Civinnovate

Discover, Learn, and Innovate in Civil Engineering

Tutorial 1

Fluid properties

1. A reservoir of glycerin has a mass of 1100kg and a volume of 0.9 m^3. Calculate its weight, mass density, specific weight and specific gravity.

Solution:
Mass of glycerin (m) = 1100kg
Volume (V) = 0.9 m^3
Weight (W) = ?
Mass density (ρ) = ?
Specific weight (γ) = ?
Specific gravity (S) = ?

W = mg = 1100x9.81 = 10791N
ρ = m/V = 1100/0.9 = 1222.22 Kg/m^3
γ = ρg = 1222.22x9.81 = 11990 N/m^3
S = $\gamma_{glycerine}/\ \gamma_{water}$ = 11990/9810 = 1.22

2. A liquid compressed in a cylinder has a volume of 2000 cm^3 at 2MN/m^2 and a volume of 1990 cm^3 at 4MN/m^2. What is its bulk modulus of elasticity?

Solution:
Initial volume (V) = 2000 cm^3
Final volume (V1) = 1990 cm^3
Change in volume (ΔV) = 1990-2000 = -10 cm^3
Change in pressure(ΔP) = 4-2 = 2MN/m^2
Bulk modulus of elasticity (K) = ?

$K = -\frac{\Delta P}{\Delta V/V} = -\frac{2}{-10/2000}$ = 400 MN/m^2

3. If the bulk modulus of elasticity of water is 2.2 Gpa (GN/m^2), what pressure is required to reduce a volume by 0.8%?

Solution:
Bulk modulus of elasticity (K) = 2.2 Gpa = 2.2x10^9 Pa
Reduction in volume ($\Delta V/V$) = -0.8% = -0.008
Pressure (P) = ?

$$K = -\frac{\Delta P}{\Delta V/V}$$

$$2.2x10^9 = -\frac{P-0}{-0.008}$$

P = 17600 KPa

4. At a depth of 7.5km in the ocean, the pressure is 75Mpa. Assume a specific weight at the surface of 10 KN/m^2 and an average bulk modulus of elasticity of 2.5 Gpa for that pressure range. Find (a) the change in specific volume between the surface and 7.5km (b) the specific volume at 7.5km and (c) the specific weight at 7.5km.

Solution:
Pressure at 7.5km (P2) = 75 Mpa = 75x10^6 N/m^2
Specific weight at the surface (γ_1) = 10 KN/m^2 = 10x1000 = 10000 N/m^2
Bulk modulus of elasticity at the surface (K) = 2.5 Gpa = 2.5x10^9 N/m^2
Change in specific volume (Δv_s) = ?
Specific volume at 7.5km (vs_2) = ?
Specific weight at 7.5km (γ_2) = ?

(a) Density at the surface (ρ_1) = γ_1/g = 10000/9.81 = 1019.4 kg/m^3
Specific volume at the surface (vs_1) = $1/\rho_1$ = 1/1019.4 = 0.000981 m^3/kg
Bulk modulus in terms of specific volume is

$$K = -\frac{\Delta P}{\Delta v_s/vs_1}$$

$$2.5x10^9 = -\frac{75x10^6 - 0}{\Delta v_s/0.000981}$$

Δv_s = -0.0000294 m^3/kg

(b) $vs_2 = vs_1 + \Delta v_s$ = 0.000981-0.0000294 = 0.000951 m^3/kg
(c) Density at 7.5km (ρ_2) = $1/vs_2$ = 1/0.000951 = 1051.5 kg/m^3
$\gamma_2 = \rho_2 g$ = 1051.5x9.81 = 10315 N/m^3

5. The surface tension of mercury and water at 60^0 c are 0.47N/m and 0.0662N/m respectively. What capillary height change will occur in these two fluids when they are in contact with air in a glass tube of radius 0.30mm? Use θ = 130^0 for mercury and 0^0 for water.

Solution:
Radius of tube (r) = 0.30 mm = 0.3/1000 = 0.0003m
Surface tension for mercury (σ_m) = 0.47N/m
Surface tension for water (σ) = 0.0662N/m
θ = 130^0 for mercury and 0^0 for water
Capillary height change for mercury (hm) = ?

Capillary height change for water (h) = ?

$hm = \frac{2\sigma_m Cos\theta}{\rho_m gr} = \frac{2x0.47xCos130}{13600x9.81x0.0003}$ = -0.0149m = -14.9mm

$h = \frac{2\sigma Cos\theta}{\rho gr} = \frac{2x0.0662xCos0}{1000x9.81x0.0003}$ = 0.0445m = 44.5mm

6. In a fluid the velocity measured at a distance of 75mm from the boundary is 1.125m/s. The fluid has absolute viscosity 0.048 NS/m^2 and relative density 0.9. What is the velocity gradient and shear stress at the boundary assuming a linear velocity distribution. Also calculate kinematic viscosity.

Solution:
Change in velocity (du) =1.125-0 = 1.125m/s
Change in distance (dy) = 75 – 0 = 75mm = 75/1000 m = 0.075m
Absolute viscosity (μ) = 0.048 NS/m^2
relative density (S)= 0.9
velocity gradient (du/dy) = ?
Shear stress (τ) = ?
Kinematic viscosity (υ) = ?

du/dy = 1.125/0.075 =15 s^{-1}

$\tau = \mu \frac{du}{dy} = 0.048x15$ = 0.72N/m^2

Density of fluid (ρ) = Sxρ_{water} = 0.9x1000 = 900kg/m^3
υ = μ/ρ = 0.048/900 = 5.3x10^{-5} m^2/s

7. The velocity distribution of a viscous liquid (dynamic viscosity = 9 Poise) flowing over a fixed plate is given by u = 0.85y - y^2 (u is velocity in m/s and y is the distance from the plate in m). What are the shear stresses at the plate surface and at y=0.3 m?

Solution:
u = 0.85y - y^2
Dynamic viscosity (μ) = 9 Poise =9/10 = 0.9 NS/m^2
Shear stress (τ) at plate (for y = 0) = ?
Shear stress (τ) for y = 0.3m = ?

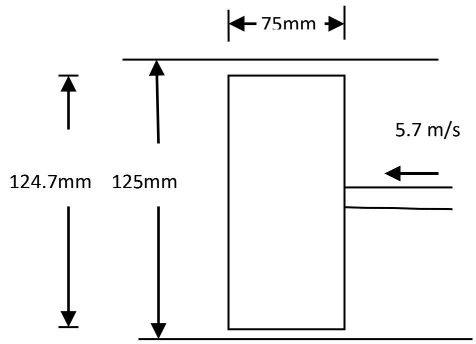
du/dy = d(0.85y - y^2)/dy = 0.85-2y

$$\tau = \mu \frac{du}{dy} = 0.9(0.85 - 2y)$$

For y = 0
Shear stress (τ) = 0.9(0.85-2x0) = 0.765 N/m^2

For y = 0.3m
Shear stress (τ) = 0.9(0.85-2x0.3) = 0.225 N/m^2

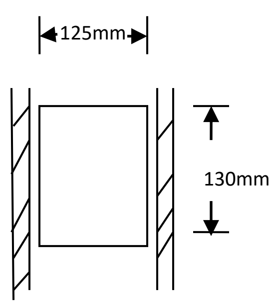
8. A piston is moving through a cylinder at a speed of 5.7m/s as shown in fig. The film of oil separating the piston from the cylinder has a viscosity of 0.95 Ns/m^2. What is the force required to maintain this motion?



Solution:
Speed of piston = 5.7m/s
Viscosity of oil (μ) = 0.95 NS/m^2
Diameter of piston (D) =124.7mm = 0.1247m
Length of piston (L) = 75mm = 0.075m
dv = 5.7m/s
dy = dr = (125-124.7)/2 mm = 0.15mm = 0.00015m
Frictional Force (F) = ?

F = Shear stress(τ) at the piston surface x surface area of piston (A)
$F = \mu \frac{du}{dy}(\pi DL) = 0.95x\frac{5.7}{0.00015}(\pi x 0.1247 x 0.075)$ = 1060.7N

9. A piston of weight 90N slides in a lubricated pipe. The clearance between piston and pipe is 0.025mm. If the piston decelerates at 0.6m/s^2 when the speed is 0.5m/s, what is the viscosity of the oil?



Solution:

Weight of piston (W) = 90N

Clearance (dy)= 0.025mm = 0.000025m

Deceleration (a) = 0.6m/s^2

Change in velocity (du) = 0.5m/s

Diameter of piston (D) = 125mm = 0.125m

Length of piston (L) = 130mm = 0.13m

Viscosity of oil (μ) = ?

Frictional force (F) = Shear stress(τ) at the piston surface x surface area of piston (A)

$F = \mu \frac{du}{dy}\,(\pi DL) = \ \mu x \frac{0.5}{0.000025}(\pi x 0.125 x 0.13) = 1021.02\mu$

Summing the forces

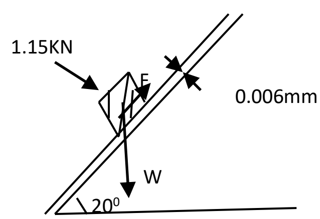
$$\sum F = ma$$

$$W - F = \frac{W}{g}a$$

$$90 - 1021.02\mu = \frac{90}{9.81}(-0.6)$$

μ = 0.094 NS/m^2

10. A square block weighing 1.15KN and 250mm on an edge slides down an incline on a film of oil 6μm thick. Assuming a linear velocity profile in the oil, calculate the terminal speed of the block. The viscosity of the oil is 0.007 NS/m^2.



Solution:

Weight of block (W) = 1.15KN =1150N

Side of block (L) = 250mm = 0.25m

Thickness (dy) = 6μm = 6x10^{-6}m

Viscosity of oil (μ) = 0.007 NS/m^2

Terminal velocity (u) = ?

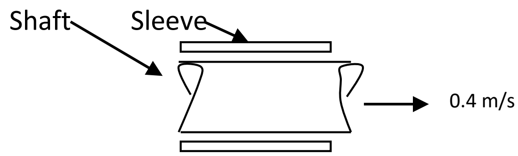
Frictional force (F) = Shear stress(τ) at block surface x surface area of the block (A)

$F = \mu \frac{du}{dy}\,(L^2) = 0.007x \frac{u}{6x10^{-6}}(0.25^2) = 72.9u$

Component of W in the direction of F is WSin20

At the terminal condition, equilibrium occurs.
F = Wsin20
72.9u = 1150xSin20
u = 5.4 m/s

11. A shaft 70mm in diameter is being pushed at a speed of 0.4m/s through a bearing sleeve 70.2mm in diameter and 250mm long. The clearance, assumed uniform, is filled with oil of kinematic viscosity 0.005 m^2/s and sp gr 0.9. Find the force exerted by the oil on the shaft.



Solution:
Diameter of shaft (D) = 70mm = 0.07m
Length of shaft (L) = 250mm = 0.25m
Change in velocity (du) = 0.4m/s
Kinematic viscosity of oil (υ) = 0.005 m^2/s
Sp gr of oil (S) = 0.9
Density of oil (ρ) = 0.9x1000 = 900kg/m^3
Dynamic viscosity (μ) = υρ = 0.005x900 = 4.5 NS/m^2
Clearance (dr) = dy = (70.2-70)/2 mm = 0.1mm = 0.0001m
Force exerted by the oil on the shaft (F) = ?

F = Shear stress(τ) at the shaft x surface area of shaft (A)
$F = \mu \frac{du}{dy}(\pi DL) = 4.5x\frac{0.4}{0.0001}(\pi x0.07x0.25)$ = 990N

12. A shaft 75mm in diameter is fixed axially and rotated inside a sleeve of diameter 75.2mm at 200rpm. The length of the shaft is 200mm. Determine the resisting torque exerted by the oil and the power required to rotate the shaft. Take viscosity of oil = 5 NS/m^2.

Solution:
Diameter of shaft (D) = 75mm = 0.075m
Radius of shaft (r) = 0.075/2 = 0.0375m
Length of shaft (L) = 200mm = 0.2m
Speed of shaft (N) = 200rpm
Dynamic viscosity of oil (μ) = 5 NS/m^2
Clearance (dr) = dy = (75.2-75)/2 mm = 0.1mm = 0.0001m
Torque exerted by the oil on the shaft (T) = ?
Power required to rotate shaft (P) = ?

Angular velocity (ω) = $\frac{2N\pi}{60} = \frac{2x200x\pi}{60}$ = 20.94rad/s
Tangential velocity (u) = $r\omega$ = 0.0375x20.94 = 0.785m/s
du = 7.85m/s

Frictional force (F) = Shear stress(τ) at the shaft x surface area of shaft = $\mu\frac{du}{dy}\ (\pi DL)$
$= 5x\frac{0.785}{0.0001}(\pi x 0.075 x 0.2)$ = 1849.6N
$T = Fr$ = 1849.6x0.0375= 69.4Nm
$P = F\ u$ = 1849.6x7.85 = 145194W = 1451.9 KW (or use P = Tω)

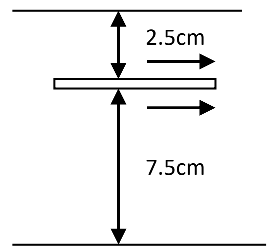
13. A cylinder of 12cm radius rotates concentrically inside a fixed cylinder of 12.6cm radius. Both cylinders are 0.3m long. Determine the viscosity of the liquid that fills the space between the cylinders if a torque of 0.9 Nm is required to maintain an angular velocity of 60rpm.

Solution:
Radius of outer cylinder (r_1) = 12.6cm = 0.126m
Radius of inner cylinder (r_2) = 12cm = 0.12m
dr =dy = 0.126-0.12 = 0.006m
Length of cylinder (L) = 0.3m
Torque (T) = 0.9Nm
N= 60rpm
Viscosity of the liquid (μ) = ?

Angular velocity (ω) = $\frac{2N\pi}{60} = \frac{2x60x\pi}{60}$ = 6.28rad/s
Tangential velocity of inner cylinder (u) = $r_2\omega$ = 0.12x6.28 = 0.75m/s
du = 0.75m/s
Frictional force (F) = Shear stress(τ) x surface area (A)
$T = F\ r_2 = \mu\frac{du}{dy}\ (2\pi r_2 L)r_2$
$0.9 = \ \mu\frac{0.75}{0.006}(2\pi x 0.120 x 0.3)x 0.120$
μ = 0.265 NS/m^2

14. A flat plate $0.3m^2$ in area moves edgewise through oil between large fixed parallels 10cm apart. If the velocity of plate is 0.6m/s and the oil has a kinematic viscosity of 0.45 stokes and specific gravity 0.8, calculate the drag force when (i) the plate is 2.5cm from one of the planes and (ii) the plate is equidistant from both the planes.

Solution:
Area of plate (A) = $0.3m^2$
Velocity of plate (u) = 0.6m/s
dv = 0.6m/s
Kinematic viscosity (υ) = 0.45 stokes = $0.45x10^{-4}$ m^2/s
Sp. gr. of fluid (S) = 0.8



Density of fluid (ρ) = 0.8x1000 = $800kg/m^3$
Dynamic viscosity (μ) = υρ = $0.45x10^{-4}$x800 = 0.036 NS/m^2

(I) dy1 = 2.5cm = 0.025m
dy2 = 10-2.5 = 7.5cm = 0.075m
Total force (F) = Force on side1 (F_1) + Force on side2 (F_2) = τ_1 A + τ_2 A = (τ_1 + τ_2) A

$F = \left(\mu\frac{du}{dy1} + \mu\frac{du}{dy2}\right)A = \left(0.036\frac{0.6}{0.025} + 0.036\frac{0.6}{0.075}\right)0.3$ = 0.345N

(I) If the plate is equidistant, $dy_1 = dy_2 = dy$ = 10/2 = 5cm = 0.05m
Total force (F) = Force on side1 (F_1) + Force on side2 (F_2) = τ_1 A + τ_2 A = (τ_1 + τ_2) A

$= \left(\mu\frac{du}{dy} + \mu\frac{du}{dy}\right)A = 2\mu\frac{du}{dy}A = \ 2x0.036x\frac{0.6}{0.05}x0.3$ = 0.26N

15. A Newtonian fluid fills the gap between a shaft and a concentric sleeve. When a force of 780N is applied to the sleeve parallel to the shaft, the sleeve attains a speed of 2m/s. If a 1400N force is applied, what speed will the sleeve attain? The temperature of the sleeve remains constant.

Solution:
Force (F1) = 780N
Velocity (u1) = 2m/s
Force (F2) = 1400N
Velocity (u2) = ?

$\tau = \mu\frac{du}{dy}$

Also, $\tau = \frac{F}{A}$

$$\mu\frac{du}{dy} = \frac{F}{A}$$

$\frac{F}{du} = \mu\frac{A}{dx}$ = const.

Therefore, $\frac{F1}{du1} = \frac{F2}{du2}$

$\frac{780}{2} = \frac{1400}{u2}$

u2 = 3.6m/s

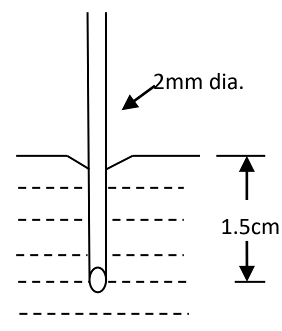
16. The surface tension of water in contact with air is 0.0725N/m. The pressure outside the droplet of water of diameter 0.02mm is atmospheric ($1.032x10^5$ N/m^2). Calculate the pressure within the droplet of water.

Solution:
Diameter of droplet = $0.02x10^{-3}$ m
Radius of droplet (r) = $0.01x10^{-3}$ m
Surface tension (σ) = 0.0725N/m
Pressure outside droplet (P_o) = $1.032x10^5$ N/m^2
Pressure within droplet (P_d) = ?

Pressure inside droplet in excess of outside pressure (P) = $\frac{2\sigma}{r} = \frac{2x0.0725}{0.01x10^{-3}}$ = 14500 N/m^2
$P_d = P+P_o$ = 14500+$1.032x10^5$ = 117700 N/m^2

17. The tip of glass tube with an internal diameter of 2mm is immersed to a depth of 1.5cm into a liquid of sp. gr. 0.85. Air is forced into the tube to form a spherical bubble just at the lower end of the tube. Estimate surface tension of liquid if the pressure in the bubble is 200Pa.

Solution:
Radius of bubble (r) = 1mm = 0.001m
Pressure inside bubble (P_i) = 200Pa
Depth of liquid (h) = 1.5cm = 0.015m
Specific weight of liquid (γ) = $sp\ gr\ x\gamma_{water}$ = 0.85x9810 = 8338.5Pa
Surface tension (σ) = ?



Pressure outside the bubble (P_o) = γh = 8338.4x0.015 = 125.07Pa

Pressure within droplet in excess of outside pressure (P) = 200-125.07 = 74.93Pa

$P = \frac{2\sigma}{r}$

$\sigma = \frac{Pr}{2} = \frac{74.93x0.001}{2}$ = 0.0375N/m

18. Three cylindrical tubes of 0.5m length are placed co-axially and the central tube is rotated at 5 rpm applying a torque of 6 Nm. Determine the viscosity of oil which fills the space between tubes. Take r_1, r_2 and r_3 as 0.15m, 0.152m and 0.154m.

Solution:

Radius of cylinder1 (r_1) = 0.15m

Radius of cylinder2 (r_2) = 0.152m

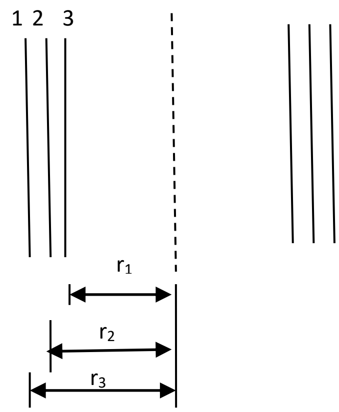
Radius of cylinder3 (r_3) = 0.154m

Length of cylinders (L) = 0.5m

Applied torque to cylinder2 (T) = 6 Nm

rpm of cylinder2 (N) = 6

Viscosity of oil (μ) =?



Torque is transmitted through oil layer from tube 2 to tube 1 and tube 3.

Angular velocity of tube 2 (ω) = $\frac{2N\pi}{60} = \frac{2x5x\pi}{60}$ = 0.5236rad/s

Tangential velocity tube2 (u_2) = $r_2\omega$ = 0.152x0.5236= 0.07958m/s

du = 0.07958m/s

F_1 = Viscous force on tube 1, F_2 = Viscous force on tube 2

For small space between cylinders, the velocity gradient may be assumed to be a straight line and average radius r can be taken.

$r_a = (r_1+r_2)/2$ = 0.151m, $r_b = (r_2+r_3)/2$ = 0.153m

$dy_1 = r_2-r_1$ = 0.002m, $dy_2 = r_3-r_2$ = 0.002m

$$T = F_1 r_a + F_2 r_b$$

$$T = \mu \frac{du}{dy_1}\,(2\pi r_a L)\,r_a + \mu \frac{du}{dy_2}\,(2\pi r_b L)\,r_b$$

$$6 = \mu \frac{0.07958}{0.002}\,(2\pi x 0.151 x 0.5)\,0.151 + \mu \frac{0.07958}{0.002}\,(2\pi x 0.153 x 0.5)\,0.153$$

μ = 1.038 NS/m^2

19. A fluid of absolute viscosity of 0.045 Pa-s and sp gr of 0.91 flows over a flat plate. The velocity of fluid at 70mm height over the plate is 1.13m/s. Calculate the shear stress at the solid boundary and at points 20mm above the plate considering (a) linear velocity distribution, and (b) parabolic velocity distribution with vertex at point 70mm away from the surface.

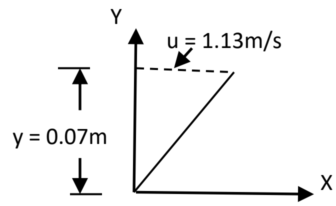
Solution:

Absolute viscosity (μ) =0.045 Pa-s

sp gr = 0.91

Velocity at 70mm from the plate = 1.13 m/s

(a) Linear velocity distribution



Equation: u = ay

a = tanθ = 1.13/0.7 = 16.1

u = 16.1y

$\frac{du}{dy} = 16.1$

As the velocity gradient is constant, shear stress is also constant throughout.

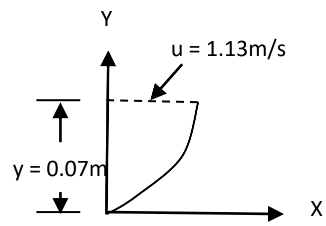
Shear stress $(\tau) = \mu\frac{du}{dy}$ = 0.045x16.1 = 0.726 Pa (throughout)

(b) Parabolic velocity distribution

Equation: $u = ay^2 + by + c$

At y = 0, u = 0

c = 0



At y = 0.07m, $\frac{du}{dy} = 0$

$\frac{du}{dy} = 2ay + b$

2ax0.07+b = 0

b = -0.14a

At y = 0.07m, u = 1.13m/s

$1.13 = ax0.07^2 + bx0.07 + c$

Substituting the values of b and c

$1.13 = ax0.07^2 - 0.14ax0.07 + 0$

a =-230.61

b = -0.14x-230.61 = 32.29

Hence, $u = -230.61y^2 + 32.29y$

$\frac{du}{dy} = -461.22y + 32.29$

At y = 0, $\frac{du}{dy} = 32.29$

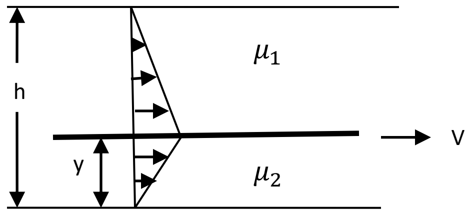
At y = 0.02m, $\frac{du}{dy} = -461.22x.02 + 32.29$ = 23.0656

Shear stress at y = 0, $(\tau_0) = \mu\frac{du}{dy}$ = 0.045x32.29 = 1.453 Pa

Shear stress at y = 0.02m, $(\tau_{0.02}) = \mu\frac{du}{dy}$ = 0.045x23.0656 = 1.038 Pa

20. Through a narrow gap of height h, a thin plate of large extent is pulled at a velocity V. On one side of the plate is oil of viscosity μ_1 and on the other side oil of viscosity μ_2. Calculate the position of the plate so that (a) the shear force in the two sides of the plate is equal, and (b) the pull required to drag the plate is minimum.

Solution:



(a) shear stress on upper face = $\tau_1 = \mu_1 \frac{du}{dy} = \mu_1 \frac{V}{h-y}$

shear stress on lower face = $\tau_2 = \mu_2 \frac{du}{dy} = \mu_2 \frac{V}{y}$

$\tau_1 = \tau_2$

$\mu_1 \frac{V}{h-y} = \mu_2 \frac{V}{y}$

$y = \frac{\mu_2 h}{\mu_1 + \mu_2}$

(b)F = pull required to drag the plate per unit area

$F = \tau_1 + \tau_2$

$F = \mu_1 \frac{V}{h-y} + \mu_2 \frac{V}{y}$

For F to be minimum,

dF/dy = 0

$\frac{dF}{dy} = \mu_1 \frac{V}{(h-y)^2} - \mu_2 \frac{V}{y^2}$

$0 = \mu_1 \frac{V}{(h-y)^2} - \mu_2 \frac{V}{y^2}$

$\frac{\mu_1}{\mu_2} = \frac{h^2 - 2hy + y^2}{y^2}$

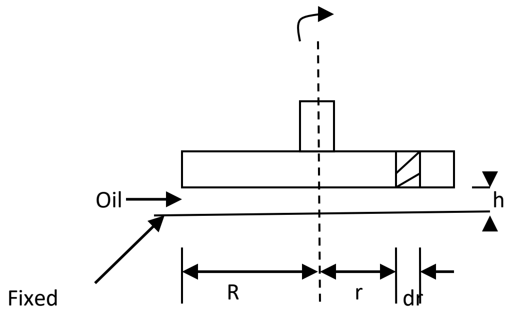
$\frac{h^2}{y^2} - \frac{2h}{y} + \left(1 - \frac{\mu_1}{\mu_2}\right) = 0$

Solving for h/y and neglecting –ve root

$\frac{h}{y} = 1 + \sqrt{\frac{\mu_1}{\mu_2}}$

$y = \frac{h}{1 + \sqrt{\frac{\mu_1}{\mu_2}}}$

21. A 120mm circular disc rotates on a table separated by an oil of film of 2mm thickness. Find the viscosity of the oil if the torque required to rotate the disc at 60 rpm is 4×10^{-4} Nm. Assume the velocity gradient in the oil film to be linear.



Solution:

Radius of disc (R) = 120/2 = 60mm = 0.06m

Thickness (h) = 2mm = 2x10^{-3}m

Torque (T) = 4x10^{-4} Nm

N = 60 rpm

Angular velocity (ω) = $\frac{2N\pi}{60} = \frac{2x60x\pi}{60}$ = 6.28rad/s

Viscosity of oil (μ) = ?

Consider an elementary ring or disc at radius r and having a width dr.

Torque on the element (dT) = Shear force x r

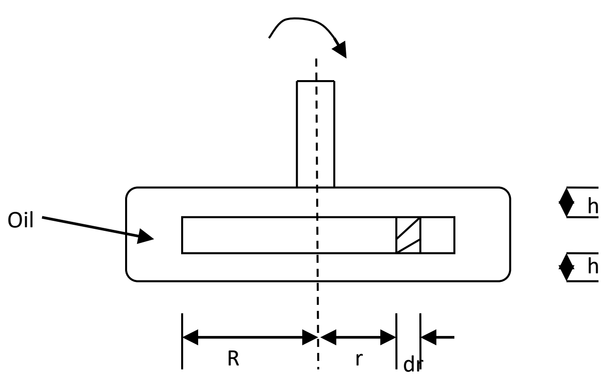
$= \tau dAr = \mu\frac{du}{dy}(2\pi rdr)r = \mu\frac{u}{h}(2\pi rdr)r = \mu\frac{r\omega}{h}(2\pi rdr)r = \frac{2\pi\mu\omega}{h}r^3dr$

Total torque (T) = $\int_o^R \frac{2\pi\mu\omega}{h}r^3dr = \frac{\pi\mu\omega R^4}{2h}$

$4x10^{-4} = \frac{\pi\mu x6.28x0.06^4}{2x2x10^{-3}}$

μ = 0.0062 Ns/m^2

22. A disk of radius R rotates at an angular velocity ω inside an oil bath of viscosity μ as shown in figure. Assuming a linear velocity profile and neglecting shear on the outer disk edges, derive an expression for the viscous torque on the disk.



