 2\pi r\, dx = 0$$

$$\boxed{\tau = \frac{-r}{2}\left(\frac{\partial P}{\partial x}\right)}$$ — linear variation (τ as function of r)

∵ Flow is Laminar

$$\tau = \mu\frac{du}{dy}$$

$$-\frac{r}{2}\left(\frac{\partial P}{\partial x}\right) = \mu\frac{du}{dy}$$

∵ $y + r = R$

$dy + dr = dR$

$dy + dr = 0$

R-constant

$dy = -dr$

$$\frac{-r}{2}\left(\frac{\partial P}{\partial x}\right) = \mu\frac{du}{(-dr)}$$

$$\frac{du}{dr} = \frac{r}{2\mu}\cdot\frac{\partial P}{\partial x}$$

Integrate.

$$u = \frac{r^2}{4\mu}\frac{\partial P}{\partial x} + C$$

At $r = R$, $u = 0$

$$C = \frac{-R^2}{4\mu}\frac{\partial P}{\partial x}$$

$$u = \frac{r^2}{4\mu}\left(\frac{\partial P}{\partial x}\right) - \frac{R^2}{4\mu}\left(\frac{\partial P}{\partial x}\right)$$

$$\boxed{u = \frac{-1}{4\mu}\frac{\partial P}{\partial x}\left(R^2 - r^2\right)}$$ — Parabolic u vs r^2

For max velocity at section,

$r = 0$, $U = U_{max}$

$$U_{max} = \frac{-R^2}{4\mu}\cdot\frac{\partial P}{\partial x}$$

Mean velocity of flow:

- consider small ring element of width dr and radius r.

$$\dot{m} = \int_0^R \rho u \,(2\pi r \cdot dr) = \rho \pi R^2 \cdot \bar{u}$$

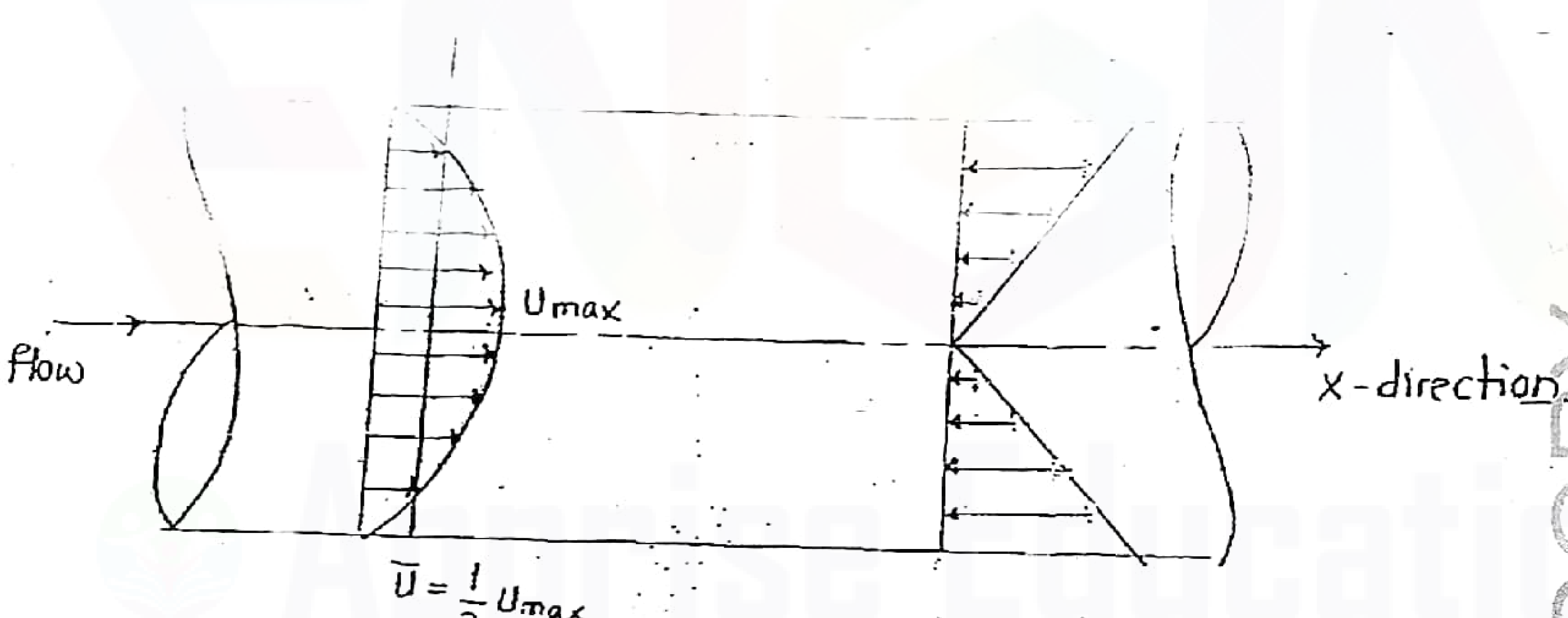
$$\bar{u} = \frac{2}{R^2}\int_0^R u \cdot r \cdot dr$$

$$\bar{u} = \frac{2}{R^2}\int_0^R \left(\frac{-1}{4\mu}\right)\left(\frac{\partial P}{\partial x}\right)(R^2 r - r^3)\, dr$$

$$\bar{u} = \frac{-R^2}{8\mu}\left(\frac{\partial P}{\partial x}\right)$$

$$\frac{\bar{u}}{U_{max}} = \frac{4}{8} = \frac{1}{2}$$

$$\bar{u} = \frac{1}{2}\, U_{max}$$



velocity distribution

shear stress distribution

Wall shear stress $(r = R)$

$$\tau_0 = \frac{-R}{2}\left(\frac{\partial P}{\partial x}\right)$$

$$\tau_0 = \frac{-D}{4}\left(\frac{\partial P}{\partial x}\right)$$

$$\frac{4\tau_0}{D} = -\frac{\partial P}{\partial x}$$

$$\frac{4\tau_0}{D}\int_0^L \partial x = \int_1^2 -\partial P$$

$$P_1 - P_2 = \frac{4\tau_0 \cdot L}{D}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{4\tau_0 \cdot L}{\rho g D} \quad \text{— head loss}$$

$$h_L = \frac{4\tau_0 L}{\rho g D} = \frac{f L V^2}{2 g D}$$

$$\tau_0 = \frac{\rho f v^2}{8}$$

$$= \frac{\rho\left(64/Re\right) v^2}{8} \qquad Re = \frac{\rho v D}{\mu}$$

$$= \frac{64\rho\, v^2}{8 Re} \qquad v = \frac{Re \cdot \mu}{\rho D}$$

$$= \frac{64\rho \cdot v^2}{Re\, 8} \times \frac{Re^2 \mu^2}{\rho^2 D^2}$$

$$\tau_0 = \frac{8\, Re\, \mu^2}{\rho D^2}$$

for max stress. $Re = 2000$ for laminar flow

$$\tau_{max} = \frac{8 \times (2000) \cdot \mu^2}{\rho D^2}$$

$$= 16000\, \mu^2$$

Q. 16 & 17
(Page 39)

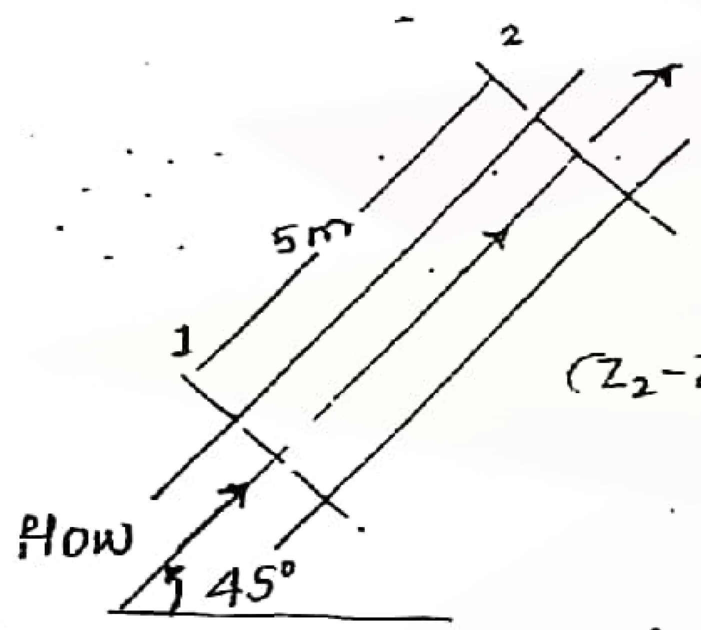
$\rho = 800\ kg/m^3$

$\mu = 0.8\ kg/m.s$

Laminar flow through pipe.

$P_1 = 435\ kN/m^2 = 435\times10^3\ Pa$

$P_2 = 200\ kN/m^2 = 200\times10^3\ Pa$



$H_1 - H_2 = h_L$

$h_L = \left(\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1\right) - \left(\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2\right)$

$(Z_2-Z_1) = 5\sin45^\circ = \frac{5}{\sqrt2}$

$= \left(\frac{P_1-P_2}{\rho g}\right) - (Z_2-Z_1)$

$V_1 = V_2$ — c/s same

$= \left(\frac{435-200}{800\times9.8}\right)\times10^3 - \left(\frac{5}{\sqrt2}\right)$

$= 26.408\ m$

$h_L = \frac{32\mu\bar{u}\cdot L}{\rho g\cdot D^2}$

$26.408 = \frac{32\times0.8\times\bar{u}\times5}{800\times9.8\times(0.1)^2}$

$\bar{u} = 16.19\ m/s$

$Q = \frac{\pi}{4}\times D^2\times\bar{u}$

$= \frac{\pi}{4}\times(0.1)^2\times16.19$

$= 0.127\ m^3/sec$

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flow is reversed (from 2 to 1)

$\therefore P_1 = 436\times10^3\ Pa$

$P_2 = ?$ for same discharge

$Q = \frac{\pi}{4}\times D^2\times V$

$= (0.1)^2\times\frac{\pi}{4}\times\bar{u}$

$\bar{u}$ will be same

head loss will be same $\therefore h_L = \frac{32\mu\bar{u}\cdot L}{\rho g D^2}$

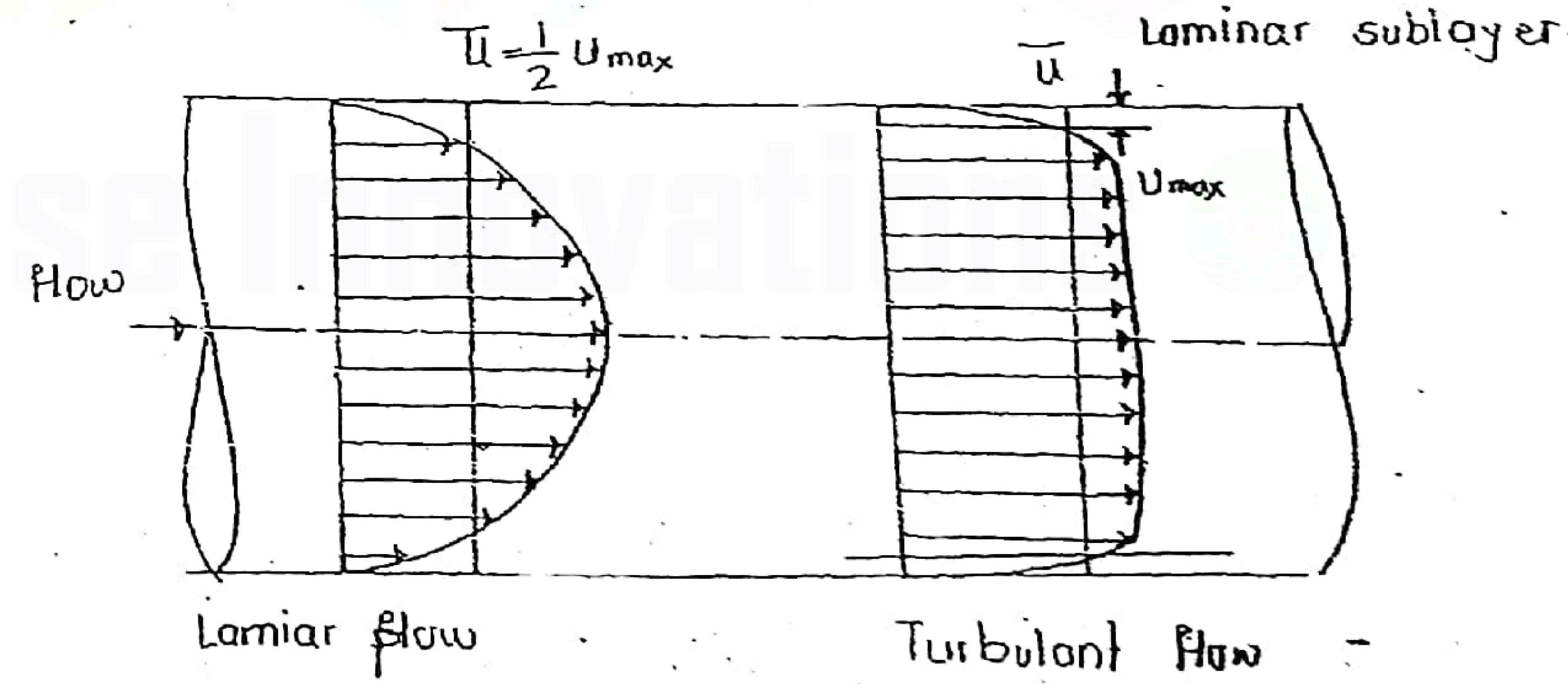
$h_L = 26.408 = H_2 - H_1$

$26.408 = \left(\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2\right) - \left(\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1\right)$

$26.408 = \frac{P_2 - 435\times10^3}{800\times9.81} + \frac{5}{\sqrt2}$

$P_2 = 614.5\times10^3\ Pa.$

Basic difference in velocity profiles of laminar and turbulent flow in pipes.



In turbulent flow because of intermixing velocity profiles are flat in nature (the layers of high momentum jump to layers of lower momentum).

Mean velocity in turbulent flow is more close to the actual profile as compared to Laminar flow.

Concept of shear velocity:

Shear stress distribution:

$$\tau = \frac{-r}{2}\left(\frac{\partial P}{\partial x}\right)$$

∵ Wall shear stress.

$$\tau_0 = \frac{-R}{2}\left(\frac{\partial P}{\partial x}\right) = \frac{-D}{4}\left(\frac{\partial P}{\partial x}\right)$$

$$\int_1^2 -\partial P = \frac{4\tau_0}{D}\int_0^L \partial x$$

$$\frac{P_1 - P_2}{\rho g} = \frac{4\tau_0 \cdot L}{D \cdot \rho g} = h_L$$

$$= \frac{fLV^2}{2gD}$$

$$\frac{\tau_0}{\rho} = \frac{fV^2}{8}$$

$$\sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{f}{8}} \cdot V = V^*$$

Since this velocity is fraction (part) of the mean velocity itself it is called Shear Velocity (Suspicious parameter)

Calculated near wall — Part of velocity.

This velocity concept is valid in both Laminar and turbulent flow (sub-layer even if doesn't exist, this concept was applicable)

Momentum correction factor (β) for Laminar flow through pipe:

Mean velocity is based on mass conservation i.e. momentum conservation is not applicable in this case. Thus momentum calculated on the basis of mass conservation gives wrong value.

∵ Velocity profile.

$$u = \frac{-1}{4\mu}\frac{\partial P}{\partial x}(R^2 - r^2)$$

$$\bar{u} = \frac{-R^2}{8\mu}\left(\frac{\partial P}{\partial x}\right)$$

Momentum correction factor is given by,

$$\beta = \frac{\text{Actual moment crossing per sec through section}}{\text{Moment crossing per sec based on mass conservation. i.e. (Mean velocity)}}$$

$$\beta = \frac{\int_0^R \rho \cdot u \cdot (2\pi r \cdot dr) \cdot u}{\underbrace{\rho(\pi R^2)\cdot \bar{u}}_{(\text{mass flow rate})} \times \underbrace{\bar{u}}_{(\text{Mean velocity})}}$$

$$= \frac{\frac{2}{R^2}\int_0^R u^2 r \cdot dr}{(\bar{u})^2}$$

$$= \frac{\frac{2}{R^2}\left(\frac{-1}{4\mu}\right)^2\left(\frac{\partial P}{\partial x}\right)^2 \int_0^R r(R^2 - r^2)^2 \cdot dr}{\left(\frac{-R^2}{8\mu}\right)^2\left(\frac{\partial P}{\partial x}\right)^2}$$

$$= \frac{8}{R^6}\int_0^R (R^4 r + r^5 - 2R^2 r^3)\cdot dr$$

$$\beta = \frac{8}{R^6}\left[\frac{R^6}{2} + \frac{R^6}{6} - \frac{R^6}{2}\right]$$

$$= \frac{8}{4}$$

$(\beta)_{Laminar} = \frac{4}{3}$ $\qquad$ $(\beta)_{Turbulent} = 1.2$

Kinetic energy correction factor (α)

$$\alpha = \frac{\text{Actual K.E. crossing through section per sec}}{\text{K.E. crossing through section based on mean velocity}}$$

$$= \frac{\frac{1}{2}\int_0^R (\rho \cdot u \cdot 2\pi r \cdot dr) \cdot u^2}{\frac{1}{2}(\rho \cdot \pi R^2 \bar{u}) \cdot \bar{u}^2}$$

$$= \frac{\frac{2}{R^2}\int_0^R u^3 r \cdot dr}{\bar{u}^3}$$

$$= \frac{\frac{2}{R^2}\left(\frac{-1}{4\mu}\right)^3 \left(\frac{\partial P}{\partial x}\right)^3 \int_0^R (R^2 - r^2)^3 \cdot r \cdot dr}{\left(\frac{-R^2}{8\mu}\right)^5 \left(\frac{\partial P}{\partial x}\right)^3}$$

$$= \frac{16}{R^8}\int_0^R (R^2 - r^2)^3 \cdot r \cdot dr$$

$$= 2$$

$(\alpha)_{Laminar} = 2$ $\qquad$ $(\alpha)_{Turbulent} = \frac{4}{3}$

PIPE FLOWS

In this section different types of energy losses are studied in pipe flows.

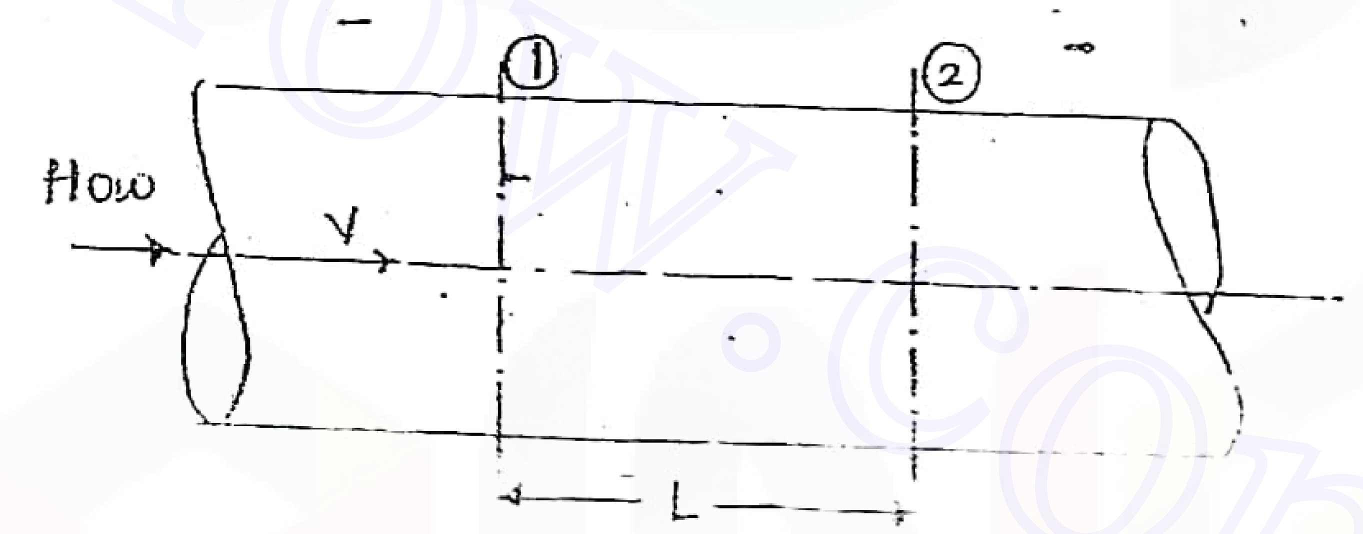
In general energy losses in pipes are divided in two different categories.

(i) Major energy loss in pipe flows (Head loss due to friction)

(ii) Minor losses in pipes.

Major energy loss in pipes :

(i) Darcy's formula.



frictional loss $= h_f = \frac{fLV^2}{2gD}$

where

f - friction factor

f' - coefficient of friction (Darcy's coeffici

$f' = \frac{f}{4}$

If $(Re)_D < 2000$, Laminar flow

$f = \frac{64}{(Re)_D}$ $\qquad$ $f' = \frac{16}{(Re)_D}$

If $(R_e)_D > 4000$, turbulent flow.

$$f = \frac{0.079 \times 4}{(R_e)_D^{1/4}} \qquad f' = \frac{0.079}{(R_e)_D^{1/4}}$$

(ii) Chezy's formula.

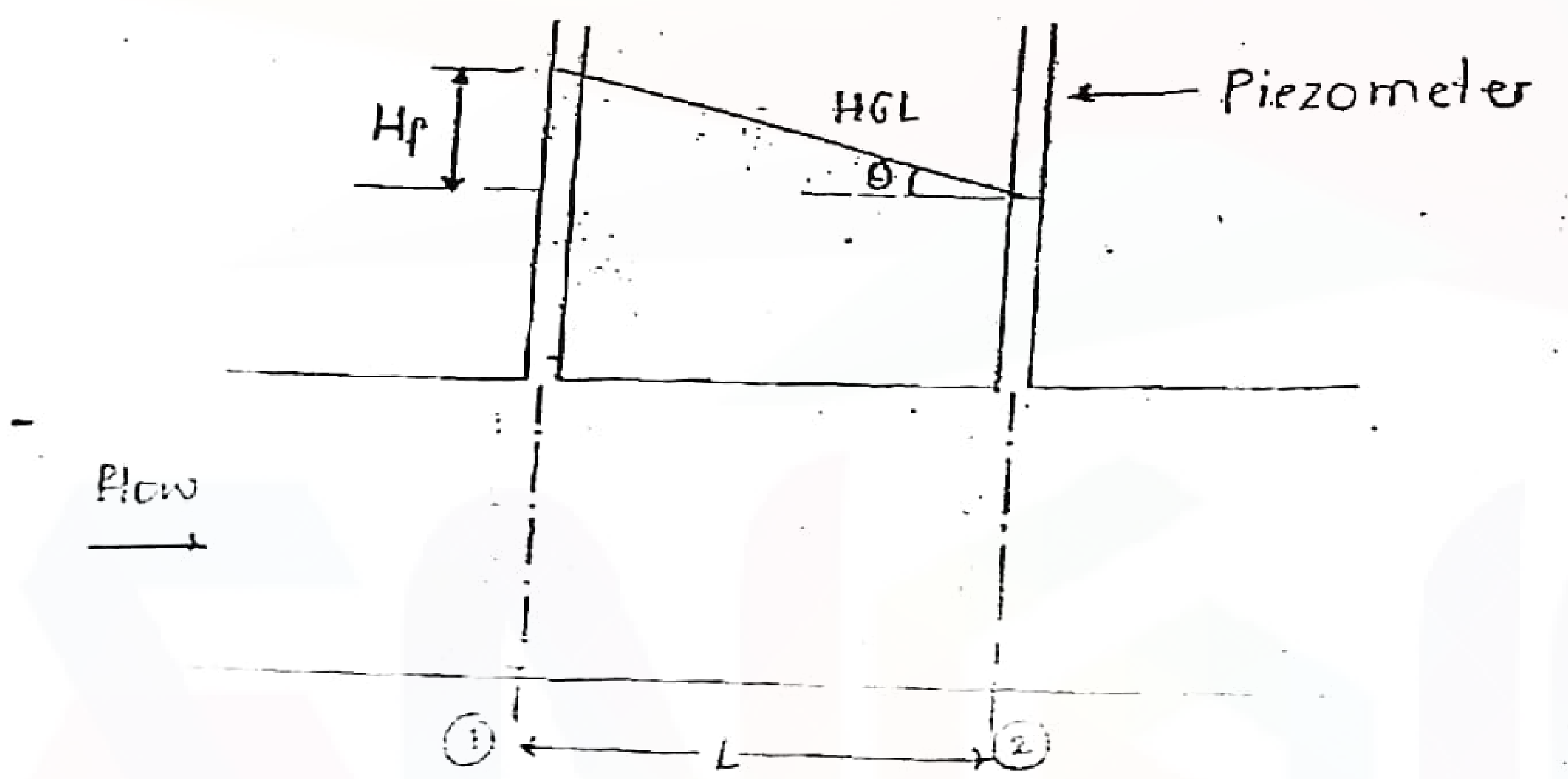
$$V = C\sqrt{mi}$$

where,

C - Chezzy's constant.

i - slope of HGL

m - Hydraulic mean depth.



$$i = \tan\theta = \frac{h_f}{L}$$

$$m = \frac{A}{P} = \frac{\pi/4\, D^2}{\pi D} = \frac{D}{4}$$

$$V = C\sqrt{D/4 \times \left(\frac{h_f}{L}\right)}$$

$$V^2 = C^2 \cdot \frac{D}{4} \times \frac{h_f}{L}$$

$$h_f = \frac{4V^2L}{C^2D}$$

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Q. What is the value of Chezzy's constant so that value of head loss is same by both the above head loss formula?

$$\frac{fLV^2}{2gD} = \frac{4V^2L}{C^2D}$$

$$C = \sqrt{\frac{8g}{f}}$$

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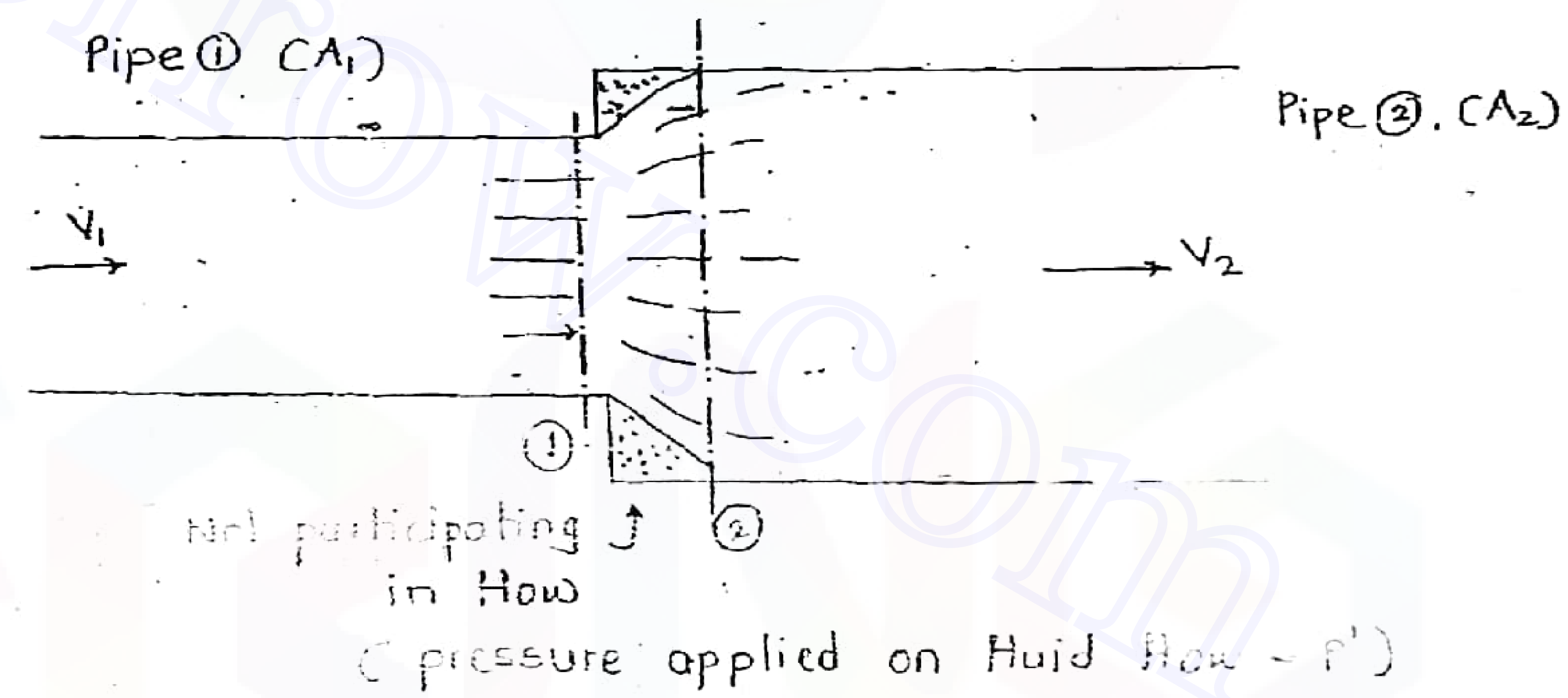
Minor energy losses in pipe flows:

(1) Head loss due to sudden enlargement in pipes:



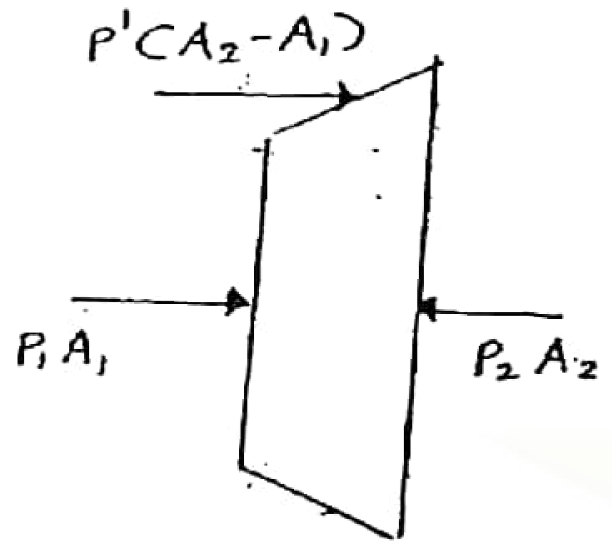
Applying energy equation at ① and ②

$$\frac{P_1}{\gamma g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma g} + \frac{V_2^2}{2g} + Z_2 + h_{Le}$$

$$h_e = \frac{P_1 - P_2}{\gamma g} + \frac{V_1^2 - V_2^2}{2g} \qquad \text{---(i)}$$

Loss due to friction is very small as section ① and ② are very close to each other.

consider fluid between ① and ②



$\therefore \dot{m} = \rho A_1 V_1 = \rho A_2 V_2$ (mass flow rate is constant)

Applying Newton's 2^{nd} law of motion in the direction of flow.

$$P_1A_1 - P_2A_2 + P'(A_2 - A_1) = \dot{m}V_2 - \dot{m}V_1$$

It has experimentally seen that.

$$P' = P_1$$

$$(P_1 - P_2)A_2 = \rho A_2 V_2 (V_2 - V_1)$$

$$\frac{P_1 - P_2}{\rho} = V_2^2 - V_1 V_2$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1 \cdot V_2}{g} \quad \text{---- (ii)}$$

from --- (i) & --- (ii)

$$h_{Le} = \frac{V_2^2 - V_1 V_2}{g} + \frac{V_1^2 - V_2^2}{2g}$$

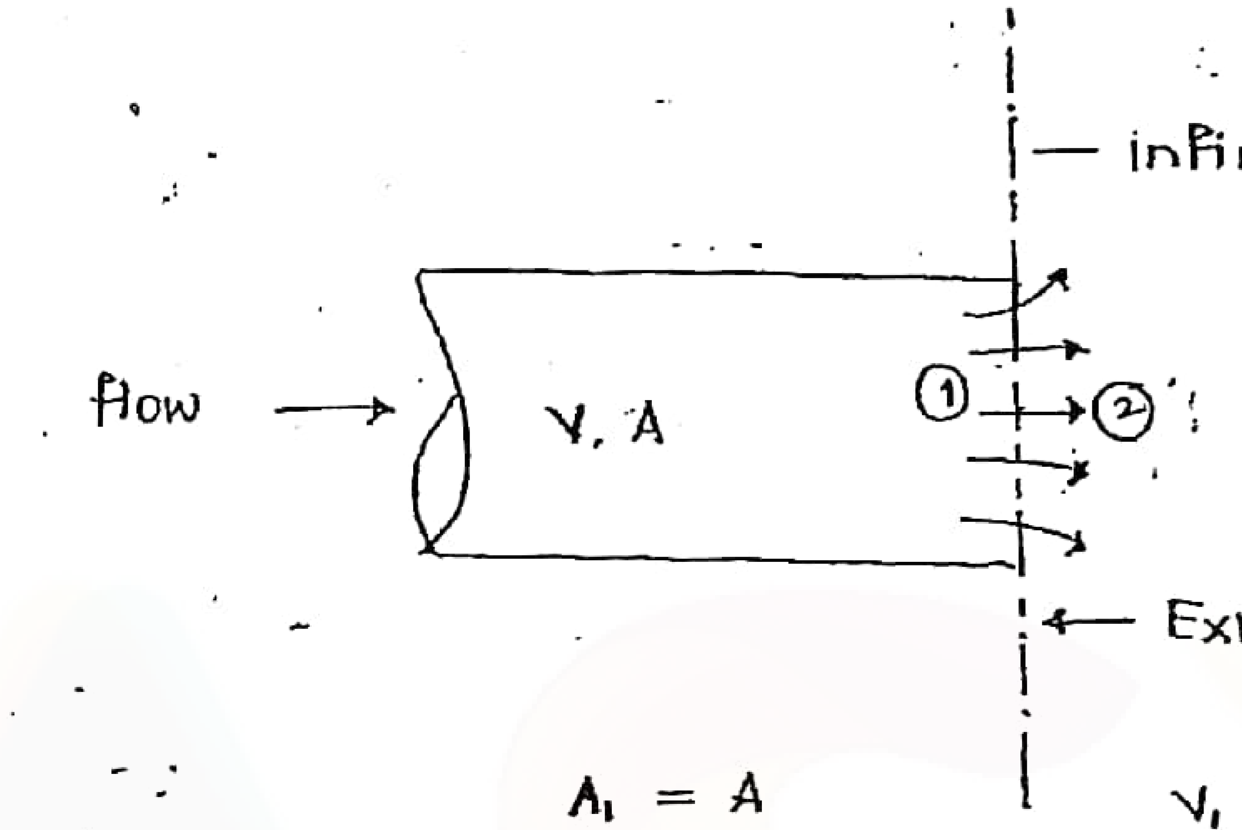
$$= \frac{2V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{2g}$$

$$= \frac{V_1^2 + V_2^2 - 2V_1V_2}{2g}$$

$$h_{Le} = \frac{(V_1 - V_2)^2}{2g}$$

Head loss due to sudden enlargement is velocity head difference square divided by 2g.

(2) Head loss at the exit of pipe:



$A_1 = A$ $\quad V_1 = V$

$A_2 = \infty$ $\quad V_2 = 0$

$\because A_1V_1 = A_2V_2$

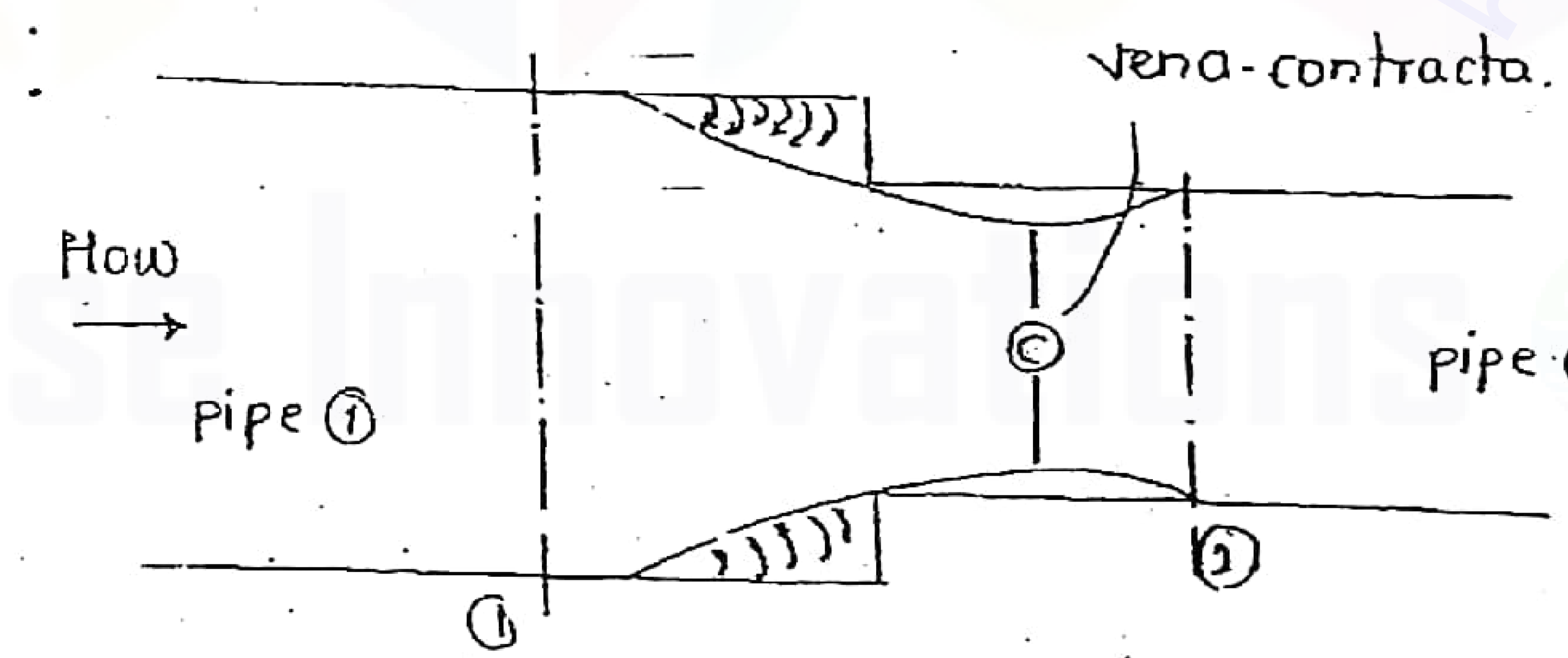
$V_2 = \frac{A_1V_1}{A_2} = 0$

$$h_{Lexit} = \frac{(V_1 - V_2)^2}{2g}$$

$$= \frac{(V_1 - 0)^2}{2g}$$

$$h_{Lexit} = \frac{V_1^2}{2g}$$

(3) Loss of head due to sudden contraction in pipes



from section © to ② there is expansion of flow due to sudden contraction at pipe section

Cause sudden ...

$$h_{Lc} = \frac{(V_c - V_2)^2}{2g}$$

$$= \left(\frac{V_c}{V_2} - 1\right)^2 \cdot \frac{V_2^2}{2g}$$

$$= \left(\frac{A_2}{A_c} - 1\right)^2 \cdot \frac{V_2^2}{2g} \qquad A_1V_1 = A_cV_c$$

$$= \left(\frac{1}{A_c/A_2} - 1\right)^2 \cdot \frac{V_2^2}{2g}$$

$$h_{Lc} = \left(\frac{1}{C_c} - 1\right)^2 \cdot \frac{V_2^2}{2g}$$

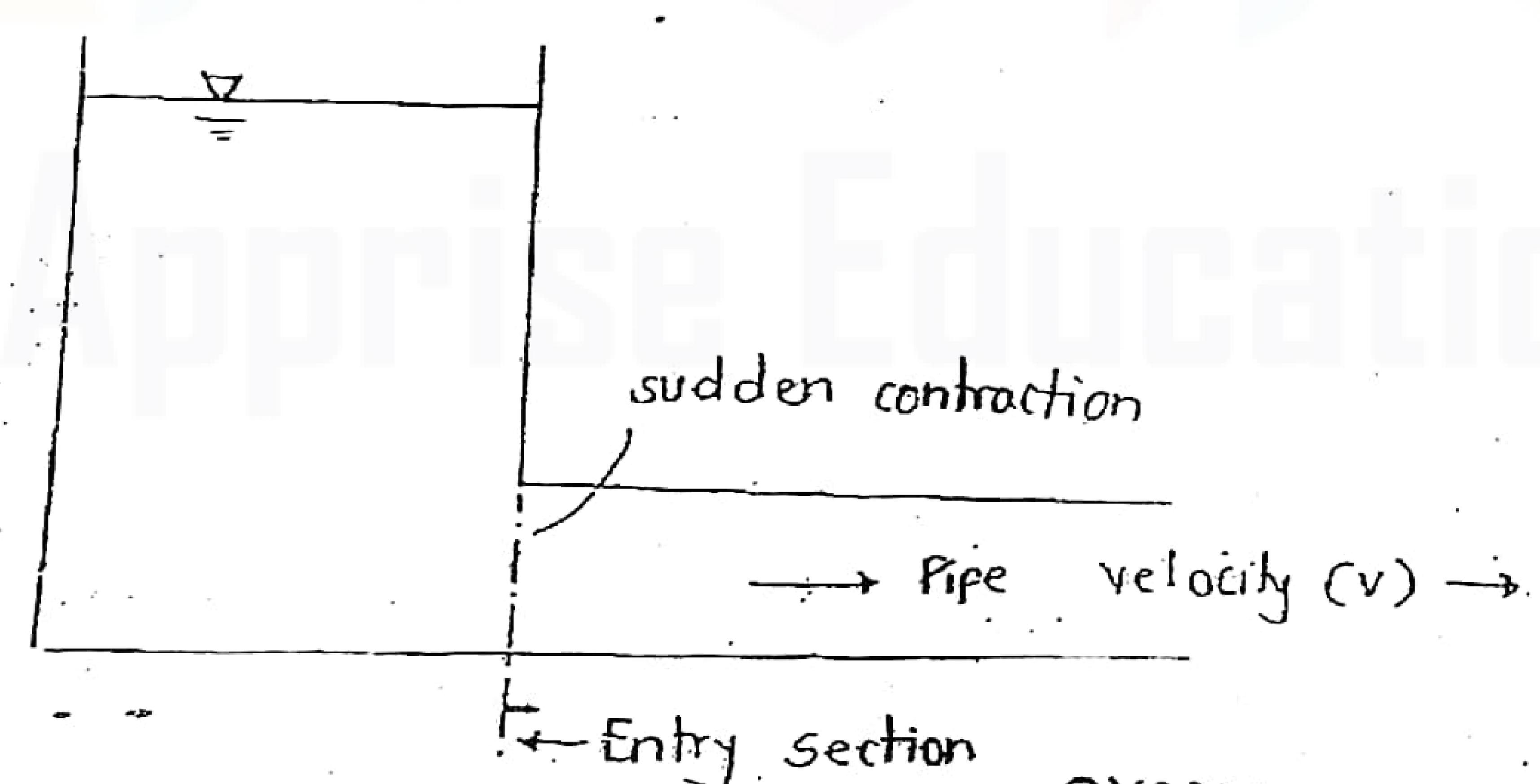
C_c - coefficient of contraction for sudden contraction.

If C_c is not given,

$$\left(\frac{1}{C_c} - 1\right)^2 \simeq 0.5$$

$$h_{Lc} = 0.5 \frac{V_2^2}{2g}$$

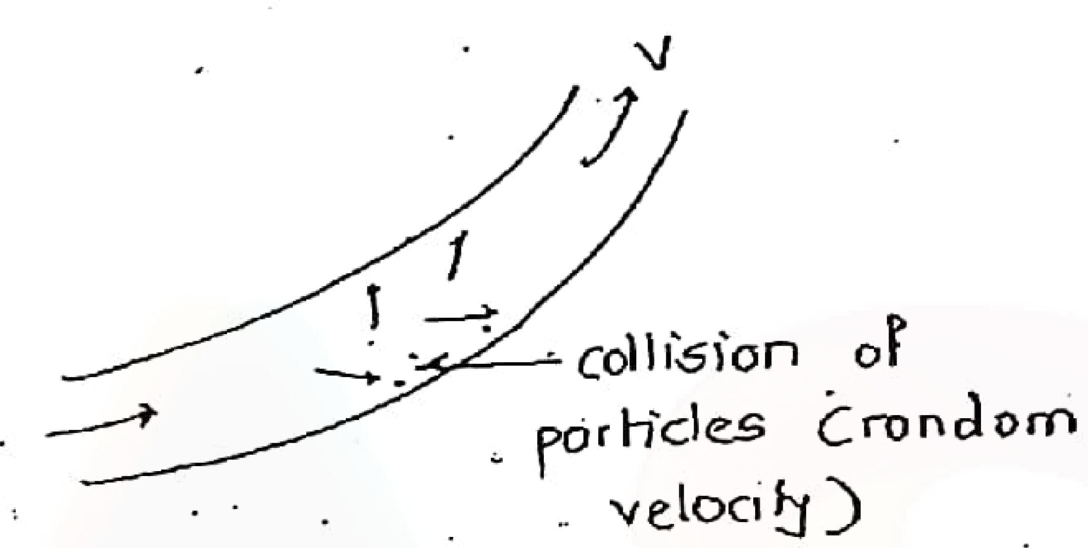
(4) Loss of head at entrance of pipe.



It is case of sudden contraction.

$$h_{L\,entry} = 0.5 \frac{V^2}{2g}$$

(5) Loss of head due to bends:



$$h_{L\,bend} = k \cdot \frac{V^2}{2g}$$

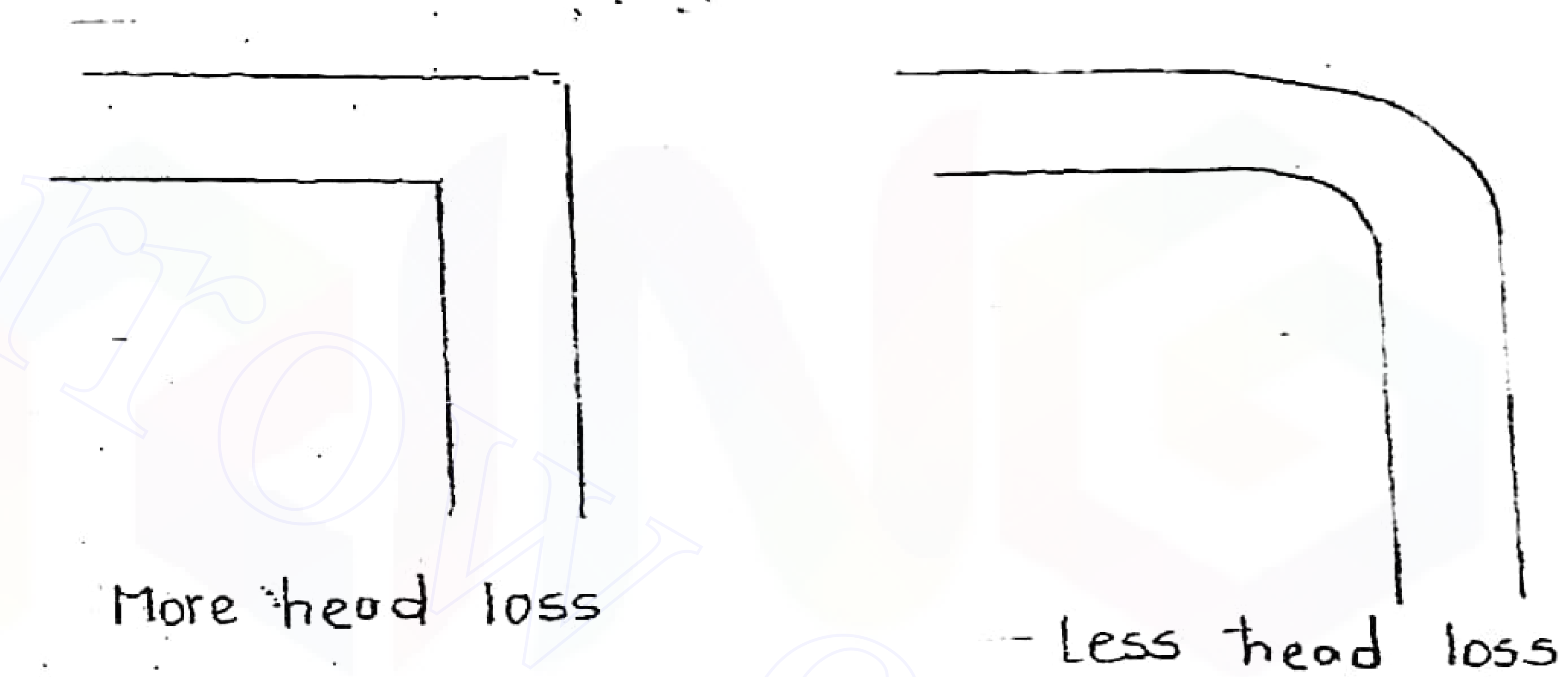
where

k - loss coefficient for the pipe bend.

k depends on following factors:

(i) Radius of pipe (R_{pipe}):

If radius of pipe is more k will be less

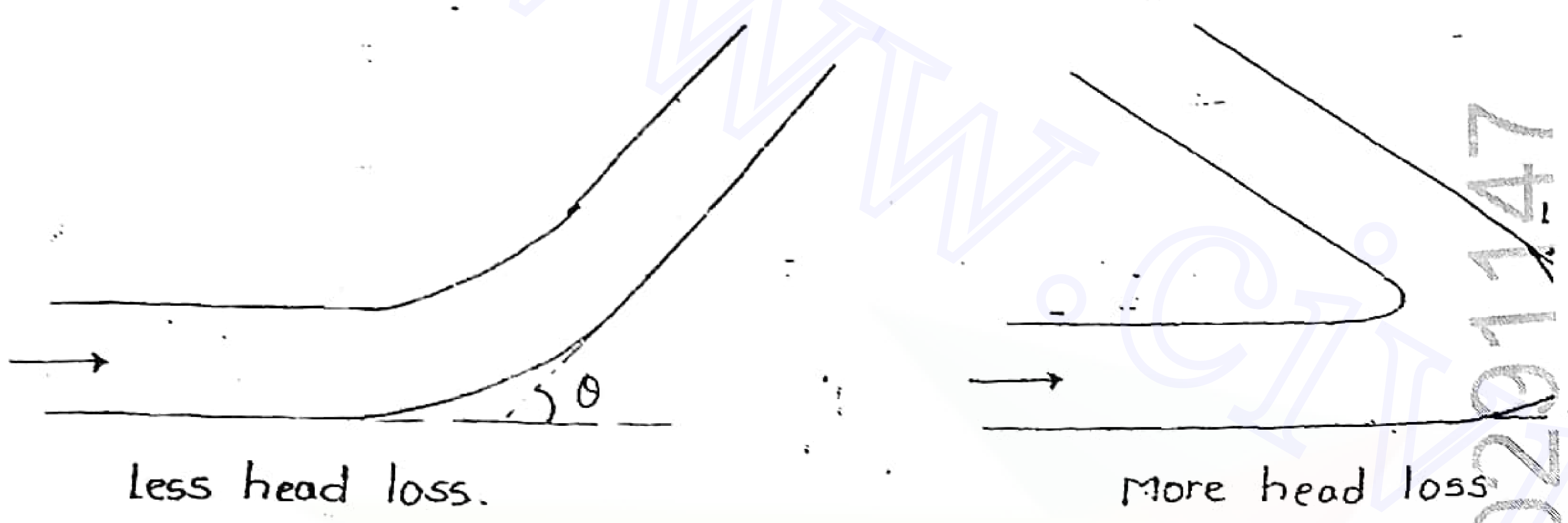


(ii) Radius of bend: (R_{bend})

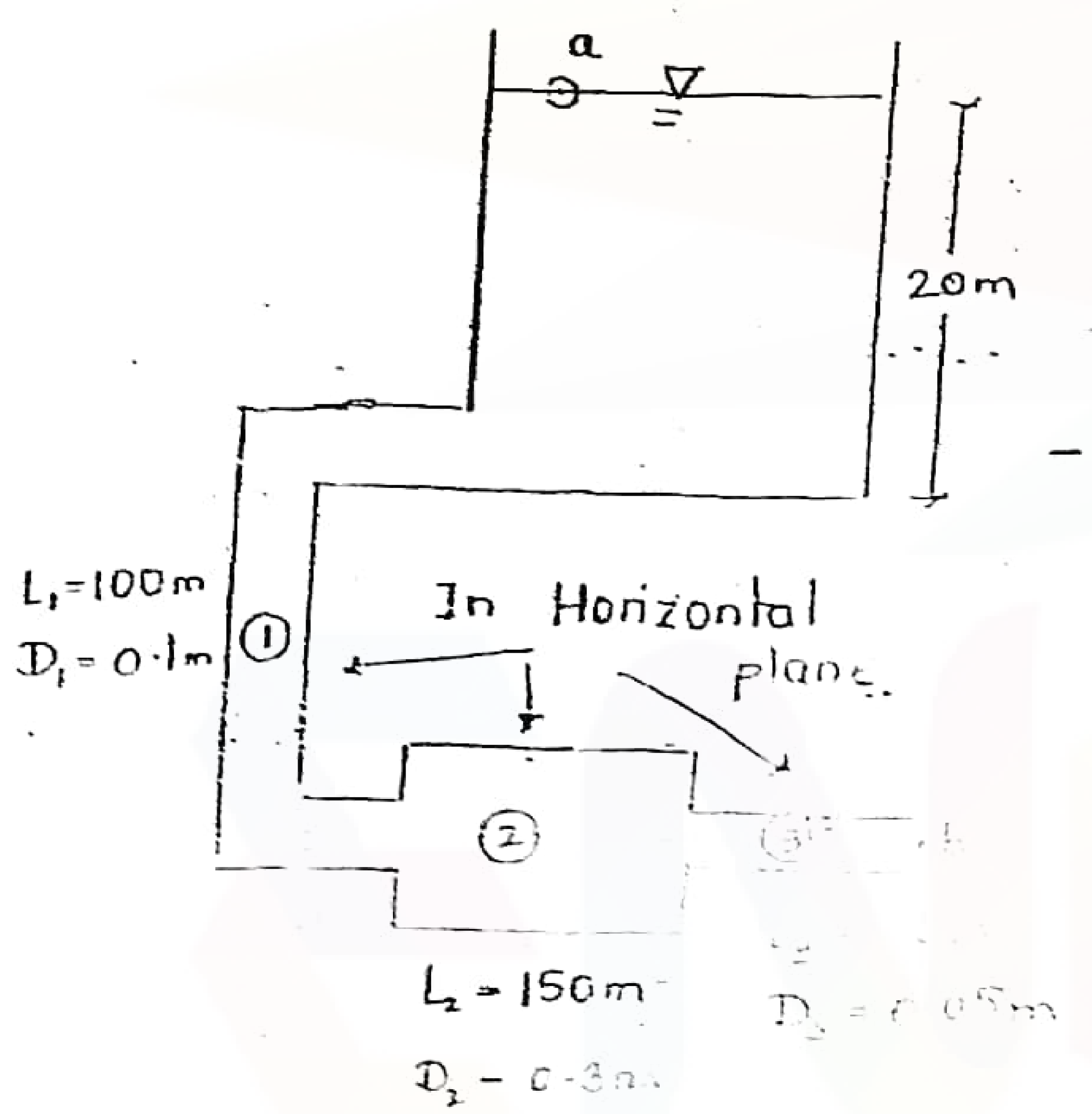
If radius of bend is more k will be less

(iii) Angle of bend (θ)

If angle of bend is more, k will be more thus head loss will be more.



Q.3 (Page 40)



$K_{bend} = 1$

$f' = 0.002$

$f = 0.002 \times 4$

$= 0.008$

$\therefore A_1V_1 = A_2V_2 = A_3V_3$

$\frac{\pi}{4}(0.1)^2 \cdot V_1 = \frac{\pi}{4}(0.3)^2 V_2 = \frac{\pi}{4}(0.05)^2 V_3$

$4V_1 = 36V_2 = V_3$

$V_1 = 9V_2$

$V_3 = 36V_2$

Applying energy equation at points ⓐ and ⓑ

$$\frac{P_{atm}}{\rho g} + \cancel{\frac{V_a^2}{2g}}^{\,0} + 20 = \frac{P_{atm}}{\rho g} + \cancel{\frac{V_b^2}{2g}}^{\,0} + 0 + h_L$$

(horizontal pl of pip)

large surface area $\therefore V_a \simeq 0$

sudden enlargement A_2 is infinite $\therefore V_2 = 0$

$h_L = 20$ m

Considering all the losses:

$$20 = \frac{(0.008)\cdot 100 \cdot V_1^2}{2\times 9.8 \times 0.1} + \frac{(0.008)\cdot 150 \cdot V_2^2}{2\times 9.8\times 0.3} + \frac{(0.008)\cdot 50\, V_3^2}{2\times 9.8\times 0.05}$$

$$+ \frac{0.5V_1^2}{2g} + \frac{V_1^2}{2g} + \frac{V_1^2}{2g} + \frac{(V_1-V_2)^2}{2g} + \frac{0.5V_3^2}{2g} + \frac{V_3^2}{2g}$$

$V_2 =$

use $V_1 = 9V_2$
$V_3 = 36V_2$

$Q = \frac{\pi}{4}(0.3)^2 \times V_2$

$=$

If Minor losses are neglected.

$$20 = \frac{0.008\times 100\, V_1^2}{2\times 9.8\times 0.1} + \frac{(0.008)150\, V_2^2}{2\times 9.8\times 0.3} + \frac{0.008\times 50\, V_3^2}{2\times 9.8\times 0.05}$$

$V_2' =$

$Q' = \frac{\pi}{4}(0.3)^2 \times V_2'$

$=$

% Error in discharge

$= \frac{|Q' - Q|}{Q} \times 100$

$=$

Total energy line :

(Energy gradient line) - - TEL or EGL

"It is the line joining the points representing the values of total head at the various cross sections of pipe in the direction of flow."

Hydraulic gradient line :
(HGL)

"It is the line joining the points representing the values / levels of piezometric heads at the various cross sections of the pipe in the direction of flow.

$$TEL = \frac{P}{\rho g} + \frac{V^2}{2g} + Z$$

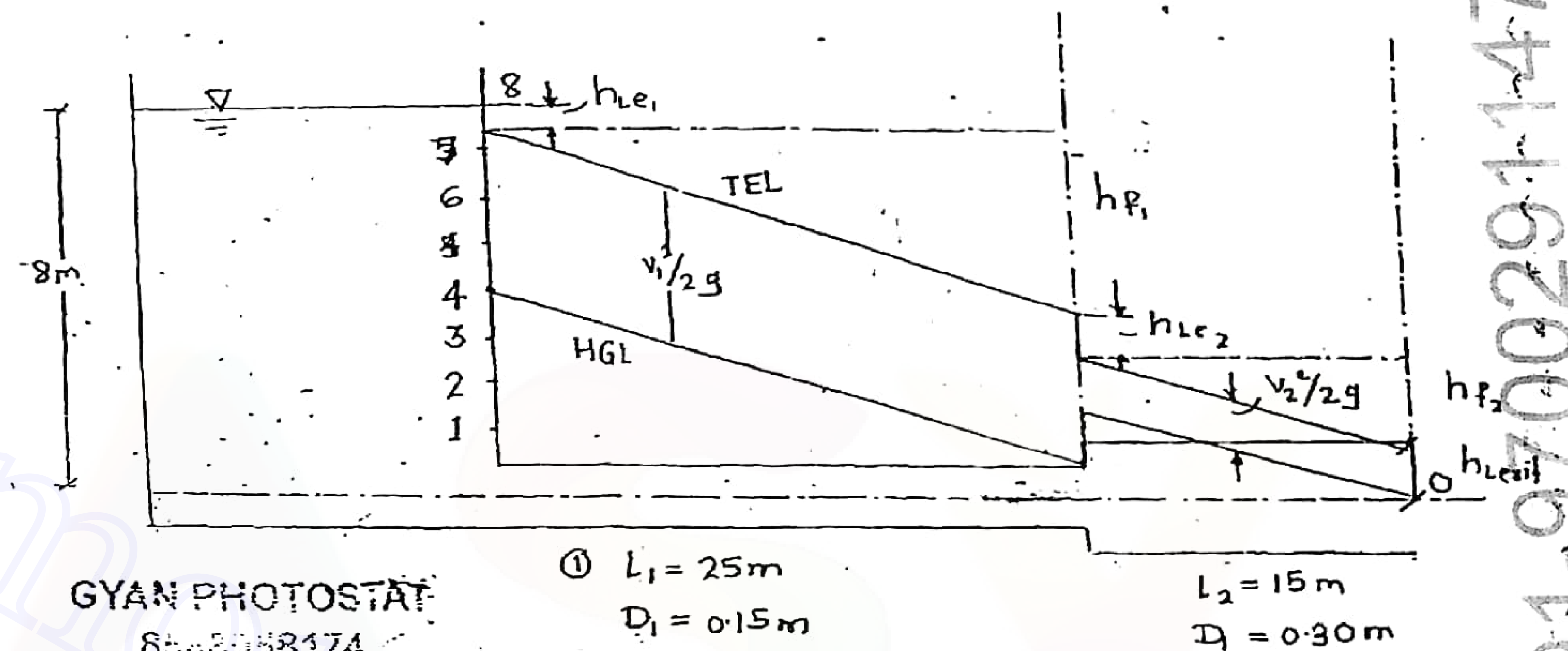
$$HGL = \left(\frac{P}{\rho g} + Z\right)$$

(i) Total energy line always goes down in direction of flow until and unless external energy is supplied in between.