Radius of disc = R

Clearance = h on both sides
Torque = T
N = rpm
Angular velocity = (ω) = $\frac{2N\pi}{60}$
Viscosity of oil = μ

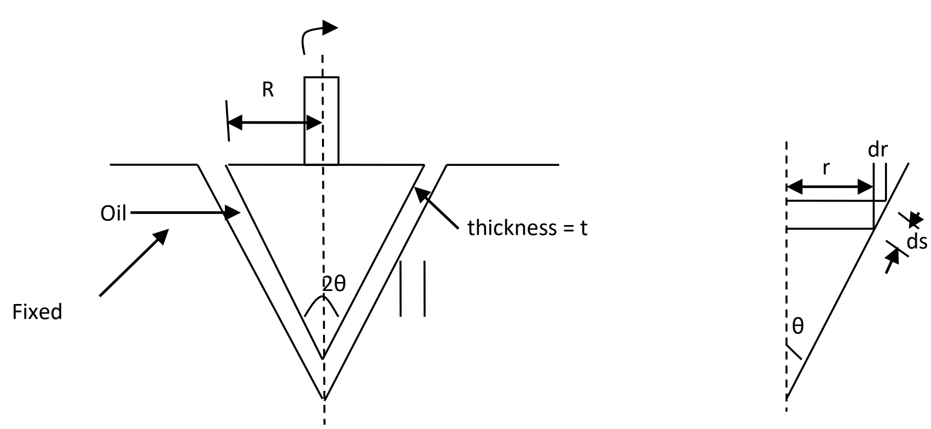
Consider an elementary ring or disc at radius r and having a width dr.

Torque on the element (dT) = Shear force x r
$= 2\tau dAr = 2\mu\frac{du}{dy}(2\pi rdr)r = 2\mu\frac{u}{h}(2\pi rdr)r = 2\mu\frac{r\omega}{h}(2\pi rdr)r = \frac{4\pi\mu\omega}{h}r^3dr$
Total torque (T) = $\int_o^R \frac{4\pi\mu\omega}{h}r^3dr = \frac{\pi\mu\omega R^4}{h}$

23. Distilled water stands in a glass tube of 10mm diameter at a height of 25mm. What is the true static height? Take surface tension of water = 0.074 N/m and angle of contact = 0^0.

Solution:
Diameter of tube (d) = 10 mm = 0.01m
Surface tension of water (σ) = 0.074N/m
θ = 0^0 for water
Capillary height change for water (h) is
$h = \frac{4\sigma Cos\theta}{\rho gd} = \frac{4x0.074xCos0}{1000x9.81x0.01}$ = 0.003m = 3mm
True static height = 25-3 = 22mm

24. A solid cone of maximum radius R and vertex angle is to rotate at an angular velocity ω . An oil of viscosity μ and thickness t fills the gap between the cone and the housing, Derive an expression for the torque required and the rate of heat dissipation in the bearing.



Solution:

Maximum radius of cone = R
thickness of oil = t
Angular velocity = (ω) = $\frac{2N\pi}{60}$
Viscosity of oil = μ

Consider an elementary area dA at radius r of the cone.

Torque on the element (dT) = Shear force x r
$= \tau dAr = \mu\frac{du}{dy}(2\pi rdr)r = \mu\frac{u}{t}(2\pi rds)r = \mu\frac{r\omega}{t}\left(2\pi r\frac{dr}{sin\theta}\right)r = \frac{2\pi\mu\omega}{tsin\theta}r^3dr$
Total torque (T) = $\int_o^R \frac{2\pi\mu\omega}{tsin\theta}r^3dr = \frac{\pi\mu\omega R^4}{2tsin\theta}$
Rate of heat dissipation = power utilized in overcoming resistance = $T\omega = \frac{\pi\mu\omega^2 R^4}{2tsin\theta}$

Tutorial 2

Fluid pressure

1. A cylinder contains a fluid at a gauge pressure of 360 KN/m^2. Express this pressure in terms of a head of (a) water, and (b) mercury of sp gr = 13.6
What would be the absolute pressure in the cylinder if atmospheric pressure is 760mm Hg.

Solution:
Pressure (P) = 360 KN/m^2 = 360x10^3 N/m^2
Head (h) = ?
$P = \rho g h$ where ρ = Density of fluid

$$h = \frac{P}{\rho g}$$

a) Head in terms of water (ρ = 1000 kg/m^3)

$h = \frac{360x10^3}{1000x9.81}$ = 36.7m

b) Head in terms of mercury
ρ = sp gr x density of water = 13.6x1000 = 13600 kg/m^3

$h = \frac{360x10^3}{13600x9.81}$ = 2.7m

Atmospheric pressure (h) = 760mmhg = 0.76m hg
Atmospheric pressure $(Patm) = \rho_{mercury} gh = 13600x9.81x0.76$ = 101396N/m^2 = 101.3KN/m^2
Absolute pressure (Pabs) = ?
Pabs = Pgauge + Patm = 360+101.3 = 461.3KN/m^2

2. What would the pressure in kN/m^2 be if the equivalent head is measured as 400mm of (a) mercury (sp gr 13.6) (b) water (c) oil specific weight 7.9 kN/m^3 (d) a liquid of density 520 kg/m^3?

Solution:
Head (h) = 400mm = 0.4m
Pressure (P) =?
$P = \rho g h$ where ρ = Density of fluid

a) In terms of mercury, ρ = sp gr x density of water = 13.6x1000 = 13600 kg/m^3
$P = \rho g h$ = 13600x9.81x0.4 = 53366 N/m^2 = 53.366 KN/m^2
b) In terms of water, ρ = 1000 kg/m^3
$P = \rho g h$ = 1000x9.81x0.4 = 3924 N/m^2 = 3.924 KN/m^2
c) In terms of oil of sp. wt. (γ_{oil}) = 7.9 kN/m^3

$P = \rho gh = \gamma_{oil}h$ = 7.9x.4 = 3.16 KN/m^2
d) In terms of liquid with ρ = 520 kg/m^3
$P = \rho gh$ = 520x9.81x0.4 = 2040 N/m^2 = 2.04 KN/m^2

3. A manometer connected to a pipe indicates a negative gauge pressure of 50mm of mercury. What is the absolute pressure in the pipe in N/m^2 if the atmospheric pressure is 1 bar?

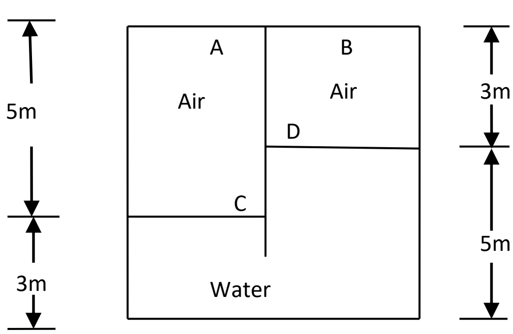
Solution:
Atmospheric pressure (Patm) = 1 bar = 1x10^5 N/m^2
Head (h) = -50mmhg = -0.05m hg
Absolute pressure (pabs) = ?

ρ of mercury = sp gr x density of water = 13.6x1000 = 13600 kg/m^3
Gauge pressure (Pgauge) $= \rho gh$ = -13600x9.81x0.05 = -6671 N/m^2
Pabs = Pgauge + Patm = -6671+1x10^5 = 93329 N/m^2 = 93.3KN/m^2

4. An open tank contains 5.7m of water covered with 2.6m of kerosene (sp wt = 8KN/m^3). Find the pressure at the interface and at the bottom of the tank.

Solution:
Height of kerosene (h) = 2.6m
Height of water (h1) = 5.7m
Sp wt of kerosene (γ) = 8KN/m^3
Pressure at interface (Pint) = ?
Pressure at bottom (Pbottom) = ?
$Pint = \gamma h$ = 8x2.6 = 20.8KN/m^2
$Pbottom = Pint + \gamma_{water}h1$ = 20.8+9.81x5.7 = 76.7 KN/m^2

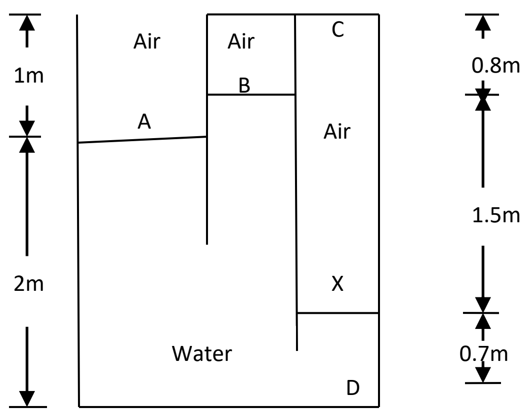
5. The closed tank in the fig. is at 20^0 C. If the pressure at point A is 96 Kpa absolute, what is the absolute pressure at point B? What percent error results from neglecting the specific weight of air? (Take sp wt of air = 0.0118 KN/m^3)



Solution:

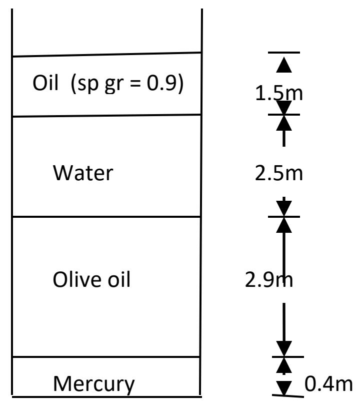
Sp wt of air (γ_{air}) = 0.0118 KN/m^3
Sp wt of water (γ_{water}) = 9.81 KN/m^3
Starting from A,
$P_A + P_{AC} - P_{CD} - P_{DB} = P_B$
$P_A + \gamma_{air} h_{AC} - \gamma_{water} h_{DC} - \gamma_{air} h_{DB} = P_B$
P_B = 96 + 0.0118x5 – 9.81x2 – 0.0118x3 = 76.404 Kpa
Neglecting air,
$P_A - \gamma_{water} h_{DC} = P_B$
P_B = 96 – 9.81x2 = 76.38 Kpa
Error = (76.404-76.38)/76.404 = 0.00031 = 0.031%

6. In the fig., the pressure at point A is 2900 N/m^2. Determine the pressures at points B, C and D. (Take density of air = 1.2 kg/m^3)



Solution:
Density of water (ρ) = 1000kg/m^3
P_A = 2900 N/m^2
Density of air (ρ_{air}) = 1.2 kg/m^3
P_B = ?, P_C =?, P_D = ?
Starting from A,
$P_B = P_A - \rho g h_{between\ A\ and\ B}$ = 2900 – 1000x9.81x0.2 = 938 N/m^2
$P_C = P_A + \rho g h_{between\ A\ and\ X} - \rho_{air} g h_{XC}$ = 2900 + 1000x9.81x1.3 -1.2x9.81x2.3 = 15626 N/m^2
$P_D = P_A + \rho g h_{between\ A\ and\ D}$ = 2900 + 1000x9.81x2 = 22520 N/m^2

7. In the fig., the absolute pressure at the bottom of the tank is 233.5 Kpa. Compute the sp gr of olive oil. Take atmospheric pressure = 101.3 Kpa.



Solution:

Absolute pressure at bottom (P_{abs}) =233.5 Kpa

Atmospheric pressure (P_{atm}) = 101.3 Kpa

Sp wt of water (γ) = 9.81 KN/m^3

Sp wt of oil (γ_{oil}) = 0.9x9.81 KN/m^3 = 8.829 KN/m^3

Sp wt of mercury (γ_m) = 13.6x9.81 KN/m^3 = 133.416 KN/m^3

Sp gr of olive oil (S) = ?

$P_{abs} = P_{atm} + P_{gauge}$

$P_{abs} = P_{atm} + P_{oil} + P_{water} + P_{olive\ oil} + P_{mercury}$

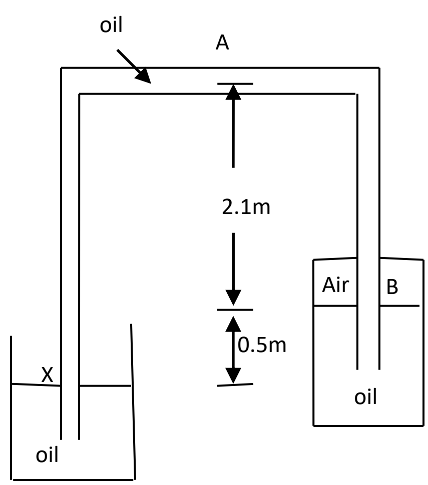
$233.5 = 101.3 + \gamma_{oil}\, h_{oil} + \gamma\, h_{water} + \gamma_{olive\ oil}\, h_{olive\ oil} + \gamma_m\, h_{mercury}$

$233.5 = 101.3 + 8.829x1.5 + 9.81x\ 2.5 + \gamma_{olive\ oil}\, x2.9 + 133.416x0.4$

$\gamma_{olive\ oil} = 14.16$ KN/m^3

$S = \frac{\gamma_{olive\ oil}}{\gamma} = \frac{14.16}{9.81} = 1.44$

8. The tube shown in the fig. is filled with oil of sp gr 0.82. Determine the pressure heads at A and B in meters of water.

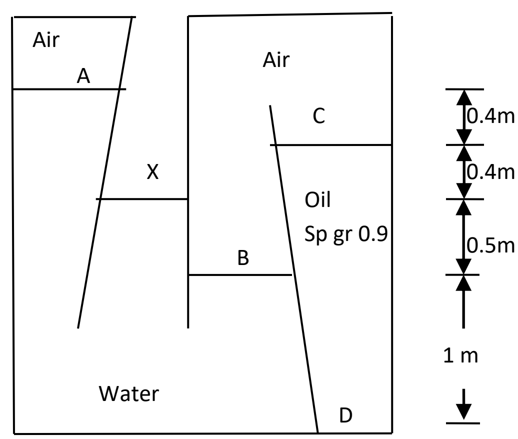


Solution:
sp gr of oil = 0.82
Sp wt of oil (γ_{oil}) = 0.82x9810 = 8044.2 N/m^3
Head in terms of water at a and B (h_A and h_B) = ?
Take atmospheric pressure to be 0 for gauge pressure.
$P_A = 0 - \gamma_{oil}\, h_{between\ X\ and\ A}$ = - 8044.2x2.6 = -20914.9 Pa

$h_A = \frac{P_A}{\gamma_{water}} = -\frac{20914.9}{9810}$ = -2.132 m
$P_B = P_A + \gamma_{oil}\, h_{between\ A\ and\ B}$ = -20914.9 + 8044.2x2.1= -4022.1 Pa

$h_B = \frac{P_B}{\gamma_{water}} = -\frac{4022.1}{9810}$ = -0.41 m

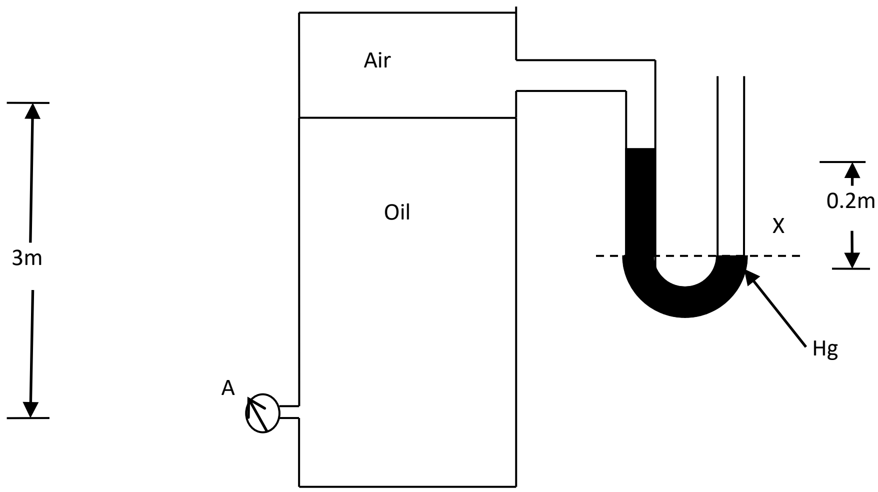
9. Calculate the pressures at A, B, C and D in the fig.



Solution:
Sp wt of water (γ) = 9810 N/m^3
sp gr of oil = 0.9
Sp wt of oil (γ_{oil}) = 0.9x9810 = 8829 N/m^3

Take atmospheric pressure to be 0 for gauge pressure.
$P_A = 0 - \gamma\, h_{between\ X\ and\ A}$ = - 9810x0.8 = 7848 Pa
$P_B = 0 + \gamma\, h_{between\ X\ and\ B}$ = 9810x0.5 = 4905 Pa
Neglecting air, P_C =P_B = 4905 Pa
$P_D = P_C + \gamma_{oil}\, h_{between\ C\ and\ D}$ = 4905 + 8829x1.9 = 21680 Pa

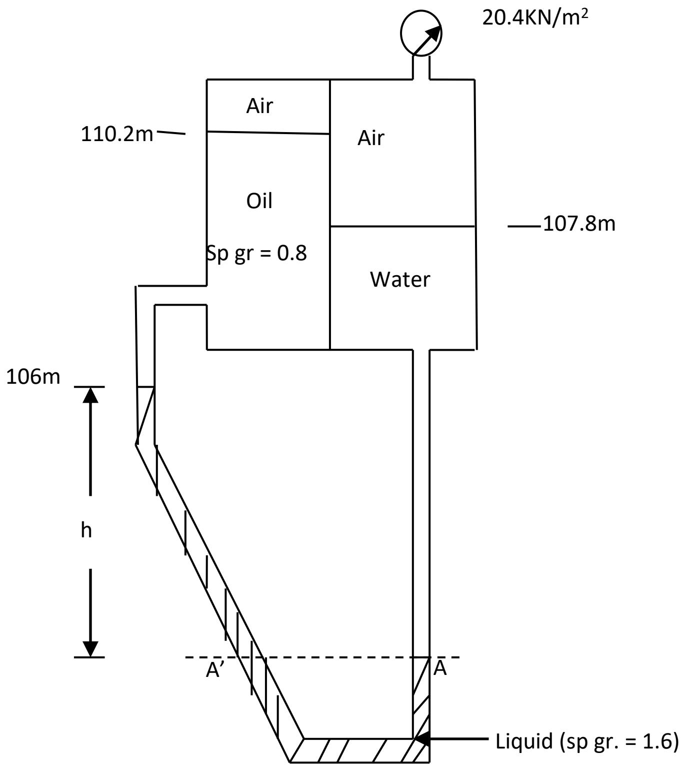
10. The tank in the fig. contains oil of sp gr 0.75. Determine the reading of gauge A in N/m^2.



Solution:
sp gr of oil = 0.75
Sp wt of oil (γ_{oil}) = 0.75x9810 = 7357.5 N/m^3
Sp wt of mercury (γ_m) = 13600x9.81 N/m^3 = 133416 N/m^3

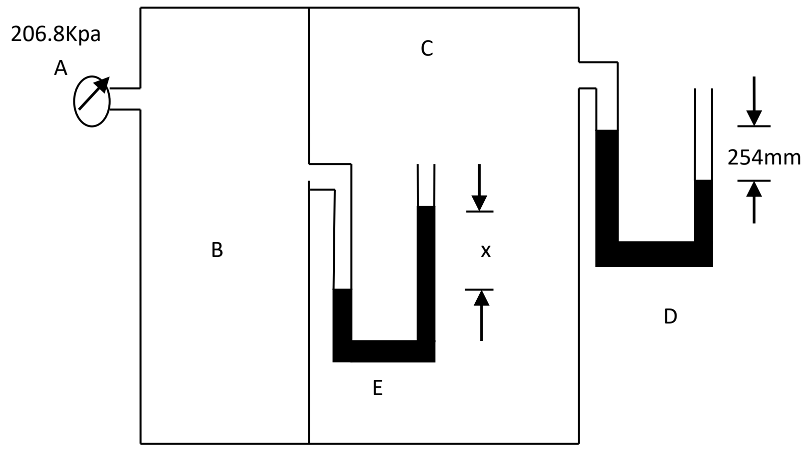
Take atmospheric pressure to be 0 for gauge pressure.
Starting from X and neglecting air,
$0 - \gamma_m h_m + \gamma_{oil}\, h_{oil} = P_A$
PA = -133416x0.2+7357.5x3 = -4610.7 N/m^2

11. In the left hand of the fig., the air pressure is -225mm of Hg. Determine the elevation of the gauge liquid in the right hand column at A.



Solution:
Air pressure at the left hand tank = -225mm Hg = -0.225m Hg
Sp wt of water (γ) = 9810 N/m^3
Sp wt of oil (γ_{oil}) = 0.8x9810 = 7848 N/m^3
Sp wt of mercury (γ_m) = 13600x9.81 KN/m3 = 133416 KN/m^3
Sp wt of liquid (γ_{liquid}) = 1.6x9810 = 15696 N/m^3
$P_{air} = -0.225\gamma_m$ = -0.225x133416 = -30018.6 N/m^2
$P_{A'} = P_A$
$P_{air} + \gamma_{oil}\, h_{oil} + \gamma_{liauid}\, h_{liquid} \ = 20400 + \gamma h_{water}$
-30018.6 + 7848x(110.2-106) + 15696 xh = 20400 + 9810 x(107.8-106+h)
h = 5.96m
Elevation at A = 106-5.96 = 100.04m

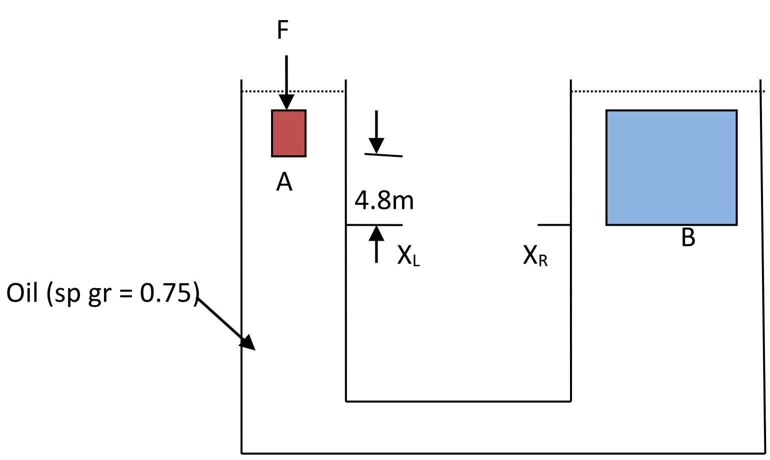
12. Compartments B and C in the fig. are closed and filled with air. The barometer reads 99.98 Kpa. When gages A and D read as indicated, what should be the value of x for gage E? (Hg in each tube)



Solution:
P_A = 206.8 KPa = 206800 Pa
Sp wt of mercury (γ_m) = 13.6x9810 N/m^3 = 133416 N/m^3

Starting from A and neglecting air,
$P_A - \gamma_m X + \gamma_m x\, 0.254 = 0$
206800 – 133416 X + 133416x0.254 = 0
X = 1.8m

13. In the fig., the areas of the plunger A and cylinder B are 38.7 cm^2 and 387 cm^2, respectively, and the weight of B is 4500 N. The vessel and the connecting passages are filled with oil of specific gravity 0.75. What force F is required for equilibrium, neglecting the weight of A?



Solution:

Area of A (A_A) =38.7 cm^2, Area of B (A_B) =387 cm^2

Weight of B (W_B) = 4500N

Sp wt of water (γ) = 9810 N/m^3

Sp wt of oil (γ_{oil}) = 0.75x9810 N/m^3 = 7357.5 N/m^3

Pressure at X_L = Pressure at X_R

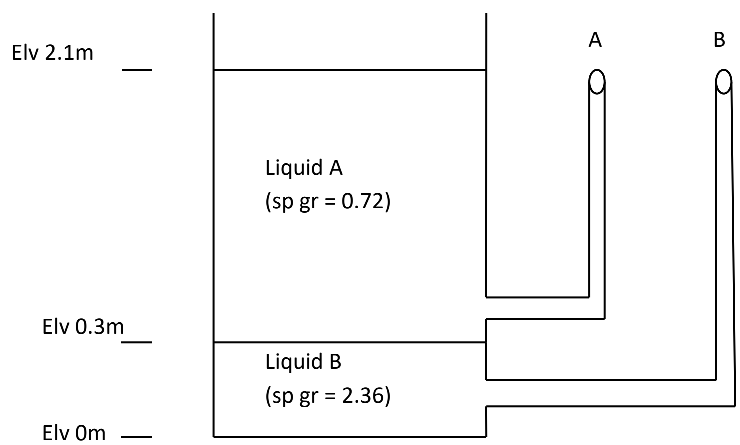
$P_A + \gamma_{oil}\, h_{oil\ from\ A\ to\ X_L} = W_B/A_B$

P_A + 7357.5x4.8 = 4500/387x10^{-4}

P_A = 80963 N/m^2

F = P_A A_A = 80963 x 38.7x10^{-4} = 313N

14. For the open tank, with piezometers attached on the side, containing two different immiscible liquids as shown in the fig., find (a) the elevation of liquid surface in piezometer A, (b) the elevation of liquid surface in piezometer B, and (c) the total pressure at the bottom of the tank.



Solution:

Sp wt of liquid A (γ_A) = 0.72x9810 N/m^3 = 7063.2 N/m^3

Sp wt of liquid B (γ_B) = 2.36x9810 N/m^3 = 23151.6 N/m^3

a) Elevation of liquid surface in piezometer A = elevation of liquid A in the tank = 2.1 m

b) Pressure due to A at the interface (P_A) = $\gamma_A\, h_{liquid\ A}$ = 7063.2 x (2.1-0.3) = 12713.8 N/m^2

Equivalent head for liquid B due to P_A is

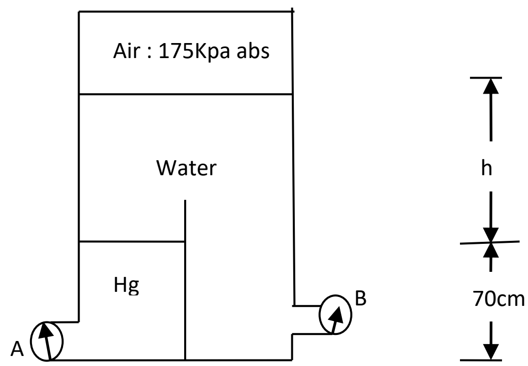
$h_A = P_A/\gamma_B$ = 12713.8/23151.6 = 0.55m

Elevation of liquid surface in piezometer B = Elv of liquid at B + h_A = 0.3 + 0.55 = 0.85m

c) Total pressure at the bottom = $\gamma_A\, h_{liquid\ A} + \gamma_B\, h_{liquid\ B}$

= 7063.2 x (2.1-0.3) + 23151.6x0.3 = 19659 N/m^2

15. In the fig., gage A reads 290Kpa abs. What is the height of water h? What does gage B read?

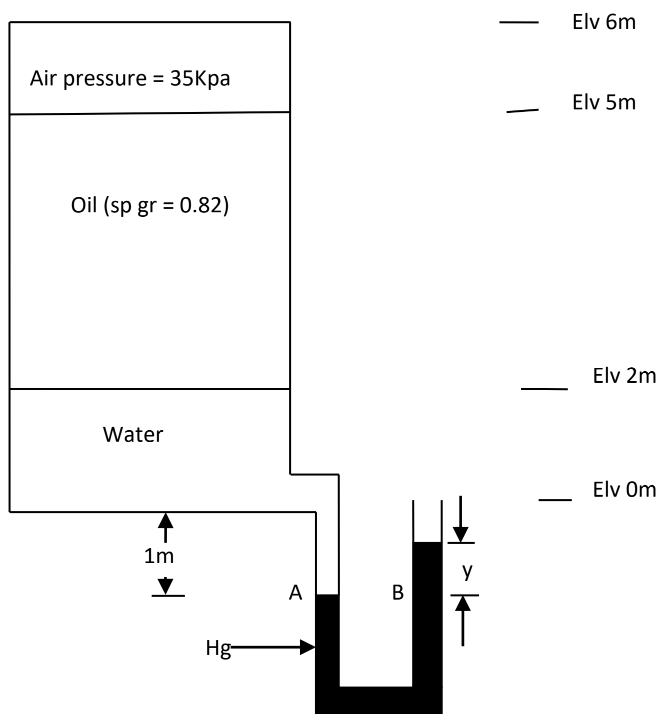


Solution:
Sp wt of water (γ) = 9.81 KN/m^3
Sp wt of mercury (γ_m) = 13.6x9.81 = 133.416 KN/m^3
P_A =290Kpa abs
h = ?
P_B =?