(ii) Hydraulic gradient line (HGL) may go up and down in the direction of flow. (sudden contraction, expansion)

(iii) HGL will always be below total energy line (TEL) and the vertical gap / difference between these two lines at any section represents the value of kinetic head at that section. Thus in the pipe of uniform diameter, hydraulic gradient line (HGL) will always be parallel to Total energy line (TEL)

Q. 4. (Page No: 40)



Considering all losses.

$$8 = \frac{0.5V_1^2}{2g} + \frac{0.04 \times 25 \times V_1^2}{2 \times 9.8 \times 0.15} + \frac{(V_1 - V_2)^2}{2 \times 9.8} + \frac{0.04 \times 15 \times V_2^2}{2 \times 9.8 \times 0.30} + \frac{V_2^2}{2 \times 9.8}$$

$$= \frac{0.5V_1^2}{2 \times 9.8} + \frac{0.04 \times 25 \times V_1^2}{2 \times 9.8 \times 0.15} + \frac{(V_1 - 0.25V_1)^2}{2 \times 9.8} + \frac{0.04 \times 15\ (0.25V_1)^2}{2 \times 9.8 \times 0.30} + \frac{(0.25V_1)^2}{2 \times 9.8}$$

$$8 = 0.413\ V_1^2$$

$$V_1 = 4.40 \text{ m/sec}$$

$$V_2 = 0.25\ V_1$$
$$= 1.1 \text{ m/sec}$$

Head losses in pipes:

(i) $h_{Le_1} = \dfrac{0.5\,V_1^2}{2g} = 0.493$ m

(ii) $h_{f_1} = \dfrac{0.04 \times 25 \times V_1^2}{2g \times 0.15} = 6.57$ m

(iii) $h_{Lc} = \dfrac{(V_1 - V_2)^2}{2g} = 0.555$ m

(iv) $h_{f_2} = \dfrac{0.04 \times 75 \times V_2^2}{2 \times 9.8 \times 0.30} = 0.123$ m

(v) $h_{Lexit} = \dfrac{V_2^2}{2g} = 0.061$ m

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Different connections in pipes:

Head loss due to friction.

$$h_f = \frac{fLV^2}{2gD}$$

$\because Q = AV = \frac{\pi}{4} D^2 V$

$$= \frac{fL}{2gD}\left(\frac{4Q}{\pi D^2}\right)^2 \qquad V = \frac{4Q}{\pi D^2}$$

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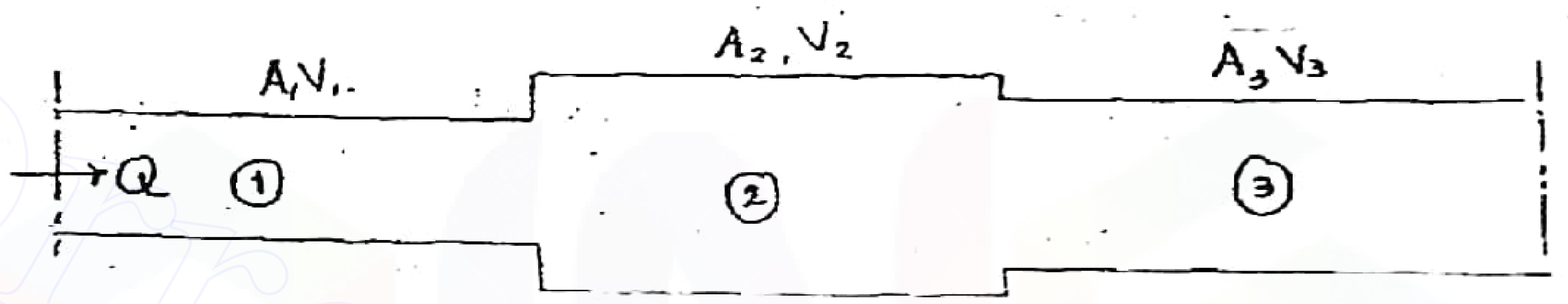
$$h_f = \frac{8fLQ^2}{\pi^2 g D^5}$$

$$h_f = r \cdot Q^2$$

where $r = \dfrac{8fL}{\pi^2 g D^5}$ - pipe property.

(i) Pipes in series connection:



$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

$\therefore$ Total head loss.

$$h_f = h_{f_1} + h_{f_2} + h_{f_3}$$

Equivalent pipe: (L and D)

It is the pipe which gives same discharge at the same head loss.

$$h_f = h_{f_1} + h_{f_2} + h_{f_3} + \ldots$$

$$\frac{8fLQ^2}{\pi^2 g D^5} = \frac{8f_1L_1Q^2}{\pi^2 g D_1^5} + \frac{8f_2L_2Q^2}{\pi^2 g D_2^5} + \ldots$$

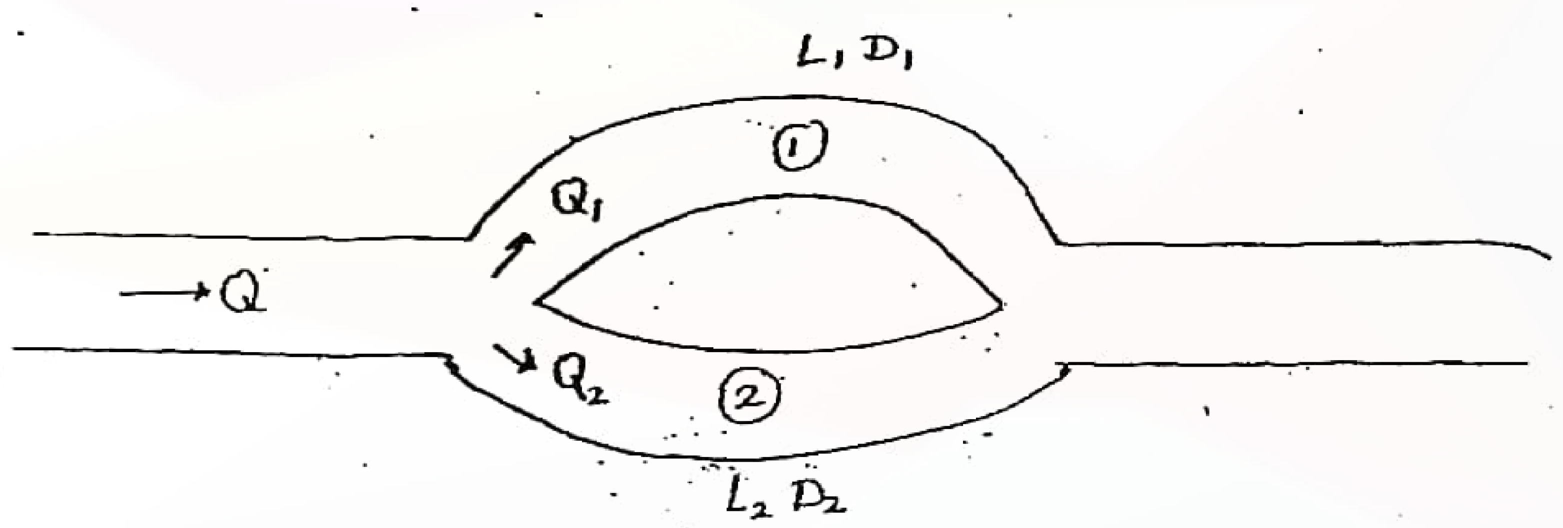
$$\frac{fL}{D^5} = \frac{f_1 L_1^2}{D_1^5} + \frac{f_2 L_2^2}{D_2^5} + \frac{f_3 L_3^2}{D_3^5} + \cdots$$

(Dupuits equation)

If $f_1 = f_2 = f_3 = \cdots = f$

$$\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \cdots$$

(2) Pipes in parallel connection :



Head loss is same.

$$h_{f_1} = h_{f_2}$$

$$\frac{8 f_1 \cdot L_1 \cdot Q_1^2}{\pi^2 \cdot g \cdot D_1^5} = \frac{8 f_2 \cdot L_2 \cdot Q_2^2}{\pi^2 \cdot g \cdot D_2^5}$$

Equivalent pipe : (L and D)

$$Q = Q_1 + Q_2 + Q_3 + \cdots$$

$$h_f = h_{f_1} = h_{f_2} = h_{f_3} = \cdots$$

$$\frac{8 f_1 L_1 Q_1^2}{\pi^2 g D_1^5} = \frac{8 f_2 \cdot L_2 \cdot Q_2^2}{\pi^2 g D_2^5} = \cdots$$

$$\frac{8 f L Q^2}{\pi^2 g D^5} = \frac{8 f_1 L_1 Q_1^2}{\pi^2 g D_1^5} + \frac{8 f_2 \cdot L_2 \cdot Q_2^2}{\pi g D_2^5} + \cdots$$

vp = separation pressure

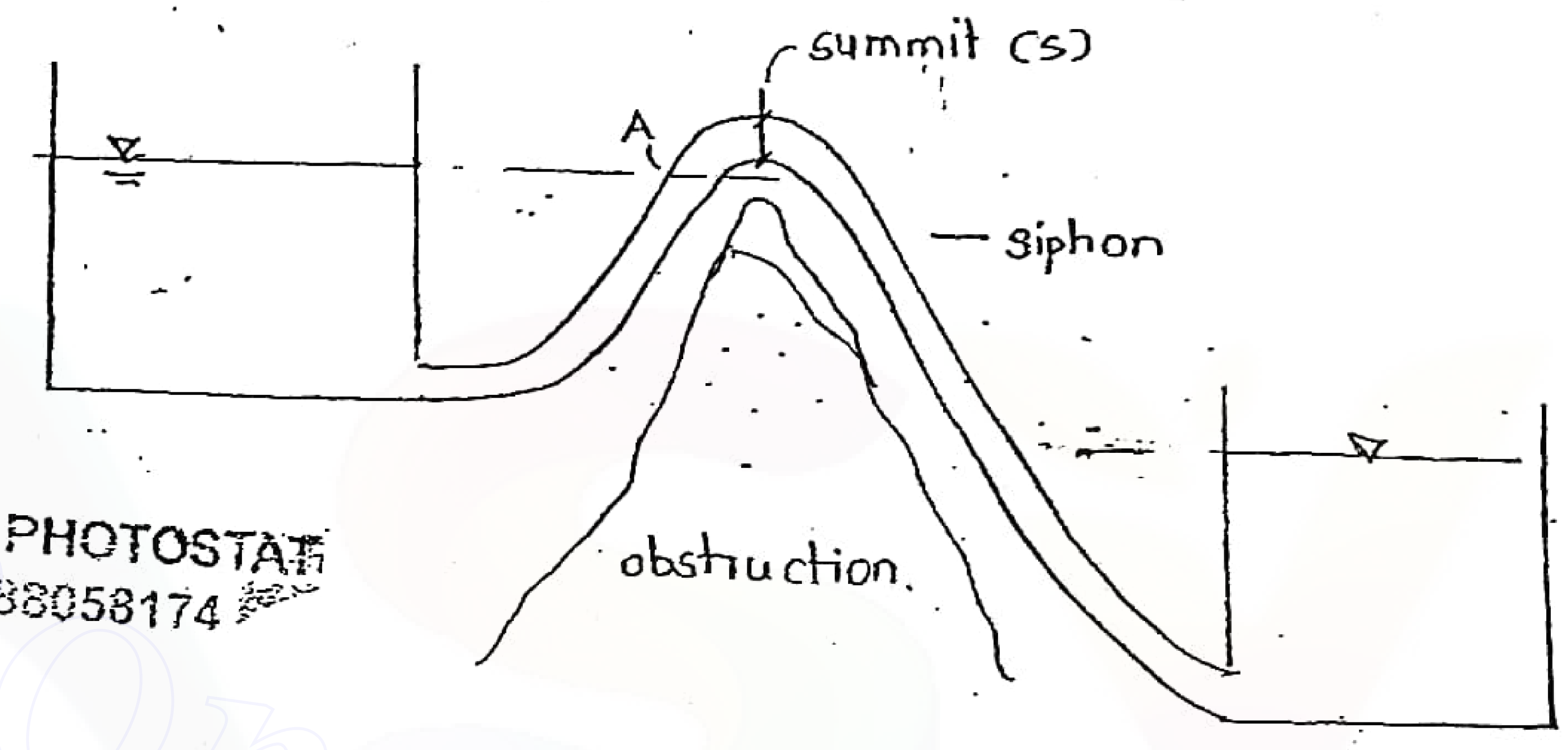
Syphon :

It is a long bent tube which is basically used to transfer the fluid from upper reservoir to lower reservoir.

erosion - surface roughness

vapour pressure should be less than ext liquid pressure

If vp > ext pr DS starts coming out



At summit point,

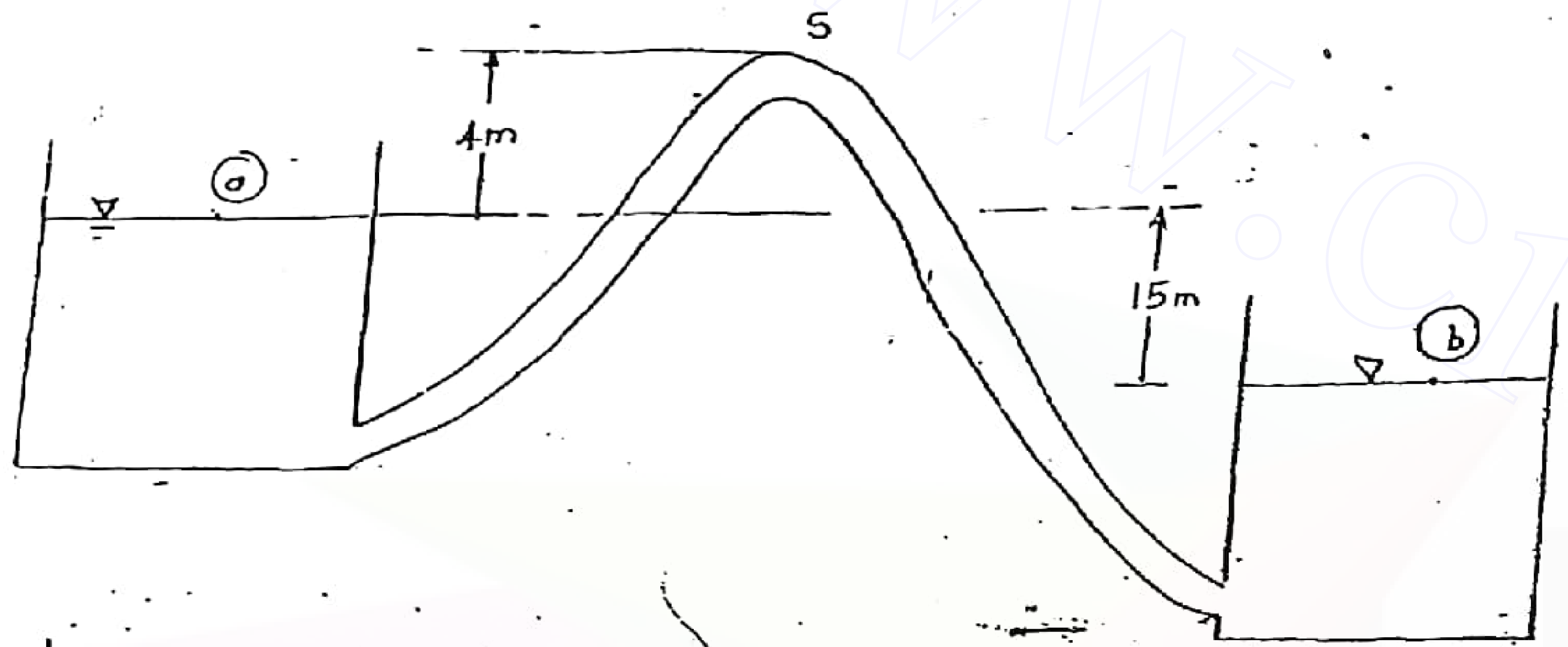
Pressure head $\geq$ Vapour pressure of liquid.

$$\frac{P_s}{\rho g} \geq \frac{P_v}{\rho g}$$

Upto point A in syphon water will rise due to pressure head. but beyond that water needs to be provided extra head for flow to the d/s reservoir. As c/s is same, velocity head cannot be increased. Datum head is already greater for summit point than point A. Thus pressure head at point A is required to be increased or pressure head at summit point can be reduced to make water flow. This pressure at summit point is reduced by creating –ve pressure on lower limb of syphon.

But this pressure head if goes below vapour pressure of water, dissolved gases starts coming out. When the are obstructed, collision of their bubbles occur causing very dangerous phenomenon (Cavitation). It is caused to surface roughness (erosion)

Q 6. (Page 40)



For syphon, $D = 0.2\,m$

$L = 600\ m$

$\left(\frac{P_v}{\rho g}\right)_{water} = 2.8\,m$ – vapour pressure head

$f' = 0.004 \quad \therefore f = 0.016$

$\frac{P_{atm}}{\rho g} = 10.3\ m$

Applying energy equ^n at points ⓐ and ⓑ

$$\frac{P_{atm}}{\rho g} + \cancel{\frac{V_a^2}{2g}}^{\,0} + 15 = \frac{P_{atm}}{\rho g} + \cancel{\frac{V_b^2}{2g}}^{\,0} + 0 + \frac{0.0016 \times 600 \times V^2}{2 \times 9.8 \times (0.2)}$$

(free surface)

$V = 2.476\ m/s$ – velocity through syphon.

Applying energy equ^n between ⓐ and S.

$$\underset{(10.3)}{\frac{P_{atm}}{\rho g}} + \cancel{\frac{V_a^2}{2g}}^{\,0} + 0 = \frac{P_s}{\rho g} + \frac{V^2}{2g} + 4 + \frac{0.0016 \times L \times (2.476)^2}{2 \times 9.8 \times (0.2)^5}$$

(free surface)

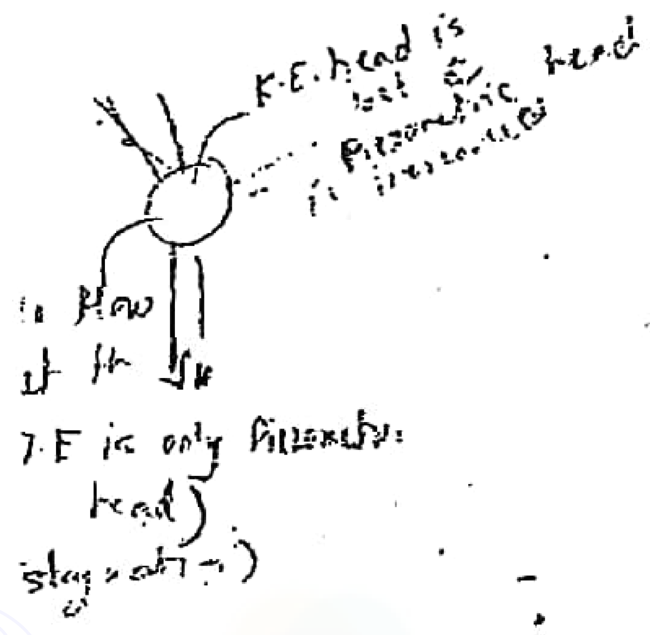
$$\frac{0.0016 \times L_{max} \times (2.476)^2}{2 \times 9.8 \times 0.2} = \left[10.3 - \frac{(2.476)^2}{2 \times 9.8} - 4\right] - \left(\frac{P_s}{\rho g}\right)_{water}$$

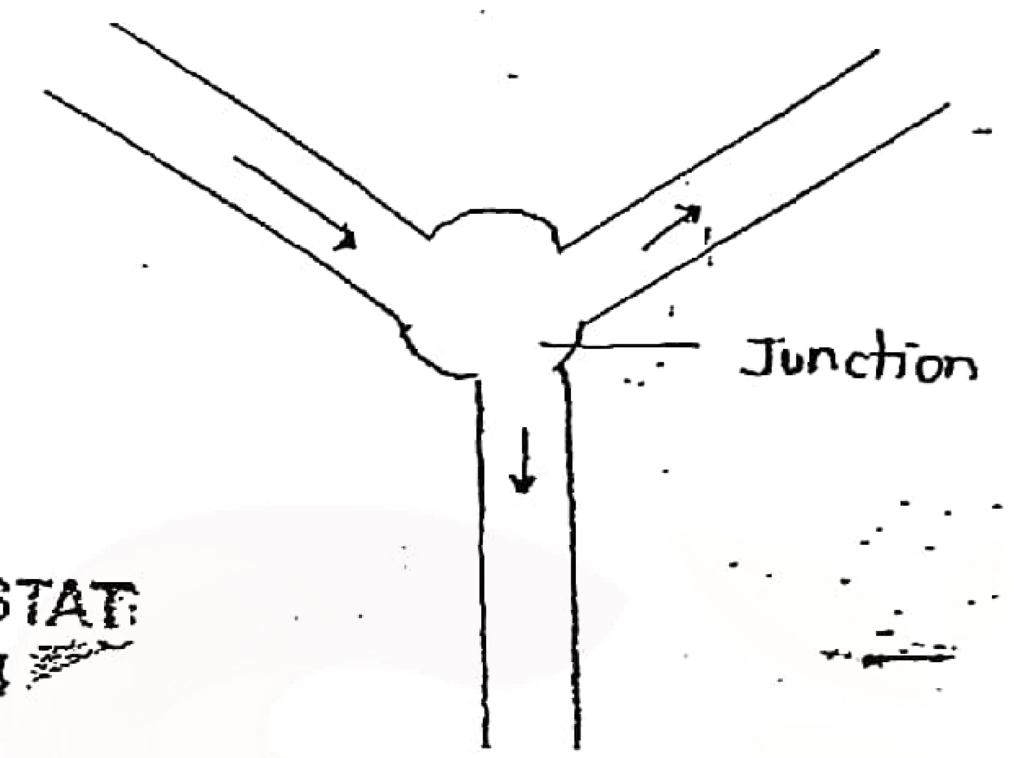
For L to be L_{max}

$\left(\frac{P_s}{\rho g}\right)_{water} = \left(\frac{P_v}{\rho g}\right)$ i.e. vapour pressure

Concept of pipe junction :

Junction is place where the pipes are connected.



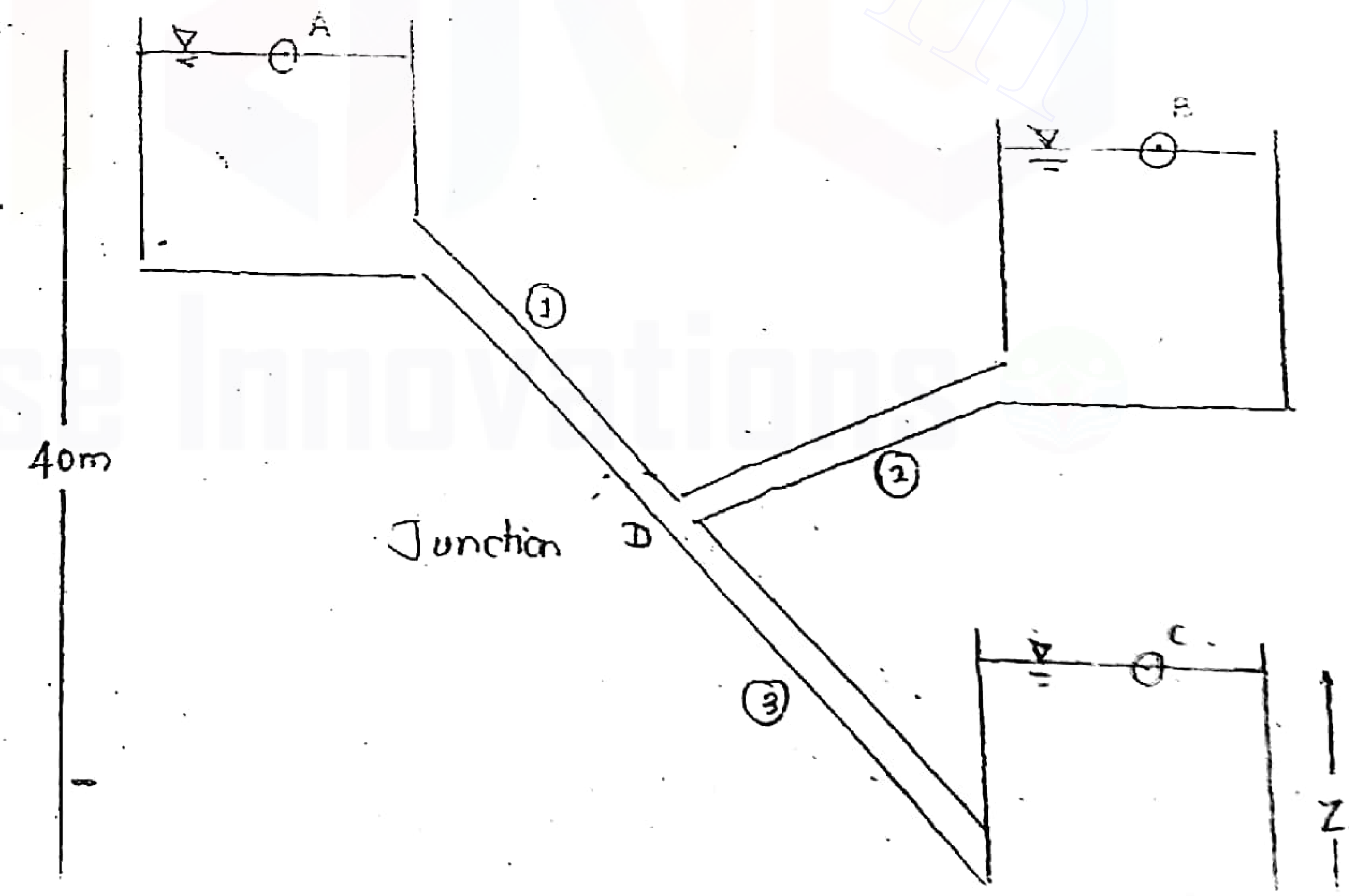


At the junction there is no flow. The K.E. head is lost at the junction (i.e. zero). Thus only piezometric head is present at junction.

T.E. = Piezometric head at junction

There is stagnation at the junction.

Q.7. (Page 40)



$L_1 = 1200$ m　　$L_2 = 600$ m　　$L_3 = 800$ m
$D_1 = 0.30$ m　　$D_2 = 0.2$ m　　$D_3 = 0.3$ m
$Q_1 = 0.06\ m^3/sec$　　$Q_2 = ?$　　$Q_3 = ?$　　& $Z_c = ?$

$f' = 0.006$　　$\therefore f = 0.024$

Points A, B, C are at free surface. K.E. of these points can (head) be neglected.
D is junction. Thus K.E. head is zero.
At A, B, C and D. TE will be datum + P.E. head i.e. only piezometric head.

$H_A = (10.3) + 40 = 50.3$ m
water head (pressure)　　datum

$H_B = (10.3) + 38 = 48.3$ m

$H_c = (10.3) + Z_c$

$\therefore$ frictional loss in pipe ① : $h_{f_1} = H_A - H_D$
$= 50.3 - \cancel{48.3} - H_D$

$H_D = 50.3 - h_{f_1}$

$$H_D = 50.3 - \frac{8 \times 0.024 \times 1200 \times (0.06)^2}{\pi^2 \times 9.8 \times (0.3)^5}$$

$= 46.77$ m

$H_B > H_D$ $\therefore$ Flow in pipe ② is from B to D.

$H_B - H_D = h_{f_2}$ - head loss due to friction in ②

$$48.3 - 46.77 = \frac{8 \times 0.024 \times 600 \times Q_2^2}{\pi^2 \times 9.8 \times (0.2)^5}$$

$Q_2 = 0.02\ m^3/sec$

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$Q_3 = Q_1 + Q_2$
$= 0.06 + 0.0202$
$= 0.0802\ m^3/s$.

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$$h_{f_3} = \frac{8 \times 0.024 \times 800 \times (0.0802)^2}{\pi^2 \times 9.8 \times (0.3)^5} = 4.20 \text{ m}$$

$H_D - H_c = h_{f_3}$

$46.77 - (10.3 + Z_c) = 4.20$

$Z_c = 32.27$ m.

Hardy - Cross Method :
(for pipe networks having many junctions)

Let assumed discharge in pipe $= Q_0$ (initial guess)

ΔQ - correction in discharge.

$\therefore$ New value of discharge,

$Q = (Q_0 + \Delta Q)$

Frictional head loss.

$h_f \propto Q^2$

$= r (Q_0 + \Delta Q)^2$

For a loop.

$\sum_{loop} h_f = 0$

$\sum_{loop} r (Q_0 + \Delta Q)^2 = 0$　　In least square method we minimise the error (ΔQ)

$\sum_{loop} (\cancel{r Q_0^2} + 2 r Q_0 \cdot \Delta Q + \cancel{\Delta Q^2}) = 0$

$\sum_{loop} r Q_0^2 + \sum_{loop} (2 r Q_0 \cdot \Delta Q) = 0$

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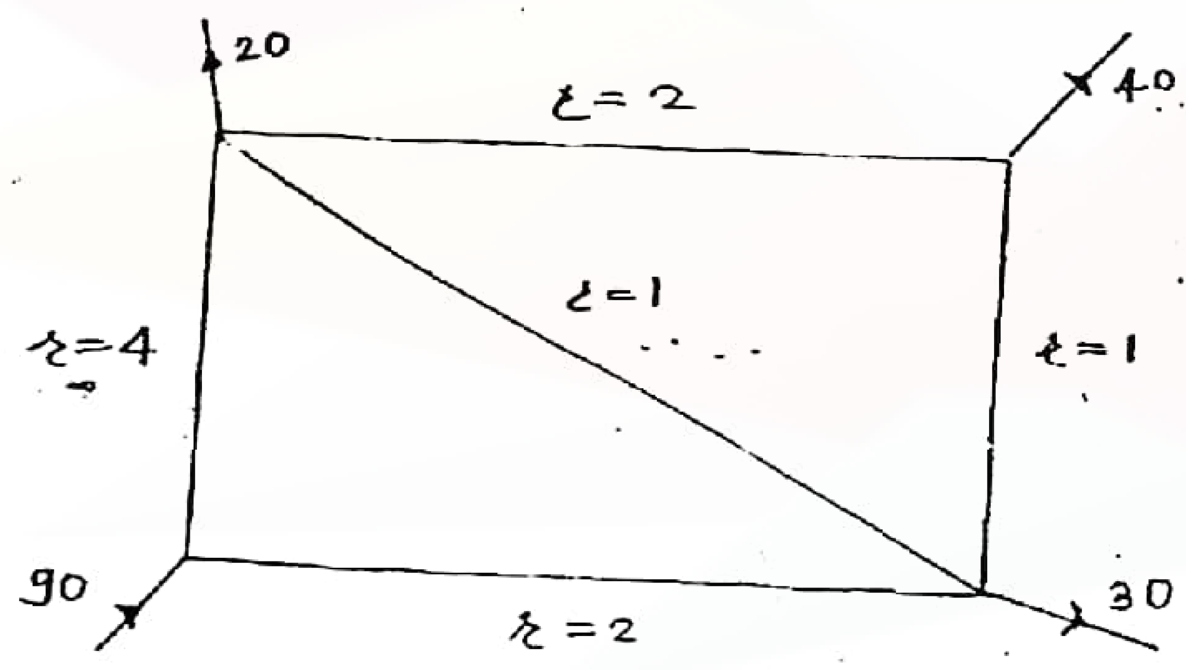
$$\Delta Q \sum_{loop} |2 r Q_o| = -\sum_{loop} r Q_o^2$$

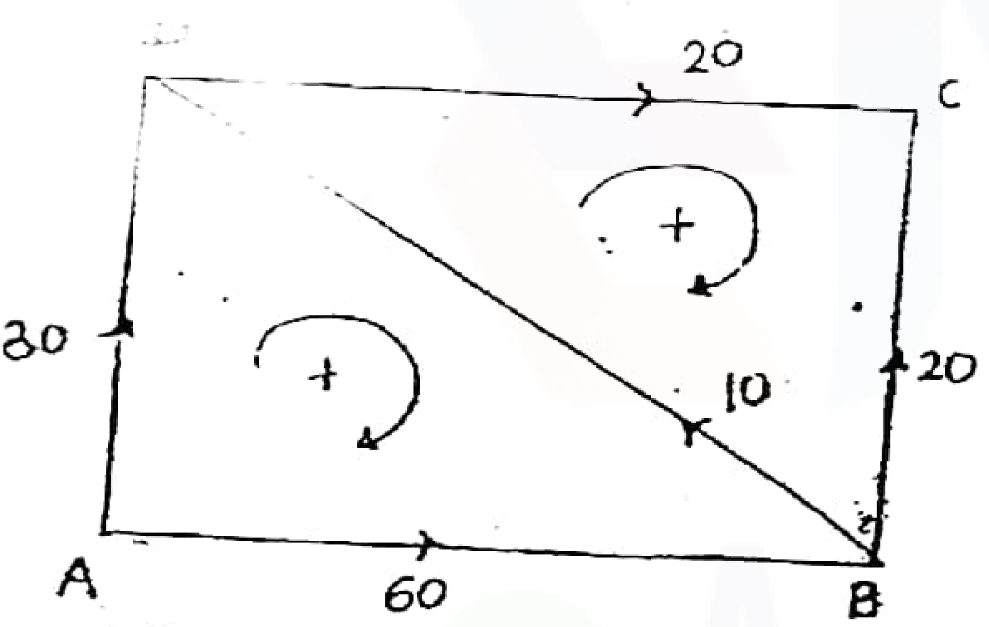
$$\Delta Q = \frac{-\sum_{loop} r \cdot Q_o^2}{\sum_{loop} |2 r Q_o|}$$

Using this ΔQ new value of Q is calculated and used for new Iteration.

Q. Find discharge in every pipe line along with its direction.



Initial guess (Iteration I)



for loop ADBA

Pipe	r	Q_o	$r \cdot Q_o^2$	$\|2 r Q_o\|$
AD	4	30	3600	240
DB	1	10	-100	20
BA	2	60	-7200	240

$$\sum r Q_o^2 = -3700$$

$$\sum |2 r Q_o| = 500$$

$$\Delta Q = \frac{-\sum r Q_o^2}{\sum |2 r Q|} = \frac{-(-3700)}{500}$$

$= 7.4$ clockwise

for loop DCBD.

Pipe	r	Q_o	$r Q_o^2$	$\|2 r Q_o\|$
DC	2	20	800	80
CB	1	20	-400	40
BD	1	10	100	20

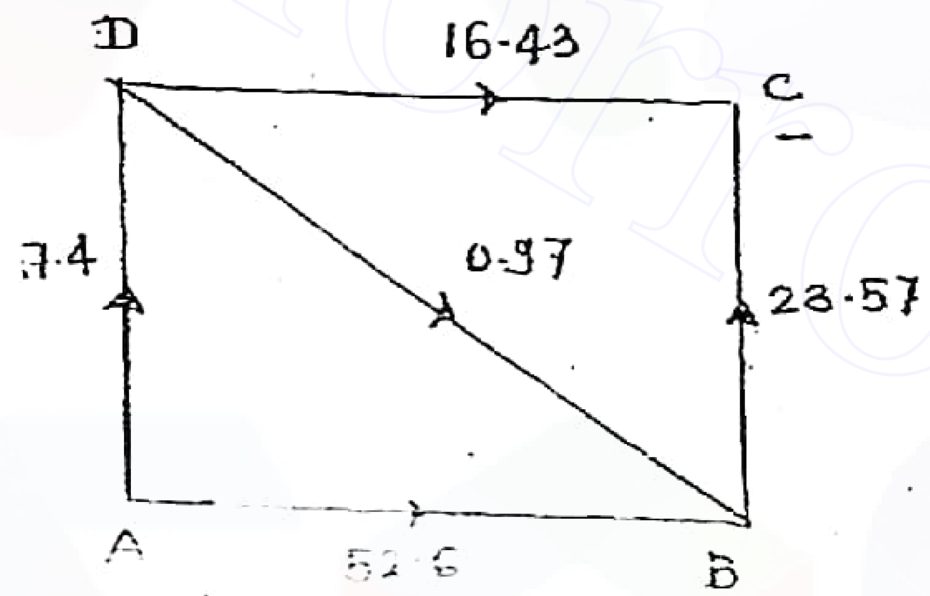
$$\sum r Q_o^2 = 500$$

$$\sum |2 r Q_o| = 140$$

$$\Delta Q = \frac{-500}{140} = -3.57$$

$= 3.57$ anticlock.

Iteration II :



For loop ADBA

Pipe	r	Q_o	$r Q_o^2$	$\|2 r Q_o\|$
AD	4	37.4	5595.04	299.2
DB	1	0.97	0.9409	1.94
BA	2	52.6	-5533.52	210.4

$\Delta Q = -0.127$

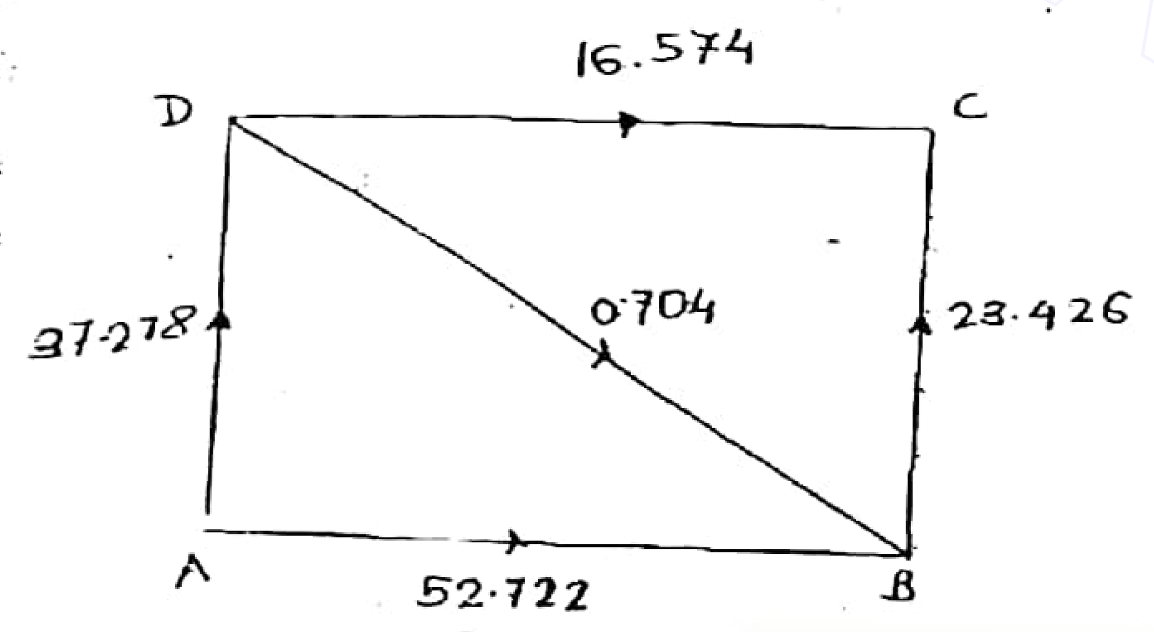
$= 0.127$ (Anticlock)

for loop DCBD

Pipe	r	Q_o	$r Q_o^2$	$\|2 r Q_o\|$
DC	2	16.43	539.88	65.72
CB	1	23.57	-555.54	47.14
BD	1	0.97	-0.9409	1.825

$\Delta Q = 0.144$ (clockwise)

Iteration III



For loop ADBA

Pipe	r	Q_0	$r Q_0^2$	$\lvert 2 r Q_0 \rvert$
AD	4	37.278		
DB	2	0		
BA	1	52.722		

$\Delta Q = 0.00204$ (clockwise)

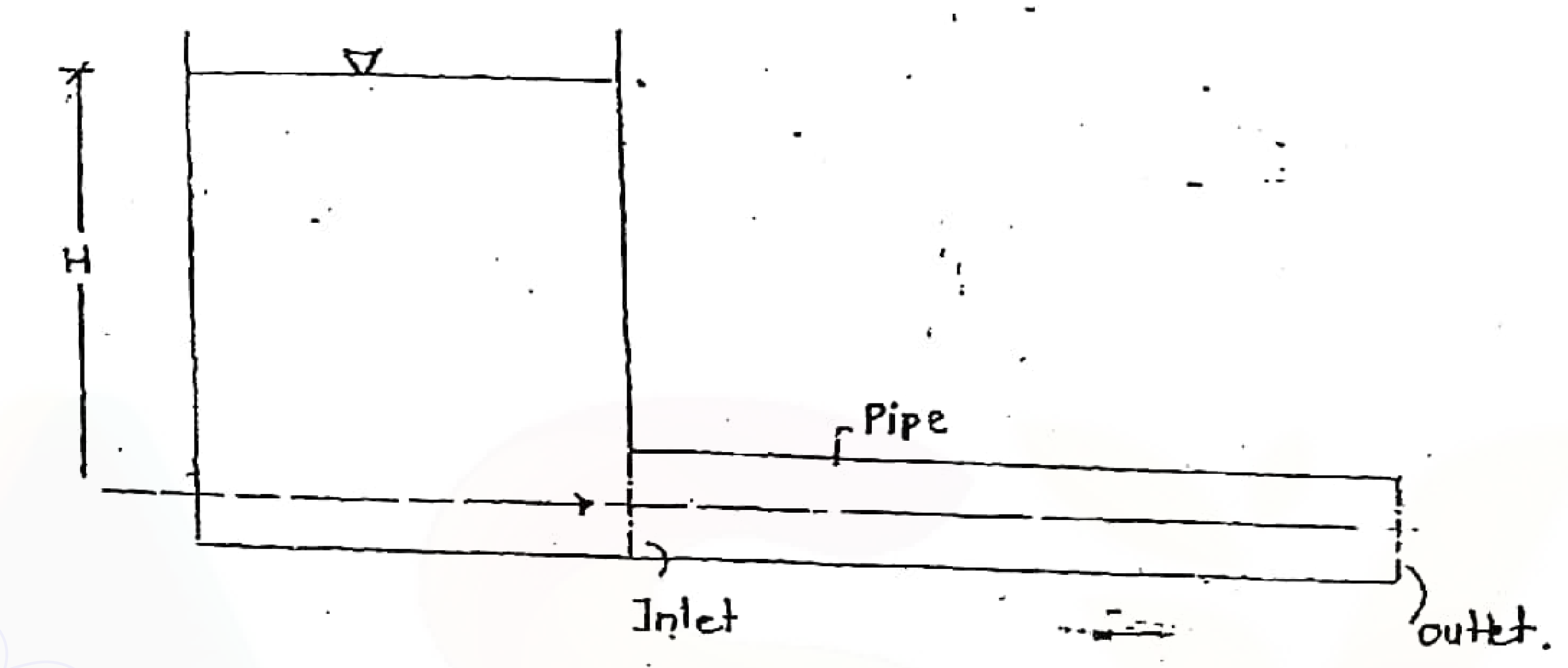
For loop DCBD.

Pipe	r	Q_0	$r Q_0^2$	$\lvert 2 r Q_0 \rvert$
DC	1			
CB	1			
BD	1			

$\Delta Q =$

Pipe	Corrected discharge	direction
AB	AB	
BC	BC	
CD	CD	
DA	DA	
DB	DB	

Power transmission through pipe :



$\therefore$ Power at inlet,

$P_{inlet} = \dot{m} g H$

$= (\rho Q) \cdot g H$

$\because$ mgH – energy

$\frac{mgH}{time}$ – power

$\therefore$ Power at exit.

$P_{outlet} = \rho Q \cdot g (H - h_f)$

$= \rho g Q \left[H - \frac{8 f L Q^2}{\pi^2 g D^5} \right]$

$= \rho g \left[QH - \frac{8 f L Q^3}{\pi^2 g D^5} \right]$

For P_{outlet} to be maximum,

$$\frac{d P_{outlet}}{dQ} = 0$$

$$\rho g \left[QH - \frac{8 f L \cdot 3 Q^2}{\pi^2 g D^5} \right] = 0$$

$\because h_f = \frac{8 f L Q^2}{\pi^2 g D^5}$

$$(H - 3 h_f) = 0$$

$$H = 3 h_f$$

$$h_f = \frac{1}{3} H$$

Efficiency of power transmission.

$$\eta_{transmission} = \frac{P_{output}}{P_{input}}$$

$$= \frac{\rho g Q \,(H - h_f)}{\rho g Q H}$$

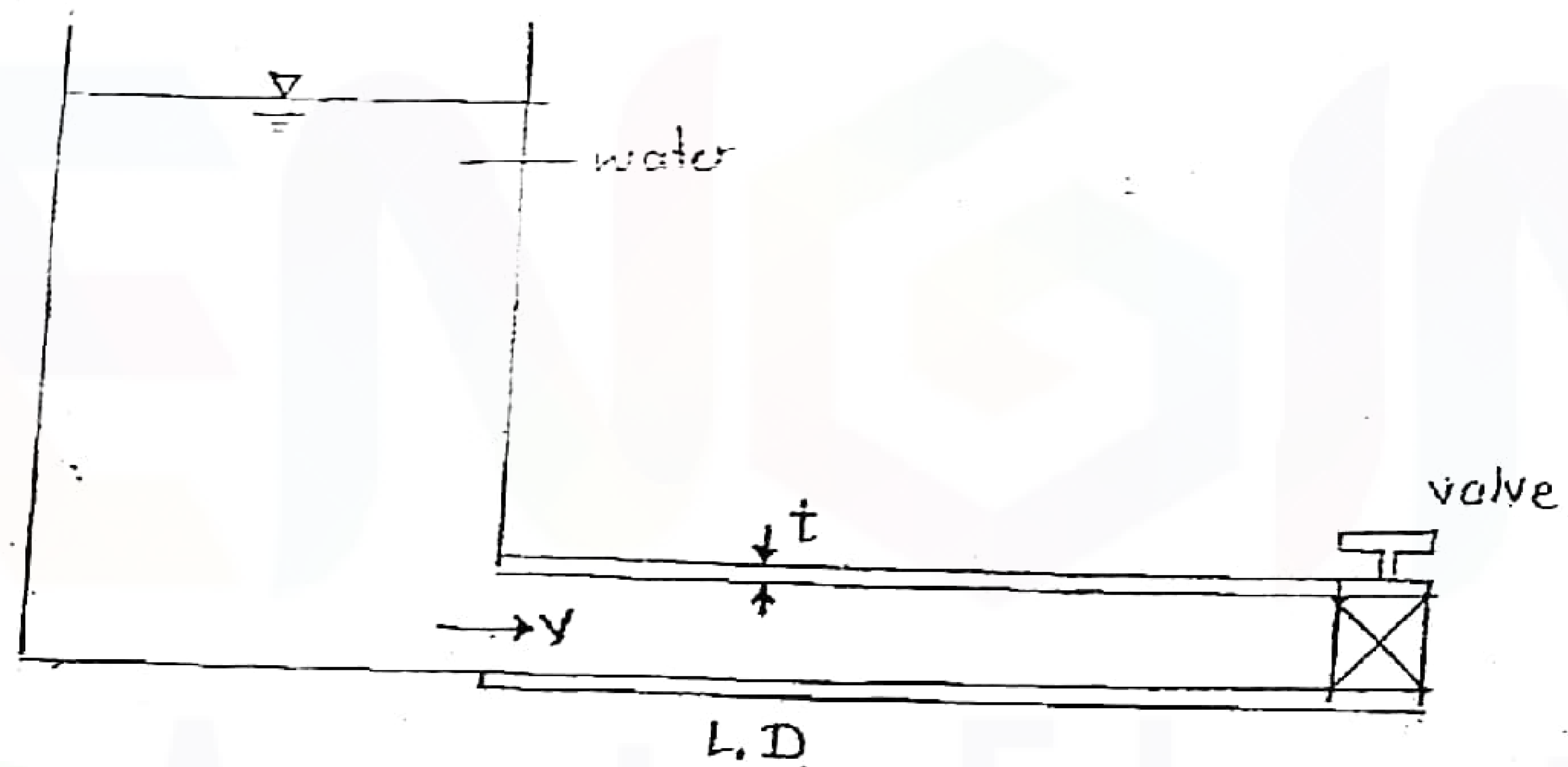
$$= 1 - \frac{h_f}{H}$$

For max. power at exit section

$$\eta_{transmission} = 1 - \frac{(H/3)}{H}$$

$= 0.66$ i.e. $66.66\ \%$

Water hammering effects in pipes :



let

L - length of pipe
D - diameter of pipe
t - thickness of pipe.
V - velocity of water in the pipe.
C - velocity of pressure wave (sound.

$$C = \sqrt{\frac{K}{\rho}}$$ K - bulk modulus of liquid

∵ Since valve is closed suddenly kinetic head at that section is reduced to zero as velocity becomes zero. Thus Pressure head increases.

∵ Pressure is hydrostatic (in all directions) it is transferred to the pipe length.

In one cycle.

Total travel of pressure wave $= L + L = 2L$

∴ The period of pressure wave $= \frac{2L}{C}$

If t_c is time taken in the closure of valve.

If $t_c > \frac{2L}{C}$ - Gradual closure.

(Before closure of valve pressure is already distributed)

If, $t_c < \frac{2L}{C}$ - Sudden closure

(i) Gradual closure of valve in rigid and elastic pipes :

After closure.

$$\text{force developed} = \frac{mV - m \times 0}{t_c}$$

$$F = \frac{mV}{t_c}$$

$$= \frac{(AL) \cdot \rho \cdot V}{t_c}$$

F

$$P = \frac{\rho V L}{t_c}$$ — Pressure rise in pipe.

(i) Sudden closure of valve in rigid pipes.

As pipe is rigid.

K.E. of water before closure = stored in strain energy of water after closure.

$$\frac{1}{2} m V^2 = \left[\frac{1}{2} P \cdot \frac{P}{K}\right](AL)$$

Expt — Energy per unit volum

Since, for any fluid,

$$\text{Volumetric stress} = \frac{\text{Force}}{\text{area}} = \frac{\text{Pressure force}}{\text{area}} = P$$

$$\text{Volumetric strain} = \frac{P}{K}, \quad (K - \text{bulk modulus})$$

$$\frac{1}{2} m V^2 = \frac{1}{2} \frac{P^2}{K} \cdot AL$$

$$\frac{1}{2} (AL \cdot \rho) V^2 = \frac{1}{2} \frac{P^2}{K} AL$$

$$P^2 = \rho K V^2$$

$$P = V\sqrt{\rho K}$$

$$= V\sqrt{\frac{\rho K \cdot \rho}{\rho}}$$

$$P = \rho V C$$ — pressure rise.

(ii) Sudden closure of valve in elastic pipes:

As pipe is elastic.

K.E. of water before closure = strain energy stored in water + strain energy stored in pipe material.

$$\frac{1}{2} m V^2 = \left(\frac{1}{2} \cdot P \cdot \frac{P}{K}\right) \cdot AL +$$

$$\left[\frac{1}{2E}\left(\sigma_L^2 + \sigma_h^2 - \frac{2\sigma_L \cdot \sigma_h}{m}\right)\right] \cdot (\pi D t \cdot l)$$

where,

σ_L - longitudinal stress $= \frac{PD}{4t}$

σ_h - Hoop's stress $= \frac{PD}{2t}$

$\frac{1}{m}$ = poisson's ratio, Take $\left(\frac{1}{m} = \frac{1}{4} = 0.25\right)$

$$\frac{1}{2}\left(\frac{\pi}{4} D^2 \cdot l\right) \cdot \rho V^2 = \frac{1}{2} \frac{P^2}{K} \cdot \left(\frac{\pi}{4} D^2\right) \cdot l + \frac{1}{2E}\left[\frac{P^2 D^2}{16 t^2} + \frac{P^2 D^2}{4 t^2} - \frac{P^2 D^2}{16 t^2}\right] (\pi D t l)$$

$$\frac{1}{2} \frac{\pi}{4} D^2 l \rho V^2 = \frac{1}{2} \frac{P^2}{K} \frac{\pi}{4} D^2 l + \frac{1}{2E} \frac{P^2 D^2}{4 t^2} \pi D t \cdot l$$

$$\rho V^2 = P^2 \left[\frac{1}{K} + \frac{D}{Et}\right]$$

$$P^2 = \frac{\rho V^2}{\left[\frac{1}{K} + \frac{D}{Et}\right]}$$

$$P = V\sqrt{\frac{\rho}{\frac{1}{K} + \frac{D}{Et}}}$$

DIMENSIONAL ANALYSIS

Aim:
(i) Development of functional relationship between different parameters.
(ii) Modelling and similitude studies.

Rayleigh's method :

Q. The time period of simple pendulum depends on length of simple pendulum and acceleration due to gravity. Derive an expression for time period of simple pendulum.

$$T \equiv l \,.\, g$$

Dependant variable — Independant variables

$$T \propto l^a . g^b$$

$$T = C\, l^a . g^b \qquad C - \text{constant}$$

$$= [L^a][M^0 L^1 T^{-2}]^b$$

$$[T] = [M^0 L^{b+a} T^{-2b}]$$

$$a + b = 0$$

$$-2b = 1$$

$$b = -1/2$$

$$a = \frac{1}{2}$$

$$\therefore T = C\sqrt{\frac{l}{g}}$$

By experiments we know that $C = 2\pi$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

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Q. Drag force of sphere in fluid depends on its dia, velocity and density and dynommic viscosity of fluid. Find the expression for drag force on the sphere.

$$F \equiv D,\, V,\, \rho,\, \mu.$$

$$F \propto D^a . V^b . \rho^c . \mu^d$$

$$F = C \cdot D^a . V^b . \rho^c . \mu^d$$

$$[M^1L^1T^{-2}] = [L^1]^a [L^1T^{-1}]^b [M^1L^{-3}]^c [M^1L^{-1}T^{-1}]^d$$

$$[M^1L^1T^{-2}] = [M^{c+d}\, L^{a+b-3c-d}\, T^{-b-d}]$$

$$c + d = 1 \quad \text{— ①} \qquad d = (1-d)$$

$$a + b - 3c - d = 1 \quad \text{— ②}$$

$$-b - d = -2 \quad \text{— ③}$$

$\therefore$ There are three equations and 4 unknowns (D, V, ρ, μ)

$$c = (1-d)$$

$$b = (2-d)$$

from — ②

$$a + (2-d) + 3(1-d) - d = 1$$

$$a = (2-d)$$

$$F = c \cdot D^{2-d} . V^{2-d} . \rho^{1-d} . \mu^d$$

$$= c \cdot (\rho V^2 D^2) . \left(\frac{\mu}{\rho V D}\right)^d$$

$$F = c \cdot (\rho V^2 D^2) . f\left(\frac{\mu}{\rho V D}\right)$$

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Note :

$$F_D = C\,(\rho V^2 D^2)\left(\frac{\mu}{\rho V D}\right)^d$$

Non-dimensional quantity $\left(\frac{1}{Re}\right)$

$$\frac{F_D}{\rho V^2 D^2} = C\cdot\left(\frac{\mu}{\rho V D}\right)^d$$

both are non-dimensional groups – π groups.

$$\pi_1 = C\cdot(\pi_2)^d$$

$$\pi_1 - C(\pi_2)^d = 0$$

$$f(\pi_1, \pi_2) = 0$$ – Buckinghum π approach.

$$\pi_1 = C\cdot f(\pi_2)$$

$$\pi_2 = C\cdot f(\pi_1)$$

Buckinghum's π - method :-

Let total number of variables = m

No. of fundamental variables = n (always ≤ 3)

Geometric variables	Kinematic variable	Dynamic variable
L, D, H ...	V, ω, N, a, g, ν ...	ρ, μ, m ...

No. of non-dimensional group's = (m – n)

i.e. $\pi_1, \pi_2, \pi_3 \ldots \pi_{(m-n)}$

According to Buckinghum - π method,

$$f(\pi_1, \pi_2, \pi_3, \ldots \pi_{m-n}) = 0$$

$$\pi_1 = f(\pi_2, \pi_3, \pi_4 \ldots \pi_{m-n})$$

$$\pi_2 = f(\pi_1, \pi_3, \pi_4 \ldots \pi_{m-n})$$

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Q. Find expression for drag force (F)

F_D, D, V, ρ, μ

No. of variables (total) = 5 = m

Fundamental variables = n = 3 (Take D, V, ρ)

(If 2 Dynamic/kinematic variables are present select only 1 from each type)

∴ No. of π - groups = (5 – 3) = 2.

To find non-dimensional group of (F_D)

$$\pi_1 = (D^a V^b \rho^c).F_D$$

$$[M^0 L^0 T^0] = [L^a][LT^{-1}]^b[ML^{-3}]^c[MLT^{-2}]$$

$$= [M^{c+1} L^{a+b-3c+1} T^{-b+2}]$$

$c + 1 = 0$ ∴ $a = -2$

$a + b - 3c + 1 = 0$ $b = -2$

$-b - 2 = 0$ $c = -1$

$$\pi_1 = \frac{F_D}{\rho V^2 D^2}$$

For non-dimensional group of (μ)

$$\pi_2 = (D^a V^b \rho^c).\mu$$

$$= [M^{c+1} L^{a+b-3c-1} T^{-b-1}]$$

$$\pi_2 = \frac{\mu}{\rho V D}$$

According to Buckinghum's π - theorem :

$$f(\pi_1, \pi_2) = 0$$

$$f\left(\frac{F_D}{\rho V^2 D^2}, \frac{\mu}{\rho V D}\right) = 0$$

$$\frac{F_D}{\rho V^2 D^2} = f\left(\frac{\mu}{\rho V D}\right) \times 0$$

$$F_D = C \cdot (\rho V^2 D^2) \cdot f\left(\frac{\mu}{\rho V D}\right) \quad \text{--- (i)}$$

Selecting fundamental variables - D, V and μ.

$$\pi_1 = (D^a V^b \mu^c)\, F_D$$

$$\pi_1 = \frac{F_D}{\mu V D}$$

$$\pi_2 = (D^a V^b \mu^c)\, \rho$$

$$= \frac{\rho V D}{\mu}$$

According to Buckingham's π theorem.

$$f(\pi_1, \pi_2) = 0$$

$$f\left(\frac{F_D}{\mu V D}, \frac{\rho V D}{\mu}\right) = 0$$

$$\frac{F_D}{\mu V D} \quad f\left(\frac{\rho V D}{\mu}\right)$$

$$F_D = C \cdot (\mu V D)\, f\left(\frac{\rho V D}{\mu}\right) \quad \text{--- (ii)}$$

$$= C\,(\mu V D)\, \frac{f\left(\frac{\rho V D}{\mu}\right)}{\left(\frac{\rho V D}{\mu}\right)}$$

$$F_D = C\,(\mu V D)\, f\left(\frac{\mu}{\rho V D}\right)$$

Different types of forces in fluid flow system:

If 'L' is the characteristic dimension of the system of flow

Inertial force.

$$F_i = m \times a$$

$$= \rho \cdot L^3 \cdot \left(\frac{V}{t}\right)$$

$$= \rho \cdot L^2 \left(\frac{L}{t}\right) \cdot V$$

$$= \rho L^2 V^2 \qquad \because \left(\frac{L}{t}\right) = V$$

This inertia force is to be balanced by the other forces in the flow.