$P_A = 175 + \gamma h + \gamma_m h_m$
290 = 175 + 9.81h+133.416x0.7
h = 2.2m

$P_B = 175 + \gamma\ (h + 0.7)$
P_B = 175 + 9.81 (2.2+0.7) = 203.4 KN/m^2

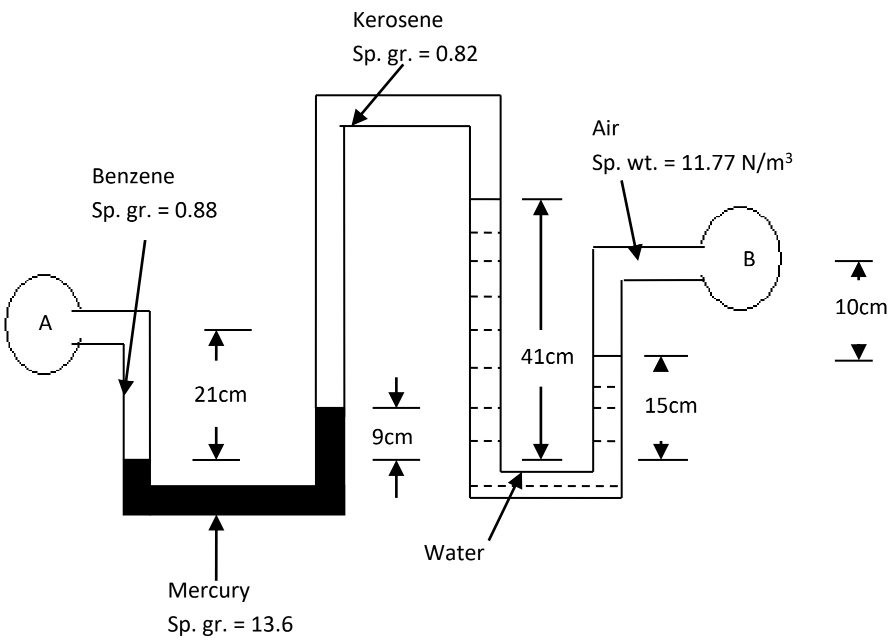
16. A manometer is attached to a tank containing three different fluids as shown in fig. What will be the difference in elevation of the mercury column in the manometer (i.e. y)?



Solution:
Sp wt of water (γ) = 9.81 KN/m^3
Sp wt of mercury (γ_m) = 13.6x9.81 = 133.416 KN/m^3
Sp wt of oil (γ_{oil}) = 0.82x9.81 = 8.0442 KN/m^3

$P_A = P_B$
$35 + \gamma_{oil} h_{oil} + \gamma h = \gamma_m \, y$
35+ 8.0442x3+9.81x3= 133.416y
y = 0.66m

17. Determine the pressure difference between two points A and B in the fig.



Solution:

Sp wt of water (γ) = 9.81 KN/m^3
Sp wt of mercury (γ_m) = 13.6x9.81 = 133.416 KN/m^3
Sp wt of benzene (γ_B) = 0.88x9.81 = 8.6328 KN/m^3
Sp wt of kerosene (γ_K) = 0.82x9.81 = 8.0442 KN/m^3
Sp wt of air (γ_{air}) = 0.01177 KN/m^3

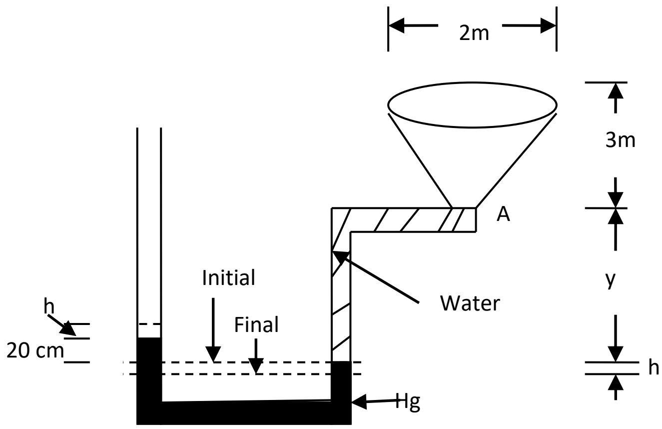
Starting from A,

$P_A + \gamma_B x0.21 - \gamma_m x0.09 - \gamma_K x(0.41 - 0.09) + \gamma(0.41 - 0.15) - \gamma_{air} x0.1 = P_B$

$P_A + 8.6328x0.21 - 133.416x0.09 - 8.0442x0.32 + 9.81x\ 0.26 - 0.01177x0.1 = P_B$

$P_A - P_B$ = 10.22 Kpa

18. Fig. below shows a conical vessel having its outlet at A to which a U-tube manometer is connected. The reading of the manometer given in the fig. shows when the vessel is empty. Find the reading of the manometer when the vessel is completely filled with water.

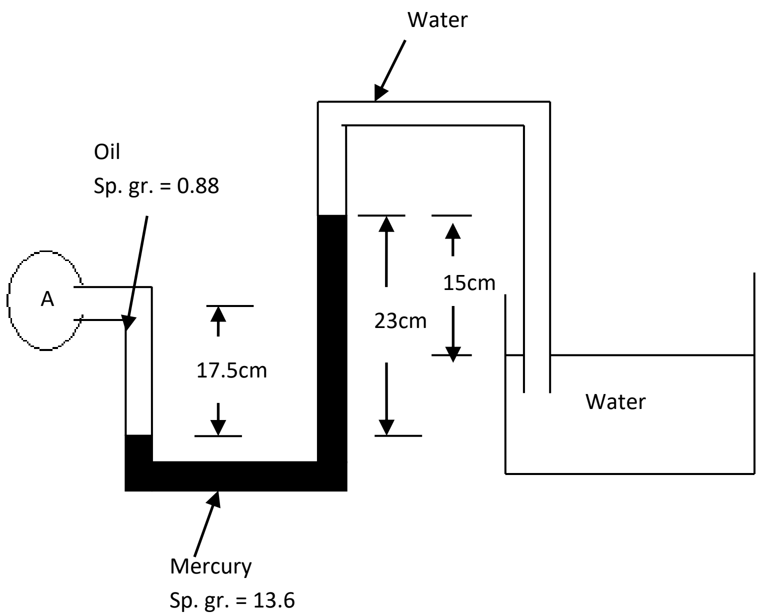


Solution:
Sp wt of water (γ) = 9.81 KN/m^3
Sp wt of mercury (γ_m) = 13.6x9.81 = 133.416 KN/m^3

When the vessel is empty, equating pressure at initial level
$\gamma y = \gamma_m x0.2$
9.81 y = 133.416x0.2
y = 2.72m

When the vessel is filled with water, let us say the mercury moves by height h
Equating pressure at final level
$\gamma(3 + y + h) = \gamma_m(h + 0.2 + h)$
$9.81(3 + 2.72 + h) = 133.416(0.2 + 2h)$
h = 0.1145m
Deflection of mercury = 2h+0.2 =2x0.1145+0.2 = 0.429m

19. Compute the absolute pressure at point A in the fig.



Solution:

Sp wt of water (γ) = 9.81 KN/m^3
Sp wt of mercury (γ_m) = 13.6x9.81 = 133.416 KN/m^3
Sp wt of oil (γ_{oil}) = 0.88x9.81 = 8.6328 KN/m^3

Starting from A,

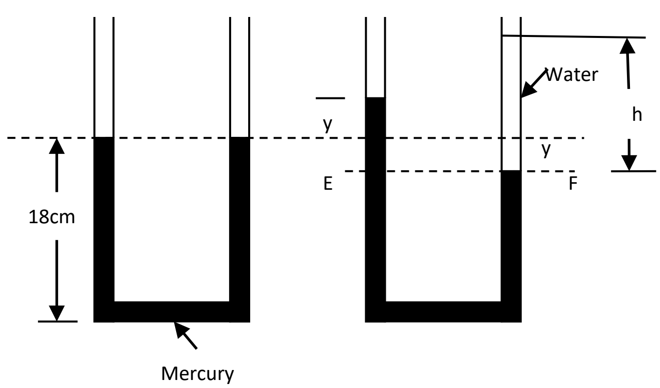
$P_A + \gamma_{oil}x0.175 - \gamma_m x0.23 + \gamma x0.15 = 0$

$P_A + 8.6328x0.175 - 133.416x0.23 + 9.81x0.15 = 0$

PA = 27.7 Kpa

Absolute pressure at A = Patm +27.7 = 101.3+27.7 = 129Kpa

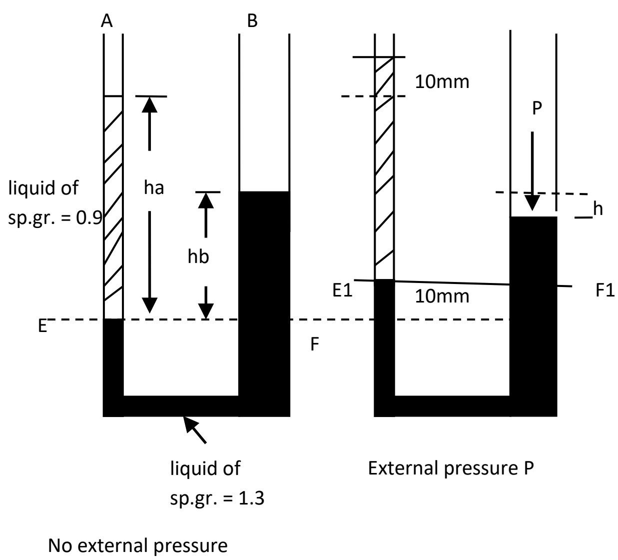
20. The fig. shows a 1cm diameter U-tube containing mercury. If now 20cc of water is poured into the right leg, find the levels of the free liquid surfaces in the two tubes.



Solution:
With the addition of 20ml of water,
Drop in mercury level in right leg = y
Rise in mercury level in left leg = y
Depth of water column = h
h= Vol of water added/Cross sectional area = $20x10^{-6}/\left(\frac{\pi}{4}x0.01^2\right)$ = 0.254m

$P_E = P_F$
$\gamma_{mercury}x2y = \gamma_{water}xh$
13.6x9810x2y = 9810x0.254
y = 0.0093m = 0.93cm
Height of free mercury level in the left leg = 18+0.93 = 18.93cm
Height of free water level in the right leg = 18-0.93+25.4 = 42.47cm

21. The diameters of the limbs A and B of a U-tube shown in fig. are 5mm and 20mm respectively. The limb A contains a liquid of sp. gr. 0.9 while the limb B contains a liquid of sp.gr. 1.3. The fig. shows the position of the liquids in the two limbs. Find what pressure should be applied on the surface of the heavier liquid in limb B so that the rise in level in the limb A is 10mm.



Solution:
Sp gr of liquid in A = 0.9
Sp. gr. of liquid in B = 1.3
Diameter of limb A = 5mm
Diameter of limb B = 20mm

a. When no external pressure is applied

$P_E = P_F$

$\gamma_A x h_a = \gamma_B x h_b$

$0.9x9810xh_a = 1.3x9810xh_b$

$h_a = 1.3h_b/0.9$ (a)

b. When external force p is applied on the surface of liquid B

Rise in limb A = 10mm

Fall in limb B = h

Level EF shifts to new position is E1F1.

Volume of liquid transferred to left limb = Volume of liquid fell in right limb

c/s area of left limbx10mm = c/s of right limbxh

$\left(\frac{\pi}{4}x(5x10^{-3})^2\right)x10x10^{-3} = \left(\frac{\pi}{4}x(20x10^{-3})^2\right)h$

h = 0.000625m

$P_{E1} = P_{F1}$

$\gamma_A x(h_a + 10x10^{-3} - 10x10^{-3}) = P + \gamma_B x(h_b - 10x10^{-3} - h)$

$0.9x9810xh_a = P + 1.3x9810x(h_b - 10x10^{-3} - 0.000625)$

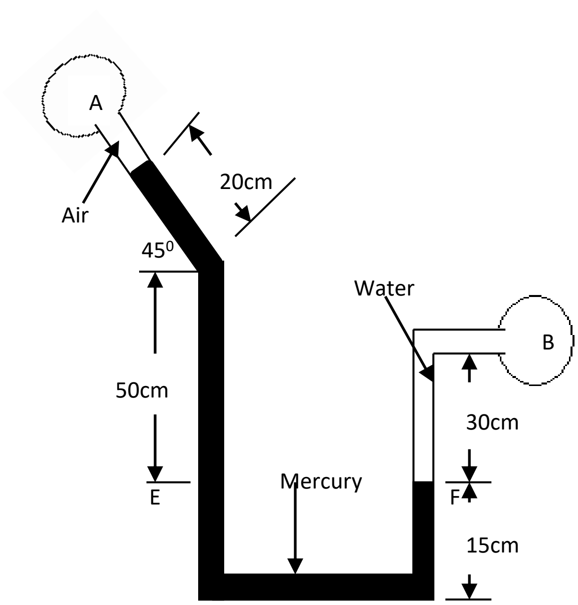
$8829h_a = P + 12753(h_b - 0.010625)$ (b)

From a and b

$8829x1.3h_b/0.9 = P + 12753(h_b - 0.010625)$

P = 135.5 N/m^2

22. Find the pressure difference between containers A and B shown in the figure.



Solution:

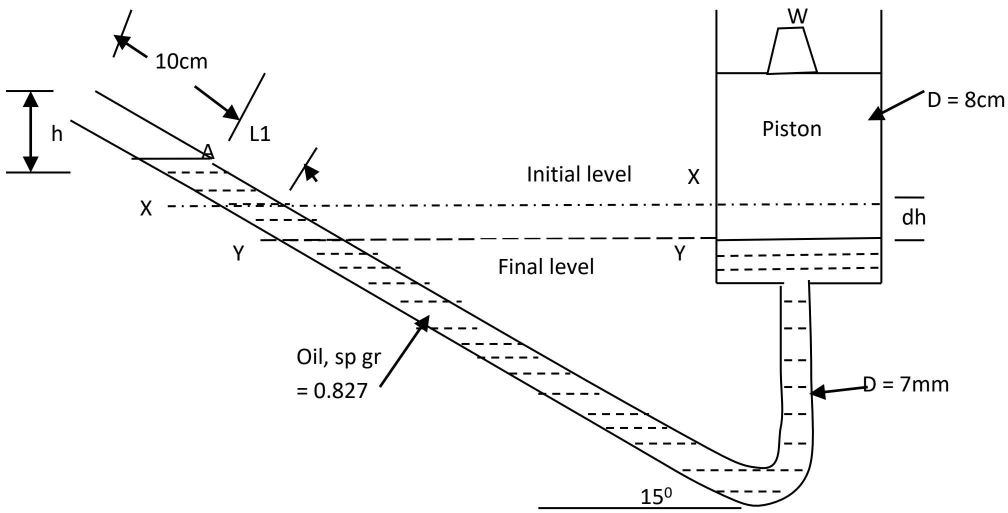
Pressure at E = pressure at F

$P_A + \gamma_{mercury}x0.20Sin45 + \gamma_{mercury}x0.5 = P_B + \gamma_{water}x0.3$

$P_A + 13.6x9810x0.20Sin45 + 13.6x9810x0.5 = P_B + 9810x0.3$

$P_A - P_B$ = -82633 N/m^2

23. An 8cm diameter piston compresses manometer oil into an inclined 7mm diameter tube, as shown in figure below. When a weight W is added to the top of the piston, the oil rises an additional distance of 10cm up the tube. How large is the weight, in N?



Solution:

Diameter of piston (D) = 8cm = 0.08m

Diameter of tube (d) = 7mm = 0.007m

h = 0.10Sin15 = 0.0258m

When the manometer is not connected to the container, the mercury in the reservoir is at original level and at level A in the tube.

Equating pressure at XX

$P_X = \gamma_m L1Sin15$ (a)

Due to compression, the fluid in the container moves down by dh and the fluid in the tube moves up by 10cm.

Volume of fluid fallen = Volume of fluid risen

$$\frac{\pi}{4}x0.08^2dh = \frac{\pi}{4}x0.007^2x0.1$$

dh = 0.000766m

Equating the pressure at new level (YY)

$\frac{W}{Area\ of\ piston} + P_X + \gamma_{air}dh = \gamma_m(h + dh) + \gamma_m L1Sin15$ (b)

From a and b

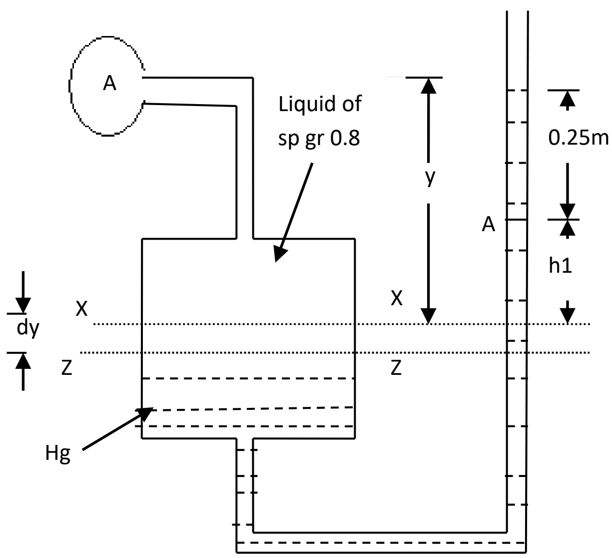
$\frac{W}{Area\ of\ piston} = \gamma_m h + (\gamma_m - \gamma_{air})dh$

Neglecting dh

$\frac{W}{\frac{\pi}{4}x0.08^2} = 0.827x9810x0.0258$

W = 1.05 N

24. Figure below shows a pipe containing a liquid of sp gr 0.8 connected to a single column micromanometer. The area of reservoir is 60 times that of the tube. The manometer liquid is mercury. Find the pressure in the pipe.



Solution:

When the manometer is not connected to the container, the mercury in the reservoir is at original level and at level A in the tube.

Equating pressure at XX

$\gamma_{oil} y = \gamma_{mercury} h1$ (a)

Due to pressure, manometric liquid in the reservoir drops by dy and it will travel a distance of 0.25m in the tube.

Volume of fluid fallen = Volume of fluid risen

Ady=ax0.25

60ady = ax0.25

dy = 0.0041m

Equating pressure at new level (ZZ)

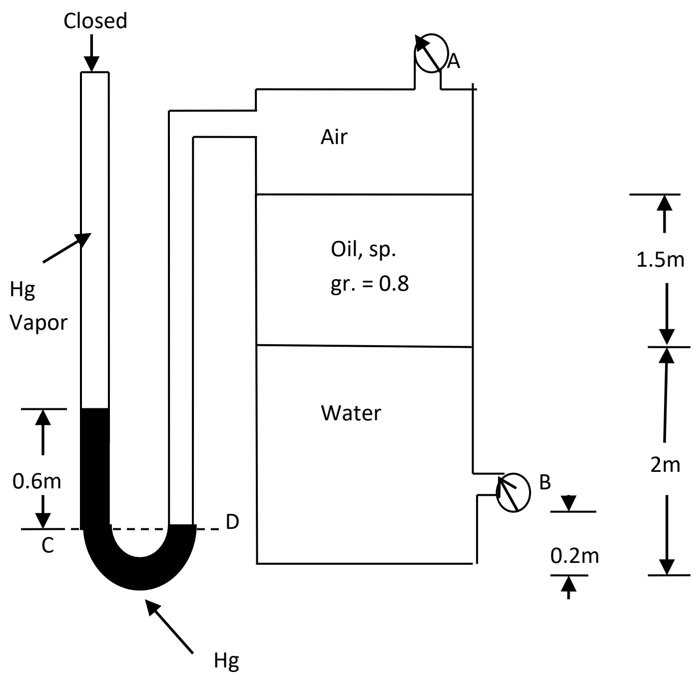
$P_A + \gamma_{oil}(y + dy) = \gamma_{mercury}(0.25 + h1 + dy)$ (b)

From a and b

$P_A + \gamma_{mercury} h1 = \gamma_{mercury} h1 + \gamma_{mercury}(0.25 + dy) - \gamma_{oil} dy$

$P_A = 13.6x9810(0.25 + 0.0041) - 0.8x9810x0.0041$ = 33868 N/m^2

25. Find the gauge readings at A and B if the atmospheric pressure is 755mmHg.



Solution:

Atm. pr. = $\gamma_{mercury}\ x\ 755x10^{-3} = 13.6x9810x755x10^{-3}$ = 100729 N/m^2

Neglect pressure due to mercury vapor and air (small)

Writing pressure equation for gauge A,

$$P_C = P_D = P_A$$

$$\gamma_{mercury}\ x\ 0.6 = P_A$$

P_A = 13.6x9810x0.6 = 80049.6 N/m^2 (abs. pr.)

Gauge pressure at A = Abs. pr. - Atm. pr. = 80049.6-100729 = -20679.5 N/m^2

Writing pressure equation for gauge B starting from gauge A

$$P_A + \gamma_{oil}x1.5 + \gamma_{water}x1.8 = P_B$$

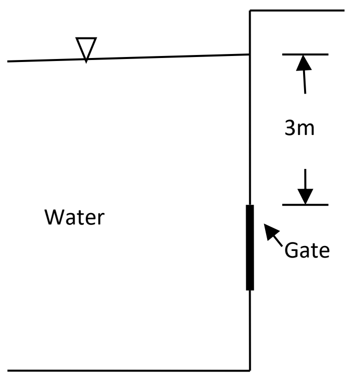
$$-20679.5 + 0.8x9810x1.5 + 9810x1.8 = P_B$$

P_B = 8750.5 N/m^2

Gauge pressure at B = 8750.5 N/m^2

Tutorial 3
Hydrostatic force on submerged bodies

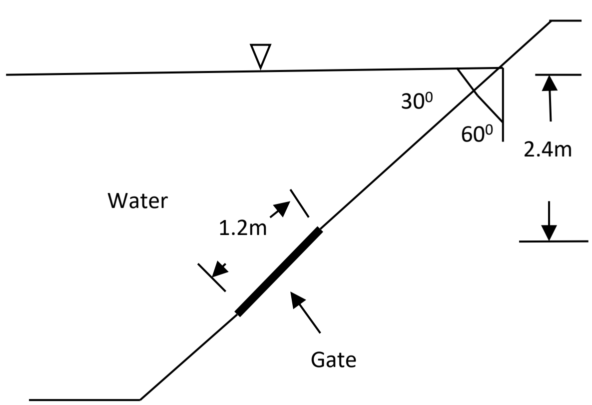
1. A vertical rectangular gate, 1.4m high and 2 m wide, contains water on one side. Determine the total resultant force acting on the gate and the location of c.p.



Solution:
Area (A) = 2x1.4 = 2.8 m^2
Location of CG ($\bar{y}$) = (3+1.4/2) = 3.7m
Resultant force on gate (F) = ?
Cp (y_p) = ?

$F = \gamma A\bar{y}$ = 9810x2.8x3.7 = 101631 N = 101.631 KN
M.I. about CG $(I_G) = \frac{1}{12}x2x1.4^3$ = 0.457 m^4
$y_p = \bar{y} + \frac{I_G}{A\bar{y}} = 3.7 + \frac{0.457}{2.8x3.7}$ =3.74m

2. An inclined rectangular gate (1.5m wide) contains water on one side. Determine the total resultant force acting on the gate and the location of c.p.

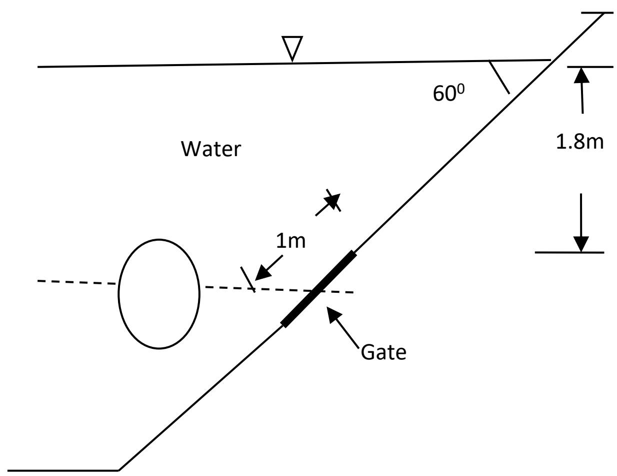


Solution:

Area (A) = 1.5x1.2 = 1.8 m^2
Location of CG ($\bar{y}$) = (2.4+1.2Sin30/2) = 2.7m
Resultant force on gate (F) = ?
Cp (y_p) = ?

$F = \gamma A\bar{y}$ = 9810x1.8x 2.7 = 47676N = 47.676 KN
M.I. about CG $(I_G) = \frac{1}{12}x1.5x1.2^3$ = 0.216 m^4
$y_p = \bar{y} + \frac{I_G Sin^2\theta}{A\bar{y}} = 2.7 + \frac{0.216Sin^2 30}{1.8x2.7}$ = 2.71m

3. An inclined circular with water on one side is shown in the fig. Determine the total resultant force acting on the gate and the location of c.p.

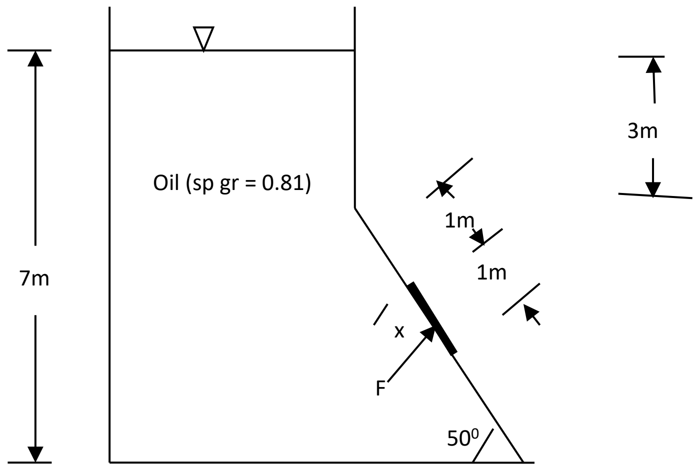


Solution:

Area (A) = $\pi x\frac{1^2}{4}$ = 0.785 m^2
Location of CG ($\bar{y}$) = (1.8+1.0Sin60/2) = 2.23m
Resultant force on gate (F) = ?
Cp (y_p) = ?

$F = \gamma A\bar{y}$ = 9810x0.785x2.23 = 17173N = 17.173 KN
M.I. about CG $(I_G) = \frac{\pi}{64}x1^4$ = 0.049 m^4
$y_p = \bar{y} + \frac{I_G Sin^2\theta}{A\bar{y}} = 2.23 + \frac{0.049Sin^2 60}{0.785x2.23}$ = 2.25m

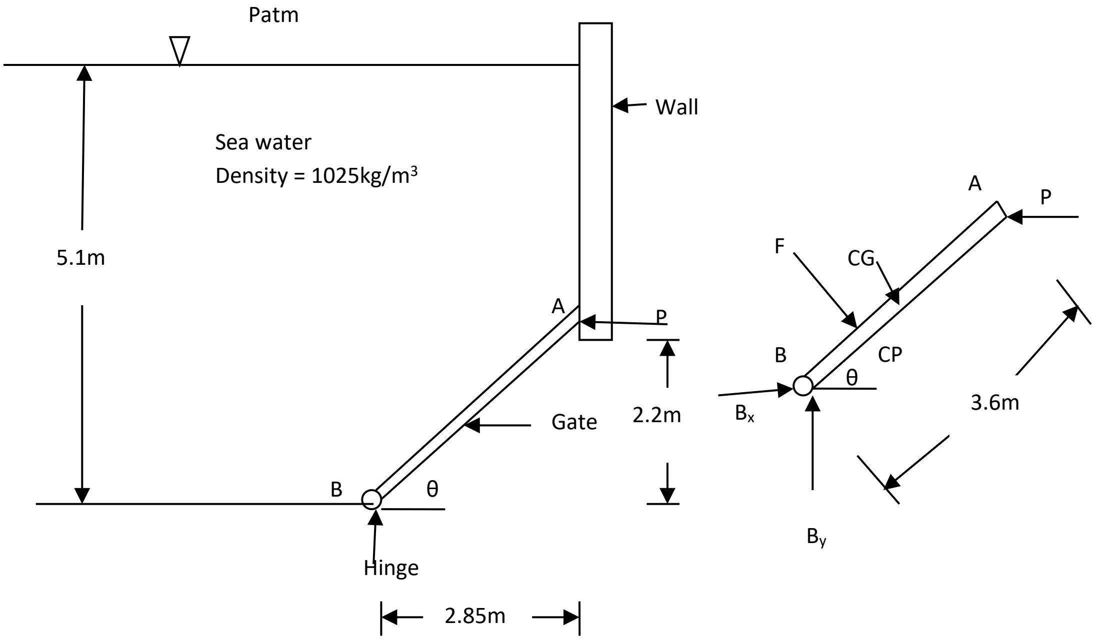
4. Gate AB in the fig. is 1m long and 0.7m wide. Calculate force F on the gate and position X of c.p.