(i) Viscous force (F_v)

$$F_v = \underbrace{\mu \left(\frac{V}{L}\right)}_{\text{stress}} \cdot \underbrace{A}_{\text{scale}}$$

$$= \mu \cdot \frac{V}{L} \cdot L^2$$

$$= \mu V \cdot L$$

(ii) Pressure force (F_p)

$$F_p = \Delta p \cdot A$$

$$= \Delta p \cdot L^2$$

(iii) Gravity force (F_g)

$$F_g = mg$$

$$= (\rho \cdot L^3) \cdot g$$

$$= \rho L^3 g$$

(iv) Surface tension force : (F_T)

$$F_T = \sigma L$$

(v) Compressibility force (Elastic force) (F_E) :

$$F_e = K \cdot A \quad \because K - \text{bulk modulus}$$

$$= \frac{\text{vol. stress}}{\text{vol. strain}}$$

stress

$$= K L^2$$

dimensionless

Non-dimensional numbers in fluid flow system :

1. Reynold's Number:

It is defined as ratio of inertia forces to viscous forces. This number plays very important role in those areas where viscosity effects are predominant.

e.g. Boundary layer flows like pipe flows.

$$Re = \frac{F_i}{F_v}$$

$$= \frac{\rho V^2 L^2}{\mu V L}$$

$$Re = \frac{\rho V L}{\mu}$$

2. Euler's Number : (E_u)

It is defined as square root of ratio of inertia force to pressure force. This number becomes highly significant in those areas where pressure forces are severly dominant.

e.g. pipe flows (pressure energy used to increase total head)

$$E_u = \sqrt{\frac{F_i}{F_p}}$$

$$= \sqrt{\frac{\rho V^2 L^2}{\Delta p \cdot L^2}}$$

$$E_u = \frac{V}{\sqrt{\Delta P/\rho}}$$

Note:

$$\because E_u = \frac{\rho V^2}{\Delta P}$$

$$E_u = \frac{1/2 \, \rho V^2}{\Delta P}$$

(Sir Sengel) (defined as ratio of energy per unit volume)

$$E_u = \frac{\sqrt{\Delta P/\rho}}{V}$$

(David Fox)

3. Froude's Number:

It is defined as the square root of the ratio of inertia force to gravity force. It becomes important in those regions where gravity forces are predominant.

e.g. open channel flows, flow through spillways, rivers, canals

$$F_r = \sqrt{\frac{F_i}{F_g}}$$

$$= \sqrt{\frac{\rho V^2 L^2}{\rho L^3 \cdot g}}$$

$$F_r = \frac{V}{\sqrt{gL}}$$

4. Weber Number:

It is defined as square root of the ratio of inertia force to surface tension force. It becomes important in capillary flows.

$$W_b = \sqrt{\frac{F_i}{F_T}}$$

$$= \sqrt{\frac{\rho L^2 V^2}{\sigma L}}$$

$$= \frac{V}{\sqrt{\sigma/\rho L}}$$

5. Mach Number:

It is defined as square root of the ratio of inertia force to compressibility force (elastic force). This number becomes important in compressible flow system.

$$M_a = \sqrt{\frac{F_i}{F_e}}$$

$$= \sqrt{\frac{\rho L^2 V^2}{K \cdot L^2}}$$

$$= \frac{V}{\sqrt{K/\rho}}$$

$$= \frac{V}{C}$$

$\sqrt{\frac{K}{\rho}} = C$ - velocity of sound

Modelling and Similitude:

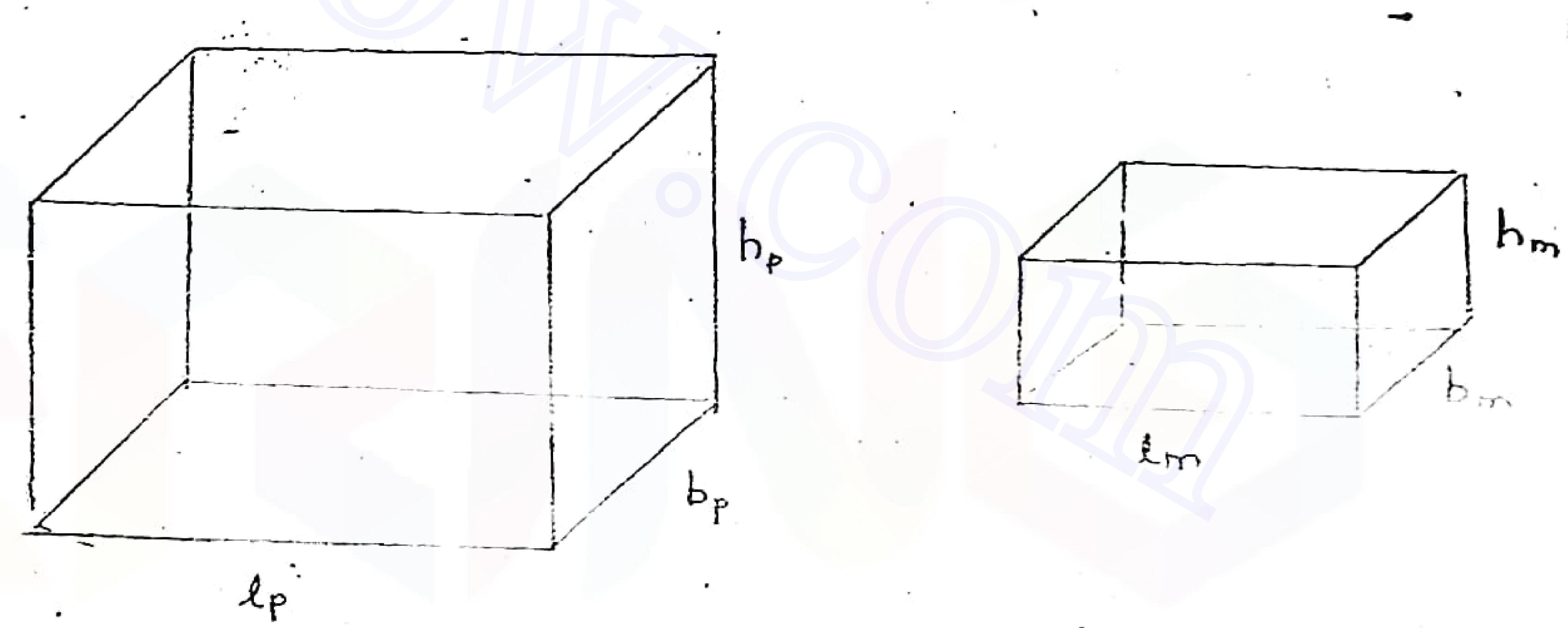
It is branch of science which deals with the studies of prototypes by doing experiments on their models.

For this purpose to be fulfilled, there should be existance of very good similarity between the models and their prototypes.

Different types of similarities between Models and their prototype:

(i) Geometric similarity:

The models and the prototypes are said to be geometrically similar if every dimension in prototype is reduced to same scale in their models.



Prototype (P) (Model (m))

$$\frac{l_M}{l_P} = \frac{b_m}{b_p} = \frac{h_m}{h_p} = \cdots\cdots = L_r$$

It is called 'Length scale ratio' or scale ratio.

(ii) Kinematic similarity :

— It is similarity of velocity and acceleration between the model and its prototype.

$$\frac{(V_1)_m}{(V_1)_p} = \frac{(V_2)_m}{(V_2)_p} = \cdots\cdots \frac{V_m}{V_p} - \text{constant} = V_r$$

It is called velocity scale ratio

$$V_r = \frac{L_r}{t_r}$$

$$\frac{(a_1)_m}{(a_1)_p} = \frac{(a_2)_m}{(a_2)_p} = \cdots\cdots \frac{a_m}{a_p} = a_r \quad - \text{accl}^n \text{ scale ratio}$$

$$a_r = \frac{V_r}{t_r} = \frac{L_r}{t_r^2}$$

(iii) Dynamic similarity :

It is the similarity of forces between the models and prototype.

$$\frac{(F_v)_m}{(F_v)_p} = \frac{(F_p)_m}{(F_p)_p} = \frac{(F_g)_m}{(F_g)_p} = \frac{(F_e)_m}{(F_e)_p}$$

Different Model laws :

Every model is constructed on the basis of different model laws. There are in general 5 model laws. (similarity of non-dimensional number.)

(i) Reynold's model law :

$$(Re)_m = (Re)_p$$

(ii) Euler's model law :

$$(Eu)_m = (Eu)_p$$

(iii) Froude's Model law :

$$(Fr)_m = (Fr)_p$$

(iv) Weber Model law :

$$(Wb)_m = (Wb)_p$$

(v) Mach Model law :

$$(Ma)_m = (Ma)_p$$

Monday
2nd December 2013

Q. A model is constructed on the basis of Reynold's model law. Calculate velocity scale ratio, discharge scale ratio and [illegible] in terms of length scale ratio and kinematic viscosity scale ratio.

— According to Reynold's model law, —

$$(Re)_m = (Re)_p$$

$$\frac{V_m \cdot L_m}{\nu_m} = \frac{V_p \cdot L_p}{\nu_p}$$

$$V_r = \frac{V_m}{V_p} = \left(\frac{\nu_m}{\nu_p}\right)\left(\frac{L_p}{L_m}\right)$$

$$= \frac{\nu_r}{L_r}$$

$$Q_r = A_r \cdot V_r$$

$$= L_r^2 \cdot \frac{\nu_r}{L_r}$$

$$Q_r = L_r \cdot \nu_r$$

Q. If the Reynold's model law and Froude model law both are applicable, find velocity scale ratio, discharge scale ratio and kinematic viscosity scale ratio in terms of length scale ratio.

According to Froude's model law,

$$(F_r)_M = (F_r)_P$$

$$\frac{V_M}{\sqrt{g L_M}} = \frac{V_P}{\sqrt{g L_P}}$$

$$V_r = \sqrt{\frac{L_M}{L_P}}$$

$$V_r = \sqrt{L_r}$$

$\because$ Both the model laws are applicable.

$$\sqrt{L_r} = \frac{\nu_r}{L_r}$$

$$\nu_r = (L_r)^{3/2}$$

and

$$Q_r = A_r \cdot V_r$$

$$= L_r^2 \sqrt{L_r}$$

$$= (L_r)^{5/2}$$

Q. 9 (Page 45):

15 Marks

Prototype (ship in water)	Model (in air)
$L_P = 300$ m	$L_r = \frac{1}{100}$
$\rho_P = 1030$ kg/m³	$V_m = 30$ m/s
$V_P = ?$	$(F_D)_M = 60$ N
$(F_D)_P = ?$	$\rho_M = 1.24$ kg/m³
$\nu_P = 0.012 \times 10^{-4}$ m²/s	$\nu_M = 0.018 \times 10^{-4}$ m²/s

Reynold's model law,

$$(R_e)_M = (R_e)_P$$

$$\frac{V_M \cdot L_M}{\nu_M} = \frac{V_P \cdot L_P}{\nu_P}$$

$$V_r = \frac{\nu_r}{L_r}$$

$$= \frac{0.018}{0.012} \times 100 = 150$$

$$\frac{V_M}{V_P} = 150$$

$$V_P = \frac{30}{150} = 0.2 \text{ m/sec}$$

$$(F_D)_r = \mu_r \cdot V_r \cdot L_r$$

$$= \rho_r \cdot \nu_r \cdot V_r \cdot L_r$$

$$= \frac{1.24}{1030} \times \frac{0.018}{0.012} \times \left(\frac{30}{0.2}\right)\left(\frac{1}{100}\right)$$

$$= 2.708 \times 10^{-3}$$

$$\frac{(F_D)_M}{(F_D)_P} = 2.708 \times 10^{-3}$$

$$(F_D)_P = \frac{60}{2.708 \times 10^{-3}}$$

$$= 22150.54 \text{ N}.$$

Q. 11 (Page 45)

25 Marks

Prototype (water)	Model (air)
$L_P = 1.5$ m	$L_m = 0.3$ m
$V_P = 3.5$ m/s	$V_M = 25$ m/sec
$(F_D)_P = ?$	$(\rho_{air})_M = ?$
$(\mu_w) = 1.0 \times 10^{-3}$ Pa·sec	$(F_D)_M = 10$ N

∴ ρ_{water}, μ_{air} and μ_{water} are not affected by pressure their values can be taken as it is.

but for ρ_{air} (which highly depends on pressure) actual ρ_{air} will be different.

∵ Reynold's model law.

$$(R_e)_m = (R_e)_p$$

$$\frac{\rho_m \cdot V_m \cdot L_m}{\mu_m} = \frac{\rho_p \cdot V_p \cdot L_p}{\mu_p}$$

$$\rho_r = \frac{\mu_r}{V_r \cdot L_r}$$

$$= \frac{1.90 \times 10^{-5}}{1.0 \times 10^{-3}} \times \left(\frac{3.5}{35}\right) \times \frac{15}{0.3}$$

$$\frac{\rho_m}{\rho_p} = 9.5 \times 10^{-3}$$

$$\rho_m = 9.5 \times 10^{-3} \times 998 \text{ kg/m}^3$$

$$= 9.48 \text{ kg/m}^3$$

Ideal gas equation

$$P = \rho R T$$

$$\frac{P_1}{P_2} = \frac{\rho_1}{\rho_2}$$

$$\frac{1 \text{ atm}}{(P_{air})_m} = \frac{1.17}{9.48}$$

$$(P_{air})_m = 8.103 \text{ atm}$$

$$(F_D)_r = \mu_r \cdot V_r \cdot L_r$$

$$= \frac{1.9 \times 10^{-5}}{1.0 \times 10^{-3}} \times \frac{35}{3.5} \times \frac{6.30}{1.5}$$

$$= 0.038$$

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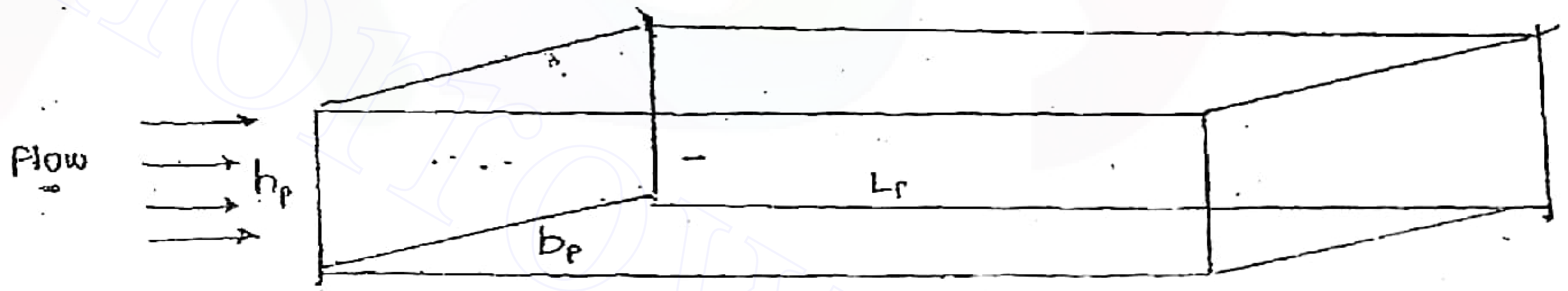
Concept of distorted models:

Sometimes in flow systems likes rivers, canals and spillways, as compared to vertical dimension horizontal dimension is very very large. Therefore different scale ratios are selected for the horizontal dimension and vertical dimension. Thus, the models having different scale ratios are known as distorted models. These models are not geometrically similar with the prototypes.

These models are made on the basis of Froude's model law (gravity force is dominent).



Prototype (p)

Here, $h_p << < b_p$ and L_p

$(L_r)_H$ - scale ratio in horizontal direction

$$(L_r)_H = \frac{L_m}{L_p} = \frac{b_m}{b_p}$$

$(L_r)_V$ - scale ratio in vertical direction

$$(L_r)_V = \frac{h_m}{h_p}$$

(i) Area scale ratio:

$$A_r = \frac{A_m}{A_p} = \frac{h_m \cdot b_m}{h_p \cdot b_p}$$

$$A_r = (L_r)_V \cdot (L_r)_H$$

(ii) Velocity scale ratio :

$$(F_R)_M = (F_R)_P$$

$$\frac{V_M}{\sqrt{g \cdot h_M}} = \frac{V_P}{\sqrt{g\, h_P}}$$

h - characteristic dimension

$$V_r = \sqrt{(L_r)_v}$$

(iii) Discharge scale ratio

$$Q_r = \frac{Q_M}{Q_P}$$

$$= \frac{A_M \cdot V_M}{A_P \cdot V_P}$$

$$= A_r \cdot V_r$$

$$= (L_r)_H \cdot (L_r)_v \cdot \sqrt{(L_r)_v}$$

$$Q_r = (L_r)_H \cdot (L_r)_v^{3/2}$$

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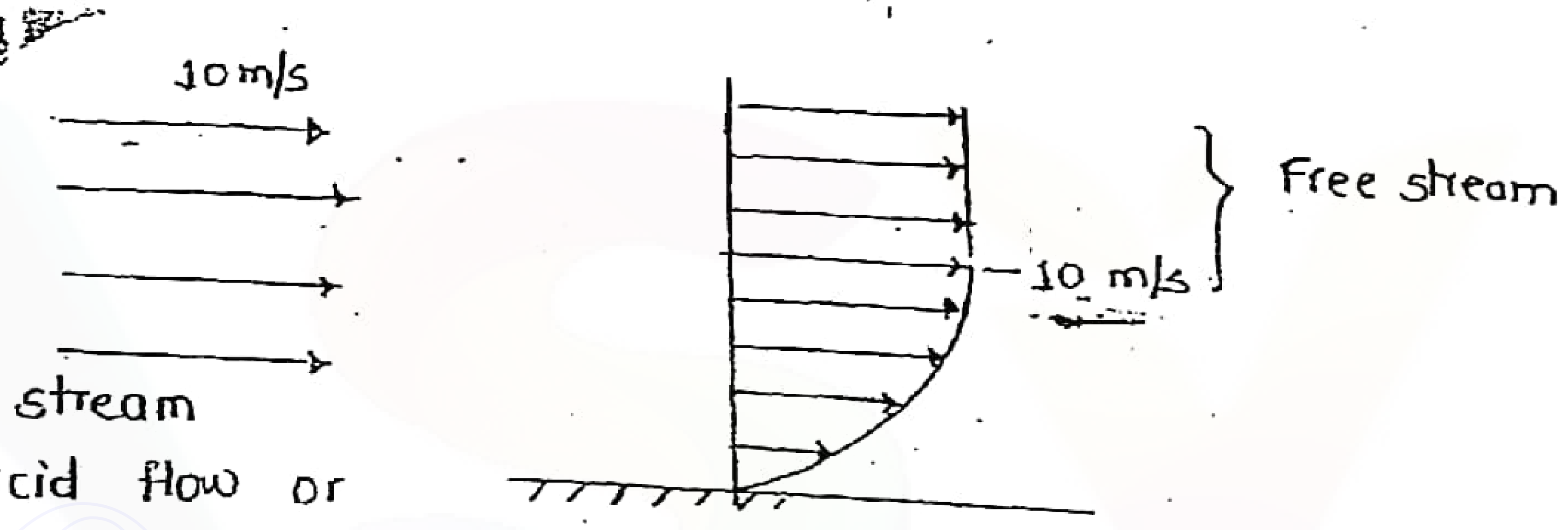
FLOW PAST SUBMERGED BODIES

The aim is when fluid is flowing over some body how much force is exerted by the fluid on the body ?

GYAN PHOTOSTAT
8588058174

$U_\infty \rightarrow$ velocity of free ~~surface~~ stream w.r.t. body

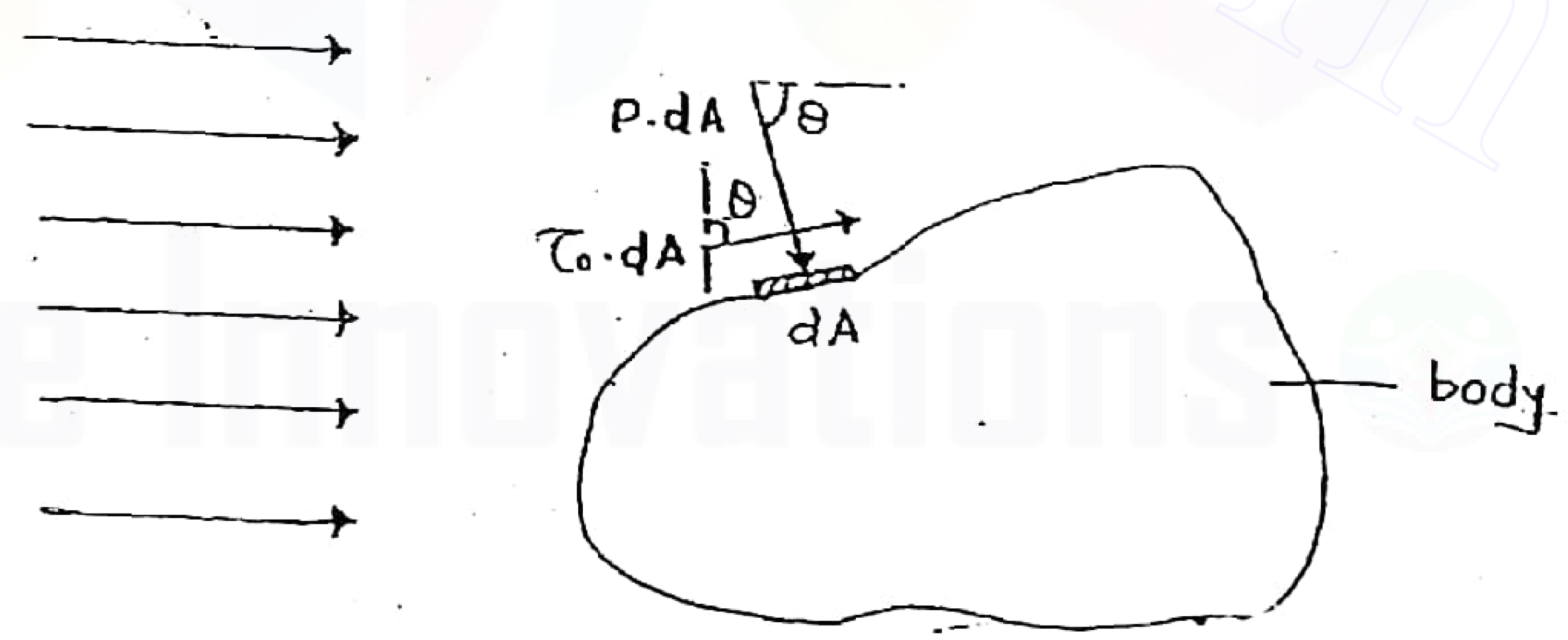


Free stream
(Inviscid flow or
shear stress is zero)

Force exerted by fluid flow on the body

In the direction of flow (Drag force)

In the transverse direction of flow (Lift force)



Total drag force,

$$F_D = \oint P \cdot dA \cdot \cos\theta + \oint \tau_o \cdot dA \cdot \sin\theta$$

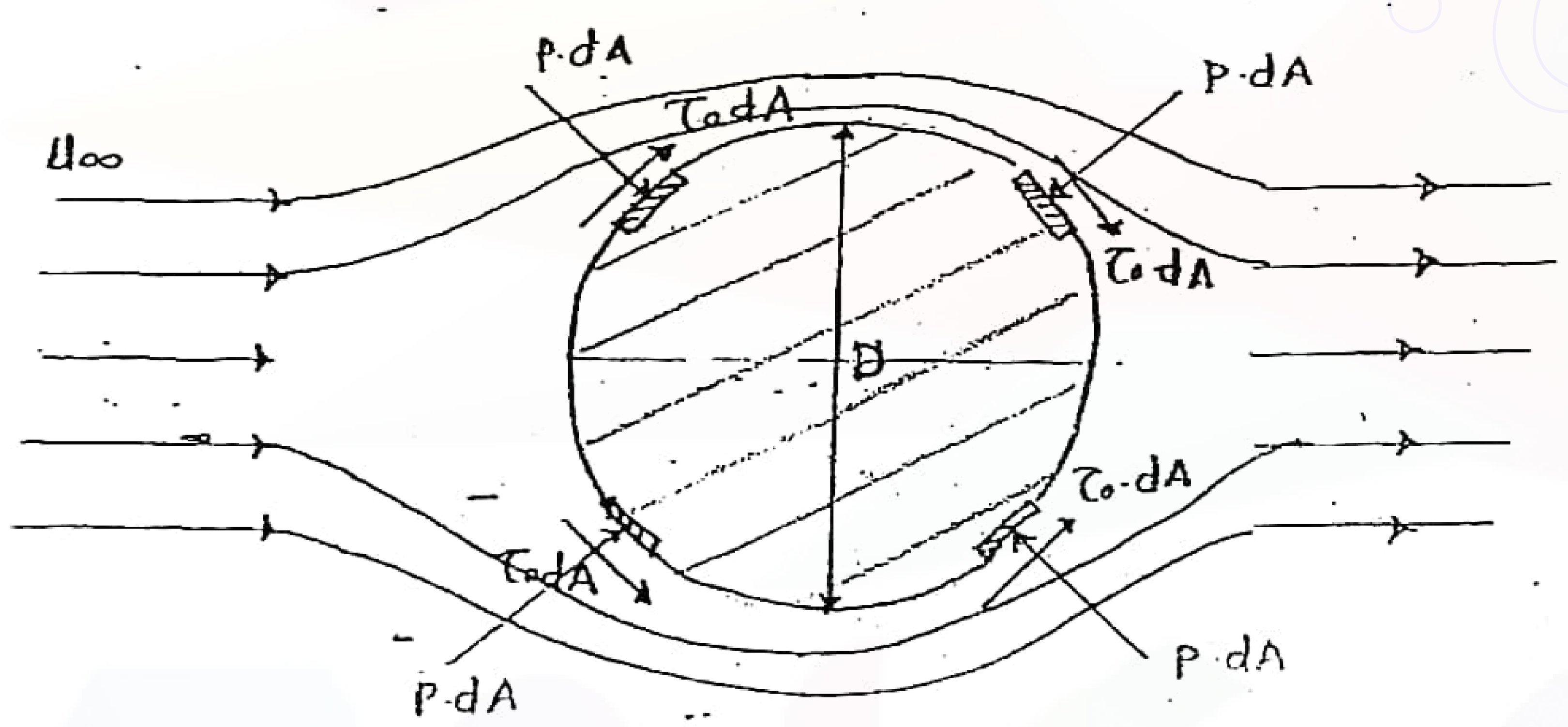
+91-9700291147

Total Lift drag.

$$F_L = \oint P \cdot dA \cdot \sin\theta + \oint \tau_0 \cdot dA \cdot \cos\theta$$

The skin-friction drag on any body can never be zero. It can only be zero in the case of zero shear resistance by the body (smooth body having no No slip condition)

Flow over circular cylinder :



Plan.

Pressure drag = 0
shear friction drag ≠ 0] Total drag ≠ 0 (F_D)

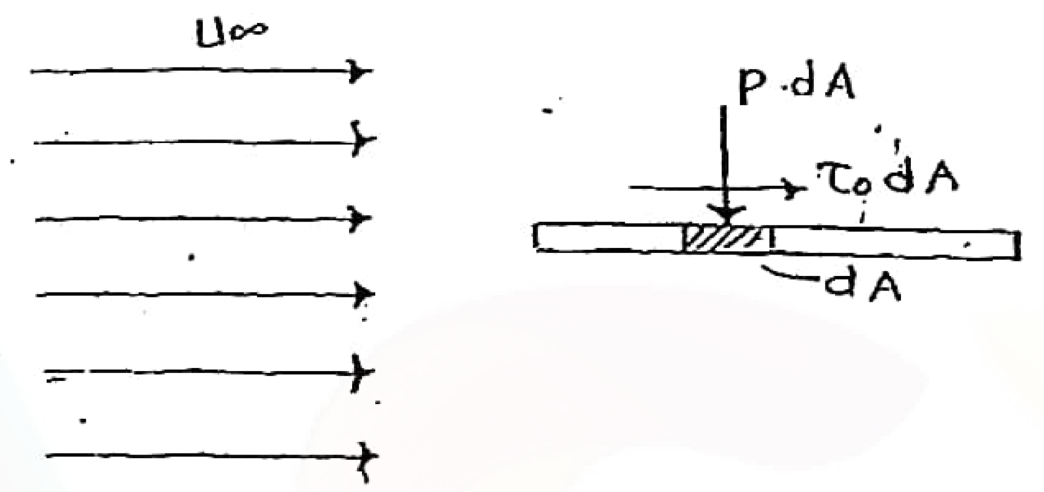
Pressure lift = 0
skin-friction lift = 0] Total lift = 0 (F_L)

There can be a drag without lift but there can not be a lift without drag.

(i) Streamline Bodies :

The bodies whose shape coincides with streamlines are known as Streamline bodies.

(TT Mayor Material of negligible shear resistance - Swim suit)

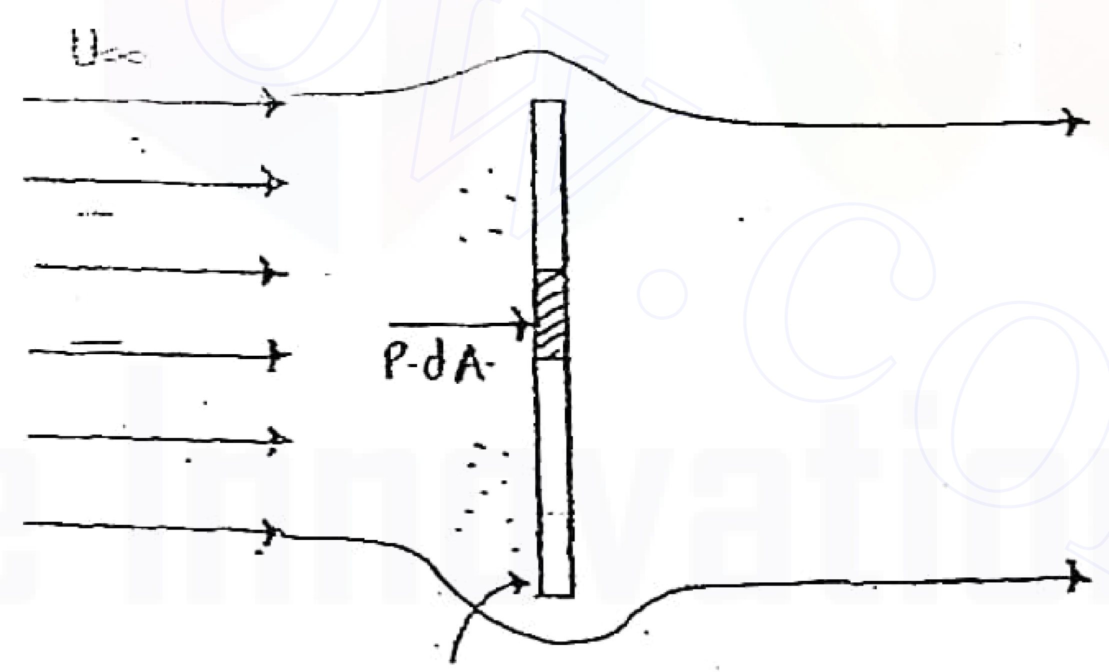


For streamline bodies, pressure drag is minimum. For the perfectly streamline -100% coinciding with streamline, the pressure drag is zero.

The shear friction drag can be minimised by the material of less coefficient of viscosity but pressure drag can only be reduced by making streamline body.

(ii) Bluff bodies :

The bodies whose shape do not coincide with the streamlines are called Bluff bodies and in these bodies the pressure drag is excessively high.