Solution:
Sp. wt of oil (γ) =0.81x9810 = 7946 N/m3
Area (A) = 0.7x1 = 0.7 m^2
Location of CG ($\bar{y}$) = (3+1Sin50+1Sin50/2) = 4.15m
Resultant force on gate (F) = ?
x = ?

$F = \gamma A\bar{y}$ = 7946x0.7x4.15 = 23083 N = 23.08 KN
M.I. about CG $(I_G) = \frac{1}{12}x0.7x1^3$ = 0.058 m^4
Vertical distance of CP from free surface
$y_p = \bar{y} + \frac{I_G Sin^2\theta}{A\bar{y}} = 4.15 + \frac{0.058Sin^2 50}{0.7x4.15}$ = 4.161m
Vertical distance between CP from CG = 4.161-(3+1Sin50)= 0.395m
x = 0.395/sin50= 0.515m

5. The gate in the fig. is 1.2m wide, is hinged at point B, and rests against a smooth wall at A. Compute (a) the force on the gate due to sea water pressure, (b) the horizontal force exerted by the wall at point A, and (c) the reaction at hinge B.



Solution:

Sp wt of sea water (γ) = 1025x9.81 = 10055 N/m^3

Area (A) = 1.2x3.6 = 4.32 m^2

Location of CG ($\bar{y}$) = (5.1-2.2)+2.2/2 = 4.0m

a) Resultant force on gate (F) = ?

$F = \gamma A\bar{y}$ = 10055x4.32x4.0 = 173750 N = 173.75 KN

b) Force P = ?

M.I. about CG $(I_G) = \frac{1}{12} x1.2x3.6^3$ = 4.665 m^4

Vertical distance of CP from free surface

$y_p = \bar{y} + \frac{I_G Sin^2\theta}{A\bar{y}} = 4.0 + \frac{4.665x(2.2/3.6)^2}{4.32x4}$ = 4.1m

Vertical distance between B and CP = 5.1-4.1 = 1m

Location of F from B = 1/sinθ = 1.636m

Taking moment about B,

Px2.2-173.75x1.636 = 0

P = 129.2KN

c) Reactions at hinge, B_x and B_y = ?

$\sum F_x = 0$

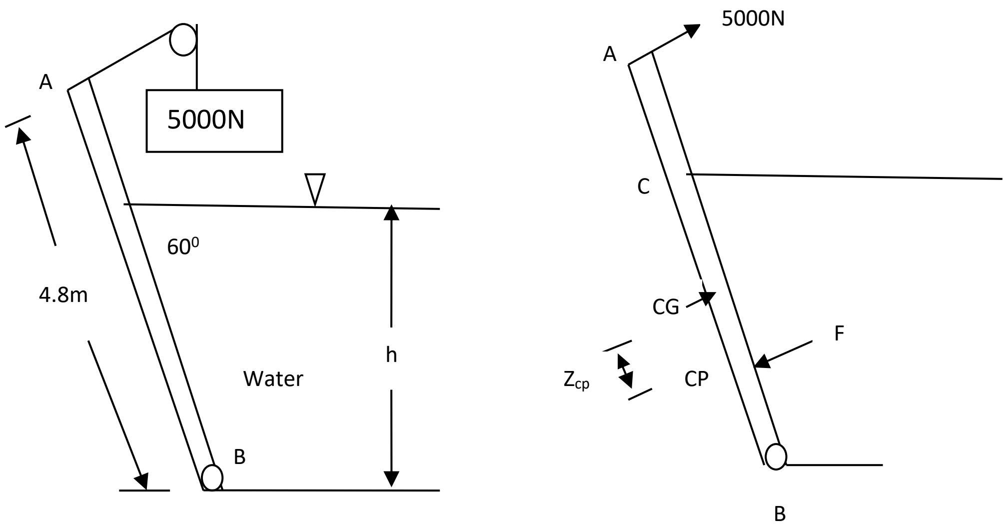
B_x – 129.2 + 173.75x2.2/3.6 = 0

B_x = 23.02 KN

$\sum F_y = 0$

B_y - 173.75x2.85/3.6 = 0
B_y = 137.55 KN

6. Gate AB in fig. is 4.8m long and 2.4m wide. Neglecting the weight of the gate, compute the water level h for which the gate will start to fall.

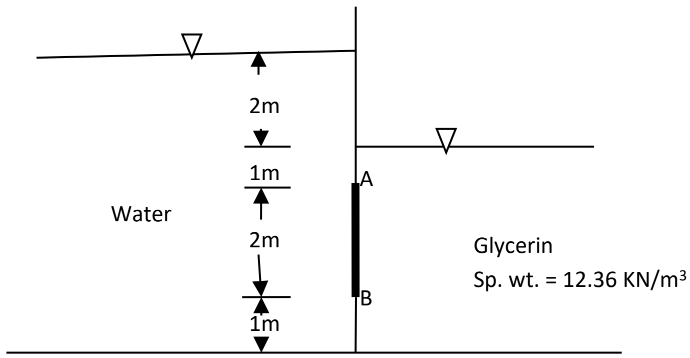


Solution:
Area (A) = 2.4xh/Sin60 = 2.77h m^2
Location of CG ($\bar{y}$) = h/2 m
Resultant force on gate (F) is
$F = \gamma A\bar{y}$ = 9810x2.77hxh/2 = 13587h^2 N

M.I. about CG $(I_G) = \frac{1}{12}x2.4x(\frac{h}{sin60})^3$ = 0.308h^3 m^4
Vertical distance of CP from free surface
$y_p = \bar{y} + \frac{I_G Sin^2\theta}{A\bar{y}} = h/2 + \frac{0.308h^3\ Sin^2 60}{2.77hxh/2}$ = 0.667h
Distance of F from B = (h-0.667h)/Sin60=0.384h

Taking moment about B,
5000x4.8-13587h^2x0.384h = 0
h = 1.66m

7. Find the net hydrostatic force per unit width on rectangular panel AB in the fig. and determine its line of action.

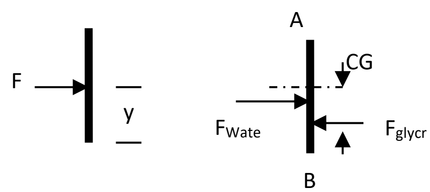


Solution:

Area (A) = 2x1 = 2 m^2

Location of CG ($\bar{y}$) = 2+1+2/2 = 4 m for water side

Location of CG ($\overline{y1}$) = 1+2/2 = 2 m for glycerin side



Resultant force on gate (F) = ?

$F = \gamma A\bar{y}$

Force due to water (F_{water}) = $\gamma A\bar{y}$ = 9.81x2x4 = 78.49KN

Force due to glycerin (F_{glyc}) = $\gamma A\overline{y1}$ =12.36x2x2 = 49.44KN

Net force (F) = 78.49-49.44 = 29.04KN

M.I. about CG $(I_G) = \frac{1}{12}x1x2^3$ = 0.666 m^4

Distance of F_{water} from CG $\left(y_{cp1}\right) = \bar{y} + \frac{I_G}{A\bar{y}} = 4 + \frac{0.666}{2x4}$ = 4.083m

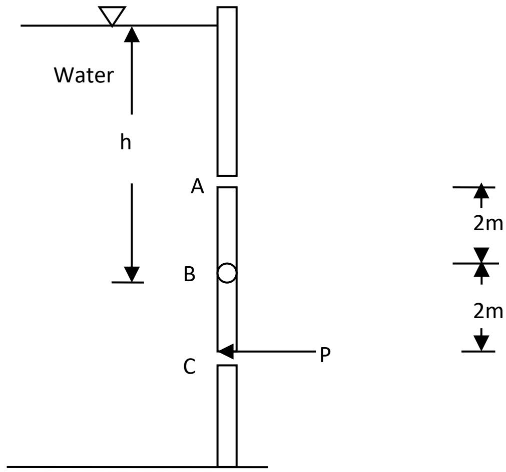
Distance of F_{glyc} from CG $\left(y_{cp2}\right) = \bar{y} + \frac{I_G}{A\overline{y1}} = 2 + \frac{0.666}{2x2}$ = 2.166m

Taking moment about B,

29.04y = 78.49x(5-4.083)-49.44x(3-2.166)

y = 0.945m

8. Circular gate ABC in the fig. is 4m in diameter and is hinged at B. Compute the force P just sufficient to keep the gate from opening when h is 8m.



Solution:

Area (A) = $\pi x \frac{4^2}{4}$ = 12.56 m^2

Location of CG ($\bar{y}$) = 8m

P = ?

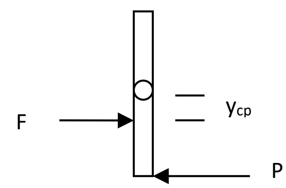
Resultant force on gate (F)

$F = \gamma A\bar{y}$ = 9810x12.56x8 = 985708 N = 985.708KN

M.I. about CG $(I_G) = \frac{\pi}{64} x 4^4$ = 12.56 m^4

Position of CP from free surface

$y_{cp} = \bar{y} + \frac{I_G}{A\bar{y}} = 8 + \frac{12.56}{12.56x8}$ = 8.125 m

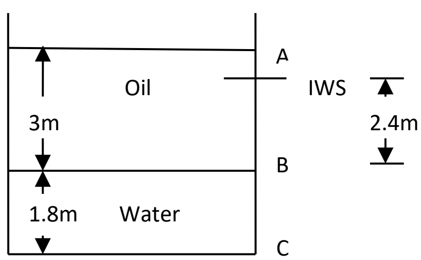


Taking moment about B,

985.7x0.125-Px2=0

P = 61.6KN

9. The tank in the fig. contains oil (sp gr = 0.8) and water as shown. Find the resultant force on side ABC and its point of application. ABC is 1.2m wide.



Solution:

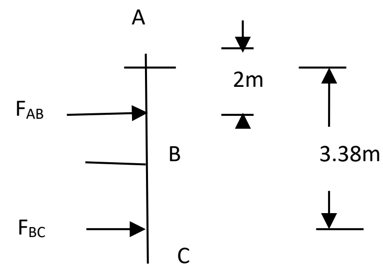
Sp wt of oil (γ_{oil}) = 0.8*9810 N/m^3 = 7848 N/m^3

Area (A1) = 1.2x3 = 3.6 m^2

Area (A2) = 1.2x1.8 = 2.16 m^2
Location of CG for AB ($\overline{y1}$) = 3/2 =1.5m
Resultant force on ABC = ?

Force on AB
$F_{AB} = \gamma_{oil} A1\overline{y1}$ =7848x3.6x1.5 = 42379N
Pt. of application of F_{AB} = (2/3)x3 = 2m below A

Force on BC
Water is acting on BC and any superimposed liquid can be converted to an equivalent depth of water.
Equivalent depth of water for 3m of oil = $\frac{\gamma_{oil}\, h_{oil}}{\gamma} = \frac{7848x3}{9810}$ = 2.4m
Employ an imaginary water surface of 2.4m.
Location of CG for IWS ($\overline{y2}$) = 2.4+1.8/2 = 3.3m



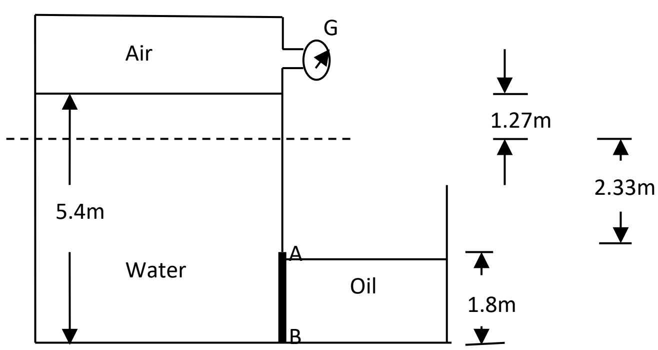
$F_{BC} = \gamma A2\overline{y2}$ = 9810x2.16x3.3 = 69925N
Point of application of F_{BC} from A is

$\bar{h} = \overline{y2} + \frac{I_G}{A\overline{y2}} = 3.3 + \frac{\frac{1}{12}x1.2x1.8^3}{2.16x3.3}$ =3.38 m
i.e. 3.38+0.6 = 3.98m from A

Total force on side ABC (F) = 42379+69925 = 112304N = 112.304 KN
Taking moment about A,
112304 y = 42379x2+69925x3.98
y = 3.23m
F acts at 3.23m below A.

(Alternative method: Solve by drawing pressure diagram. Force = Area of pressure diagram x width. Take moment to find position of resultant force.)

10. Gate AB in the fig. is 1.25m wide and hinged at A. Gage G reads -12.5KN/m^2, while oil (sp gr = 0.75) is in the right tank. What horizontal force must be applied at B for equilibrium of gate AB?

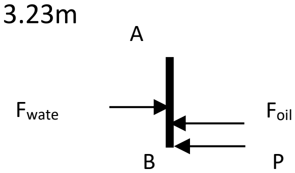


Solution:
Sp wt of oil (γ_{oil}) = 0.75*9810 N/m^3 = 7357.5 N/m^3
Area (A) = 1.25x1.8 = 2.25 m^2

Location of CG for right side ($\overline{y1}$) = 1.8/2 =0.9m

Force on AB at the right side
$F_{oil} = \gamma_{oil} A\overline{y1}$ =7357.5x2.25x0.9 = 14899N
Pt. of application of F_{oil} = (2/3)x1.8 = 1.2m from A

Left side
For the left side, convert the negative pressure due to air to equivalent head in water.
Equivalent depth of water for -12.5KN/m^2 pressure = $\frac{P}{\gamma} = \frac{-12.5}{9.81}$ = -1.27m
This negative pressure head is equivalent to having 1.27m less water above A.
Location of CG from imaginary water surface ($\overline{y2}$) = 2.33 + 1.8/2 = 3.23m

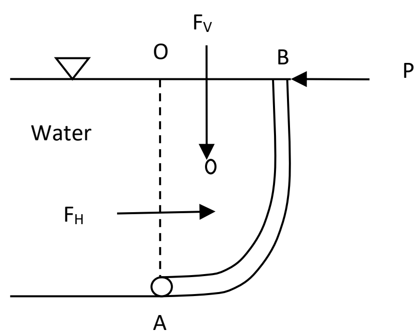


$F_{water} = \gamma A\overline{y2}$ = 9810x2.25x3.23 = 71294N
Point of application of F_{water} from A is

$\bar{h} = \bar{y} + \frac{I_G}{A\bar{y}} = 3.23 + \frac{\frac{1}{12}x1.25x1.8^3}{2.25x3.23}$ =3.31m from IWS
i.e. 3.31-2.33 = 0.98m from A

Taking moment about A
Px1.8 + 14899x1.2-71294x0.98 = 0
P = 28883 N = 28.883 KN

11. The gate AB shown is hinged at A and is in the form of quarter-circle wall of radius 12m. If the width of the gate is 30m, calculate the force required P to hold the gate in position.

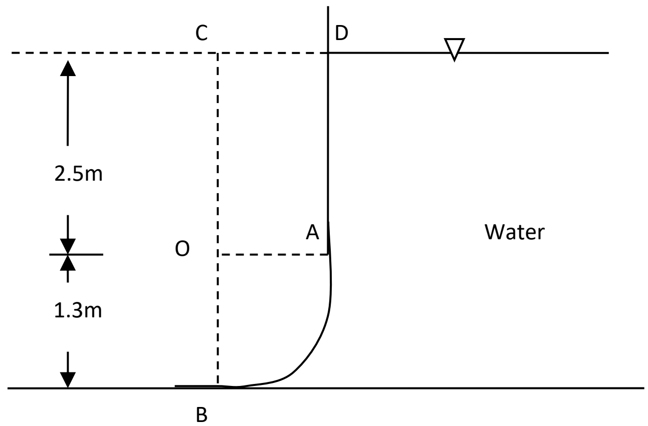


Solution:
Horizontal force $(F_H) = \gamma A\bar{y}$ = 9810x(30x12)x12/2 = 21189600 N = 21189.6 KN (right)
F_H acts at a distance of 12x1/3 =4m above the hinge A.
Vertical force (F_v) = Weight of volume of water vertically above AB = $\gamma\ Volume_{AOB}$

$$= 9810x\left[\frac{\pi x12^2}{4}x30\right]$$ = 33284546 N = 33284.546 KN (downward)

F_V acts at a distance of $4r/3\pi$ = 4x12/3x3.1416 = 5.1m from the vertical AO.
Taking moment about A,
Px12= 21189.6x4+33284.546x5.1
P = 21209 KN

12. The water is on the right side of the curved surface AB, which is one quarter of a circle of radius 1.3m. The tank's length is 2.1m. Find the horizontal and vertical component of the hydrostatic acting on the curved surface.



Solution:

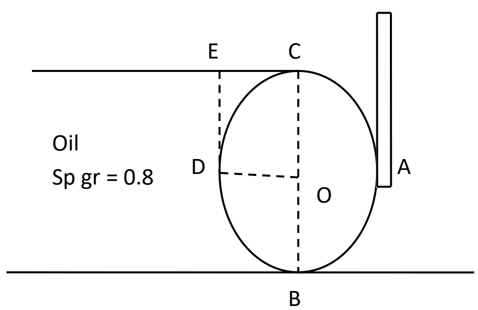
Horizontal force $(F_H) = \gamma A\bar{y}$ = 9810x(1.3x2.1)x(2.5+1.3/2) = 84361 N = 84.361 KN (right)

Vertical force (Fv) = Weight of imaginary volume of water vertically above AB

$= \gamma\ [Volume_{AOB} + Volume_{AOCD}]$

$= 9810x\left[\frac{\pi x1.3^2}{4}x2.1 + 2.5x1.3x2.1\right]$ = 94297 N = 94.297KN (downward)

13. The 1.8m diameter cylinder in the fig. weighs 100000N and 1.5m long. Determine the reactions at A and B, neglecting friction.



Solution:

Horizontal force $(F_H) = \gamma A\bar{y}$ = 0.8x9810x(1.8x1.5)x(1.8/2) = 19071 N (right)

Vertical force (F_v) = Weight of volume of water vertically above BDC

Vertical force $(F_v) = (F_v)_{DB} - (F_v)_{DC} = \gamma\ [Volume_{BDECO} - Volume_{DECD}]$

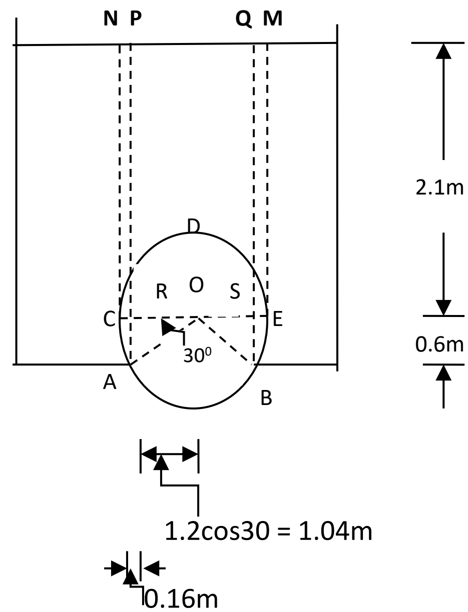
$= \gamma Volume_{BCD}$

$=0.8x9810x\left[\frac{\pi x 0.9^2}{2}x1.5\right]$ = 14978 N (up)

Reaction at A = F_H = 19071 N (left)

Reaction at B = Weight of cylinder – F_V = 100000-14978 = 85022N (up)

14. In the fig., a 2.4m diameter cylinder plugs a rectangular hole in a tank that is 1.4m long. With what force is the cylinder pressed against the bottom of the tank due to the 2.7m depth of water?



Solution:

Water is above the curve portion CDE, whereas it is below the curve portion AC and BE. For AC and BE, imaginary weight of water vertically above them is considered and the vertical force on theses part acts upwards.

Net vertical force= $(F_V)_{CDE}$ (down) – $(F_V)_{AC}$ (up) – $(F_V)_{BE}$ (up)

= Weight of volume of water vertically above CDE- imaginary Weight of volume above arc AC- imaginary weight of volume above arc BE

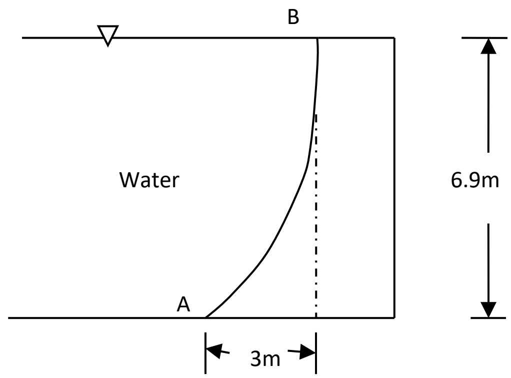
$=\gamma\ [Volume_{\ above\ CDE} - Volume_{\ above\ AC} - Volume_{\ above\ BB}]$

$$= 9810x[(volume\ rectangle\ CEMN - volume\ semicircle\ CDE) - (volume\ rectangleCRPN + volume\ sectorCOA - volume\ triangleROA) - (volume\ rectangleSEMQ + volume\ sectorBOE - volume\ triangleBOS)]$$

$$=9810x\left[\left(2.4x2.1x1.4 - \frac{\pi x1.2^2}{2}x1.4\right) - \left(0.16x2.1x1.4 + \frac{30}{360}x\pi x1.2^2x1.4 - \frac{1}{2}1.04x.6x1.4\right) - \left(0.16x2.1x1.4 + \frac{30}{360}\pi x1.2^2x1.4 - \frac{1}{2}1.04x.6x1.4\right)\right]$$

= 27139N (down)

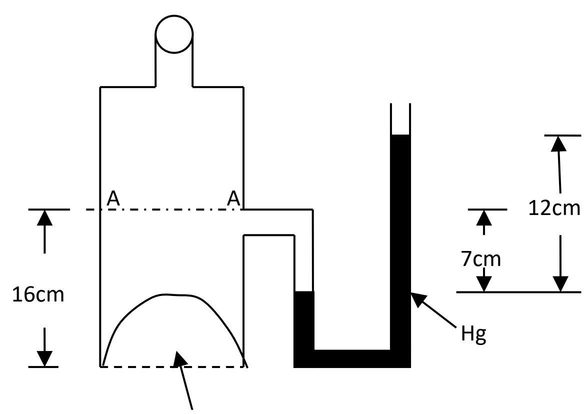
15. A dam has a parabolic profile as shown in the fig. Compute the horizontal and vertical components of the force on the dam due to the water. The width of dam is 15m. (Parabolic area = 2/3(b*d)



Solution:
Horizontal force $(F_H) = \gamma A\bar{y}$ = 9810x(15x6.9)x6.9/2 = 3502906 N = 3502.906 KN (right)

Vertical force (Fv) = Weight of volume of water vertically above AB = $\gamma\ Volume_{\ above\ AB}$
$= 9810\ x\frac{2}{3}x3x6.9x15$ = 2030670 N = 2030.67 KN (down)

16. The bottled liquid (sp gr = 0.9) in the fig. is under pressure, as shown by the manometer reading. Compute the net force on the 50mm radius concavity in the bottom of the bottle.

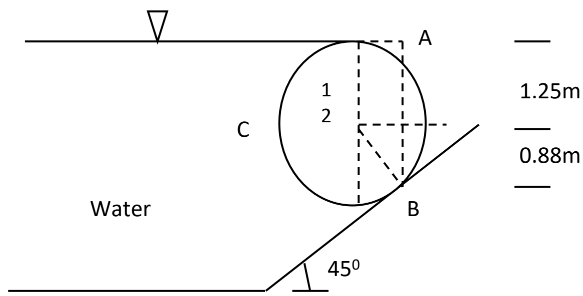


Solution:
From symmetry, F_H = 0
Manometeric equation for pressure,
$P_{AA} + \gamma x0.07 = \ \gamma_{Hg}x0.12$
P_{AA} + 0.9x9810x0.07 = 13.6x9810x0.12
P_{AA} = 15392 N/m^2

F_V = P_{AA} A_{bottom}+ Weight of liquid below AA = P_{AA} A_{bottom}+ $\gamma\ Volume_{below\ AA}$

= $P_{AA}A_{bottom}+\gamma\left[Volume_{cylinder\ of\ height\ 16cm} - Volume_{hemisphere\ of\ radius\ 50mm}\right]$
= $15392x\pi x0.05^2 + 0.9x9810[\pi x0.05^2x0.16 - \frac{1}{2}x\frac{4}{3}x\pi x0.05^3]$
= 129.7N (down)

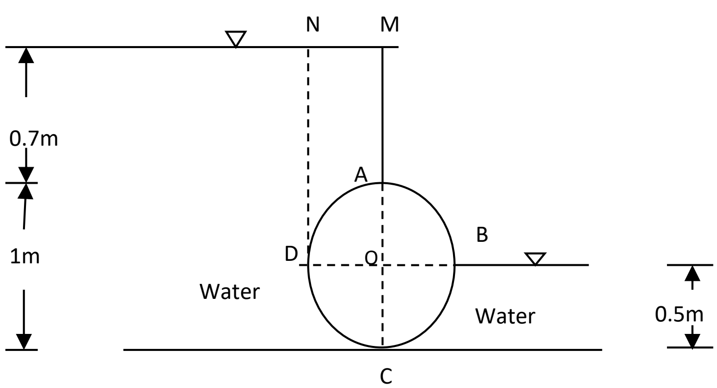
17. The cylinder in the fig. is 1.5m long and its radius is 1.25m. Compute the horizontal and vertical components of the pressure force on the cylinder.



Solution:
AB = 1.25+1.25Sin45 = 1.25+0.88 = 2.13m
Horizontal force $(F_H) = \gamma A\bar{y}$ = 9810x(2.13x1.5)x(2.13/2) = 33380 N = 33.38 KN (right)

Vertical force (Fv) = Weight of volume of water vertically above ABC
= γ $[Volume\ 1 + Volume2 + Volume3 + Volume4]$
$$= 9810x\left[\frac{1}{2}x\pi x1.25^2x1.5 + 0.88x1.25x1.5 + 0.5x0.88x0.88x1.5 + \frac{1}{8}x\pi x1.25^2x1.5\right]$$
= 67029N = 670.29 KN (up)

18. The 1m diameter log (sp gr = 0.82) divides two shallow ponds as shown in the fig. Compute the net horizontal and vertical reactions at point C, if the log is 3.7m.



Solution:
Horizontal force on ADC $(F_{H1}) = \gamma A1\overline{y1}$ = 9810x3.7x1x1.2= 43556 N (right)

Horizontal force on BC $(F_{H2}) = \gamma A2\overline{y2}$ = 9810x3.7x0.5x0.5/2= 4537 N (left)

Vertical force on ADC (F_{V1}) = Weight of volume of water vertically above ADC

Vertical force (F_{V1}) =$(F_v)_{MNDCOAM}$ (up)- $(F_v)_{MNDAM}$ (down)

$= \gamma Volume_{AOCD}$

$= 9810x\frac{1}{2}x\pi x0.5^2x3.7$ = 14254N (up)

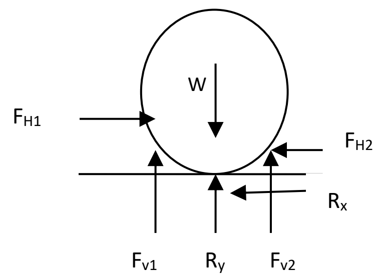
Vertical force on BC (F_{V2}) = Weight of volume of water (imaginary) vertically above BC

$= \gamma\ Volume_{BOC}$

$= 9810x\frac{1}{4}x\pi x0.5^2x3.7$ = 7127N (up)

Weight of log (W) = $\gamma_{log}\ Volume_{log}$

$= 0.82x9810x\pi x0.5^2x3.7$ = 23376N (down)



Horizontal reaction at C (R_x)

$-R_X + F_{H1} - F_{H2} = 0$

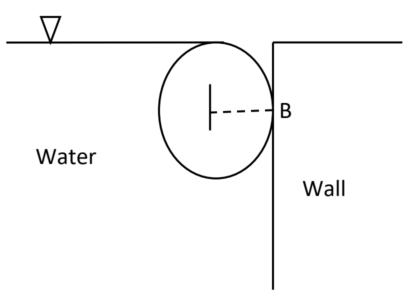
$R_x = F_{H1} - F_{H2}$ = 43556 – 4537 = 39019N (left)

Vertical reaction at C (R_y)

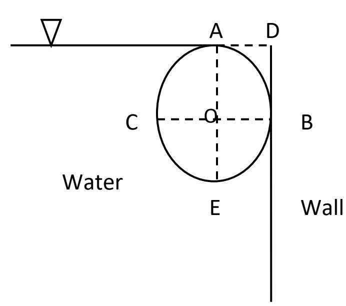
$R_y + F_{V1} + F_{V2} - W = 0$

R_y = 23376-14254-7127 = 1995N (up)

19. The 0.9m diameter cylinder in the fig. is 7m long and rests in static equilibrium against a frictionless wall at point B. Compute the specific gravity of the cylinder.



Solution:



Vertical force (F_V) = Weight of volume of water vertically above ADBECA

= (F_v) on semi-circle ACE + (F_v) on quadrant BE

For BE, imaginary weight of fluid vertically above it is considered

$$= \gamma\ [Volume_{AOEC} + Volume_{BOE} + Volume_{ADBO}]$$

$$= 9810\left[\frac{1}{2}\pi x 0.45^2 x 7 + \frac{1}{4}\pi x 0.45^2 x 7 + 0.45 x 0.45 x 7\right] = 46670\text{N (up)}$$

The reaction at B is purely horizontal.

Weight of cylinder (W) = F_V

W = 46670N

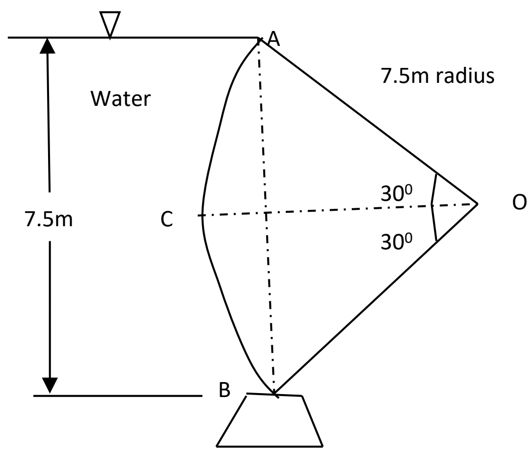
$\gamma_{cyl}\ Volume_{cyl} = 46670$

$\gamma_{cyl}\ \pi x 0.45^2 x 7 = 46670$

γ_{cyl} = 10480 N/m^3

Sp gr of cylinder = $\frac{\gamma_{cyl}}{\gamma} = \frac{10480}{9810}$ = 1.07

20. Find the horizontal and vertical forces per m of width on the tainter gate shown in the fig.



Solution:

Horizontal force $(F_H) = \gamma A\bar{y}$ = 9810x(7.5x1)x7.5/2 = 275906 N = 275.906 KN (right)

F_H acts at a distance of 7.5x2/3 = 5m from water surface.

Vertical force (Fv) = Weight of imaginary volume of water vertically above ABCA

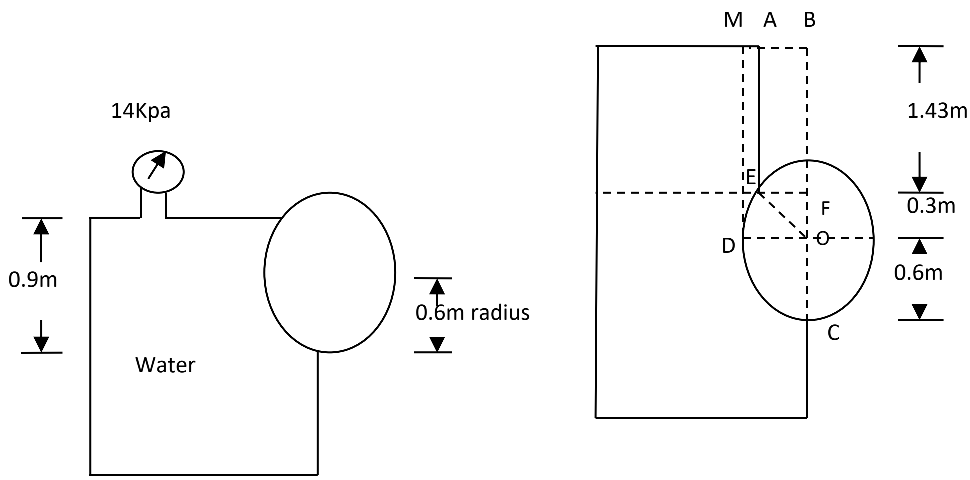
$$= \gamma\ [Volume_{sectorAOBC} - Volume_{triangleAOB}]$$

$$= 9810x\left[\frac{60}{360} x\pi x 7.5^2 x 1 - 0.5x7.5x(7.5Cos30)x1\right]$$

= 49986N = 49.986KN (up)

F_V acts through the centroid of the segment ABCA.

21. The tank whose cross section is shown in fig. is 1.2m long and full of water under pressure. Find the components of the force required to keep the cylinder in position, neglecting the weight of the cylinder.



Solution:

Pressure =14KPa

Equivalent head of water = $\frac{P}{\gamma} = \frac{14000}{9810}$ = 1.43m

Apply 1.43m water above the cylinder.

Horizontal force $(F_H) = \gamma A\bar{y}$ = 9810x(0.9x1.2)x(1.43+0.9/2) = 19918 N = 19.918 KN (right)

Sin(OEF) = 0.3/0.6

<OEF = 30^0 = <EOD

EF = 0.6Cos30 = 0.52m

Vertical force $(F_v) = (F_v)_{MABFOCDEM} - (F_v)_{MAED}$

= Weight of volume of water vertically above ABFOCDEA

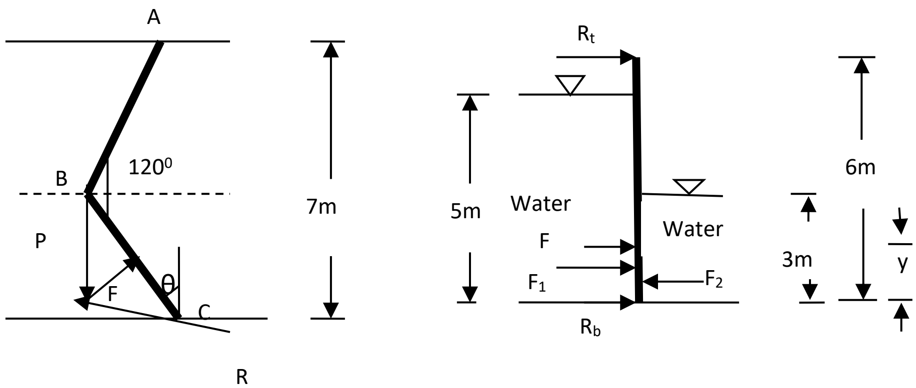
$= \gamma\ [Volume_{ABFE} + Volume_{triangleEOF} + Volume_{sector\ EOD} + Volume_{quadrant\ COD}]$

$= 9810x\left[0.52x1.43x1.2 + 0.5x0.3x0.52x1.2 + \frac{30}{360}x\pi x0.6^2x1.2 + \frac{1}{4}x\pi x0.6^2x1.2\right]$

= 14110 N = 14.11 KN (up)

Forces required to keep the cylinder in positions are: 19.918KN to the right and 14.11KN to the up.

22. Each gate of a lock is 6m high and is supported by two hinges placed on the top and the bottom. When the gates are closed, they make an angle of 120^0. The width of the lock is 7m. If the water levels are 5m and 2m at upstream and downstream respectively, determine the magnitude of forces on the hinge due to the water pressure.



Solution:

F = Resultant water force, P = Reaction between gates, R = total reaction at hinge

θ = 30^0

Width of lock = 3.5/cos30 = 4.04m

Resolving forces along gate

Pcosθ = Rcosθ i.e. P=R (a)

Resolving forces normal to gate

PSinθ +RSinθ = F (b)

From a and b

P = F/2Sinθ

Horizontal force on upstream side $(F_1) = \gamma A1\overline{y1}$ = 9810x4.04x5x5/2= 495405 N

F1 acts at 5/3m = 1.66 from bottom

Horizontal force on downstream side $(F_2) = \gamma A2\overline{y2}$ = 9810x4.04x3x3/2= 178346N

F2 acts at 3/3 = 1m from bottom

F = F_1– F_2 = 495405-178346 = 317059 N

Taking moment about the bottom to find the point of application of F,

317059y = 495405x1.66-178346x1

y = 2.03m

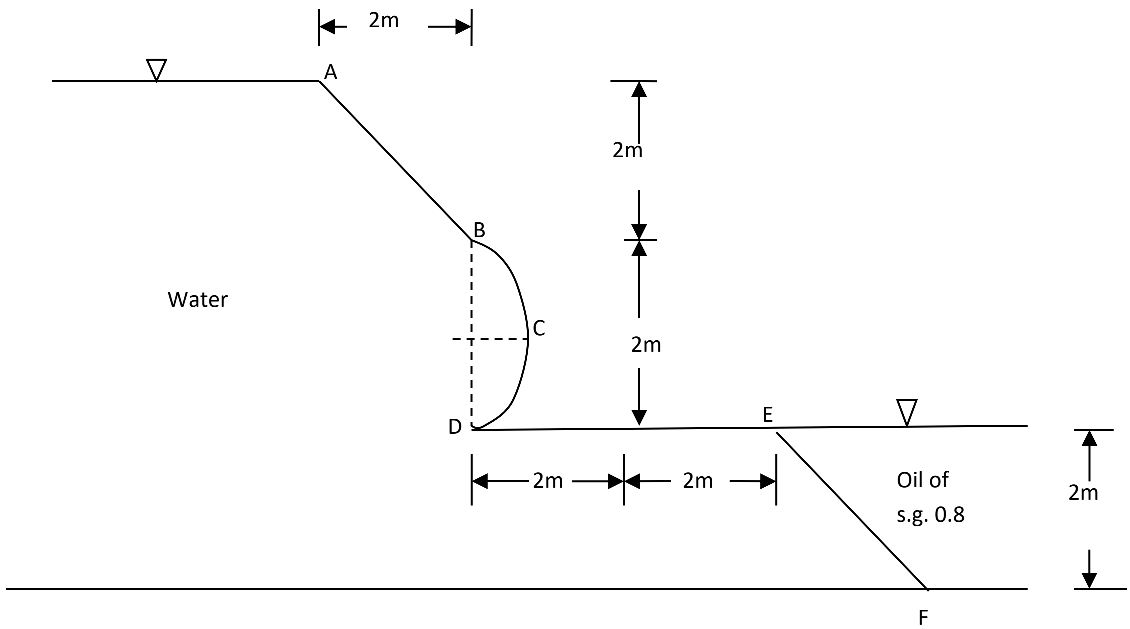
P = F/2Sinθ = 317059/2sin30 =317059N

R = P = 317059N

Taking moment about bottom hinge

R_t x6 =317059x2.03
R_t = 107272N = 107.272KN
R_b = R – R_t = 317059-107272 = 209787N = 209.787KN

23. Find the net horizontal and vertical forces acting on the surface ABCDEF of width 5m as shown in the figure below. BCD is a half circle.



Solution:
AB = 2/Sin45^0 = 2.8284m
EF = 2/Sin45^0 = 2.8284m

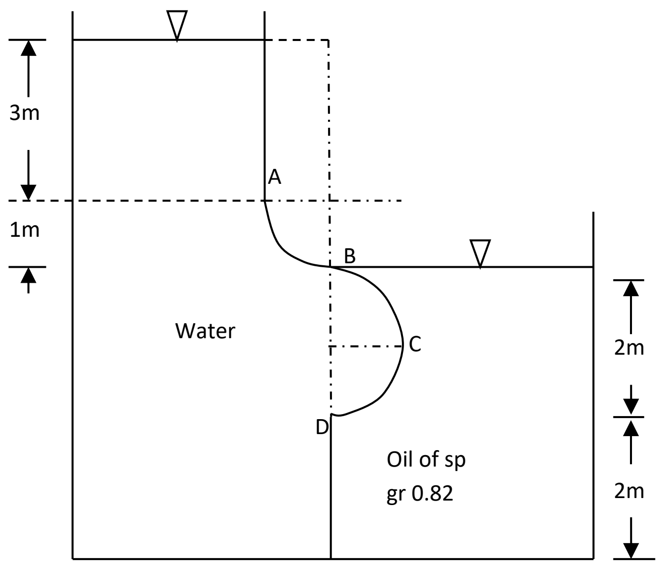
Pressure force on inclined surface AB $(F_1) = \gamma_{water} A_1 \overline{y_1}$ = 9810x(2.8284x5)x1 = 138733N which is perpendicular to AB
$F_{1x} = F_1$ Cos45^0 = 138733 Cos45^0 = 98099N (right)
$F_{1y} = F_1$ Sin45^0 = 138733 Sin45^0 = 98099N (up)
For curved surface BCD
$F_{2x} = \gamma_{water} A_2 \overline{y_2}$ = 9810x(2x5)x3 = 294300 N (right)
$F_{2y} = \gamma_{water} V_{above\ BCD} = 9810x\left(\frac{1}{2}\pi x\frac{2^2}{4}\right)x5$ = 77048N (down)

Pressure force on EF due to water $(F_3) = \gamma_{water} A_3 \overline{y_3}$ = 9810x(2.8284x5)x5 = 693665N which is perpendicular to EF
$F_{3x} = F_3$ Cos45^0 = 693665 Cos45^0 = 490495N (right)
$F_{3y} = F_3$ Sin45^0 = 693665 Sin45^0 = 490495N (up)

Pressure force on EF due to oil $(F_4) = \gamma_{oil} A_4 \overline{y_4}$ = 0.8x9810x(2.8284x5)x1 = 110986N which is perpendicular to EF

$F_{4x} = F_4 \cos45^0$ = 1109955 Cos45^0 = 78479N (left)
$F_{4y} = F_4 \sin45^0$ 1109955 Sin45^0 =78479N (down)
Net horizontal force = 98099+294300+490495-78479N = 804415N (right)
Net vertical force = 98099-77048+490495-78479N = 433067N (up)

24. Calculate the pressure force on the curved surface ABCD as shown in the figure below. AB is a quadrant of radius 1m and BCD is a semi-circle of radius 1m. Take width of curve = 5m.



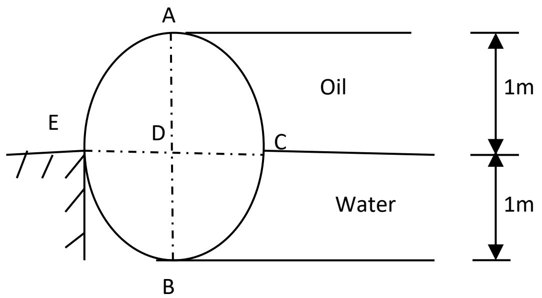
Solution:
Horizontal force on AB $(F_{1x}) = \gamma_{water} A_1 \overline{y_1}$ = 9810x(1x5)x3.5 = 171675N (right)
Vertical force on AB $(F_{1y)} = \gamma_{water} V_{above\ AB\ (imaginary)} = 9810x\left(\frac{1}{4}\pi x1^2 + 3x1\right)x5$ = 185674N (up)

Horizontal force on BCD from the left side $(F_{2x}) = \gamma_{water} A_2 \overline{y_2}$ = 9810x(2x5)x5 = 490500N (right)
Vertical force on BCD from the left side $(F_{2y)} = \gamma_{water} V_{above\ BCD} = 9810x\left(\frac{1}{2}\pi x1^2\right)x5$ = 77048N (down)

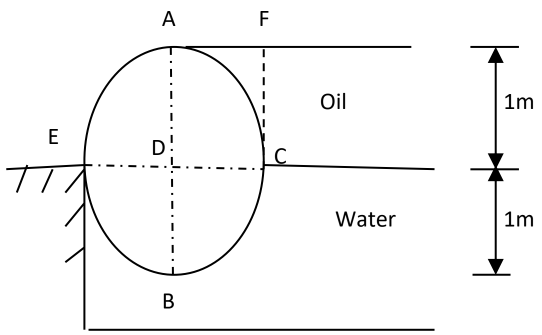
Horizontal force on BCD from the right side $(F_{3x}) = \gamma_{oil} A_2 \overline{y_3}$ = 0.82x9810x(2x5)x1 = 80442N (left)
Vertical force on BCD from the right side $(F_{3y)} = \gamma_{oil} V_{above\ BCD} = 0.82x9810x\left(\frac{1}{2}\pi x1^2\right)x5$ = 63179N (up)

Net horizontal force = 171675+490500-80442 = 581733N = 581.733KN (right)
Net vertical force = 185674-77048+63179 = 171805N = 171.805KN (up)

25. Find the weight of the cylinder (dia. =2m) per m length if it supports water and oil (sp gr = 0.82) as shown in the figure. Assume contact with wall as frictionless.



Solution:



Downward force on AC due to oil (FV_{AC}) = Weight of oil supported above curve AC

$= \gamma_{oil}\ Volume\ of\ oil\ above\ AC$

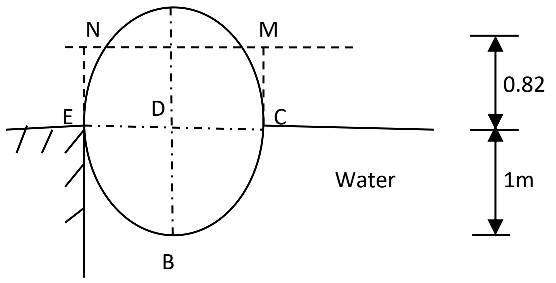
$= \gamma_{oil}\ (Volume_{AFCD} - Volume_{quadrant\ ACD})$

$= 0.82x9810\left(1x1x1 - \frac{1}{4}\pi x1^2 x1\right)$ = 1726N

Pressure at C due to 1m oil (P) = $\gamma_{oil}\ x1$ = 0.82x9810x1 = 8044.2 Pa

Equivalent head of water due to 1m oil = $\frac{P}{\gamma} = \frac{8044.2}{9810}$ = 0.82m

Apply 0.82m water above EC.



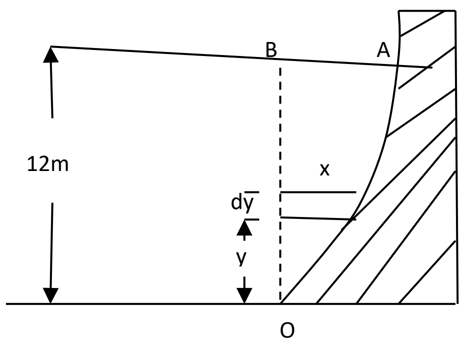
Upward vertical force on CBE (FV_{CBE}) = Weight of water above CBE

$= \gamma(Volume_{semi\ circle\ CBE} + Volume_{CMNE})$

$$= 9810\left(\frac{1}{2}\pi x 1^2 x 1 + 0.82 x 2 x 1\right)$$ = 31498N

Weight of cylinder = FV_{CBE} - FV_{AC} = 31498-1726 = 29772 N

26. Find the magnitude and direction of the resultant pressure force on a curved face of a dam which is shaped according to the relation y = x^2/6. The height of water retained by the dam is 12m. Assume unit width of the dam.



Solution:

The equation of the dam

y = x^2/6

$x = \sqrt{6y}$

Consider an element of thickness dy and length x at a distance y from the base.

Area of element = xdy

Area of OAB = $\int_0^{12} xdy = \int_0^{12} \sqrt{6y}dy$

$= \sqrt{6} x \frac{2}{3} \left|y^{3/2}\right|_0^{12}$ = 67.882 m^2

Horizontal force $(F_x) = \gamma A\bar{y}$ = 9810x(12X1)x6 = 706320N

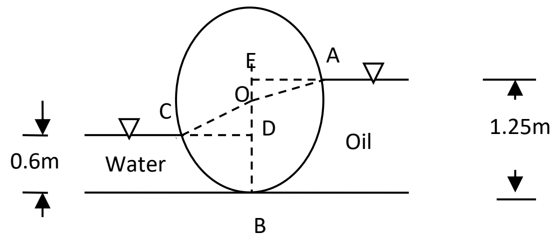
Vertical force (F_y) = Weight of water vertically above dam OA

= $\gamma\ Vol_{OAB} = \gamma\ Area_{OAB} L$ = 9810x67.882x1 = 665922N

Resultant force $(F_R) = \sqrt{F_{x+}^2 F_y^2}$ = 970742N = 970.742KN

Direction of resultant force = $Tan^{-1}\frac{F_y}{F_x}$ = $Tan^{-1}\frac{665922}{706320}$ = 43.31^0

27. A cylinder, 2m in diameter and 3m long weighing 3KN rests on the floor of the tank. It has water to a depth of 0.6m on one side and liquid of sp gr 0.7 to a depth of 1.25m on the other side. Determine the magnitude and direction of the horizontal and vertical components of the force required to hold the cylinder in position.



Solution:

OA= OB = OC = 1m, BD = 0.6m

OD = 1-0.6 = 0.4m

CD = $(1^2 - 0.4^2)^{1/2}$= 0.9165m

$< COD = tan^{-1}\left(\frac{0.9165}{0.4}\right)$= 66.4^0

OE = 1.25-1 = 0.25m

$< AOE = cos^{-1}\left(\frac{0.25}{1}\right)$= 75.5^0

<AOB = 180-75.5 = 104.5^0

AE = 0.25 tan75.5 = 0.96m

Weight of cylinder = 3KN = 3000N

Net horizontal force $(F_H) = (F_H)_{AB} - (F_H)_{CB} = \gamma_{oil}A1\overline{y1} - \gamma A2\overline{y2}$

= 0.7x9810x1.25x3x1.25/2-9810x0.6x3x0.6/2 = 10797N (left)

Net vertical force (F_V)= Weight of volume of oil vertically above AB + Weight of volume of water vertically above BC = Fv_{AB} (up) + Fv_{BC} (up)

= $\gamma_{oil}\ Volume_{AEB} + \gamma Volume_{BDC}$

$= \gamma_{oil}(Volume\ of\ sector\ AOB + Volume\ of\ \Delta AOE) + \gamma(Volume\ of\ sector\ BOC\ - Volume\ of\ \Delta COD)$

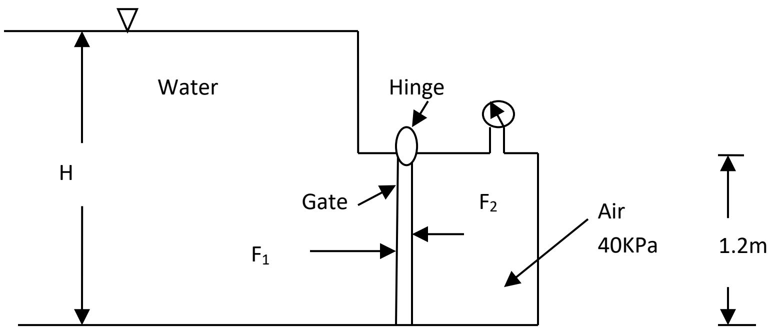
=$9810x0.7\left(\frac{104.5}{360}x\pi x1^2x3 + 0.5x0.25x0.96x3\right) + [9810x\ \left(\frac{66.4}{360}x\pi x1^2x3 - 0.5x0.9165x0.4x3\right)]$

= 32917N (up)

The components to hold the cylinder in place are 10797 N to the right and 32917-3000 = 29917N down.

Additional problems on hydrostatic force

For the system shown in figure, calculate the height H of water at which the rectangular hinged gate will just begin to rotate anticlockwise. The width of gate is 0.5m.



Solution:

Force due to water $(F_1) = \gamma A\bar{y}$

$$= 9810x(1.2x0.5)x(H - 0.6) = 5886(H - 0.6)$$

CP of F_1

M.I. about CG $(I_G) = \frac{1}{12}x0.5x1.2^3$ = 0.072m^4

Vertical distance of CP of F_1 from free surface

$$y_{p1} = \bar{y} + \frac{I_G}{A\bar{y}} = (H - 0.6) + \frac{0.072}{1.2x0.5(H-0.6)} = \frac{(H-0.6)^2+0.12}{(H-0.6)}$$

Force due to air pressure (F_2)= PA = 40x1000x1.2x0.5 =24000N, which acts at a distance of H-0.6 from the free surface.

Taking moment about hinge,

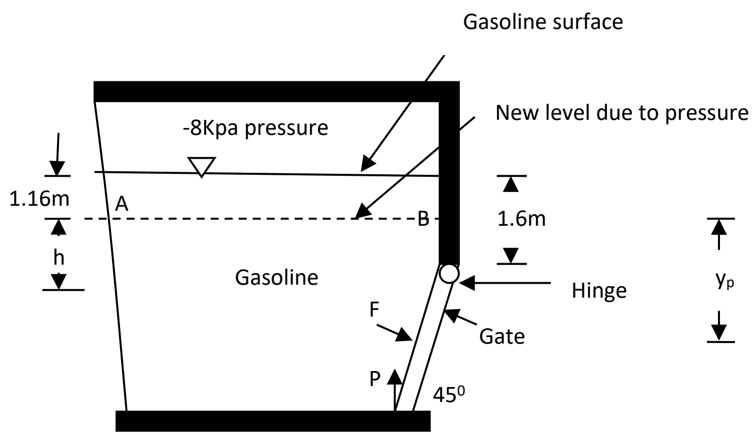
$$F_1\left[y_{p1} - (H - 1.2)\right] = F_2x0.6$$

$$5886(H - 0.6)\left[\frac{(H-0.6)^2+0.12}{(H-0.6)} - (H - 1.2)\right] = 24000x0.6$$

$$(H - 0.6)^2 + 0.12 - (H - 1.2)(H - 0.6) = 2.446$$

H = 1.6m

A 3m square gate provided in an oil tank is hinged at its top edge. The tank contains gasoline (sp. gr. = 0.7) up to a height of 1.6m above the top edge of the plate. The space between the oil is subjected to a negative pressure of 8 Kpa. Determine the necessary vertical pull to be applied at the lower edge to open the gate.