Perfectly bluff bodies

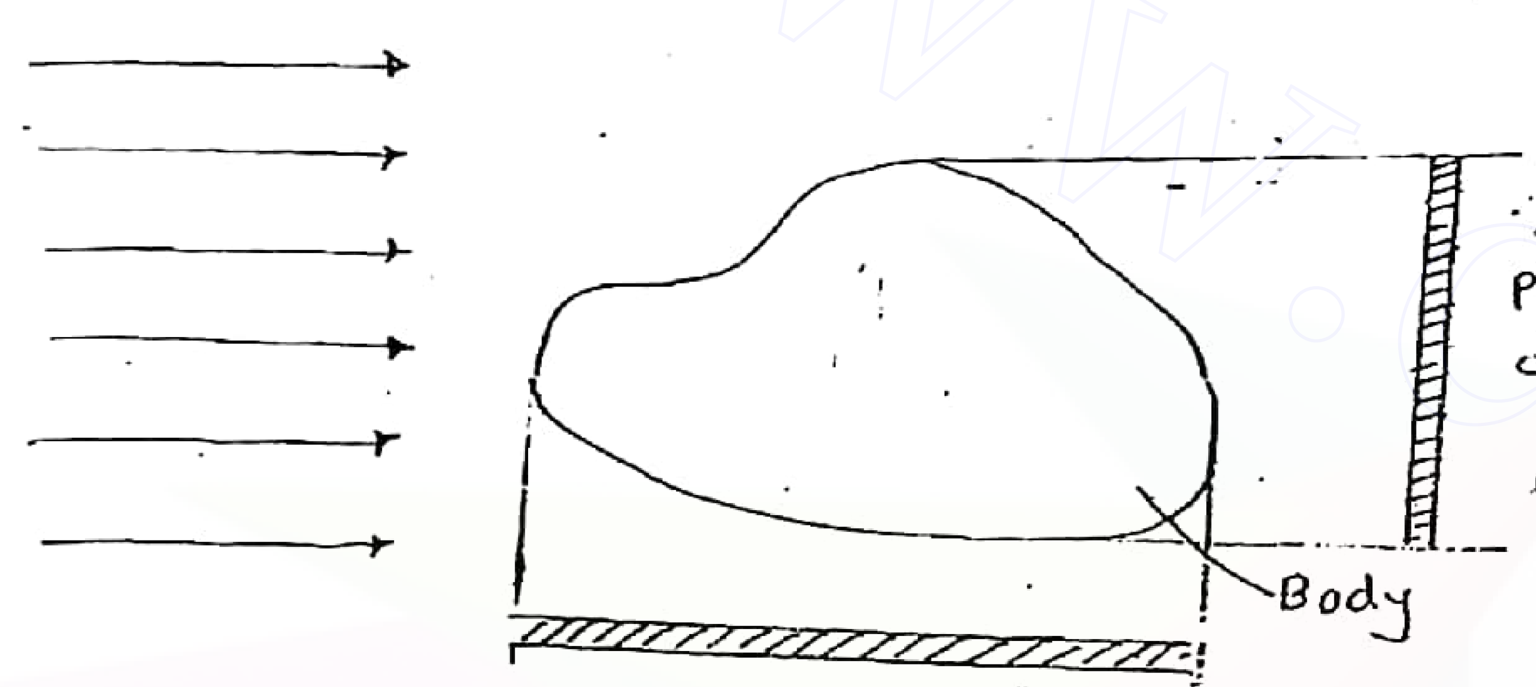
Pressure drag is maximum ($\theta = 0$) for the perfectly bluff bodies.

The drag and lift are beneficial according to the

Coefficient of drag (C_D) and coefficient of lift (C_L)

Finding C_D, C_L (research) for a bridge — severe weather

stoke's law $R_e < 0.2$



Area projected in direction of flow (max for bluff bodies)

Area projected in direction transverse to the flow. (max. for streamline bodies)

A - max. projected area for the body

Coefficient of drag (C_D):

It is drag force per unit area (A - max. projected area) per unit of energy stored per unit volume of flow.

$$C_D = \frac{F_D}{\frac{1}{2} \cdot \rho \cdot A \cdot U_\infty^2}$$

$$F_D = C_D \cdot \frac{1}{2} \cdot \rho \cdot A \cdot U_\infty^2$$

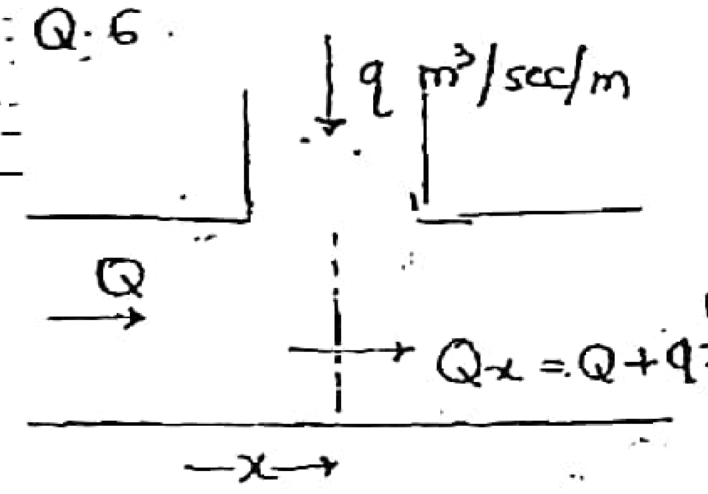
Coefficient of lift (C_L)

It is the lift force per unit area (max. projected area) per unit energy stored per unit volume of flow.

$$C_L = \frac{F_L}{\frac{1}{2} \cdot \rho \cdot A \cdot U_\infty^2}$$

$$F_L = C_L \cdot \frac{1}{2} \cdot \rho \cdot A \cdot U_\infty^2$$

Q.6



$$\dot m_{in} - \dot m_{out} = \dot m_{sto}$$

$$Q_x - (Q_x + dQ_x) = 0$$

$$Q_x - \left(Q_x + \frac{\partial Q_x}{\partial x} dx\right) = 0$$

$$\frac{\partial Q_x}{dx} = 0$$

$$\frac{\partial}{\partial x}(Q + qx) = 0$$

$$\frac{\partial Q}{\partial x} + q = 0$$

Q.12

$F = ma$ — dia. constant.

$$= m\left(v\frac{\partial v}{\partial x} + \frac{\partial v}{\partial t}\right)$$

$$= m \cdot \frac{\partial v}{\partial t}$$

$$= \frac{m}{A} \cdot \frac{\partial Q}{\partial t}$$

$m = c/s \times \text{length} \cdot \rho$

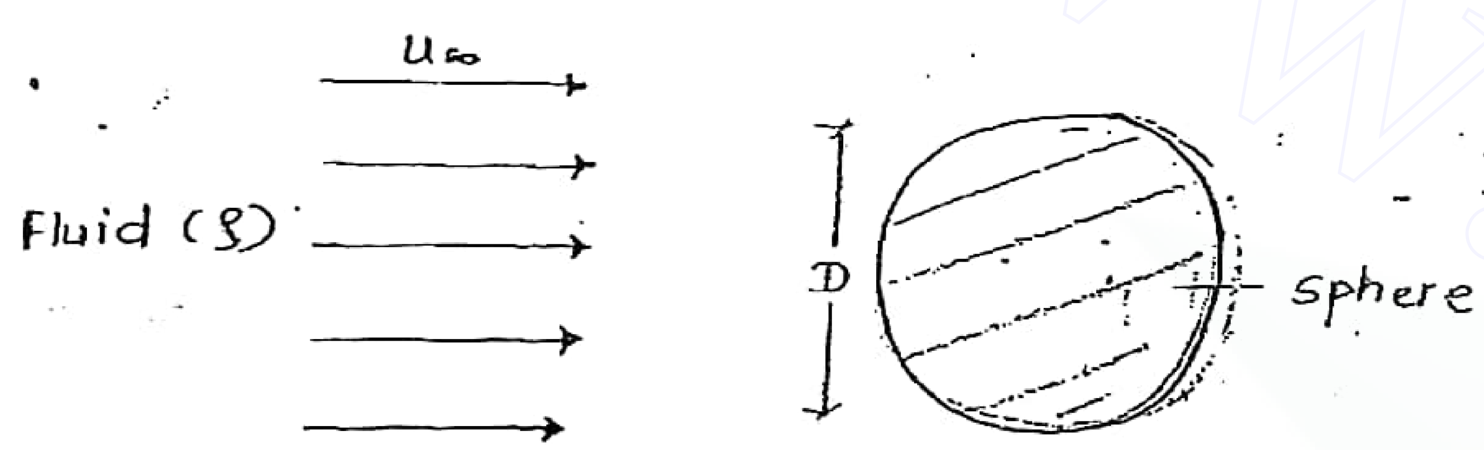
$$= \frac{A \cdot L \rho}{A} \cdot \frac{(25 - 100)\times 10^{-3}}{3}$$

$$\Delta P = \frac{F}{A} = \frac{L \times 1000 \times (-75 \times 10^{-3})}{3A}$$

$$\frac{\Delta P}{L} = \frac{1000 \times (-75 \times 10^{-3})}{3 \times \frac{\pi}{4} \times (0.2)^2}$$

$$= -796 \ \text{Pa/m}$$

Drag on sphere :



Characteristic dimension = D

$\therefore$ Reynold's number, $(R_e)_D = \dfrac{\rho\, U_\infty D}{\mu}$

$\therefore$ Projected area (circle) $= \dfrac{\pi D^2}{4}$

(i) Stokes law :

For $(Re)_D \leq 0.2$ only

Coefficient of drag, $C_D = \dfrac{24}{R_e}$

$\therefore$ Drag force, (F_D) :

$$F_D = C_D \cdot \frac{1}{2} \cdot \rho \cdot A \cdot U_\infty^2$$

$$= \frac{24}{Re} \cdot \frac{1}{2} \times \rho \times \frac{\pi D^2}{4} \times U_\infty^2$$

$$= \frac{24}{\rho U_\infty D} \cdot \mu \cdot \frac{1}{2} \rho \cdot \frac{\pi D^2}{4} U_\infty^2$$

$$F_D = 3\pi \mu D U_\infty$$

(ii) If $0.2 < Re \leq 5$

$$C_D = \frac{24}{(R_e)_D}\left(1 + \frac{3}{16\, Re}\right)$$

(iii) $5 < Re \leq 1000$

$$C_D = 0.4$$

(iv) $1000 < (R_e)_D \leq 100000$

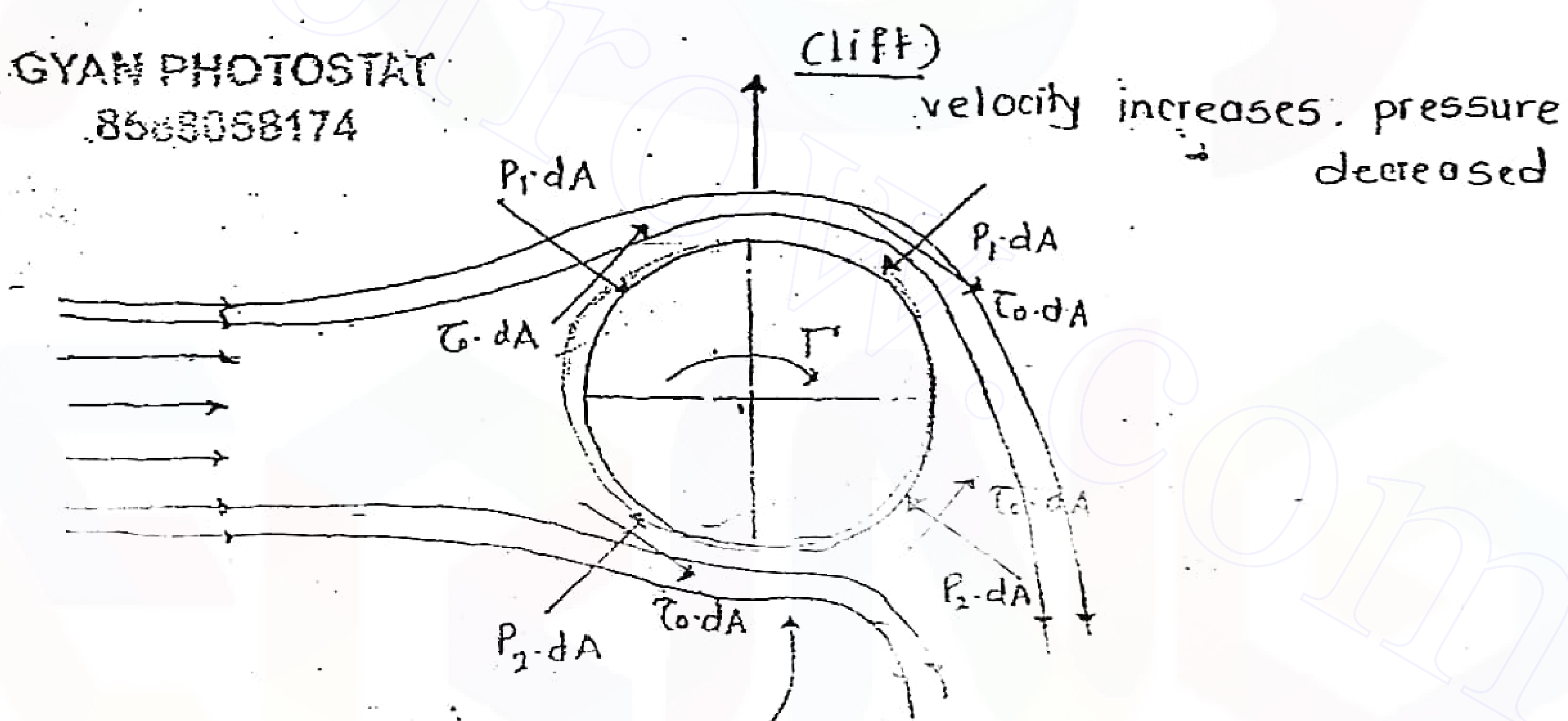
$$C_D = 0.5$$

(v) If $(Re)_D > 100000$

$$C_D = 0.2$$

Magnus Effect :

"When the cylinder is rotated in uniform stream of fluid, there will be the automatic generation of lift force." This effect was seen by Magnus. Therefore ~~it is~~ known as Magnus effect and in general this effect is true with every object, whether the object is symmetrical or assymmetrical.



Due to circulation Γ, the velocity on upper side is increased (directions of flow and circulation are same) and on lower side is decreased. Thus pressure on lower side is increased which creats the lift.

Kutta - Juokowski Lift theorem :

Lift on cylinder :

$$F_L = \rho \cdot U_\infty \cdot L \cdot \Gamma$$

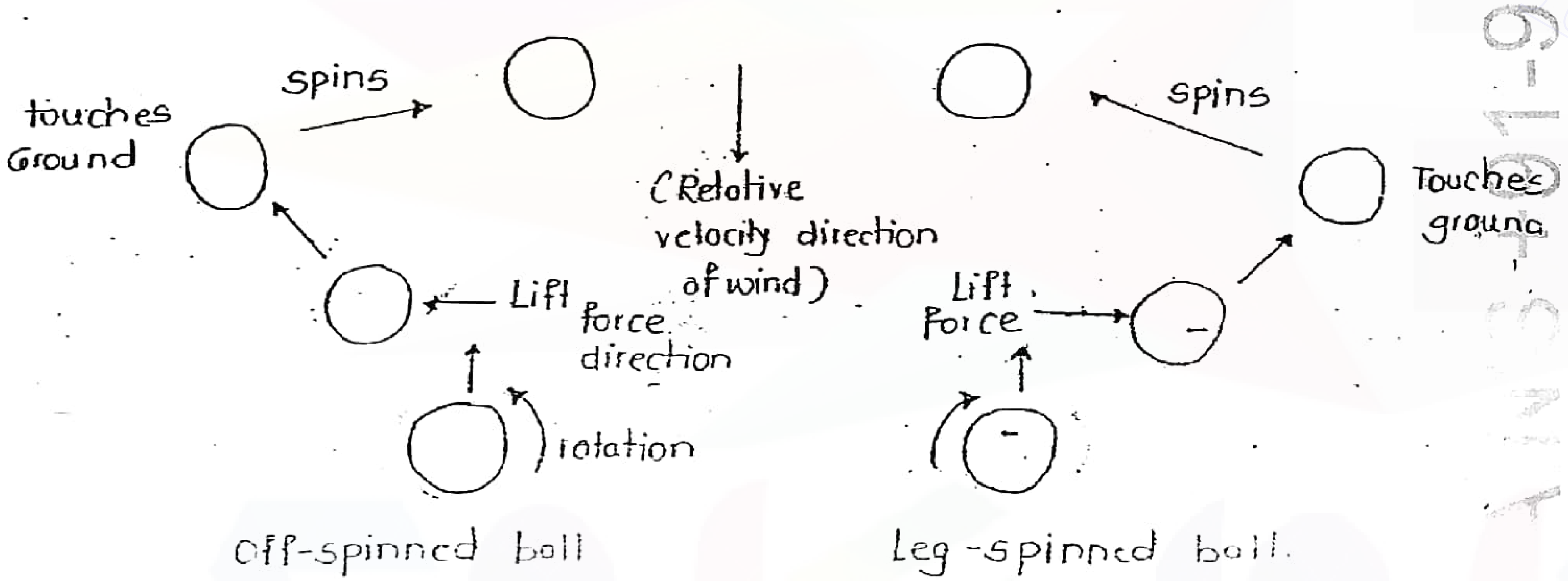
where,

L - length of cylinder

Γ - circulation given to cylinder.

Applications of Magnus effects :

(i) Spinning of ball in cricket :



When the ball touches the ground to conserve the tangential momentum, it spins at an angle of reflection of collision and strikes the stumps.

(ii) Rotation of the ball in ping-pong (Table-tenis)

(iii) Ball rotation in football.

The spin amount in case of Cricket ball depends upon the properties of medium with with ball collides (pitch conditions - dampness, roughness etc.) along with the magnitude of rotation given by Bowler.

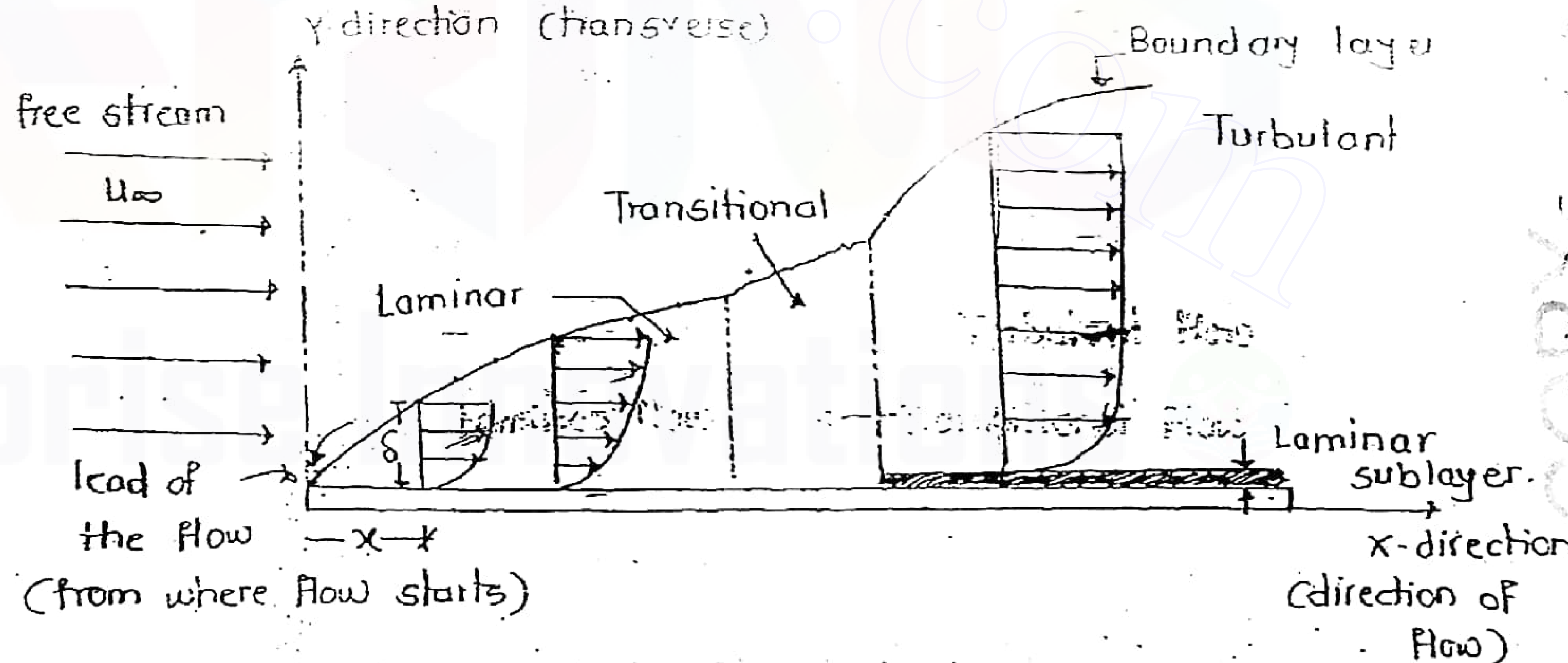
BOUNDRY LAYER THEORY

(Sir Prandll - 1904)

When a flow is developed over some surface, then the velocity of layer contacting with surface goes to zero value, with respect to surface because of maximum shear stress at the surface. Such a fundamental condition in fluid mechanics is known as no slip condition. But as we move away from the surface in the transverse direction of flow the viscous shear stresses are continuously decreasing and velocity is continuously increasing. Hence there is development of velocity gradient in the transverse direction of flow. After travelling certain distance, the viscous shear stresses are almost zero and velocity reaches to its max. value and after that velocity gradient in transverse direction will be absolutely finished.

Therefore the flow near the surface where the viscous shear stresses are pre-dominent is known as "Boundary layer flow."

Development of flow over flat plate :



U_∞ - velocity of free stream w.r.t. surface.

u - local velocity of stream w.r.t. surface.

x - characteristic dimension

Reynold's number,

$$(R_e)_x = \frac{\rho U_\infty \cdot x}{\mu} = \frac{U_\infty \cdot x}{\nu}$$

If $(R_e)_x < 3 \times 10^5$ (Laminar)

$> 5 \times 10^5$ (Turbulent Flow)

For numericals assume 5×10^5 as upper critical limit for Laminar flow.

Boundary layer thickness (δ):

It is distance travelled from surface in transverse direction of flow after which the velocity becomes 99% of the free stream velocity.

i.e. $u = 0.99\, U_\infty$

(i) As x increases thickness of boundary layer increases.

Because due to viscous shear stresses velocity profile is lost in the direction of flow and the velocity profiles are continuously changing in the direction of flow

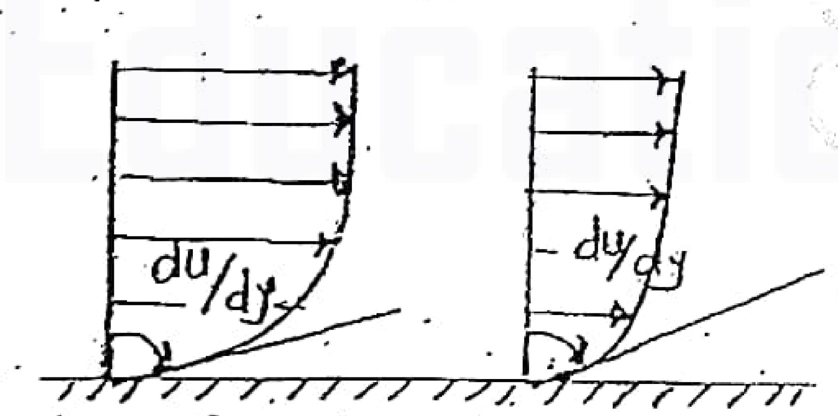
$$u = f(x, y)$$

(i) External flows can never be fully developed

Because, the velocity profile is continuously changing w.r.t. x

(ii) Surface shear stress :

$$\tau_o = \mu \left(\frac{du}{dy}\right)_{y=0}$$



Here, $\frac{dy}{dy}$ is function of x. (decreasing function)

As x increases $\frac{du}{dy}$ decreases.

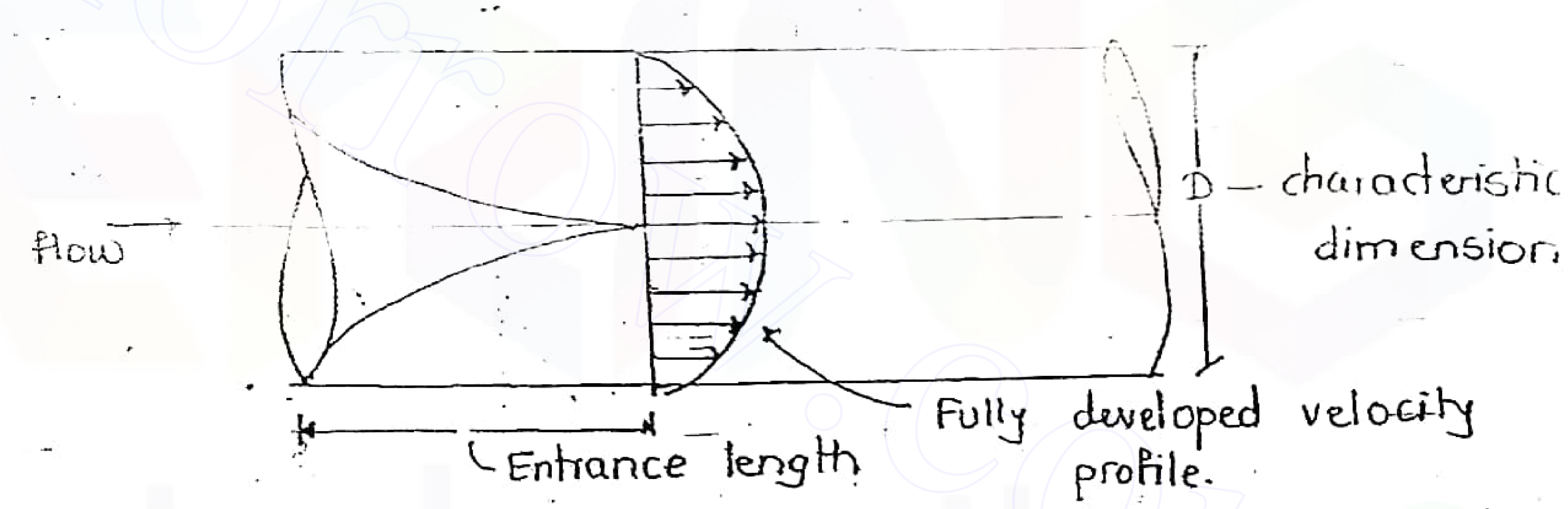
(iv) $\delta_{Turbulent} >>> \delta_{Laminar}$.

Because of intermixing of layers in turbulant flow and big energy loss the velocity profile is flatter (i.e. velocity gradient is nearly lost) Thus the boundary layer thickness is more (increasing with high rate) tho in turbulant zone as compared to Laminar zone of flow.

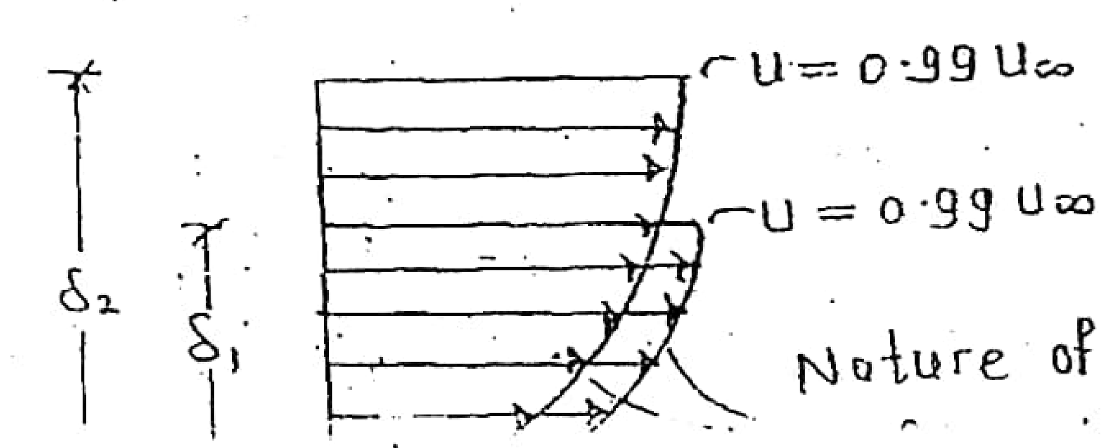
Internal flows (pipe flow) are fully developed flows. Once the flow is fully developed, velocity profile is not changing in the direction of flow i.e. u is not function of x.

In case of pipe flows (internal flows) boundary starts developing from pipe surface towards the centre and reaches the maximum value of boundary layer thickness equal to the radius of pipe and becomes constant. Thus flow is fully developed and constant velocity profile is attained.

The length required for development of pipe flow (fully) is called Entrance length.

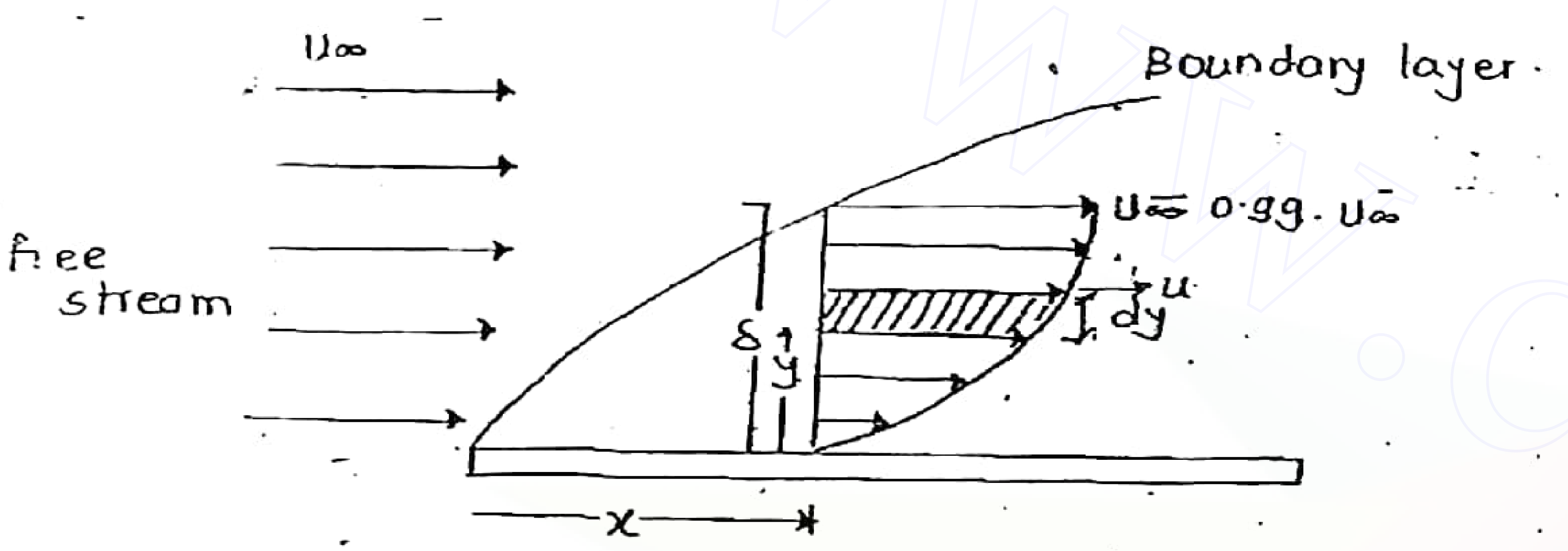


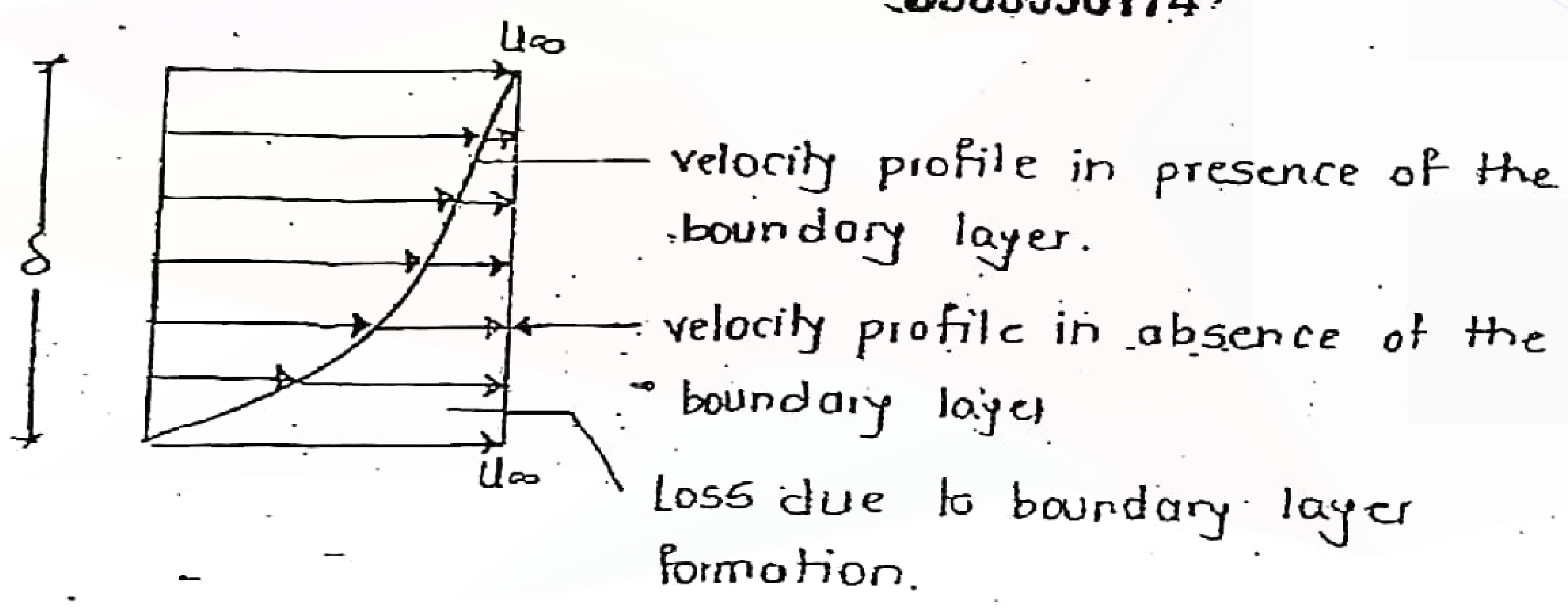
Incase of boundary layer flows velocity profile goes on reducing because of increase in viscous shear stress along x.