Solution:

Head of oil equivalent to -8 Kpa pressure = $\frac{p}{\gamma_{oil}} = \frac{-8000}{0.7x9810}$ = -1.16m

This negative pressure will reduce the oil surface by 1.16m. Let AB = new level. Make calculation by taking AB as free surface.

h = 1.6-1.16= 0.44m

$\bar{y} = (1.6 - 1.16) + \frac{1}{2}x3Sin45$ = 1.5m

Hydrostatic force $(F) = \gamma_{oil} A\bar{y}$ = 0.7x9810x(3x3)x1.5 = 92704.5N

CP of F

M.I. about CG $(I_G) = \frac{1}{12}x3x3^3$ = 6.75m^4

Vertical distance of CP of F_1 from free surface

$y_p = \bar{y} + \frac{I_G Sin^2\theta}{A\bar{y}} = 1.5 + \frac{6.75Sin^2 45}{(3x3)x1.5}$ = 1.75m

Vertical distance between the hinge and F = 1.75-0.44 = 1.31m

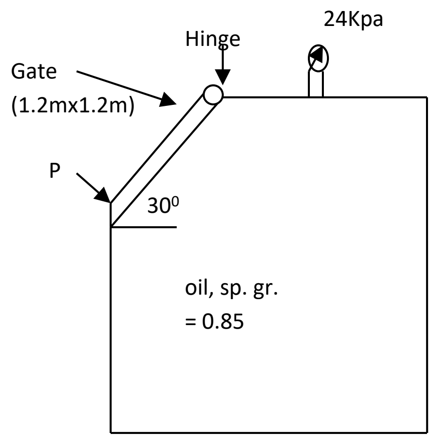
Taking moment about the hinge

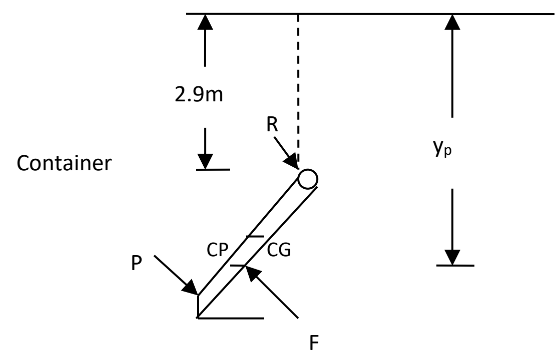
$Px3sin45 = Fx\frac{1.31}{sin45}$

$Px3sin45 = 92704.5x\frac{1.31}{sin45}$

P = 80962N

There is an opening in a container shown in the figure. Find the force P and the reaction at the hinge (R).





Solution:

Equivalent head of oil due to 24 Kpa pressure = $\frac{p}{\gamma_{oil}} = \frac{24000}{0.85x9810}$ = 2.9m of oil

Apply 2.9m of oil above the hinge.

$\bar{y} = 2.9 + \frac{1}{2}x1.2Sin30$ = 3.2m

Hydrostatic force $(F) = \gamma_{oil}A\bar{y}$ = 0.85x9810x(1.2x1.2)x3.2= 38424N

CP of F

M.I. about CG $(I_G) = \frac{1}{12}x1.2x1.2^3$ 0.1728m^4

Vertical distance of CP of F_1 from free surface

$y_p = \bar{y} + \frac{I_G Sin^2\theta}{A\bar{y}} = 3.2 + \frac{0.1728Sin^2 30}{(1.2x1.2)x3.2}$ = 3.209m

Vertical distance between the hinge and F = 3.209-2.9 = 0.309m

Taking moment about the hinge

$Px1.2 = 38424x\frac{0.309}{sin30}$

P = 19788N

R+P=F

R = 38424-19788 = 18636N

Tutorial 4

Buoyancy and floatation

1. A rectangular pontoon has a width of 6m, length of 10m and a draught of 2m in fresh water. Calculate (a) weight of pontoon, (b) its draught in seawater of density 1025 kg/m^3 and (c) the load that can be supported by the pontoon in fresh water if the maximum draught permissible is 2.3m.

Solution:

a) Weight of pontoon = Weight of water displaced

$$W = \gamma_{water}V_{water\ displaced} = 9810x6x10x2 = 1177200\ N = 1177.2\ KN$$

b) Draught in sea water (D) = ?

Weight of pontoon = Weight of sea water displaced

$$1177200 = \gamma_{sea\ water}V$$

$$1177200 = 1025x9.81x6x10xD$$

$$D = 1.95m$$

c) D_{max} = 2.3m

Load that can be supported in fresh water (P) = ?

Total upthrust (F_B) = Weight of water displaced

$$= \gamma_{water}V_{water\ displaced} = 9810x6x10x2.3 = 1353780N = 1353.78KN$$

$P = F_B - W$ = 1353.78-1177.2= 176.58 KN

2. A steel pipeline carrying gas has an internal diameter of 120cm and an external diameter of 125cm. It is laid across the bed of a river, completely immersed in water and is anchored at intervals of 3m along its length. Calculate the buoyancy force per meter run and upward force on each anchorage. Take density of steel = 7900 kg/m^3.

Solution:

Buoyant force per m = Weight of water displaced per m

$$= \gamma_{water}V = 9810x\frac{\pi}{4}x1.25^2x1 = 12039N/m$$

Buoyant force for 3m (F_{B3}) = 12039x3 = 36117N

Weight for 3 m of pipe (W_3) = $3x\gamma_{steel}V_{steel}$

$$= 3x7900x9.81x\frac{\pi}{4}x(1.25^2 - 1.20^2) = 22369N$$

Upward force on each anchorage = $F_{B3} - W_3$ = 36117-22369 = 13748N

3. A wooden block of width 2m, depth 1.5m and length 4m floats horizontally in water. Find the volume of water displaced and the position of center of buoyancy. The specific gravity of wooden block is 0.7.

Solution:

Weight of block = Weight of water displaced

$\gamma_{wood}V_{wood} = \gamma_{water}V_{water\ diplaced}$

0.7x9810x2x1.5x4 = 9810$V_{water\ displaced}$

$V_{water\ displaced}$ = 8.4m^3

Finding depth of immersion (h)

Weight of block = Weight of water displaced

$\gamma_{wood}V_{wood} = \gamma_{water}V_{water\ diplaced}$

0.7x9810x2x1.5x4 = 9810x2x4xh

h = 1.05m

Position of center of buoyancy = h/2 = 0.525m from bottom

4. A piece of wood of sp gr 0.65 is 80mm square and 1.5m long. How many Newtons of lead weighing 120KN/m^3 must be fastened at one end of the stick so that it will float upright with 0.3m out of water?

Solution:

Length of wood = 1.5m

Length of wood in water = 1.5-0.3 = 1.2m

Total weight of wood and lead = Weight of water displaced

$\gamma_{wood}V_{wood} + \gamma_{lead}V_{lead} = \gamma_{water}V_{water\ diplaced}$

$0.65x9810x0.08^2x1.5 + \gamma_{lead}V_{lead} = 9810[0.08^2x1.2 + V_{lead}]$

$61.214 + 120000V_{lead} = 75.34 + 9810V_{lead}$

V_{lead} = 0.000128 m^3

Weight of lead = $\gamma_{lead}V_{lead}$ = 120000x0.000128 = 15.38N

5. A block of wood floats in water with 40mm projecting above the water surface. When placed in glycerin of sp gr 1.35, the block projects 70mm above the surface of that liquid. Determine the sp gr of wood.

Solution:

A = Area of block

h = Height of block

S = sp gr of block

Weight of wooden block (W) = $\gamma_{wood}V_{wood} = Sx9810xAh$ (a)

Weight of water displaced (W_W) = $\gamma_{water}V_{water\ diplaced}$ = 9810xA(h-0.04) (b)

Weight of glycerin displaced (W_G) = $\gamma_{glycerin}V_{glycerin\ diplaced}$ = 1.35x9810(h-0.07) (c)

Here, W = W_W = W_G

Equating a and b

Sx9810xAh= 9810xA(h-0.04)

$S = \frac{h-0.04}{h}$

Equating b and c

9810A(h-0.04) = 1.35x9810A (h-0.07)

h = 0.155m

$S = \frac{h-0.04}{h} == \frac{0.155-0.04}{0.155}$ =0.74

6. A rectangular open box, 7.6m by 3m in plan and 3.7m deep, weighs 350KN and is launched in fresh water. (a) How deep will it sink? (b)If the water is 3.7m deep, what weight of stone placed in the box will cause it to rest on the bottom?

Solution:

a) Depth of immersion (h) = ?

Weight of block = Weight of water displaced

$350000 = \gamma_{water} V_{water\ diplaced}$

350000 = 9810x7.6x3xh

h = 1.56m

b) Weight of block and stone = Weight of water displaced

$350000 + Weight_{stone} = \gamma_{water} V_{water\ diplaced}$

$350000 + Weight_{stone} = 9810x\ 7.6x3x3.7$

$Weight_{stone}$ = 477572N = 477.57KN

7. A stone weighs 500 N in air and 200N in water. Determine the volume of stone and its specific gravity.

Solution:

Weight in air-Weight in water = Buoyant force

Buoyant force (F_B) =500-200 = 300N

F_B = Weight of water displaced

$300 = \gamma_{water} V_{water\ diplaced}$

$300 = 9810x V_{water\ diplaced}$

$V_{water\ diplaced}$ = 0.0306m^3

Volume of stone (V) = 0.306m^3

Specific weight of stone $(\gamma_{stone}) = \frac{Weight\ in\ air}{V} = \frac{500}{0.0306}$ = 16340N/m^3

Specific gravity of stone $= \frac{\gamma_{stone}}{\gamma_{water}} = \frac{16340}{9810}$ = 1.66

8. A metallic body floats at the interface of mercury of specific gravity 13.6 and water in such a way that 30% of its volume is submerged in mercury and 70% in water. Find the density of the metallic body.

Solution:

Volume of metallic body = V

Density of metallic body = ρ

Weight of metallic body = Weight of fluid displaced = Buoyant force due to mercury + Buoyant force due to water

$\rho g V = \rho_{mercury} g V_{mercury\ diplaced} + \rho_{water} g V_{water\ diplaced}$

$\rho x 9.81 x V = 13.6 x 1000 x 9.81 x 0.3V + 1000 x 9.81 x 0.7V$

ρ = 4780 kg/m^3

9. A wooden block 4mx1mx0.5m is floating in water. Its specific gravity is 0.76. Find the volume of the concrete of specific gravity 2.5, that may be placed on the block which will immerse the (a) block completely in water and (b) block and concrete completely in water.

Solution:

a) Immersion of the block completely in water

Total weight of block and concrete = Weight of water displaced

$\gamma_{wood} V_{wood} + \gamma_{concrete} V_{concrete} = \gamma_{water} V_{water\ diplaced}$

$0.76 x 9810 x 4 x 1 x 0.5 + 2.5 x 9810 x V_{concrete} = 9810 x 4 x 1 x 0.5$

$V_{concrete}$ = 0.192m^3

b) Immersion of the block and concrete completely in water

Total weight of block and concrete = Weight of water displaced

$\gamma_{wood} V_{wood} + \gamma_{concrete} V_{concrete} = \gamma_{water} V_{water\ diplaced}$

$0.76 x 9810 x 4 x 1 x 0.5 + 2.5 x 9810 x V_{concrete} = 9810 x [4 x 1 x 0.5 + V_{concrete}]$

$V_{concrete}$ = 0.32m^3

10. Determine the specific weight and volume of an object that weighs 10N in water and 12N in oil of specific gravity 0.8.

Solution:

Weight of object = W

Volume of object = V

Weight of object – Weight in water = Buoyant force due to water

$W - 10 = \gamma_{water} V_{water\ diplaced}$

$W - 10 = 9810V$

$W - 10 = 9810V$ (a)

Weight of object – Weight in oil = Buoyant force due to oil

$W - 10 = \gamma_{oil} V_{oil\ diplaced}$

$W - 12 = 0.8 x 9810 x V$

$W - 12 = 7848V$ (b)

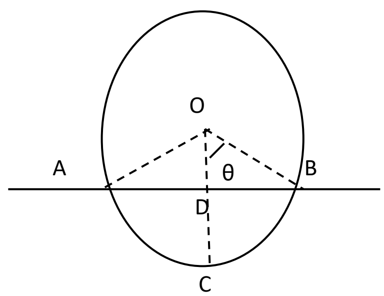
Solving a and b,

V = 0.001019 m^3

W = 10 + 9810x0.001019 = 19.996N

Specific weight of body $(\gamma_b) = \frac{W}{V} = \frac{19.996}{0.001019}$ = 19623 N/m^3

11. To what depth will a 2.4m diameter log 5m long and sp gr 0.4 sink in fresh water?



Solution:

Radius (r) = 1.2m

Depth of floatation = DC = ?

DB = 1.2Cosθ, DO =1.2Sinθ

Weight of log = Weight of water displaced

$\gamma_{log}V_{log} = \gamma_{water}V_{water\ diplaced}$

$0.4x9810x\pi x1.2^2x5 = 9810[V_{sector\ OABC} - V_{2\ traingles}]$

$9810x9.04 = 9810\left[\frac{2\theta}{360}x\pi x1.2^2x5 - 2x0.5x1.2cos\theta x1.2sin\theta\right]$

$9.04 = 0.1256\theta - 0.72sin2\theta$

Solving by trial and errors for θ

$\theta = 80$, Right side = 9.8

$\theta = 75$, Right side = 9.06

$\theta = 74.9$, Right side = 9.04

Take $\theta = 74.9$

Depth of floatation (DC) = OC – OD = 1.2 – 1.2Sin74.9 = 0.041m

12. What fraction of the volume of solid piece of metal of sp gr 7.2 floats above the surface of a container of mercury of sp. gr. 13.6?

Solution:

V = Volume of metal

V' = Volume of mercury displaced

Weight of body = Weight of mercury displaced

$\gamma_{body} V_{body} = \gamma_{mercury} V_{mercury\ diplaced}$

7.2x9810xV = 13.6x9810xV'

V'/V = 0.53

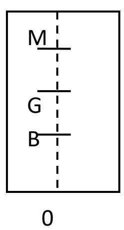
Fraction of volume above mercury = 1-0.53 = 0.47

13. A uniform body of size 4mx2mx1m floats in water. What is the weight of the body if the depth of immersion is 0.6m? Also determine the meta-centric height.

Solution:

Weight of body = Weight of water displaced

$= \gamma_{water} V_{water\ diplaced}$ = 9810x4x2x0.6 = 47088 N



Position of center of buoyancy (OB) = 0.6/2 = 0.3m

Position of cg (OG) = 1/2 = 0.5m

$MB = \frac{I}{V} = \frac{\frac{1}{12}x4x2^3}{4x2x0.6}$ = 0.556m

BG = 0.5-0.3 = 0.2m

GM = MB-BG = 0.556-0.2 = 0.356m

14. A solid cylinder of diameter 3m has a height of 2m. Find the meta-centric height of cylinder when it is floating in water with its axis vertical. The specific gravity of cylinder is 0.7.

Solution:

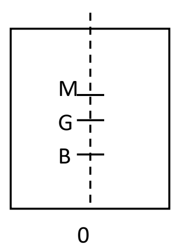
h = depth of immersion

Weight of body = Weight of water displaced

$\gamma_{cyl} V_{cyl} = \gamma_{water} V_{water\ diplaced}$

$0.7x9810x\pi x1.5^2x2 = 9810x\pi x1.5^2xh$

h = 1.4m



Position of center of buoyancy (OB) = 1.4/2 = 0.7m

Position of cg (OG) = 2/2 = 1m

BG = 1-0.7 = 0.3m

$MB = \frac{I}{V} = \frac{\frac{1}{4}x\pi x1.5^4}{\pi x1.5^2x1.4}$ = 0.4017m

GM = MB-BG = 0.4017-0.3 = 0.1017m

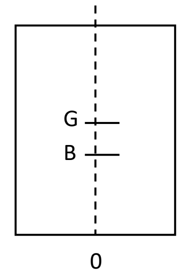
15. A solid wood cylinder has a diameter of 0.6m and a height of 1.2m. The sp.gr. of the wood is 0.6. If the cylinder is placed vertically in oil of sp.gr. 0.85, would it be stable?

Solution:
h = depth of immersion
Weight of body = Weight of oil displaced

$\gamma_{cyl}V_{cyl} = \gamma_{oil}V_{oil\ diplaced}$

$0.6x9810x\pi x0.3^2x1.2 = 0.85x9810x\pi x0.3^2xh$

h = 0.847m



Position of center of buoyancy (OB) = 0.847/2 = 0.4235
Position of cg (OG) = 1.2/2 = 0.6m
BG = 0.6-0.4235 = 0.1765m

$MB = \frac{I}{V} = \frac{\frac{1}{4}x\pi x0.3^4}{\pi x0.3^2x0.847}$ = 0.0265m

GM = MB-BG = 0.0265-0.1765 = -0.15m
As metacentric height GM is negative, the body is unstable.

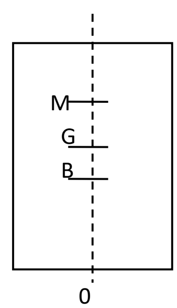
16. A body of size 3mx2mx2m floats in water. Find the limit of weight of the body for stable equilibrium.

Solution:
S = Sp gr of plastic
h = depth of immersion
Weight of body = Weight of water displaced

$\gamma_{body}V_{body} = \gamma_{water}V_{water\ diplaced}$

$Sx9810x3x2x2 = 9810x3x2xh$

h =2S



Position of center of buoyancy (OB) = 2S/2 = S
Position of cg (OG) = 2/2 = 1m
BG = OG-OB =1-S

$MB = \frac{I}{V} = \frac{\frac{1}{12}x3x2^3}{3x2x2S} = \frac{0.166}{S}$

GM = MB-BG = $\frac{0.166}{S} - 1 + S$

For stable equilibrium, GM > 0

$\frac{0.166}{S} - 1 + S > 0$

S^2-S+0.166>0

Values of S are 0.21 and 0.79

Lower limit of Weight = $\rho_{body} g V_{body}$ = 0.21x1000x9.81x3x2x2 = 24721N = 24.72KN

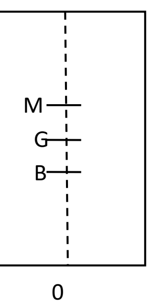
Upper limit of weight = $\rho_{body} g V_{body}$ = 0.79x1000x9.81x3x2x2 = 92999N = 92.99KN

17. A wooden cylinder of specific gravity 0.6 and circular in cross-section is required to float in oil of specific gravity 0.8. Calculate the ratio of length to diameter for the cylinder so that it will just float upright in water.

Solution:

Length of cylinder = L

Diameter of cylinder = D



h = depth of immersion

Weight of body = Weight of oil displaced

$\gamma_{cyl} V_{cyl} = \gamma_{oil} V_{oil\ diplaced}$

$0.6x9810x\frac{\pi}{4}xD^2xL = 0.8x9810x\frac{\pi}{4}xD^2xh$

h = 0.75L

Position of center of buoyancy (OB) = 0.75L/2 = 0.375L

Position of cg (OG) = L/2 = 0.5L

BG = OG-OB =0.5L-0.375L = 0.125L

$$MB = \frac{I}{V} = \frac{\frac{1}{64}x\pi xD^4}{\frac{\pi}{4}xD^2x0.75L} = \frac{D^2}{12L}$$

GM = MB-BG $= \frac{D^2}{12L} - 0.125L$

For stable equilibrium, GM > 0

$\frac{D^2}{12L} - 0.125L > 0$

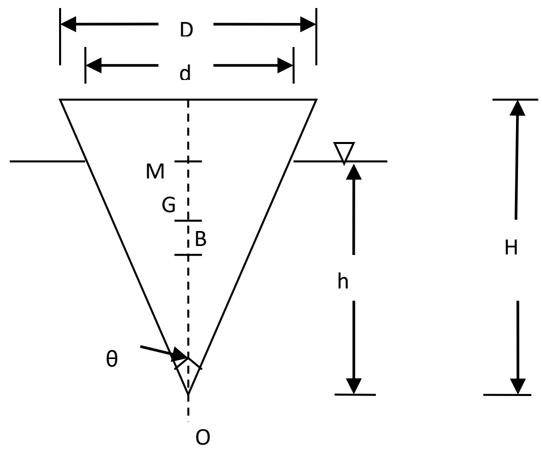
$D^2-1.5L^2>0$

$L^2/D^2 <0.667$

L/D<0.8167

18. A solid cone of specific gravity 0.7 floats in water with its apex downwards. Determine the least apex angle of the cone for equilibrium.

Solution:
D = Dia. Of cone
d =Dia. Of cone at water surface
h = Depth of immersion
H = Height of cone
2θ = Apex angle
R = Radius of cone
r = radius of cone at water surface



Finding depth of immersion (h)
Weight of cone = Weight of water displaced

$\gamma_{cone} V_{cone} = \gamma_{water} V_{water\ diplaced}$

$0.7x9810x\frac{1}{3}x\pi x R^2 x H = 9810x\frac{1}{3}x\pi x r^2 x h$

$h = \frac{0.7R^2H}{r^2}$

$h = \frac{0.7(HTan\theta)^2H}{(hTan\theta)^2}$

h = 0.8879H

OG = $\frac{3}{4}H$ = 0.75H

OB = $\frac{3}{4}h$ = 0.75x0.8879H = 0.6659H

BG = OG-OB = 0.75H-0.6659H = 0.0841H

$MB = \frac{I}{V} = \frac{\frac{1}{4}x\pi x r^4}{\frac{1}{3}x\pi x r^2 x h} = \frac{0.75r^2}{h}$

$= \frac{0.75(hTan\theta)^2}{h} = 0.75hTan^2\theta = 0.75x0.8879HTan^2\theta = 0.6659HTan^2\theta$

GM = MB-BG = $0.6659HTan^2\theta - 0.0841H$

For stable equilibrium GM>0

$0.6659HTan^2\theta - 0.0841H > 0$

$Tan^2\theta - 0.1263 > 0$

$Tan\theta > 0.3553$

$\theta > 19.56^0$

Least apex angle = 2θ = 2x19.56 = 39.12^0

19. A cylindrical buoy 1.8m in diameter, 1.2m high and weighing 10.5 KN floats in salt water of density 1025 kg/m^3. Its CG is 0.45m from the bottom. If a load of 3KN is placed on the top, find the maximum height of the CG of this load above the bottom if the buoy is to remain in stable equilibrium.

Solution:

G = CG of buoy

G1 = CG of load 3KN

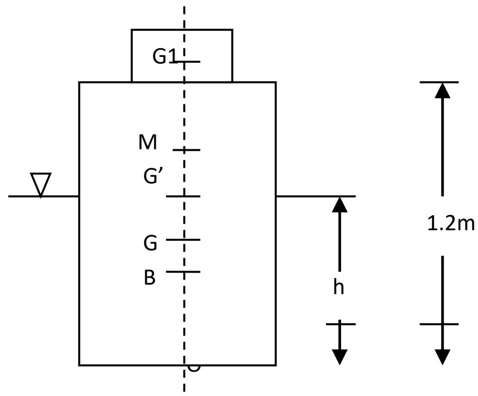
G' = Combined CG of load and buoy

h = depth of immersion

B = CB

OG =0.45m

OG1=?



Finding depth of immersion (h)

Weight of load and buoy = Weight of water displaced

$10500 + 3000 = \rho_{salt\ water} g V_{water\ diplaced}$

$13500 = 1025x9.81x\pi x0.9^2 xh$

h = 0.53m

OB = 0.53/2 = 0.265m

$MB = \frac{I}{V} = \frac{\frac{1}{64}x\pi x1.8^4}{\frac{\pi}{4}x1.8^2 x0.53}$ = 0.38m

BG' = OG'-OB = OG'-0.265

G'M = MB-BG' = 0.38- OG'+0.265 = 0.645-OG'

For stable equilibrium, G'M>0

0.645-OG'>0

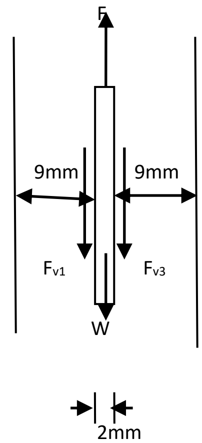
OG' < 0.645m

Taking moments about O,

3x10^{3}xOG1 + 10.5x10^{3}x0.45 = (3x10^3+10.5x10^3)x0.645

OG1 = 1.3275m

20. A plate of metal 1.1mx1.1mx2mm is to be lifted up with a velocity of 0.1m/s through an infinitely extending gap 20mm wide containing an oil of sp. gr. 0.9 and viscosity 2.1NS/m^2. Find the force required to lift the plate assuming the plate to remain midway in the gap. Assume the weight of the plate to be 30N.



Solution:
Velocity of plate (u) = 0.1m/s
Sp. gr. of oil = 0.9
Sp. wt. of oil (γ) = 0.9x9810 = 8829N/m^3
Viscosity of oil (μ) = 2.1NS/m^2
Clearance on both sides ($dy_1 = dy_2 = dy$) = 9mm = 0.009m
Weight of plate = 30N
Force required to lift the plat (F) = ?

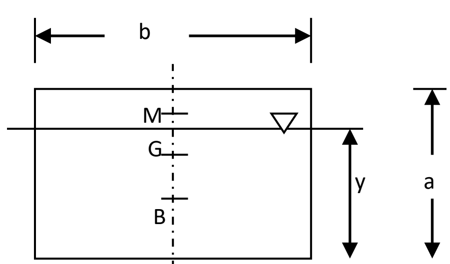
Upthrust on the plate = γxvol. of plate = 8829x1.1x1.1x2/1000 = 21.37N

Viscous force on the plate = Viscous force on left + Viscous force on right
= $\tau_1 A + \tau_2 A = (\tau_1 + \tau_2) A$
$= \left(\mu \frac{du}{dy} + \mu \frac{du}{dy}\right) A = 2\mu \frac{du}{dy} A = 2x2.1x\frac{0.1}{0.009}x1.1x1.1$ = 56.47N

Total force required to lift the plate = Weight-Upthrust + Viscous force = 30-21.37+56.47 = 65.1N

21. A block of wood of rectangular cross-section of sides a and b, and of length L has relative density of S. If the block is to float in water with its longest axis horizontal and the length a vertical, show that for stable equilibrium $\frac{b}{a} > \sqrt{6S(1-S)}$.

Solution:



G = CG, B = Center of buoyancy, M = Metacenter

O

For the block to float

Wt. of block = Wt. of water displaced

$\gamma_{block}V_{block} = \gamma_w V_{water\ diplaced}$

$\gamma_w S\ abL = \gamma_w\ byL$

y = Sa

$OB = \frac{Sa}{2}, OG = \frac{a}{2}$

$BG = \frac{a}{2} - \frac{Sa}{2} = \frac{a}{2}(1-S)$

$BM = \frac{I}{V} = \frac{\frac{1}{12}Lb^3}{bLy} = \frac{\frac{1}{12}Lb^3}{bLSa} = \frac{b^2}{12Sa}$

Metacentric height $(GM) = \frac{I}{V} - BG = \frac{b^2}{12Sa} - \frac{a}{2}(1-S)$

For stable equilibrium, GM>0

$\frac{b^2}{12Sa} - \frac{a}{2}(1-S) > 0$

$\frac{b^2}{12Sa} > \frac{a}{2}(1-S)$

$\frac{b}{a} > \sqrt{6S(1-S)}$

22. Consider a homogeneous right circular cylinder of length L, radius R, and specific gravity S, floating in water (S = 1) with its axis vertical. Show that the body is stable is $\frac{R}{L} = [2S(1-S)]^{1/2}$.

Solution:

Length of cylinder = L

Radius of cylinder = R

Specific gravity of cylinder = S

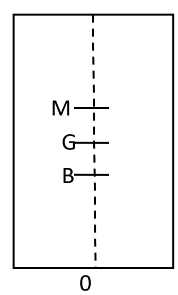
h = depth of immersion

Weight of cylinder = Weight of water displaced

$\gamma_{cyl}V_{cyl} = \gamma_{water}V_{water\ diplaced}$

$Sx9810x\pi xR^2xL = 9810x\pi xR^2xh$

h = SL



Position of center of buoyancy (OB) = SL/2

Position of CG (OG) = L/2

BG = OG-OB = $\frac{L}{2}(1-S)$

$MB = \frac{I}{V} = \frac{\frac{1}{4}x\pi xR^4}{\pi xR^2xh} = \frac{R^2}{4h} = \frac{R^2}{4SL}$

GM = MB-BG $= \frac{R^2}{4SL} - \frac{L}{2}(1-S)$

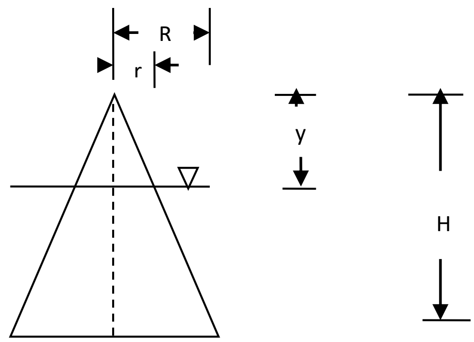
For stable equilibrium, GM $\geq$ 0

$\frac{R^2}{4SL} - \frac{L}{2}(1-S) = 0$

$\frac{R}{L} = [2S(1-S)]^{1/2}$

23. If a solid conical buoy of height H and relative density S floats in water with axis vertical and apex upwards, show that the height above the water surface of the conical buoy is equal to $H(1-S)^{1/3}$.