Nature of the velocity

Different losses in flow because of boundary layer formation.





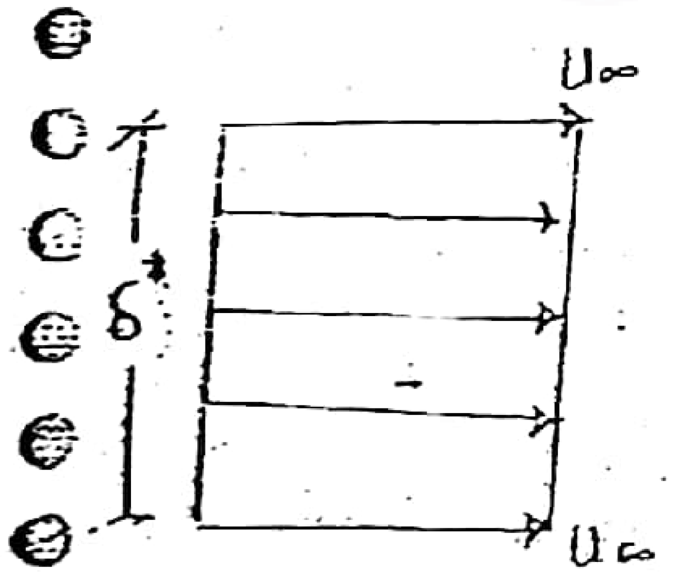
(i) Loss in mass flow rate ($\dot m$)

(Displacement thickness - δ^*)

It is thickness of free stream which is lost in terms of mass flow rate ($\dot m$) because of boundary layer formation.

$\dot m$ crossing in the presence of B.L. $= \int_0^\delta (\rho \cdot u \cdot dy)$

absence of B.L. $= \int_0^\infty (\rho U_\infty \, dy)$



Loss in mass flow rate ($\dot m$) $= \int_0^\infty (\rho U_\infty - \rho u) \, dy$

$$\rho \delta^* U_\infty = \int_0^\infty (\rho U_\infty - \rho u) \, dy$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

(ii) Momentum thickness (θ):

($\dot m v$) crossing in presence of B.L. $= \int_0^\delta (\rho \cdot u \cdot dy) \cdot u$

absence of B.L. $= \int_0^\delta (\rho u \cdot dy) \cdot U_\infty$

Loss in ($\dot m v$) $= \int_0^\delta (\rho u U_\infty - \rho u^2) \cdot dy$

$$(\rho U_\infty \cdot \theta) \cdot U_\infty = \int_0^\delta (\rho u \cdot U_\infty - \rho u^2) \cdot dy$$

$$\theta = \int_0^\delta \frac{u}{U_\infty} \left(1 - \frac{u}{U_\infty}\right) dy$$

(iii) Energy thickness (δ_E):

K.E crossing in the presence of B.L. $= \frac{1}{2} \int_0^\delta (\rho u \, dy) \cdot u^2$

$= \frac{1}{2} \int (\rho u \cdot dy) \cdot U_\infty^2$

$$= \frac{1}{2} \int_0^\delta (\rho u \cdot U_\infty^2 - \rho u^3) \cdot dy$$

$$\frac{1}{2} \rho \cdot \delta_E \cdot U_\infty \cdot U_\infty^2 = \frac{1}{2} \int_0^\delta (\rho u \cdot U_\infty^2 - \rho u^3) \cdot dy$$

$$\delta_E = \int_0^\delta \frac{u}{U_\infty} \left(1 - \frac{u^2}{U_\infty^2}\right) dy$$

The ratio of displacement thickness to momentum thickness at any section is known as shape factor.

Q. The velocity profile at section in flow is given by $\frac{u}{U_s} = \frac{y}{\delta}$. Calculate displacement, momentum and energy thicknesses.

$$\delta^* = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy$$

$$= \delta - \frac{1}{\delta}\left(\frac{\delta^2}{2}\right)$$

$$= \frac{\delta}{2}$$

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

$$= \frac{\delta}{2} - \frac{\delta}{3} = \frac{\delta}{6}$$

$$\delta_E = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u^2}{U_\infty^2}\right) dy$$

$$= \frac{\delta}{2} - \frac{\delta}{4}$$

$$= \frac{\delta}{4}$$

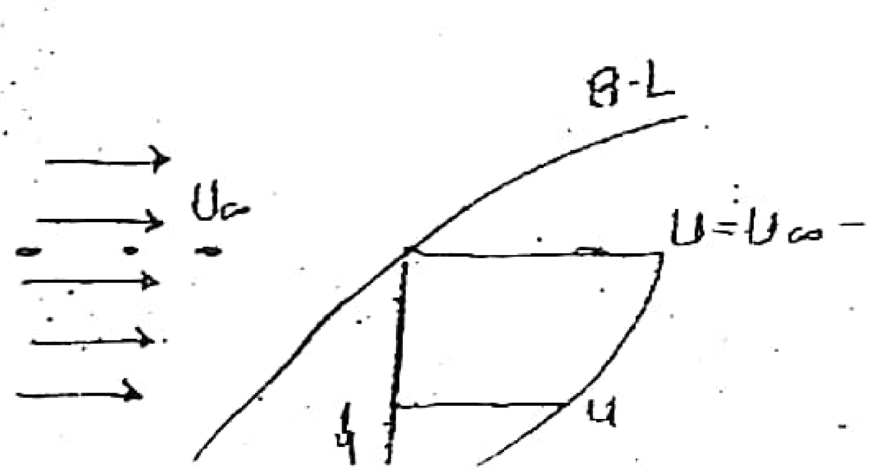
Q. Shear stress distribution at section in Laminar flow is given as $\tau = \tau_0\left(1 - \frac{y}{\delta}\right)$ calculate displacement, momentum and energy thickness.

15 Marks.

For laminar flow,

$$\mu \frac{du}{dy} = \tau_0\left(1 - \frac{y}{\delta}\right)$$ Newton's law of viscosity.

$$du = \frac{\tau_0}{\mu}\left(1 - \frac{y}{\delta}\right) dy$$



$$\int du = \int \frac{\tau_0}{\mu}\left(1 - \frac{y}{\delta}\right) dy$$

$$u = \frac{\tau_0}{\mu}\left(y - \frac{y^2}{2\delta}\right)$$

We know that,
at $y = \delta$, $u \approx U_\infty$

$$U_\infty = \frac{\tau_0}{\mu}\left(\delta - \frac{\delta^2}{2\delta}\right)$$

$$U_\infty = \frac{\tau_0 \cdot \delta}{2\mu}$$

$$\frac{u}{U_\infty} = \frac{\frac{\tau_0}{\mu}\left(y - \frac{y^2}{2\delta}\right)}{\frac{\tau_0 \cdot \delta}{2\mu}}$$

$$= \frac{2}{\delta}\left(y - \frac{y^2}{2\delta}\right)$$

$$\frac{u}{U_\infty} = \frac{2y}{\delta} - \frac{2y^2}{2\delta^2}$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

$$= \int_0^\delta \left(1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right) dy$$

$$= \delta - \delta + \frac{1}{3}\delta$$

$$= \frac{\delta}{3}$$

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right) - \left(\frac{4y^2}{\delta^2} + \frac{y^4}{\delta^4} - 4\frac{y^3}{\delta^2}\right)$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{5y^2}{\delta^2} + 4\frac{y^3}{\delta^2} + \frac{y^4}{\delta^4}\right) dy$$

$$= \delta - \frac{5}{3}\delta + \delta^2 + \frac{\delta}{5}$$

Q. The velocity profile at section in flow is given by $\frac{u}{U_s} = \frac{y}{\delta}$. Calculate displacement, momentum and energy thicknesses.

$$\delta^* = \int_0^\delta \left(1 - \frac{y}{\delta}\right) . dy$$

$$= \delta - \frac{1}{\delta}\left(\frac{\delta^2}{2}\right)$$

$$= \frac{\delta}{2}$$

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) . dy$$

$$= \frac{\delta}{2} - \frac{\delta}{3} = \frac{\delta}{6}$$

$$\delta_E = \int_0^\delta \frac{u}{U_\infty} . \left(1 - \frac{u^2}{U_\infty^2}\right) . dy$$

$$= \frac{\delta}{2} - \frac{\delta}{4}$$

$$= \frac{\delta}{4}$$

Q. Shear stress distribution at section in laminar flow is given as $\tau = \tau_0\left(1 - \frac{y}{\delta}\right)$. Calculate displacement, momentum and energy thickness.

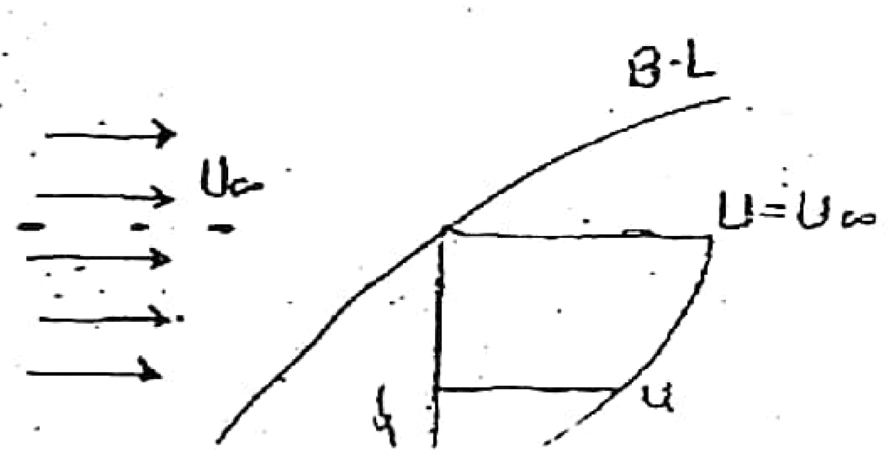
15 Marks.

For laminar flow,

$$\mu \frac{du}{dy} = \tau_0\left(1 - \frac{y}{\delta}\right)$$

Newton's law of viscosity

$$du = \frac{\tau_0}{\mu}\left(1 - \frac{y}{\delta}\right) . dy$$



$$\int du = \int \frac{\tau_0}{\mu}\left(1 - \frac{y}{\delta}\right) . dy$$

$$u = \frac{\tau_0}{\mu}\left(y - \frac{y^2}{2\delta}\right)$$

We know that,

at $y = \delta$, $u \simeq U_\infty$

$$U_\infty = \frac{\tau_0}{\mu}\left(\delta - \frac{\delta^2}{2\delta}\right)$$

$$U_\infty = \frac{\tau_0 . \delta}{2\mu}$$

$$\frac{u}{U_\infty} = \frac{\frac{\tau_0}{\mu}\left(y - \frac{y^2}{2\delta}\right)}{\frac{\tau_0 . \delta}{2\mu}}$$

$$= \frac{2}{\delta}\left(y - \frac{y^2}{2\delta}\right)$$

$$\frac{u}{U_\infty} = \frac{2y}{\delta} - \frac{2y^2}{2\delta^2}$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) . dy$$

$$= \int_0^\delta \left(1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right) . dy$$

$$= \delta - \delta + \frac{1}{3}\delta$$

$$= \frac{\delta}{3}$$

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) . dy$$

$$= \int_0^\delta \left(-\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(\frac{4y^2}{\delta^2} + \frac{y^4}{\delta^4} - 4\frac{y^3}{\delta^2}\right)$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{5y^2}{\delta^2} + 4\frac{y^3}{\delta^2} + \frac{y^4}{\delta^4}\right) . dy$$

$$= \delta - \frac{5}{3}\delta + \delta^2 + \frac{\delta}{5}$$

$$\delta_E = \int_0^{\delta} \frac{u}{U_\infty}\left(1 - \frac{u^2}{U_\infty^2}\right) dy$$

$$= \int_0^{\delta} \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right) - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(\frac{4y^2}{\delta^2} + \frac{y^4}{\delta^4} - \frac{4y^3}{\delta^3}\right)$$

$$= \int_0^{\delta} \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right) - \left[\left(\frac{8y^3}{\delta^2} + \frac{2y^5}{\delta^5} - \frac{8y^4}{\delta^4}\right) - \left(\frac{4y^4}{\delta^4} - \frac{y^6}{\delta^6} + \frac{4y^5}{\delta^5}\right)\right] dy$$

$$= \int_0^{\delta} \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2} - \frac{8y^3}{\delta^2} + \frac{2y^5}{\delta^5} + \frac{12y^4}{\delta^4} - \frac{y^6}{\delta^6}\right) dy$$

$$= \delta - \frac{\delta}{3} - 2\delta^2 + \frac{\delta}{3} + \frac{12\delta}{5} - \frac{\delta}{7}$$

$$= \frac{114}{7}\delta - 2\delta^2$$

Local and average skin friction coefficient:

Local skin friction ($C_{f,x}$)

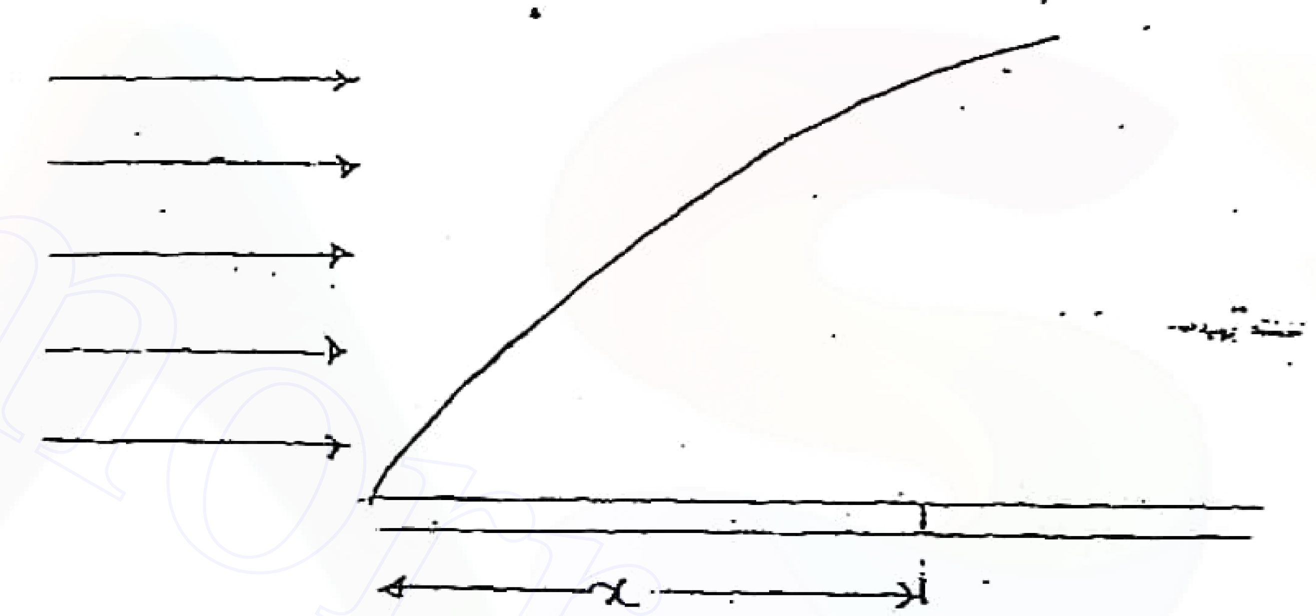
It is shear stress at distance 'x' from leading edge per unit of K.E. per unit volume of free stream that crosses the point.

$$C_{fx} = \frac{\tau_0}{\quad}$$

$$\tau_0 = f(x)$$

$$C_{dfx} = f(x)$$

Avg. skin friction coefficient.
(Coefficient of drag) - C_D or $\overline{C_{fx}}$



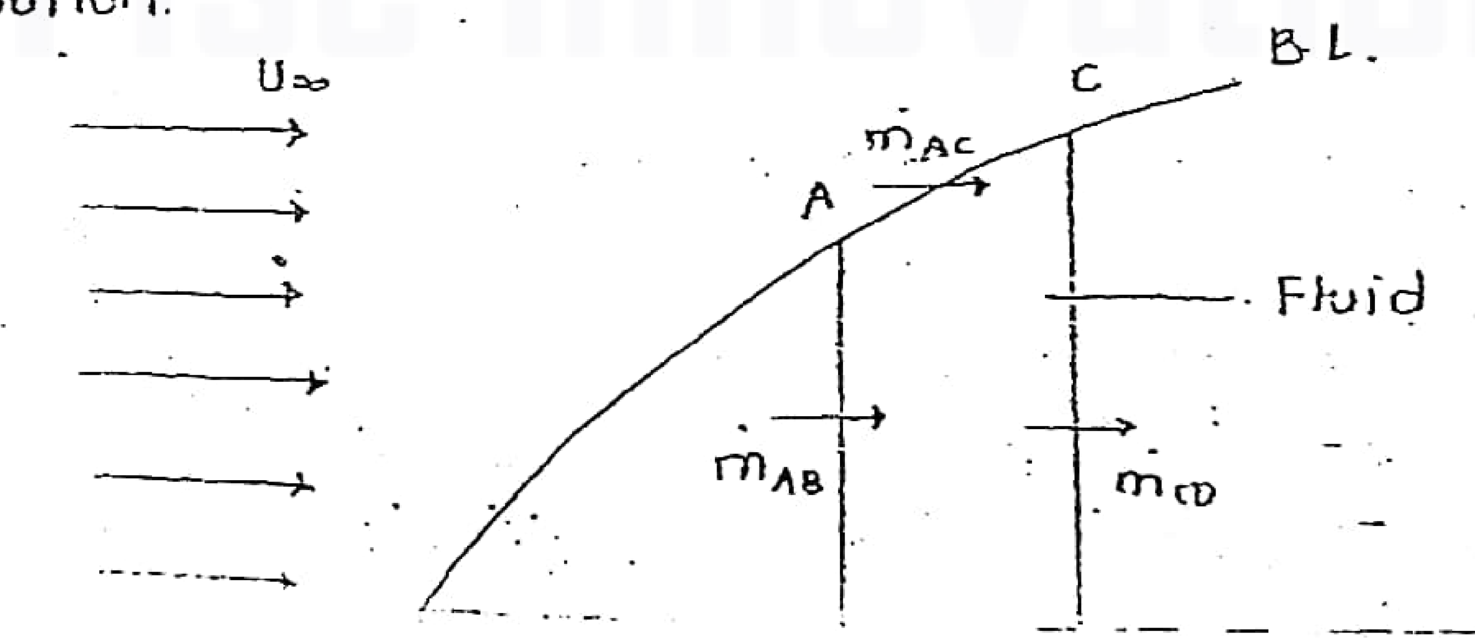
$$\overline{C_{fx}} = C_D = \frac{1}{x}\int_0^x C_{fx}\, dx$$

$$F_{drag} = \overline{C_{dfx}} \cdot \frac{1}{2}\rho U_\infty^2 \cdot A$$

Coefficient of drag is average value of local skin friction

Von-Karman Integral momentum equation inside boundary level:

Covered the complete section of boundary layer without leaving single layer. Thus it is called complete integration equation.



$$\delta_E = \int_0^{\delta} \frac{u}{U_\infty}\left(1 - \frac{u^2}{U_\infty^2}\right) dy$$

$$= \int_0^{\delta}\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right) - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(\frac{4y^2}{\delta^2} + \frac{y^4}{\delta^4} - \frac{4y^3}{\delta^3}\right)$$

$$= \int_0^{\delta}\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right) - \left[\left(\frac{8y^3}{\delta^2} + \frac{2y^5}{\delta^5} - \frac{8y^4}{\delta^4}\right) - \left(\frac{4y^4}{\delta^4} - \frac{y^6}{\delta^6} + \frac{4y^5}{\delta^5}\right)\right] dy$$

$$= \int_0^{\delta}\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2} - \frac{8y^3}{\delta^2} + \frac{2y^5}{\delta^5} + \frac{12y^4}{\delta^4} - \frac{y^6}{\delta^6}\right) dy$$

$$= \delta - \frac{\delta}{3} - 2\delta^2 + \frac{\delta}{3} + \frac{12\delta}{5} - \frac{\delta}{7}$$

$$= \frac{114}{7}\delta - 2\delta^2$$

Local and average skin friction coefficient:

Local skin friction (C_{fx})

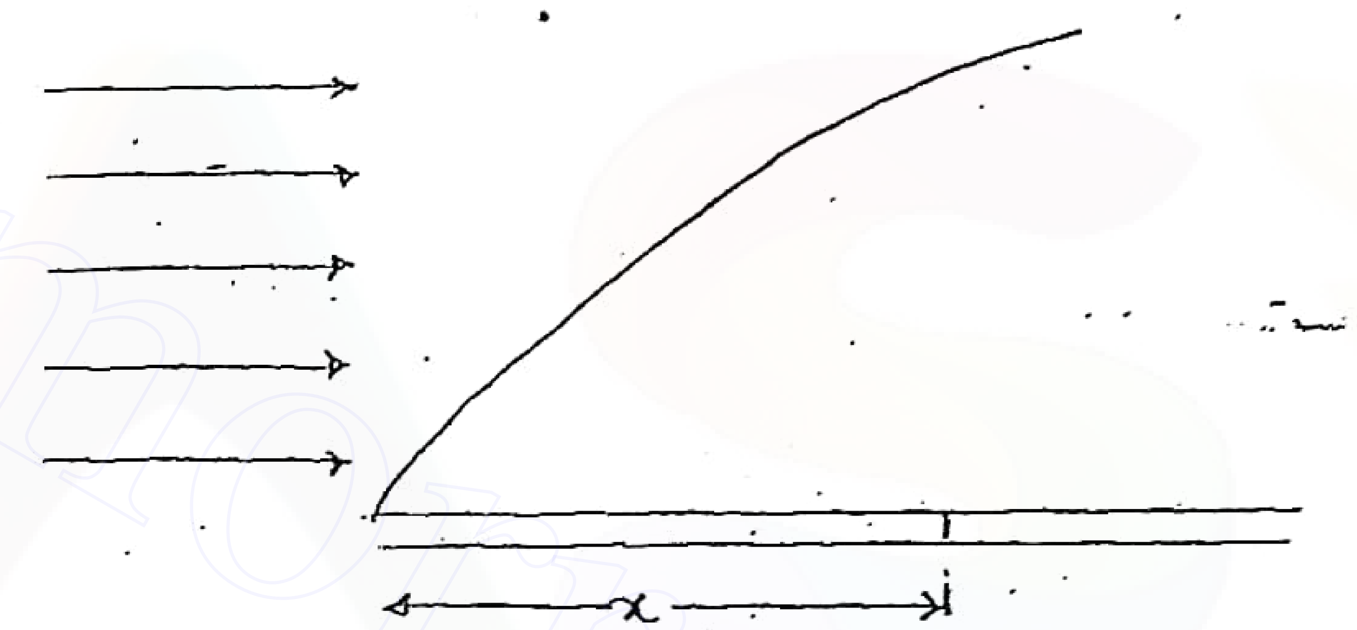
It is shear stress at distance 'x' from leading edge per unit of K.E. per unit volume of free stream that crosses the point.

C_{f} τ_0

$$\tau_0 = f(x)$$
$$C_{dfx} = f(x)$$

Avg. skin friction coefficient.

(coefficient of drag) - C_D or $\overline{C_{fx}}$



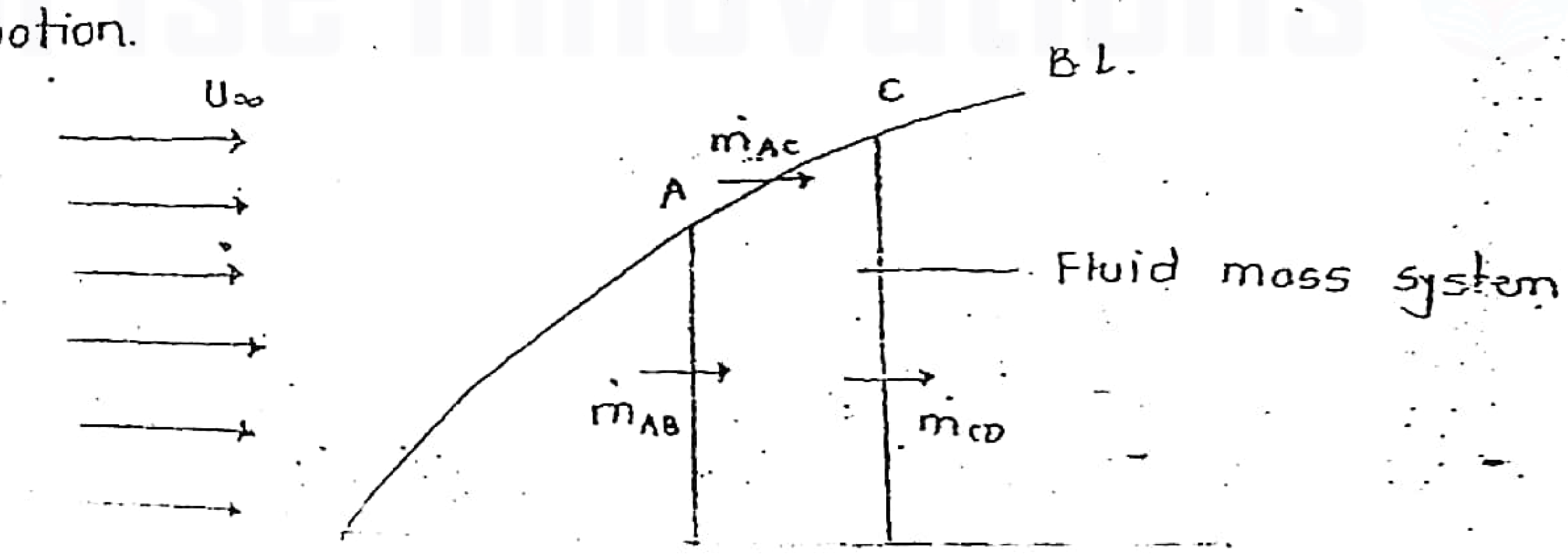
$$\overline{C_{fx}} = C_D = \frac{1}{x}\int_0^{x} C_{fx}\, dx$$

$$F_{drag} = \overline{C_{dfx}} \cdot \frac{1}{2}\rho\, U_\infty^2 \cdot A$$

Coefficient of drag is average value of local skin friction.