Solution:
R = Radius of cone
r =Radius of cone at water surface
y = Height above the water surface
H = Height of cone



Weight of cone = Weight of water displaced

$\gamma_{cone} V_{cone} = \gamma_{water} V_{water\ diplaced}$

$Sx\gamma_{water} x \frac{1}{3}\pi R^2 H = \gamma_{water} x \left(\frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 y\right)$

$R^2 SH = R^2 H - r^2 y$

$y = \frac{R^2}{r^2} H(1-S)$ (a)

From similar triangles,

$\frac{R}{r} = \frac{H}{y}$ (b)

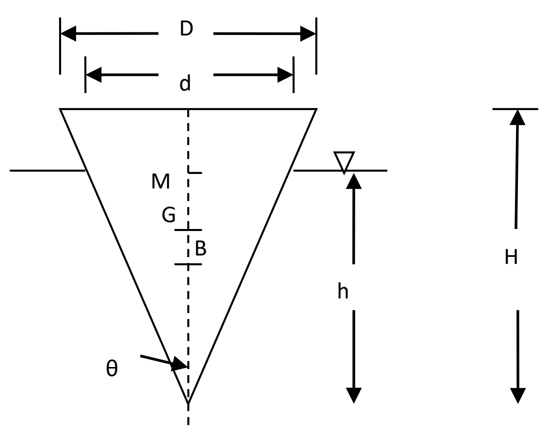
From a and b

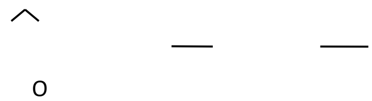
$y = \frac{H^2}{y^2} H(1-S)$

$y = H(1-S)^{1/3}$

24. A cone of base radius R and height H floats in water with the vertex downwards. If θ is the semi-vertex angle of the cone and h is the depth of immersion, show that for stable equilibrium $Sec^2\theta > H/h$.

Solution:
D = Dia. Of cone
d =Dia. Of cone at water surface
h = Depth of immersion
H = Height of cone
2θ = Apex angle
R = Radius of cone
r = radius of cone at water surface





$r = htan\theta$

OG = $\frac{3}{4}H$

OB = $\frac{3}{4}h$

BG = $\frac{3}{4}H - \frac{3}{4}h = 0.75(H-h)$

$MB = \frac{I}{V} = \frac{\frac{1}{4}\pi r^4}{\frac{1}{3}\pi r^2 h} = \frac{0.75r^2}{h} = \frac{0.75(htan\theta)^2}{h} = 0.75htan^2\theta$

GM $= \frac{I}{V} - BG = 0.75htan^2\theta - 0.75(H-h)$

For stable equilibrium, GM>0

$0.75htan^2\theta - 0.75(H-h) > 0$

$htan^2\theta > (H-h)$

$h(1+tan^2\theta) > H$

$Sec^2\theta > H/h$

25. A cone of base diameter d and height H floats in water with the axis vertical and vertex downwards. If the sp gr of the cone material is S, show that for stable equilibrium $H < \frac{1}{2}\left[\frac{d^2S^{1/3}}{1-S^{1/3}}\right]^{1/2}$.
If S = 0.7, what would be the minimum value of R/H for stable equilibrium?

Solution:

d = Dia. Of cone

d1 =Dia. Of cone at water surface

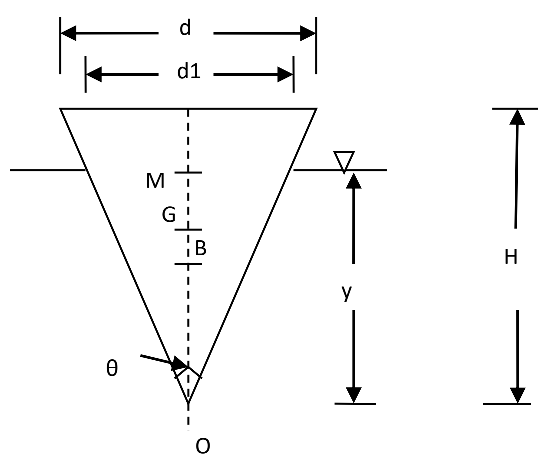
y = Depth of immersion

H = Height of cone

2θ = Apex angle

R = Radius of cone

r = radius of cone at water surface



Weight of cone = Weight of water displaced

$\gamma_{cone}V_{cone} = \gamma_{water}V_{water\ diplaced}$

$Sx\gamma_{water}x\frac{1}{3}\frac{\pi d^2}{4}H = \gamma_{water}x\frac{1}{3}\frac{\pi d1^2}{4}y$

$Sd^2H = d1^2y$ (a)

From similar triangles,

$\frac{R}{r} = \frac{H}{y}$

$\frac{d/2}{d1/2} = \frac{H}{y}$

$d1 = \frac{y}{H}d$ (b)

From a and b

$y = HS^{1/3}$ (c)

OG = $\frac{3}{4}H$

OB = $\frac{3}{4}y$

BG = $\frac{3}{4}H - \frac{3}{4}y = \frac{3}{4}(H - y) = \frac{3}{4}\left(H - HS^{1/3}\right) = \frac{3}{4}H\left(1 - S^{1/3}\right)$

$MB = \frac{I}{V} = \frac{\frac{1}{64}\pi d1^4}{\frac{1}{3}\frac{\pi d1^2}{4}y} = \frac{3d1^2}{16y} = \frac{3\left(\frac{y}{H}d\right)^2}{16y} = \frac{3d^2y}{16H^2} = \frac{3d^2HS^{1/3}}{16H^2} = \frac{3d^2S^{1/3}}{16H}$

GM = $\frac{I}{V} - BG = \frac{3d^2S^{1/3}}{16H} - \frac{3d^2S^{1/3}}{16H}$

For stable equilibrium, GM>0

$\frac{3d^2S^{1/3}}{16H} - \frac{3}{4}H\left(1 - S^{1/3}\right) > 0$

$\frac{3d^2S^{1/3}}{16H} > \frac{3}{4}H\left(1 - S^{1/3}\right)$

$H < \frac{1}{2}\left[\frac{d^2S^{1/3}}{1-S^{1/3}}\right]^{1/2}$

Numerical

$H < \frac{1}{2}\left[\frac{d^2S^{1/3}}{1-S^{1/3}}\right]^{1/2}$

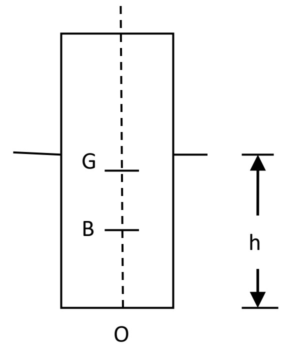
$H < \frac{d}{2}\left[\frac{S^{1/3}}{1-S^{1/3}}\right]^{1/2}$

$H < R\left[\frac{S^{1/3}}{1-S^{1/3}}\right]^{1/2}$

$\frac{R}{H} > \left[\frac{1-S^{1/3}}{S^{1/3}}\right]^{1/2}$

Minimum value of $\frac{R}{H} = \left[\frac{1-S^{1/3}}{S^{1/3}}\right]^{1/2} = \left[\frac{1-0.7^{1/3}}{0.7^{1/3}}\right]^{1/2}$ = 0.355

26. A cylindrical buoy 1.25m in diameter and 1.8m high has a mass of 770kg. Show that it will not float with its axis vertical in sea water of density 1025 kg/m^3. If one end of vertical chain is fastened to the base, find the pull required just to keep the buoy vertical. The CG of the buoy is 0.9m from its base.



Solution:
a) h = depth of immersion
Weight of body (W) = mg = 770x9,81 = 7553.7N
Weight of body = Weight of water displaced

$7553.7 = \rho_{sea\ water} g V_{water\ diplaced}$

$7553.7 = 1025x9.81x\frac{\pi}{4}x1.25^2xh$

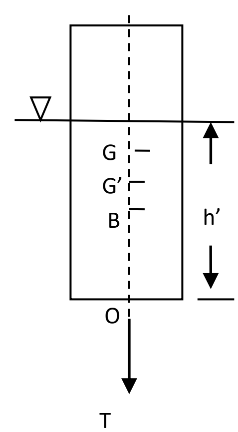
h = 0.612m

Position of center of buoyancy (OB) = 0.612/2 = 0.306m
Position of CG (OG) = 1.8/2 = 0.9m
BG = 0.9-0.306 = 0.594m

$MB = \frac{I}{V} = \frac{\frac{1}{64}x\pi x1.25^4}{\frac{\pi}{4}x1.25^2x0.612}$ = 0.16m

GM = MB-BG = 0.16-0.594 = -0.434m
As metacentric height GM is negative, the body is unstable and it will not float with its axis vertical.

b) T =pull in chain
Net upthrust (R) = T+W =(T+7553.7)



Finding new depth of immersion (h')
Net upthrust = Weight of water displaced

$R = \rho_{sea\ water} g V_{water\ diplaced}$

$R = 1025x9.81x\frac{\pi}{4}x1.25^2xh'$

h' = 0.00008R

OB = 0.00008R/2 = 0.00004(T+7553.7)
OG = 0.9m
G' = new CG due to T and W (lowered due to T)

$MB = \frac{I}{V} = \frac{\frac{1}{64}x\pi x1.25^4}{\frac{\pi}{4}x1.25^2x0.00008R} = \frac{1220.7}{(T+7553.7)}$

Taking moment about O (W passing through G and R through G')
W OG = R OG'
7553.7x0.9 = (T+7553.7)OG'

$OG' = \frac{6798.33}{T+7553.7}$

BG' = OG'-OB = $\frac{6798.33}{T+7553.7} - 0.00004(T + 7553.7)$

GM = MB-BG' = $\frac{1220.7}{(T+7553.7)} - \frac{6798.33}{T+7553.7} + 0.00004(T + 7553.7)$

$= \frac{-5577.63 - 0.00004(T+7553.7)^2}{T+7553.7}$

For stable equilibrium, GM>0

$$\frac{-5577.63 + 0.00004(T + 7553.7)^2}{T + 7553.7} > 0$$

T>4254.8N

27. A hollow cylinder of external radius R and internal radius r, height h and sp. gr. S floats in a liquid of sp. gr. S_0. Show that for its stable equilibrium

$$h \leq S_0\sqrt{\frac{R^2 + r^2}{2S(S_0 - S)}}$$

Solution:

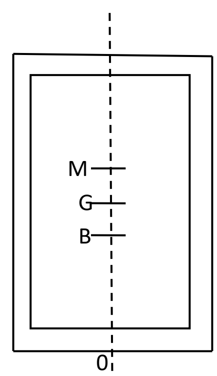
Height of cylinder = h

Specific gravity of liquid = S_0

Specific gravity of cylinder = S

h' = depth of immersion

Weight of cylinder = Weight of liquid displaced



$\gamma_{cyl}V_{cyl} = \gamma_{liquid}V_{water\ diplaced}$

$Sx9810x\pi x(R^2 - r^2)xh = 9810xS_0x\pi x(R^2 - r^2)xh'$

$h' = \frac{Sh}{S_0}$

Position of center of buoyancy (OB) = $\frac{Sh}{2S_0}$

Position of CG (OG) = h/2

BG = OG-OB = $\frac{h}{2} - \frac{Sh}{2S_0} = \frac{h}{2S_0}(S_0 - S)$

$MB = \frac{I}{V} = \frac{\frac{1}{4}x\pi x(R^4 - r^4)}{\pi x(R^2 - r^2)xh'} = \frac{R^2 + r^2}{4\frac{Sh}{S_0}} = \frac{S_0(R^2 + r^2)}{4Sh}$

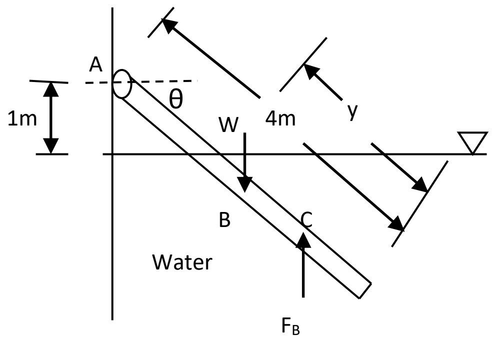
GM = MB-BG $= \frac{S_0(R^2+r^2)}{4Sh} - \frac{h}{2S_0}(S_0 - S)$

For stable equilibrium, GM $\geq$ 0

$\frac{S_0(R^2+r^2)}{4Sh} - \frac{h}{2S_0}(S_0 - S) \geq 0$

$h \leq S_0\sqrt{\frac{R^2+r^2}{2S(S_0-S)}}$

28, The wooden beam shown in the figure is 200mmx200mm and 4m long. It is hinged at A and remains in equilibrium at θ with the horizontal. Find the inclination θ. Sp. gr. of wood = 0.6.



Solution:

Length of beam immersed under water = y

Weight of beam (W) = $\gamma_{beam} V_{beam}$ = 0.6x9810x(0.2x0.2x4) = 941.76N

W acts at a distance of 2m from A.

Buoyant force on the beam (F_B) = Weight of water displaced = $\gamma_{water} V_{water\ dispalced}$

= 9810x(0.2x0.2xy) = 392.4y

FB acts at a C.

$AC = 4 - \frac{y}{2}$

Taking moment about hinge A,

$941.76x2cos\theta = 392.4y\left(4 - \frac{y}{2}\right)cos\theta$

$y^2 - 8y + 9.6 = 0$

y = 6.5, 1.47

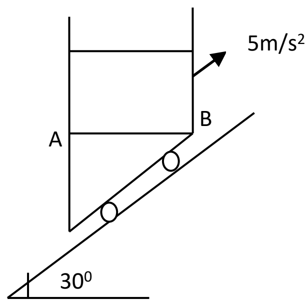
As y = 6.5 is not possible, y =1.47

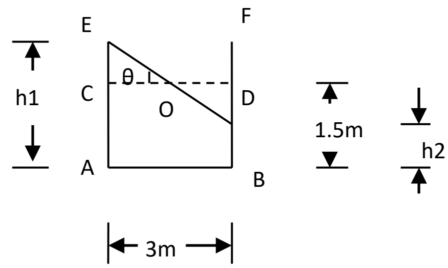
$sin\theta = \frac{1}{4-1.47}$

θ = 23.3^0

Tutorial 5
Relative equilibrium

1. An open rectangular tank 3m long and 2m wide is filled with water to a depth of 1.5m. Find the slope of the water surface when the tank moves with an acceleration of $5 m/s^2$ up a 30^0 inclined plane. Also calculate the pressure on the bottom at both ends.





Solution:
Inclination of surface (α) = 30^0
Acceleration (a) = 5 m/s^2
Slope of water surface (θ) = ?
Pressure at A (P_A) = ?, Pressure at B (P_B) = ?

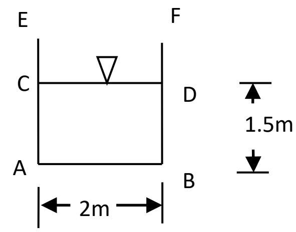
Components of acceleration
$a_x = 5\cos 30 = 4.33\ m/s^2$
$a_z = 5\sin 30 = 2.5\ m/s^2$

$$tan\theta = \frac{a_x}{g+a_z} = \frac{4.33}{9.81+2.5}$$
$\theta = 19.38^0$

Depth of water at rear end (h1) = 1.5+(3/2)xtan19.38 = 2.03m
Depth of water at front end (h2) = 1.5-(3/2)xtan19.38 = 0.97m

$$P_A = \gamma h1\left(1+\frac{a_z}{g}\right) = 9810x2.03\left(1+\frac{2.5}{9.81}\right) = 24989\ N/m^2$$
$$P_B = \gamma h2\left(1+\frac{a_z}{g}\right) = 9810x0.97\left(1+\frac{2.5}{9.81}\right) = 11941\ N/m^2$$

2. A rectangular tank 2m long, 1.5m wide and 1.5m deep is filled with oil of specific gravity 0.8. Find the force acting on the bottom of the tank when (a) the vertical acceleration $5m/s^2$ acts upwards (b) the vertical acceleration $5m/s^2$ acts downwards.



Solution:

a) Acceleration (a_z) = 5 m/s^2 (vertically upwards)

Depth of oil (h) = 1.5m

Force acting on the bottom (F_{AB}) = ?

$P_A = \gamma_{oil} h \left(1 + \frac{a_z}{g}\right) = 0.8x9810x1.5\left(1 + \frac{5}{9.81}\right)$ = 17772 N/m^2

F_{AB} = P_A x Area at bottom = 17772x2x1.5 = 53316N

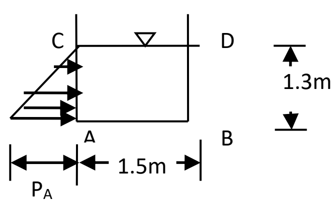
b) Acceleration (a_z) = -5 m/s^2 (vertically downwards)

Force acting on the bottom (F_{AB}) = ?

$P_A = \gamma_{oil} h \left(1 - \frac{a_z}{g}\right) = 0.8x9810x1.5\left(1 - \frac{5}{9.81}\right)$ = 5772 N/m^2

F_{AB} = P_{AB} x Area at bottom = 5772x2x1.5 = 17316N

3. An open cubical tank with each side 1.5m contains oil of specific weight 7.5KN/m^3 up to a depth of 1.3m. Find the forces acting on the side of the tank when it is being moved with an acceleration of 4m/s^2 in vertically upward and downward direction.



a) Acceleration (a_z) = 4 m/s^2 (vertically upwards)

Depth of oil (h) = 1.3m

Force acting on side (F1) = ?

$P_A = \gamma_{oil} h \left(1 + \frac{a_z}{g}\right) = 7500x1.3\left(1 + \frac{4}{9.81}\right)$ = 13725.5 N/m^2

F_1= Area of pressure diagram x width = 0.5x13725.5x1.3x1.5 = 13382N

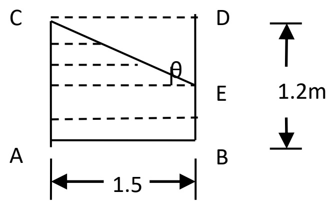
b) Acceleration (a_z) = -4 m/s^2 (vertically downwards)

Depth of oil (h) = 1.3m
Force acting on side (F2) = ?

$P_A = \gamma_{oil} h \left(1 + \frac{a_z}{g}\right) = 7500x1.3\left(1 - \frac{4}{9.81}\right)$ = 5774.46 N/m^2

F_2= Area of pressure diagram x width = 0.5x5774.46x1.3x1.5 = 5630N

4. An open rectangular tank 1.5mx1mx1.2m high is completely filled with water when at rest. Determine the volume spilled after the tank acquired a linear uniform acceleration of 0.6 m/s^2 in the horizontal direction.

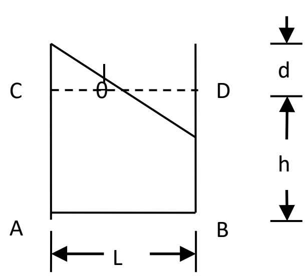


Solution:
Acceleration (a_x) = 0.6 m/s^2
Volume of water spilled=?

Slope of surface = tanθ = a_x/g = 0.6/9.81 = 0.0611
θ = 3.5^0
DE = 1.5tan3.5 =0.091m

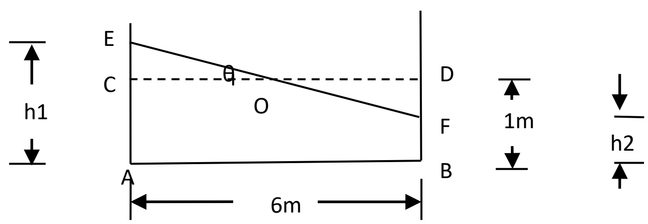
Volume of water spilled=Area of traingeCDEx width
= 0.5x1.5x0.091x1 = 0.06825 m^3

5. What distance must the sides of a tank be carried above the surface of water contained in it if the tank is to undergo a uniform horizontal acceleration of 3m/s^2 without spilling any water? (0.1529L)



Solution:
d =?
Slope of surface = $\tan\theta = a_x/g = 3/9.81 = 0.306$
$\theta = 17^0$
d = 0.5Ltan17 =0.153L
The tank must be carried 0.153L above the surface of water.

6. A rectangular tank 6m long, 2m wide and 2m deep contains water to a depth of 1m. It is accelerated horizontally at 2.5 m/s^2 in the direction of its length. Determine
a) Slope of the free surface
b) Maximum and minimum pressure intensities at bottom
c) Total force due to water acting on each end of tank. Check the difference between these forces by calculating the inertia force of the accelerated mass.



Solution:
Acceleration (a_x) = 2.5 m/s^2
Depth of water (h) = 1m

a) Slope of free surface (θ) = ?
Slope of surface = $\tan\theta = a_x/g = 2.5/9.81 = 0.255$
$\theta = 14.3^0$

b) Max pressure (P_A) =?
Min. pressure (P_B) = ?
h1= 1+3tanθ = 1.7646m
h2= 1-3tanθ = 0.2353m

$P_A = \gamma h1$ = 9810x1.7646 = 17310.7 N/m^2
$P_B = \gamma h2$ = 9810x0.2353 = 2308.3 N/m^2

c) Force on each side (F_{AE}, F_{BD}) = ?

F_{AE} = Area of pressure diagram x width = 0.5x17310.7x1.764x2 = 30546N

F_{BD} = Area of pressure diagram x width = 0.5x2308.3x0.2353x2 = 543N

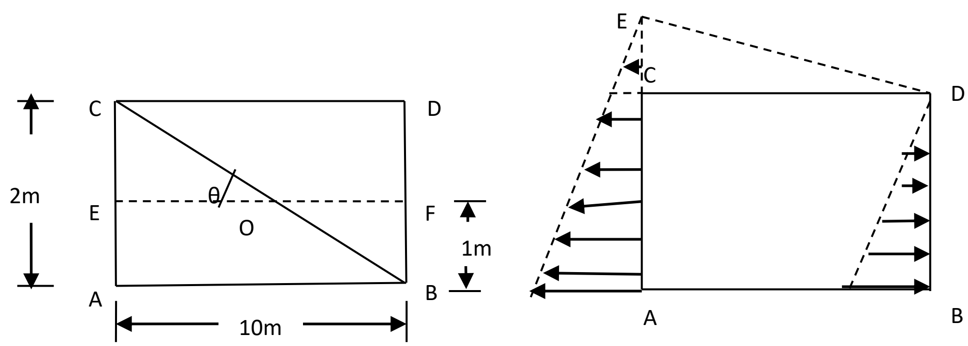
Difference in force (F_D) = 30546-543= 30003N

Force needed (F) = mass of water x linear acceleration = $\rho x\ Volume\ of\ water\ x a_x$

= 1000x(6x2x1)X2.5 = 30000N

F $\approx F_D$

7. An oil tanker 3m wide, 2m deep and 10m long contains oil of density 800 kg/m^3 to a depth of 1m. Determine the maximum horizontal acceleration that can be given to the tanker such that the oil just reaches its top end. If the tanker is closed and completely filled with the oil and accelerated horizontally at 3m/s^2, determine the total liquid thrust (hydrostatic force) on the front and rear end.



h2

Solution:

a) Maximum horizontal acceleration (a_x) = ?

When the oil touches the top end, the water surface rises 1m at the left side and falls 1m at the right side.

Maximum permissible slope $(\tan\theta) = \frac{1}{5}$ = 0.2

θ =11.3^0

$tan\theta = \frac{a_x}{g}$

$0.2 = \frac{a_x}{9.81}$

a_x = 1.962m/s^2

b) a_x = 3m/s^2

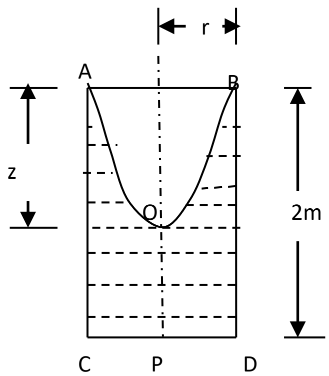
When the tanker is completely filled and closed, there will be pressure built up at the rear end equivalent to the virtual oil column CE that would assume a slope of a_x/g = 3/9.81 = 0.306.

CE = 10tanθ1 = 10x0.306 = 3.06m

Pressure at B = $\gamma x2$ = 9810x2 = 19620N/m^2
Pressure at A = $\gamma x(2+3.06)$ = 9810x5.06 = 49638.6N/m^2
Virtual pressure at C = $\gamma x3.06$ = 9810x3.06 = 30018.6N/m^2

Force at front end = Area of pressure diagram for BDx width
= 0.5x19620x2x3 = 58860N
Force at rear end = Area of pressure diagram for ACx width
= 0.5x(49638.6+30018.6)x2x3 = 238972N

8. An open circular cylinder of 1m diameter and 2m depth is completely filled with water and rotated about its axis about 45 rpm. Determine the depth at the axis and amount of water spilled. Also find the speed of rotation at which the central axial depth is zero.



Solution:
Radius (r) = 0.5m
rpm (N) = 45
Angular velocity $(\omega) = \frac{2N\pi}{60} = \frac{2x45x\pi}{60}$ = 4.712 rad/s
a) Depth at axis (PO) =?
Amount of water spilled = ?

$z = \frac{r^2\omega^2}{2g} = \frac{0.5^2 4.712^2}{2x9.81}$ =0.283m
PO = 2-0.283 = 1.717m

Amount of water spilled = Volume of paraboloid AOB
$= \frac{1}{2}x\pi r^2 z = \frac{1}{2}x\pi x0.5^2 x0.283$ = 0.111m^3

b) When O touches P, z becomes 2m.
speed of rotation (N) =?

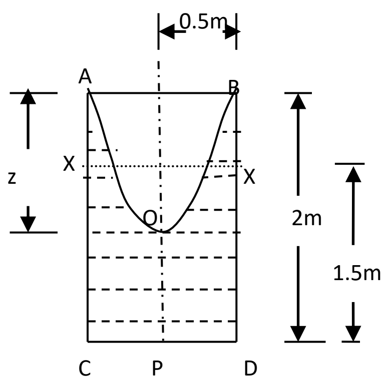
$z = \frac{r^2\omega^2}{2g}$

$2 = \frac{0.5^2\omega^2}{2x9.81}$

ω = 12.53 rad/s

$N = \frac{60\omega}{2\pi} = \frac{60x12.53}{2\pi}$ = 120rpm

9. An open circular vessel is 1m in diameter and 2m height. It contains water filled to a depth of 1.5m. If the cylinder rotates about its vertical axis, (a) what constant angular velocity can be obtained without spilling, (b) what is the pressure intensity at the center and at the corner of the bottom if ω = 6 radians/seconds.



Solution:

Radius (r) = 0.5m

a) Angular velocity (ω) =

If no water is spilled,

Volume above XX = volume of paraboloid AOB

$\pi x0.5^2x0.5 = \frac{1}{2}x\pi x0.5^2xz$

z = 1m

When the water surface just touches the top rim,

Rise of liquid at the edge above XX = Fall of liquid at the center below XX = z/2 = 0.5m

$z = \frac{r^2\omega^2}{2g}$

$1 = \frac{0.5^2\omega^2}{2x9.81}$

ω = 8.858 rad/s

$N = \frac{60\omega}{2\pi} = \frac{60x8.858}{2\pi}$ = 85 rpm

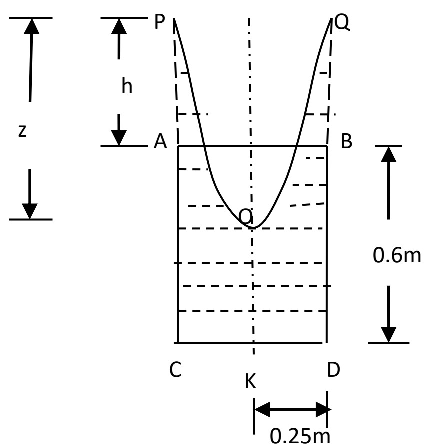
b) For ω = 6 rad/s

$z = \frac{r^2\omega^2}{2g} = \frac{0.5^2 6^2}{2g}$ = 0.46m

Origin O is 0.46/2 = 0.23m below XX.
The value of PO and CA are
PO = h1= 1.5-0.23 = 1.27m
CA = h2= 1.5+0.23 = 1.73m

Pressure at center (P_P) = $\gamma h1$ = 9810x1.27 = 12459 Pa
Pressure at corner (P_c) = $\gamma h2$ = 9810x1.73 =16971 Pa

10. A cylindrical vessel of 0.5 m diameter and 0.6 m height is completely filled with water under a pressure of 9.81 KN/m^2. It is rotated at 300 rpm about its vertical axis. Determine the pressure at point adjacent to the wall of the vessel.