Von-Karman Integral momentum equation inside boundary level:

Covered the complete section of boundary layer without leaving single layer. Thus it is called complete integration equation.



$$\therefore (\dot{m}_{AB})_{Entry} = \int_0^\delta \rho u \cdot dy$$

$$(\dot{m}_{CD})_{Exit} = \dot{m}_{AB} + \frac{\partial \dot{m}_{AB}}{\partial x} \cdot dx$$

$\because$ Conservation of mass :

$$(\dot{m}_{AB})_{entry} + (\dot{m}_{AC})_{entry} = (\dot{m}_{CD})_{exit}$$

$$(\dot{m}_{AC})_{entry} = \dot{m}_{AB} + \frac{\partial \dot{m}_{AB}}{\partial x} dx - \dot{m}_{AB}$$

$$= \frac{\partial}{\partial x}\left[\int_0^\delta \rho u \cdot dy\right] \cdot dx$$

Momentum :

$$(\dot{m}V_{AB})_{entry} = \int_0^\delta \rho u^2 \cdot dy$$

$$(\dot{m}V_{CD})_{exit} = (\dot{m}V)_{AB} + \frac{\partial \dot{m}V_{AB}}{\partial x} \cdot dx$$

$$(\dot{m}V_{AC})_{entry} = \frac{\partial}{\partial x}\left[\int \rho u \cdot dy \cdot U_\infty\right] \cdot dx$$

Newton's 2nd law in direction

$$(\dot{m}V)_{CD} - (\dot{m}V)_{AB} = (\dot{m}V)_{AC} = -\tau_0 \cdot dx$$

$$(\dot{m}V)_{AB} + (\dot{m}V)_{AC} - (\dot{m}V)_{CD} = \tau_0 \cdot dx$$ –ve for opposite direction

$$(\dot{m}V)_{AB} + (\dot{m}V)_{AC} - \left[(\dot{m}V)_{AB} + \frac{\partial (\dot{m}V)_{AB}}{\partial x} \cdot dx\right] = \tau_0 \cdot dx$$

$$\frac{\partial}{\partial x}\left[\int_0^\delta \rho u \cdot U_\infty \cdot dy\right] dx - \frac{\partial}{\partial x}\left[\int_0^\delta \rho u^2 dy\right] \cdot dx = \tau_0 \cdot dx$$

$$\frac{\partial}{\partial x}\left[\int_0^\delta (\rho u \cdot U_\infty - \rho u^2)\, dy\right] = \tau_0$$

$$\frac{\partial}{\partial x}\left[\int_0^\delta \frac{\rho u \cdot U_\infty}{\rho U_\infty^2}\left(1 - \frac{u}{U_\infty}\right) \cdot dy\right] = \frac{\tau_0}{\rho U_\infty^2}$$

$$\frac{\tau_0}{\rho U_\infty^2} = \frac{\partial \theta}{\partial x}$$

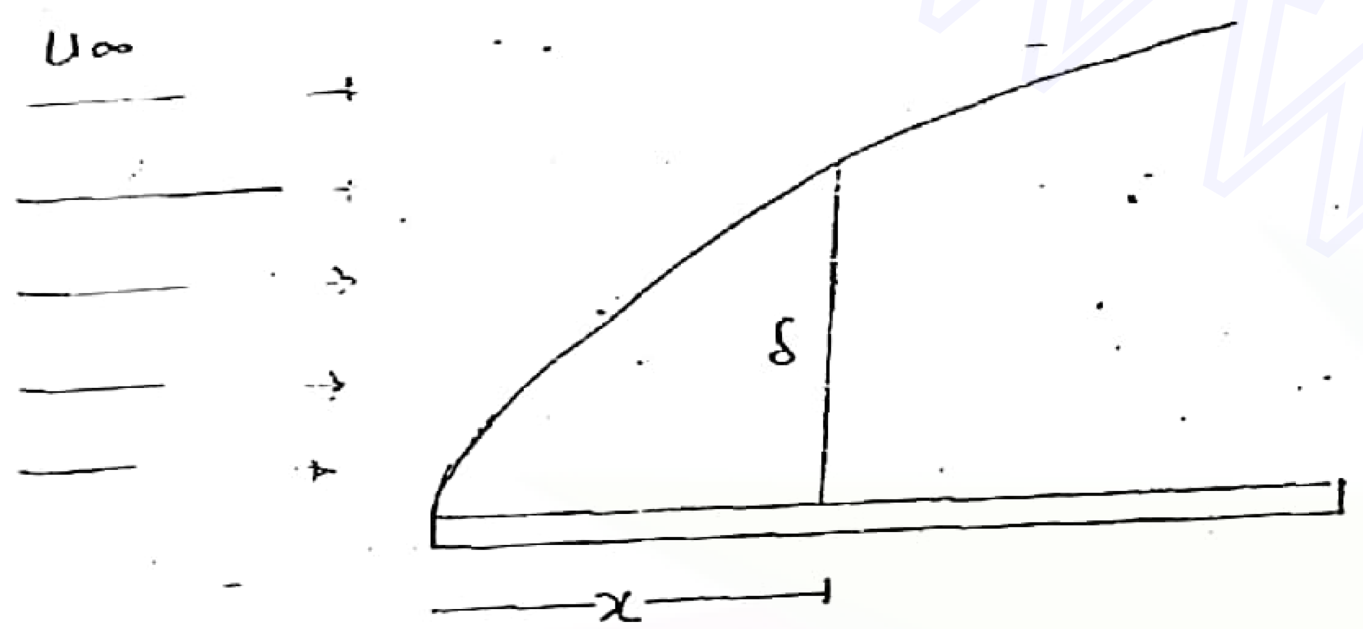
– Von-Karmann's integrated momentum equation.

where.

θ – momentum thickness

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) \cdot dy$$

Exact solution of Laminar boundary layer over flat plate (similarity transformation technique) Blasius



At a distance, x from leading edge of flow

$$(Re)_x = \frac{\rho U_\infty x}{\mu}$$

$$\delta = \frac{5x}{\sqrt{Re_x}}$$

$$C_{f,x} = \frac{0.664}{\sqrt{Re_x}}$$

Ph.D. results of Blassius

$$C_{fx} = \frac{0.664}{\sqrt{Re_x}} \qquad \text{i.e.} \ \propto \frac{1}{\sqrt{x}}$$

$$= \frac{C}{\sqrt{x}} \qquad C - \text{constant}$$

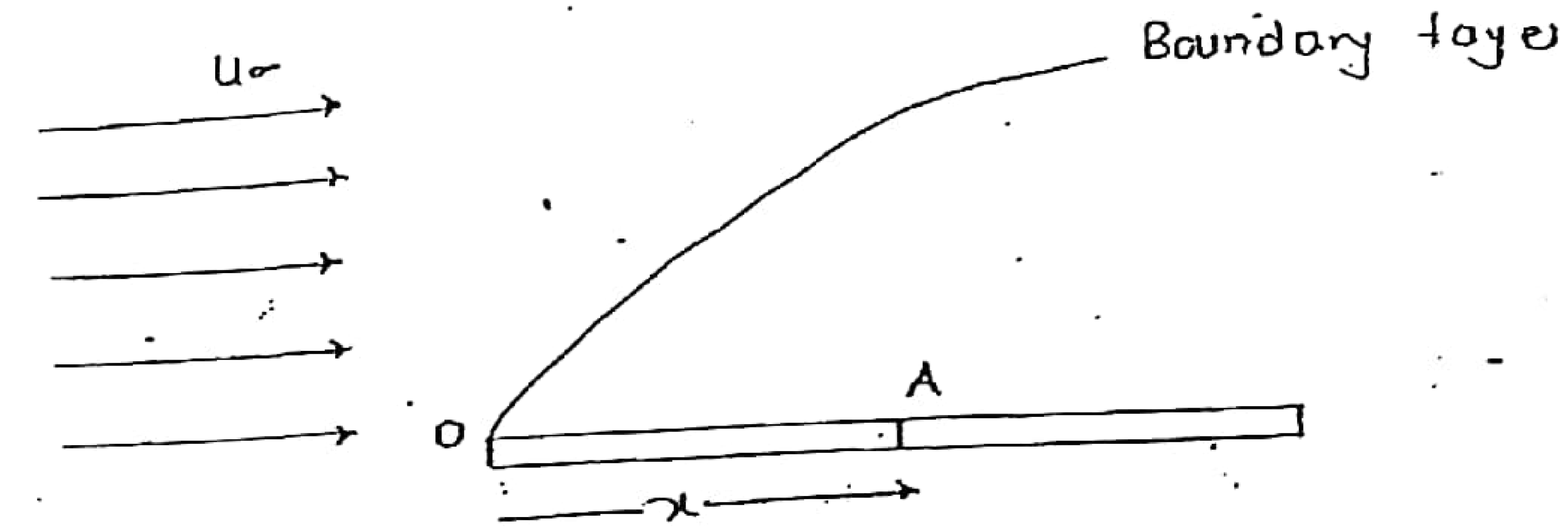
$$\overline{C_{fx}} = C_D = \frac{1}{x}\int_0^x C_{fx}\, dx$$

$$= \frac{1}{x}\int_0^x \frac{C}{\sqrt{x}}\, dx$$

$$= \frac{C}{x} \cdot 2\sqrt{x}$$

$$= \frac{2C}{\sqrt{x}}$$

$$\overline{C_{fx}} = 2 \cdot C_{fx}$$



$$F_D = \overline{C_{fx}} \times \frac{1}{2} \rho U_\infty^2 \times A$$

To find drag force (F_D) over length of flow, average skin friction coefficient is twice the local skin friction coefficient at point A. (for length OA) - It is valid only for $Re < 5\times10^5$ (laminar flow)

Von-Karmann solution of the Laminar boundary layer over a flat plate:

The most appropriate velocity profile in Laminar boundary layer flow over flat plate is cubic (According to Von-Karman)

$$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \qquad (1)$$

Momentum thickness (θ):

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

$$\theta = \frac{39\,\delta}{280}$$

Boundary shear stress (τ_o)
(Laminar flow)

$$\tau_o = \mu \left(\frac{du}{dy}\right)\Big|_{y=0}$$

$$= \mu \; U_\infty \left[\frac{3}{2\delta} - \frac{1}{2\delta^3}\cancel{3y^2}^{\,0}\right]_{y=0}$$

$$\tau_o = \frac{3\mu U_\infty}{2\delta}$$

Applying Von-Karmo's integrated momentum equation.

$$\frac{\tau_o}{\rho U_\infty^2} = \frac{\partial \theta}{\partial x}$$

$$\frac{3\mu U_\infty}{2\delta \cdot \rho \cdot U_\infty^2} = \frac{39}{280}\frac{\partial \delta}{\partial x}$$

$$\int_0^\delta \delta \cdot \partial\delta = \frac{140}{13}\frac{\mu}{\rho U_\infty}\int_0^x \partial x$$

$$\frac{\delta^2}{2} = \frac{140}{13}\frac{\mu x}{\rho \cdot U_\infty \; x}$$

$$= \left(\frac{280}{13}\right)\frac{x^2}{Re_x}$$

$$\delta = \frac{\sqrt{\frac{280}{13}} \cdot x}{\sqrt{Re_x}}$$

$$\delta = \frac{4.64\,x}{\sqrt{Re_x}}$$

C_{fx} :

$$C_{fx} = \frac{\tau_o}{\frac{1}{2}\rho U_\infty^2}$$

$$= \frac{3\mu U_\infty}{2\delta \cdot \frac{1}{2}\rho U_\infty^2}$$

$$= \frac{3\mu}{\sqrt{\frac{280}{13}} \cdot x \cdot \rho U_\infty}$$

$$= \frac{3\sqrt{13/280}}{\sqrt{Re_x}}$$

$$= \frac{0.646}{\sqrt{Re_x}}$$

$\overline{C_{fx}}$:

$$\overline{C_{fx}} = C_D = 2 \times C_{fx}$$

$$= \frac{1.292}{\sqrt{Re_x}}$$

This are approximated results given by Von Karman considering the velocity profile as cubic, given in —①

Von-Karman solution of Turbulent boundary layer flow over flat plate :

The most appropriate velocity profile in turbulent boundary layer flow over flat plate is following " one seventh power law.

$$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{1/7}$$

and

$$\tau_o = (0.0228) \cdot \rho U_\infty^2 \cdot \left(\frac{\nu}{U_\infty \cdot \delta}\right)^{1/4}$$

$$\delta = \frac{0.375\,x}{(Re_x)^{1/5}}$$

$$Cf_x = \frac{0.059}{(Re_x)^{1/5}}$$

$$Cf_x = \frac{0.059}{(Re_x)^{1/5}} \propto \frac{1}{x^{1/5}} = \frac{C}{x^{1/5}}$$

$$\overline{Cf_x} = \frac{1}{x}\int_0^x Cf_x \cdot dx$$

$$= \frac{C}{x}\int_0^x \frac{1}{x^{1/5}} \cdot dx$$

$$= \frac{5}{4} \cdot \frac{C}{x^{1/5}}$$

$$\overline{Cf_x} = \frac{5}{4} \cdot Cf_x$$

$$\overline{Cf_x} = \frac{0.059}{(Re_x)^{1/5}} \cdot \frac{5}{4}$$

$$\frac{0.073}{(Re_x)^{1/5}}$$

For numericals,

Laminar flow over plate

Exact solⁿ $\delta = \frac{5x}{\sqrt{Re_x}}$ $\overline{Cf_x} = \frac{0.664}{\sqrt{Re_x}} \times 2$

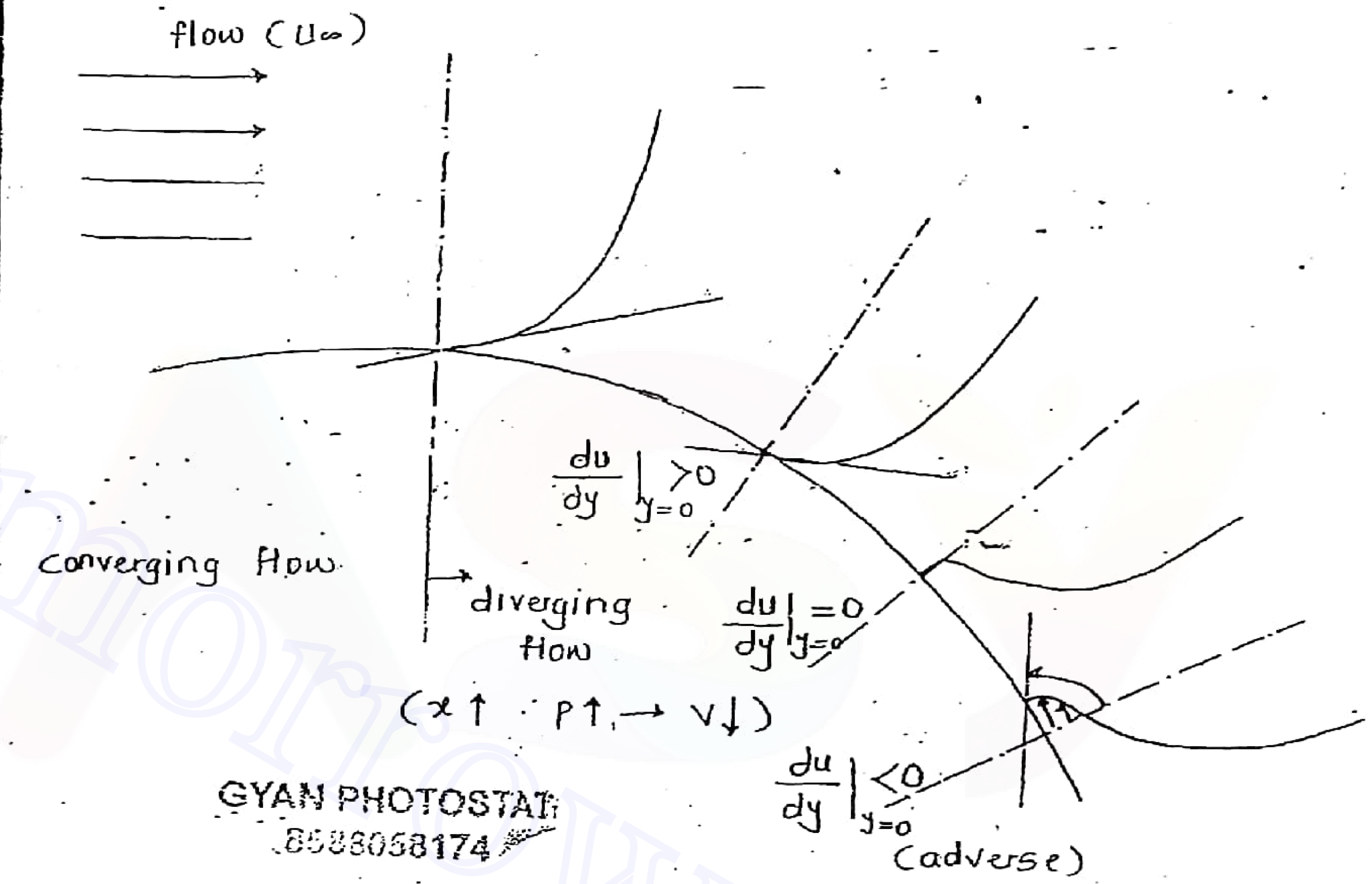
Approx. solⁿ. $\delta = \frac{4.64\,x}{\sqrt{Re_x}}$, $\overline{Cf_x} = \frac{0.646}{\sqrt{Re_x}} \times 2$

Turbulent flow over flat plates

Approx. solⁿ. $\delta = \frac{0.375\,x}{(Re_x)^{1/5}}$ $\overline{Cf_x} = \frac{0.073}{(Re_x)^{1/5}}$

No exact solⁿ available.

Boundary level flow seperation and its control :



The main cause for the flow seperation is Adverse pressure gradient.

$$\frac{\partial P}{\partial x} > 0 \qquad x \text{ — direction of flow}$$

If $\frac{\partial u}{\partial y}\Big|_{y=0} > 0$. Flow is not seperated

$\frac{\partial u}{\partial y}\Big|_{y=0} = 0$ Flow is on verge of separation

$\frac{\partial u}{\partial y}\Big|_{y=0} < 0$ Flow is seperated.

Methods to prevent flow seperation:

(i) By supplying fluid through pump.

(ii) By suction of fluid through vaccum pumps

(iii) By suction of fluid through porous medium. (Husk)

(iv) By making the bodies close to streamline shapes.

Q. Water is flowing over a flat plate of length 2m and width 1.5. The velocity of flow is 1 m/sec. Find the total drag force on the plate, Dynamic viscosity of water is 9.80×10^{-4} Pa.sec.

water - $\rho = 1000 \ kg/m^3$

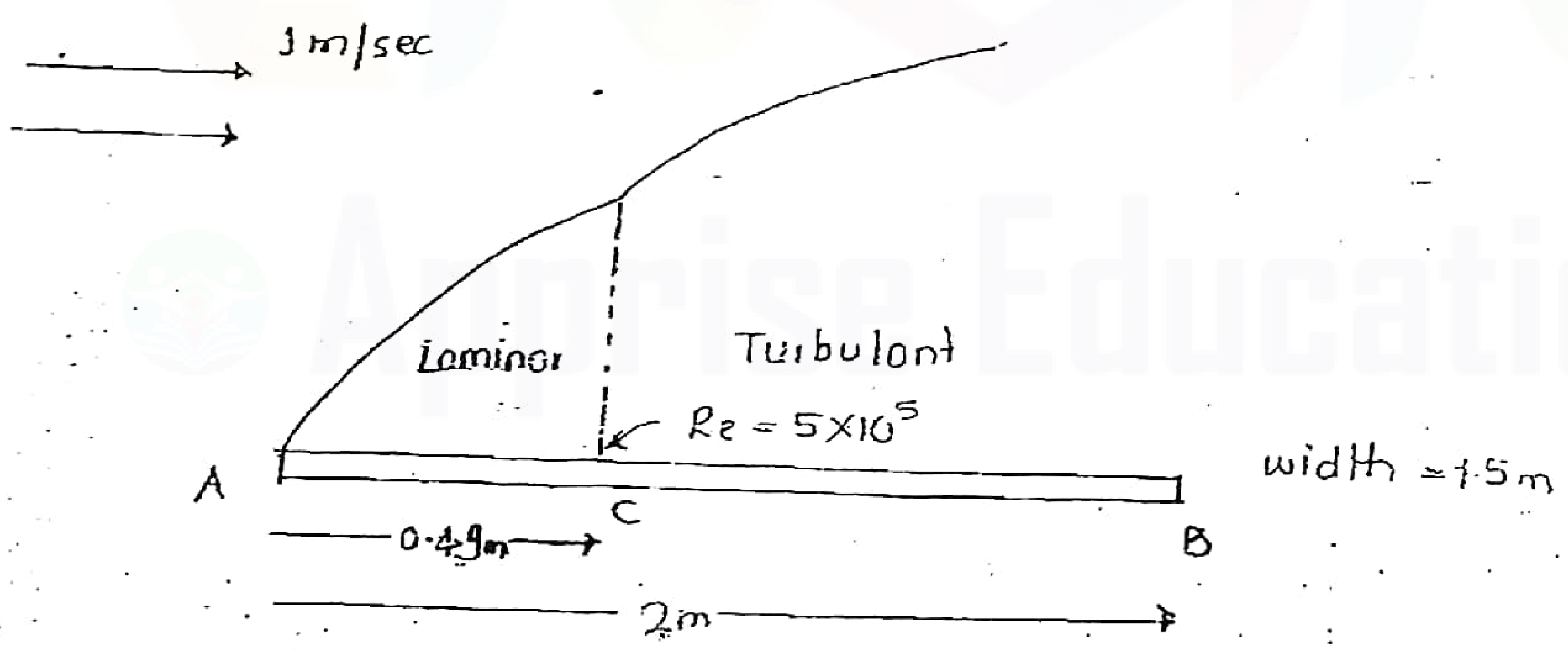
$\mu = 9.8 \times 10^{-4}$ Pa.sec

$U_\infty = 1$ m/sec.

To find $x_{critical}$:

$$(Re)_{critical} = 5\times10^5 = \frac{1000 \times 1 \times x_{critical}}{9.8\times10^{-4}}$$

$$x_{critical} = 0.49 \ m$$



AC (Laminar)

$$(F_D)_{AC-laminar} = \frac{0.664}{\sqrt{5\times10^5}} \times 2 \times \frac{1}{2} \times 1000 \times (1)^2 \times 0.49 \times 1.5$$

$$C_{fx} \times \left(\frac{1}{2} \times \rho \times U_\infty^2\right) \times \text{Area.}$$

$=$

For CB (Turbulent)

$$(F_D)_{CB-turbulent} = (F_D)_{AB-turbulent} - (F_D)_{AC-turbulent}$$

$$= \left[\frac{0.073}{(2.03\times10^6)^{1/5}} \times \frac{1}{2} \times 1000 \times 1^2 \times 2 \times 1.5\right]$$

$$- \left[\frac{0.073}{(5\times10^5)^{1/5}} \times \frac{1}{2} \times 1000 \times 1^2 \times 0.49 \times 1.5\right]$$

$=$

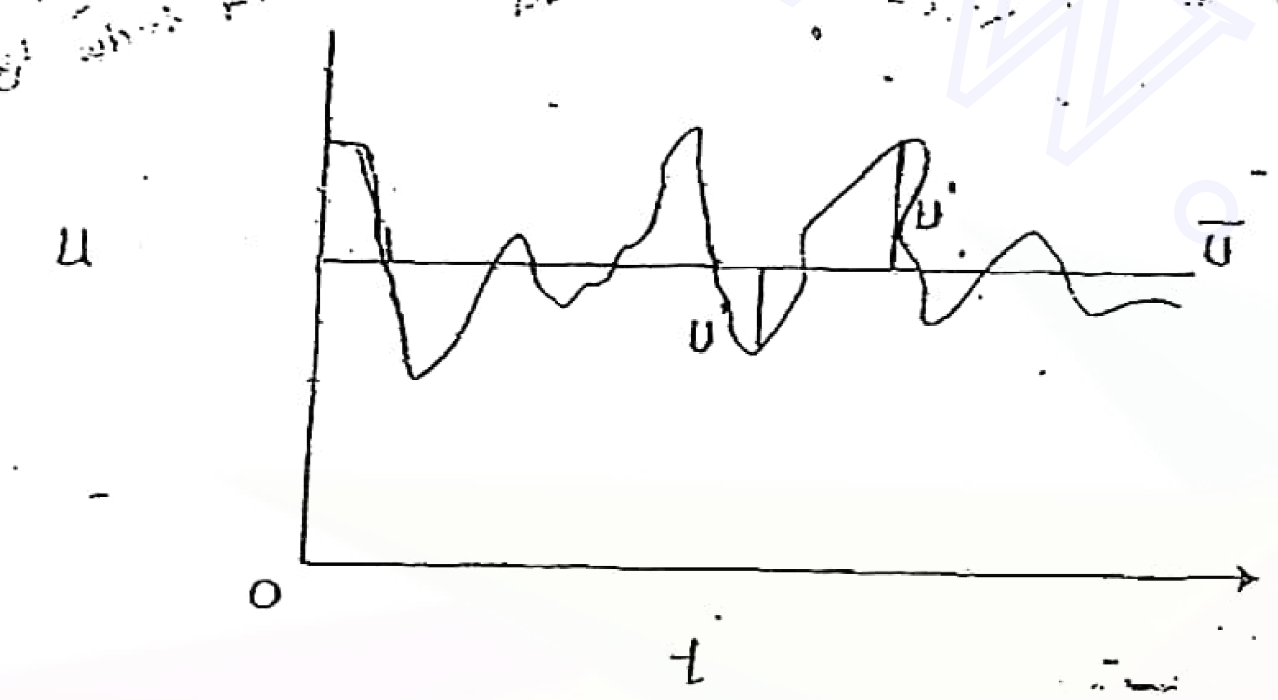
Total drag force

$$F_D = (F_D)_{AC-laminar} + (F_D)_{CB-turbulent}$$

$=$

The drag force will be doubled if the flow is on the both sides of the plate.

TURBULANT FLOWS



In turbulant flows,

$$u = \bar{u} + u'$$

$$v = \bar{v} + v'$$

$$w = \bar{w} + w'$$

where,

u', v', w' - velocity fluctuating components in X, Y & Z-directions.

Magnitude or level of turbulance

$$= \sqrt{\frac{u_1'^2 + v_1'^2 + w'^2}{3}}$$

(It is also called Root mean square fluctuating component)

Boussinesq's Theory :

The turbulant shear stress between the layers of fluid will be

$$\tau = \underbrace{\mu \frac{du}{dy}}_{} + \eta \frac{du}{dy}$$

where. Already given by Newton (μ - property of fluid)

η - Eddy (false) viscosity (property of flow)

According to Boussinesq

$$\eta \frac{du}{dy} >>>>> \mu \frac{du}{dy} \quad \text{in turbulant flows}$$

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8588058174

$$\tau = \eta \frac{du}{dy}$$

But the experimental determination of 'η' was not possible at that time.

Reynold's theory :

The turbulant flow shear stress is

$$\tau = \rho \cdot u' \cdot v'$$

where,

ρ - density of fluid

u', v' - fluctuating velocity components in X & Y

But experimental determination of u' and v' was not possible at that time.

Prandtle's mixing length theory :

Prandtle's mixing length (l)

It is distance travelled by fluid particle in jumping from one layer to adjacent layer. till it reaches the velocity of that adjacent layer.

$$u' \simeq v' \simeq l \cdot \frac{du}{dy}$$

By Karman's contril

$$l = k \cdot y$$

where

k - Karman's constant (universal)

$$k = 0.4$$

$$l = 0.4y$$

By Prandtle's theory

$$u' \simeq v' \simeq 0.4y \frac{du}{dy}$$

$$\tau = \rho u'v'$$

$$= \rho \cdot \left(0.4y \cdot \frac{du}{dy}\right)^2$$

$$0.4y \cdot \frac{du}{dy} = \sqrt{\frac{\tau}{\rho}} = V^* \text{ - shear velocity}$$

$$\int du = 2.5 V^* \int \frac{dy}{y}$$

$$u = 2.5 V^* \cdot \ln y + c$$ - local velocity

(logarithmic profile)

To find 'c'

By no slip condition.

At $y = y'$ $u = 0$

$$c = -2.5 V^* \ln y'$$

$$u = 2.5 V^* \ln\left(\frac{y}{y'}\right)$$

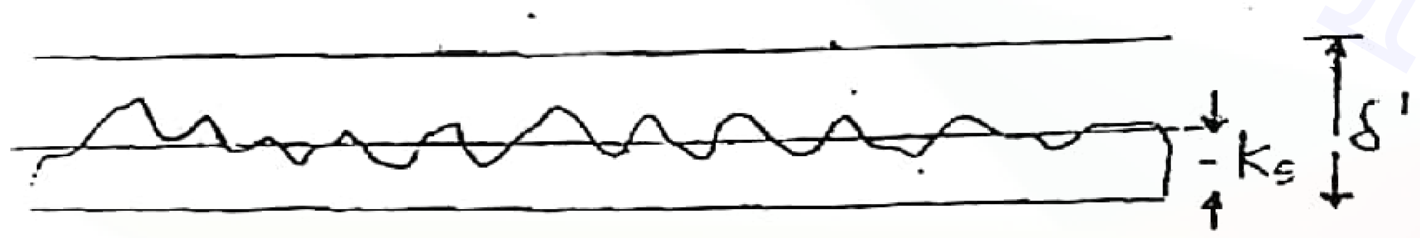
$$\frac{u}{V^*} = 2.5 \ln\left(\frac{y}{y'}\right)$$

$$\frac{u}{V^*} = 5.75 \log_{10}\left(\frac{y}{y'}\right)$$

...ically smooth boundary :

-- k_s - surface surface

δ' - laminar sub-layer thickness



If $\frac{k_s}{\delta'} < 0.25$ - smooth boundary

...'s experimental results,

$$y' = \frac{\delta'}{107} = \frac{11.6\,\nu}{V^* \cdot 107}$$

$$\frac{u}{V^*} = 5.75 \log_{10}\left(\frac{y}{\frac{11.6\,\nu}{V^* \times 107}}\right)$$

$$= 5.75 \log_{10}\left(\frac{y \cdot V^*}{\nu}\right) + 5.75 \log_{10}\left(\frac{107}{11.6}\right)$$

$$\frac{u}{V^*} = 5.75 \log_{10}\left(\frac{y \cdot V^*}{\nu}\right) + 5.55$$

u is function of y only.

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Hydrodynamically rough boundary :

If $\frac{k_s}{\delta'} > 6$ - rough surface

By Nikurde's experimental results,

$$y' = \frac{k_s}{30}$$

$$\frac{u}{V^*} = 5.75 \log\left(\frac{y}{k_s/30}\right)$$

$$\frac{u}{V^*} = 5.75 \log\left(\frac{y}{k_s}\right) + 8.5$$

Note :

(i) $\frac{u}{V^*} = 5.75 \log_{10}\left(\frac{y \cdot V^*}{\nu}\right) + 5.55$ (for smooth pipes)

(ii) $\frac{u}{V^*} = 5.75 \log_{10}\left(\frac{y}{k_s}\right) + 8.5$ (for rough pipes)

(iii) $\frac{u}{V^*} - \frac{\bar{u}}{V^*} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75$ (for both)

Friction factor in Turbulent flows :

For rough pipes :

$$\frac{u}{V^*} - \frac{\bar{u}}{V^*} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75$$

$$5.75 \log_{10}\left(\frac{y}{k_s}\right) + 8.5 - \frac{\bar{u}}{V^*} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75$$

$$\frac{\bar{u}}{V^*} = 5.75 \log_{10}\left(\frac{R}{k_s}\right) + 4.75$$

We know that

$$\sqrt{\frac{\tau}{\rho}} = \sqrt{\frac{f}{8}}\cdot \bar{u} = V^* \text{ - shear velocity}$$

$$\frac{\bar{u}}{V^*} = \sqrt{\frac{8}{f}}$$

$$\sqrt{\frac{8}{f}} = 5.75 \log_{10}\left(\frac{R}{K_s}\right) + 4.75$$

Q.
CPage

$\bar{u} = 2.6$ m/s

$u_{max} = 3.17$ m/s

for rough pipe, $f = ?$

$$\frac{u}{V^*} - \frac{\bar{u}}{V^*} = 5.75 \log\left(\frac{y}{R}\right) + 3.75$$

$$\frac{U_{max} - \bar{u}}{V^*} = 3.75 \qquad \text{at } y = R, \; u = U_{max}$$

$$\frac{3.17 - 2.6}{V^*} = 3.75$$

$V^* = 0.152$ m/sec

$$\frac{\bar{u}}{V^*} = \sqrt{\frac{8}{f}}$$

$$\frac{2.6}{0.152} = \sqrt{\frac{8}{f}}$$

$f = 0.027$

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Q. In turbulent flow through pipe find the distance from the centre where the velocity is same as mean velocity.

$u = \bar{u}$

$$\frac{u - \bar{u}}{V^*} = 5.75$$

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The end

fluid mechanics

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