Solution:
P = 9.81 KP = 9810Pa
Radius (r) = 0.25m
Rpm(N) = 300
Angular velocity $(\omega) = \frac{2N\pi}{60} = \frac{2x300x\pi}{60}$ = 31.41 rad/s
$z = \frac{r^2\omega^2}{2g} = \frac{0.25^2 31.41^2}{2g}$ = 3.14m

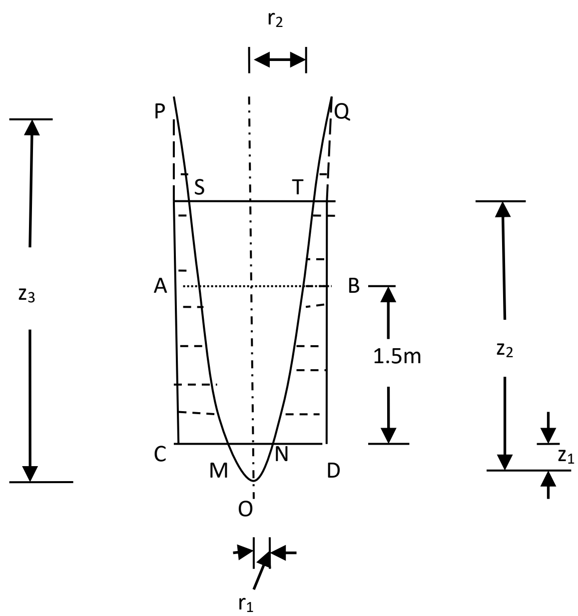
If the vessel is completely filled with water under a pressure and rotated about its vertical axis, the liquid will rise with a virtual height of h.

Volume above AB = volume of paraboloid POQ
$\pi x 0.25^2 x h = \frac{1}{2} x \pi x 0.25^2 x 3.14$
h = 1.57m

CP=h1= 0.6+1.57 = 2.17m

Pressure at C = $P + \gamma h1$ = 9810+9810X2.17 = 31098 Pa

11. A closed cylindrical vessel of 1m diameter and 2m height contains water filled to a depth of 1.5m. If the vessel is rotating at 20radians/sec, how much of the bottom of the vessel is uncovered?



Solution:
Radius (r) = 0.5m
Angular velocity (ω) = 20 rad/s

$z_3 = \frac{r^2\omega^2}{2g} = \frac{0.5^2 20^2}{2g}$ = 5.1m

$z_1 = \frac{r_1^2\omega^2}{2g} = \frac{r_1^2 20^2}{2g}$ = $20.38r_1^2$ (a)

$z_2 = \frac{r_2^2\omega^2}{2g} = \frac{r_2^2 20^2}{2g}$ = $20.38r_2^2$ (b)

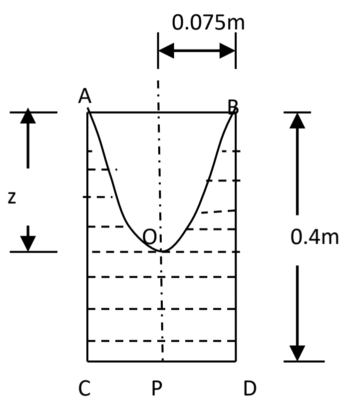
Also, $z_2 = z_1+1.5 = 20.38r_1^2 + 2$ (c)
Equating b and c,
$20.38r_1^2 + 2= 20.38r_2^2$ (d)
$r_2^2 - r_1^2 = 0.098$ (e)

Volume of air above AB = Volume of paraboliod (POQ-MON)
$\pi x 0.5^2 x 0.5 = \frac{1}{2} x \pi x r_2^2 z_2 - \frac{1}{2} x \pi x r_1^2 z_1$ (f)
From a , b and f

$0.25 = r_2^2 x 20.38 r_2^2 - r_1^2 x 20.38 r_1^2$
$r_2^4 - r_1^4 = 0.0122$ (g)
solving e and g
$(r_2^2 - r_1^2)(r_2^2 + r_1^2) = 0.0122$
$0.098 (r_2^2 + r_1^2) = 0.0122$
$r_2^2 + r_1^2 = 0.1244$
$r_2^2 - r_1^2 = 0.098$ (from e)
Solving above two equations
$r_1 = 0.115$m

Area uncovered = $\pi r_1^2 = \pi x 0.115^2$ = 0.0415m^2

12. A 400mm high open cylinder and 150mm in diameter is filled with water and rotated about its vertical axis at an angular speed of 33.5 rad/s. Determine (a) the depth of water in the cylinder when it is brought to rest, and (b) the volume of water that remains in the cylinder if the speed is doubled.



Solution:
Radius (r) = 0.075m
Angular velocity (ω) = 33.5 rad/s

a) Depth of water at rest (d) = ?

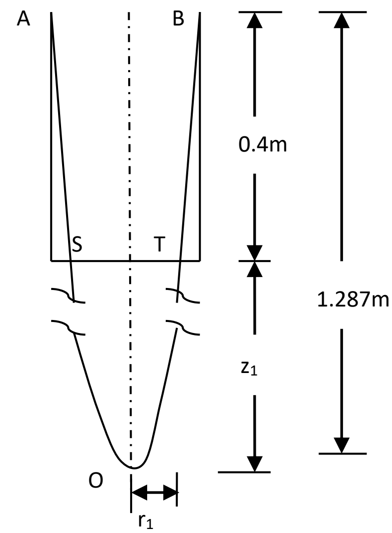
$z = \frac{r^2\omega^2}{2g} = \frac{0.075^2 33.5^2}{2x9.81}$ =0.32m

Amount of water spilled = Volume of paraboloid AOB

$= \frac{1}{2} x \pi r^2 z = \frac{1}{2} x \pi x 0.075^2 x 0.32$ = 0.002827m^3

Original volume of water $= \pi r^2 h = \pi x 0.075^2 x 0.4$ = 0.007069 m^3
Remaining volume of water (V_r) = 0.007069-0.002827 = 0.004242 m^3
$V_r = \pi r^2 d$

$0.004242 = \pi x 0.075^2 x d$
d = 0.24m

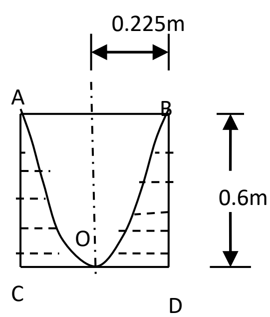


b) If the speed is doubled,
Angular velocity (ω) = 2x33.5 = 67 rad/s
$z = \frac{r^2\omega^2}{2g} = \frac{0.075^2 67^2}{2x9.81}$ = 1.287m
z_1 = 1.287-0.4 = 0.887m

$z_1 = \frac{{r_1}^2\omega^2}{2g}$
$0.887 = \frac{{r_1}^2 67^2}{2g}$
r_1 = 0.062m

Volume of water spilled = Volume of paraboloid (AOB-SOT)
$= \frac{1}{2} x\pi x 0.075^2 x 1.287 - \frac{1}{2} x\pi x 0.062^2 x 0.887$ = 0.006016 m^3
Original volume of water = 0.007069 m^3
Volume of water left = 0.007069-0.006016 = 0.00105 m^3

13. A cylindrical tank is spun at 300 rpm with its axis vertical. The tank is 0.6m high and 45cm diameter and is completely filled with water before spinning. Calculate (a) the speed at which the water surface will just touch the top rim and center bottom of the tank, and (b) the level to which the water will return when the tank stops spinning and the amount of water lost.



Solution:
Radius (r) = 0.225m
a) When the water surface touches the top rim and center bottom,

z = 0.6m

$z = \frac{r^2\omega^2}{2g}$

$0.6 = \frac{0.225^2\omega^2}{2g}$

ω = 15.25 rad/s

$N = \frac{60\omega}{2\pi} = \frac{60x15.25}{2\pi}$ = 146 rpm

b) Amount of water lost = ?

Depth of water after rest (d) = ?

Amount of water lost = Volume of paraboloid AOB

$= \frac{1}{2}x\pi x0.225^2x0.6$ = 0.0477m^3

Original volume of water $= \pi x0.225^2x0.6$ = 0.0954 m^3

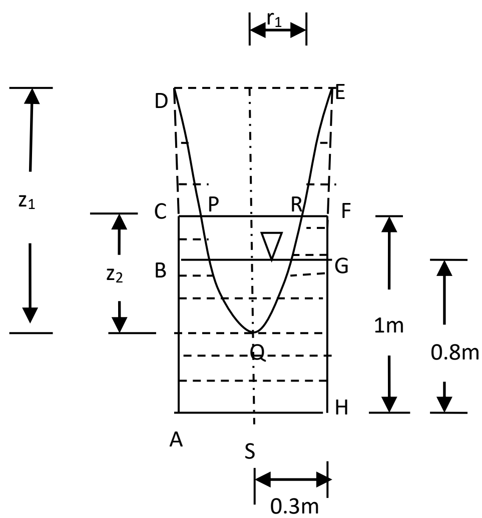
Volume of water left (V_r) = 0.0954-0.0477 = 0.0477 m^3

$V_r = \pi r^2 d$

$0.0477 = \pi x0.225^2 d$

d =0.3m

14. A cylindrical vessel closed at the top and bottom is 300mm inn diameter, 1m long and contains water up to a depth of 0.8m. The air above the water surface is at a pressure of 60 KPa. If the vessel is rotated at a speed of 250n rpm about its vertical axis, find the pressure head at the bottom of the vessel at the center point and at the edge.



Solution:

Pressure (Pa) = 60 Kpa

Head due to pressure (h) = $\frac{P_a}{\gamma} = \frac{60000}{9810}$ = 6.11m of water

Radius of cylinder (r) = 0.15m

N = 250 rpm

Pressure head at center and edge at the bottom = ?

Angular velocity $(\omega) = \frac{2N\pi}{60} = \frac{2x250x\pi}{60}$ = 26.16 rad/s

$z_1 = \frac{r^2\omega^2}{2g} = \frac{0.15^2 26.16^2}{2x9.81}$ =0.785m

$z_2 = \frac{r_1^2\omega^2}{2g} = \frac{r_1^2 26.16^2}{2x9.81} = 34.88r_1^2$ (a)

Volume of air above BG = Volume of parabola PQR

$$\pi x0.15^2 x0.2 = \frac{1}{2}x\pi x r_1^2 x z_2$$

$r_1^2 z_2 = 0.009$ (b)

Solving a and b

r_1 =0.13m, Z_2 = 0.59m

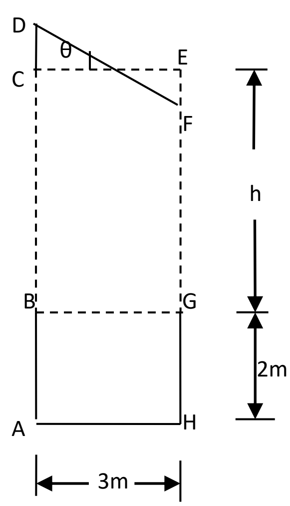
QS = 1-Z2 = 1-0.59 = 0.41m

AD = AC+CD = 1+(0.785-0.59) = 1.195m

Pressure head at center = h+QS = 6.11+0.41 = 6.52m

Pressure head at edge = h+AD = 6.11+1.195 = 7.305m

15. A closed rectangular tank full of water is 3m long, 2m wide and 2m deep. The pressure at the top of water is raised to 98.1 Kpa. If now the tank is accelerated horizontally along its length at 6m/s^2, find the forces on the front and rear ends of the tank. Check your results by Newton's law too.



Solution:
Acceleration (a_x) = 6m/s^2
Pressure (P) = 98.1Kpa

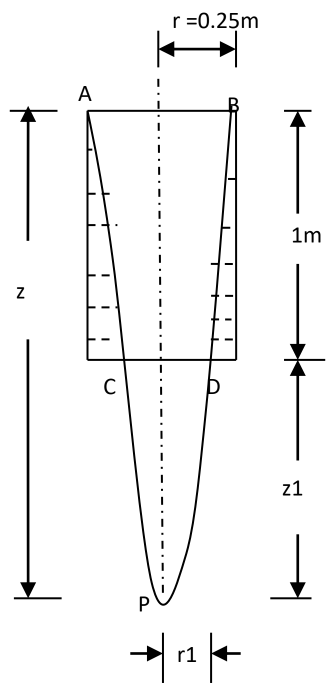
Head due to pressure (h) = $\frac{P}{\gamma} = \frac{98100}{9810}$ = 10m of water

$$tan\theta = \frac{a_x}{g} = \frac{6}{9.81}$$

CD = EF =$1.5tan\theta = 1.5x\frac{6}{9.81}$ = 0.917m
Pressure force at rear end (F_1) = $\gamma A\overline{x1}$ = 9810x(2x2)x(0.917+10+1) = 467623N
Pressure force at front end (F_2) = $\gamma A\overline{x2}$ = 9810x(2x2)x(10-0.917+1) = 395657N
Net force (F_x) = $F_1 - F_2$ = 467623 - 395657 = 71966N
According to Newton's law
F = m. a_x = ρ Volume a_x = 1000x(3x2x2)x6 = 72000 N
Hence F_x is equal to F.

16. An open cylinder tank 0.5m in diameter and 1m height is completely filled with water and rotated about its axis at 240 rpm. Determine the radius up to which the bottom will be exposed and the volume of water spilled out of the tank.

Solution:



Radius (r) = 0.25m
N = 240rpm
Angular velocity $(\omega) = \frac{2N\pi}{60} = \frac{2x240x\pi}{60}$ = 25.13 rad/s

$z = \frac{r^2\omega^2}{2g} = \frac{0.25^2 25.13^2}{2g}$ = 2.01m

Z1 = 2.01-1= 1.01m
r1 = ?
Volume of water spilled = ?

$z_1 = \frac{{r_1}^2\omega^2}{2g}$

$1.01 = \frac{{r_1}^2 25.13^2}{2g}$

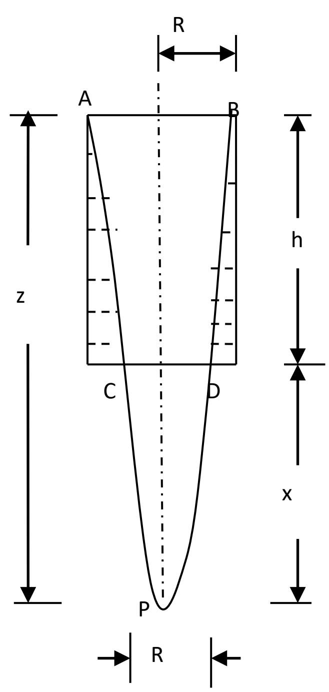
r_1 = 0.18m

Volume of water spilled = Volume of paraboloid (APB-CPD)

$= \frac{1}{2} x\pi x 0.25^2 x 2.01 - \frac{1}{2} x\pi x 0.18^2 x 1.01$ = 0.146 m^3

17. An open circular cylindrical pipe of radius R and height h is completely filled with water with its axis vertical and is rotated about its axis at an angular velocity ω. Determine the value of ω in terms of R and h such that the diameter of the exposed center portion is equal to the radius of the cylinder.

Solution:



Radius of cylinder = R
Diameter of exposed central portion = R
Radius of exposed central portion = R/2
For parabola APB

$z = \frac{R^2\omega^2}{2g}$

$x + h = \frac{R^2\omega^2}{2g}$ (a)

For parabola CPD

$x = \frac{(R/2)^2\omega^2}{2g} = \frac{R^2\omega^2}{8g}$ (b)

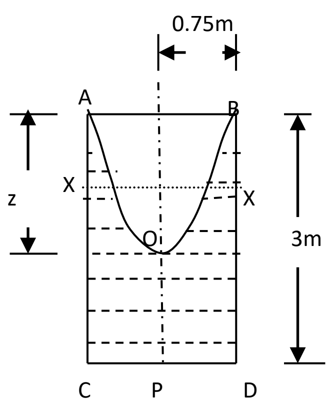
From a and b

$h = \frac{R^2\omega^2}{2g} - \frac{R^2\omega^2}{8g} = \frac{3R^2\omega^2}{8g}$

$\omega = \sqrt{\frac{8gh}{3R^2}}$

18. A cylindrical tank 1.5m in diameter and 3m in height contains water to a depth of 2.5m. Find the speed of the tank so that 20% of the original volume is spilled out.

Solution:



Radius of the tank (r) = 0.75m

After 20% of the original quantity of water spills out and if the depth is brought to rest, the depth of water will be reduced to 20%.

Depth of water when the tank comes to rest = 80% of original depth = 0.8x2.5 = 2m

Rise in water level at the end = 3-2 = 1m = fall at the center

i. e. z = 2m

$z = \frac{r^2\omega^2}{2g}$

$2 = \frac{0.75^2\omega^2}{2g}$

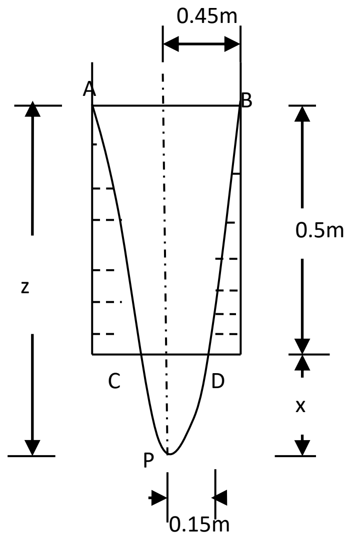
ω = 8.35 rad/s

$N = \frac{60\omega}{2\pi} = \frac{60x8.35}{2\pi}$ = 80 rpm

19. Determine the speed of rotation of a cylinder 900mm diameter when the liquid contained in it rises to 500mm height at sides and leaves a circular space 300mm diameter on the bottom uncovered. Taking

the liquid as water, calculate the total pressure on the bottom. Find also the depth when the vessel is stationary.

Solution:



Radius of cylinder (r) = 0.45m

Radius at bottom (r1) = 0.15m

For APB

$$z = \frac{r^2\omega^2}{2g}$$

$$x + 0.5 = \frac{0.45^2\omega^2}{2g} = 0.0103\omega^2 \qquad \text{(a)}$$

For CPD,

$$x = \frac{r1^2\omega^2}{2g}$$

$$x = \frac{0.15^2\omega^2}{2g} = 0.001147\omega^2 \qquad \text{(b)}$$

From a and b

ω = 7.4 rad/s, x = 0.063m

$N = \frac{60\omega}{2\pi} = \frac{60x7.4}{2\pi}$ = 70.6 rpm

Volume of water spilled = Volume of paraboloid (APB-CPD)

$= \frac{1}{2}x\pi x 0.45^2 x(0.5 + 0.063) - \frac{1}{2}x\pi x 0.15^2 x 0.063$ = 0.1768 m^3

Volume of water in the tank before rotation = $\pi x 0.45^2 x 0.5$ = 0.318 m^3

Volume of water left (V_r) =0.318-0.1768 = 0.1412 m^3

Total pressure on the bottom = Weight of water in the tank = γV_r = 9810x0.1412 = 1385N

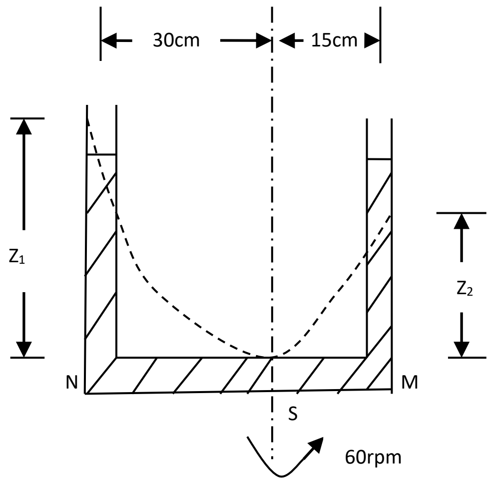
$$V_r = \pi r^2 d$$

$$0.1412 = \pi x 0.45^2 d$$

d = 0.22m

Depth of water when the vessel is stationary = 0.22m

20. A U-tube shown in figure is filled with a liquid of specific gravity 1.25 to a height of 15cm in both the limbs. It is rotated about a vertical axis 15cm from one limb and 30cm from the other. If the speed of rotation is 60rpm, find the difference in the liquid levels in the two limbs. Also find the pressure at points M and N at the base of U-tube.



Solution:

Distance from the axis of rotation to left side (r_1) = 0.3m

Distance from the axis of rotation to right side (r_2) = 0.15m

Speed of rotation (N) = 60 rpm

Angular velocity $(\omega) = \frac{2N\pi}{60} = \frac{2x60x\pi}{60}$ = 6.28 rad/s

$z_1 = \frac{r_1^2\omega^2}{2g} = \frac{0.3^2 x6.28^2}{2g}$ = 0.181m

$z_2 = \frac{r_2^2\omega^2}{2g} = \frac{0.15^2 x6.28^2}{2g}$ = 0.045m

Difference in level = 0.181-0.045 = 0.136m

Sum of Z_1 and Z_2 = 0.226

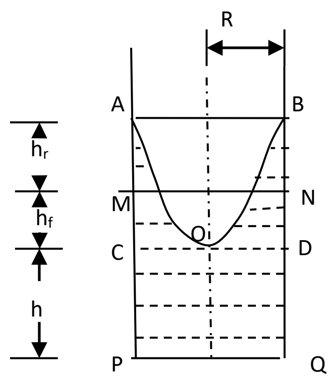
But total height of water in two limbs before rotation = 0.15+0.15 = 0.3m

The difference (0.3-0.226 = 0.074) is equally divided for two limbs.

Pressure at N = $\gamma(Z_1 + 0.074/2)$ = 1.25x9810x0.218 = 2673 Pa

Pressure at M = $\gamma(Z_2 + 0.074/2)$ = 1.25x9810x0.082 = 1005.5 Pa

21. Prove that in case of forced vortex, rise of liquid levels at the end is equal to the fall of liquid level at the axis of rotation.



Solution:
R = Radius of cylinder
MN = Water level at absolute equilibrium (original water level)
After rotation, AOB is the profile of the liquid surface.
h_r = Rise of liquid at end
h_f = Fall of liquid at the end

Volume of liquid before rotation = $\pi R^2(h + h_f)$
Volume of liquid rotation = Volume of cylinder ABQP- Volume of paraboloid AOB

$$= \pi R^2(h + h_f + h_r) - \frac{1}{2}\pi R^2(h_f + h_r)$$

Volume of liquid before rotation = Volume of liquid after rotation

$$\pi R^2(h + h_f) = \pi R^2(h + h_f + h_r) - \frac{1}{2}\pi R^2(h_f + h_r)$$

$$h_r = h_f$$

Hence, rise of liquid level at the end = fall of liquid level at the axis of rotation

Tutorial 6

Fluid Kinematics

1. Water is flowing through a pipe of 2.5cm diameter with a velocity of 0.5m/s. Compute the discharge in m^3/s and litres/s.

Solution:
Diameter of pipe (d) = 2.5cm = 0.025m
C/S Area of pipe (A) = $\frac{\pi}{4}x0.025^2$ = 0.000491 m^2
Velocity (V) = 0.5m/s
Discharge (Q) = ?

Q = AV = 0.000491x0.5 = 0.000245 m^3/s
Expressing Q in lps
Q = 0.000245x1000= 0.245 lps

2. A 30 cm diameter pipe carries oil of sp. gr. 0.8 at a velocity of 2m/s. At another section the diameter is 20cm. Compute the velocity at this section and discharge in m^3/s and kg/s.

Solution:
Diameter of pipe at section1 (d_1) = 30cm = 0.3m
Area of pipe at section1 (A_1) = $\frac{\pi}{4}x0.3^2$ = 0.07068 m^2
Velocity of pipe at section1 (V_1) = 2m/s
Diameter of pipe at section2 (d_2) = 20cm = 0.2m
Area of pipe at section1 (A_2) = $\frac{\pi}{4}x0.2^2$ = 0.0314 m^2
Velocity of pipe at section1 (V_2) =?
Discharge at section 2(Q_2) = ?

Discharge at section 1(Q_1) = A_1 V_1 = 0.07068x2 = 0.1413 m^3/s
According to continuity,
Q_2 = Q_1 = 0.1413 m^3/s
Expressing Q_2 in kg/s
Q_2 = Density x Discharge in m^3/s = 0.8x1000x0.1413 = 113 kg/s
V_2 =Q_2/A_2 = 0.1423/0.0314 = 4.5 m/s

3. Given the velocity field: $V = (6 + 2xy + t^2)i - (xy^2 + 10t)j + 25k$
What is the acceleration of a particle at (3, 0, 2) at time t = 1?

Solution:

$u = (6 + 2xy + t^2), v = (xy^2 + 10t), w = 25k$

$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$

$= (6 + 2xy + t^2)2y + (xy^2 + 10t)2x + 25kx0 + 2t$

At (3,0,2) and t = 1

$a_x = (6 + 2x3x0 + 1^2)2x0 + (3x0^2 + 10x1)2x3 + 25kx0 + 2x1$ = 62

$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$

$= (6 + 2xy + t^2)y^2 + (xy^2 + 10t)2xy + 25kx0 + 10$

At (3,0,2) and t = 1

$a_y = (6 + 2x3x0 + 1^2)0^2 + (3x0^2 + 10x1)2x3x0 + 25kx0 + 10$ = 10

$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$

$= (6 + 2xy + t^2)0 + (xy^2 + 10t)0 + 25kx0 + 0$

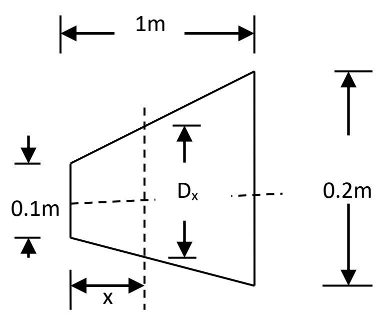
At (3,0,2) and t = 1

$a_y = (6 + 2x3x0 + 1^2)0 + (3x0^2 + 10x1)0 + 25kx0 + 0$ = 0

Acceleration $(a) = \sqrt{a_x^2 + a_y^2 + a_z^2}$ = $\sqrt{62^2 + 10^2 + 0^2}$ = 62.8

4. A conical pipe diverges uniformly from 0.1m to 0.2m diameter over a length of 1m. Determine the local and convective accelerations at the mid section assuming (a) rate of flow is 0.1 m^3/s and it remains constant, (b) at 2 sec if the rate of flow varies uniformly from 0.1 to 0.2 m^3/s in 5Sec.

Solution:



At any distance x, diameter is

$D_x = 0.1 + \left(\frac{0.2-0.1}{1}\right)x = 0.1(1 + x)$

Cross sectional area $(A_x) = \frac{\pi}{4}D_x^2 = \frac{\pi}{4}[0.1(1 + x)]^2 = 0.00785(1 + x)^2$

Velocity of flow $(u_x = u) = \frac{Q}{A_x} = \frac{Q}{0.00785(1+x)^2}$

(a) Q = 0.1 m^3/s (constant)

Local acceleration = $\frac{\partial u}{\partial t}$ = 0 (steady)

Convective acceleration = $u\frac{\partial u}{\partial x} = \frac{Q}{0.00785(1+x)^2}\frac{-2Q}{0.00785(1+x)^3}$

At mid section (x = 0.5m),

Convective acceleration $= \frac{-2x0.1^2}{0.00785^2(1+0.5)^5}$ = 42.74 m/s^2

(b) Q varies uniformly from 0.1 to 0.2 m^3/s in 5Sec

At t = 2S

Q =0.1 $+ \left(\frac{0.2-0.1}{5}\right) x2$ = 0.14 m^3/s

Local acceleration = $\frac{\partial u}{\partial t} = \frac{1}{0.00785(1+x)^2}\frac{\partial Q}{\partial t}$

At x = 0.5m, t = 2s

Local acceleration = $\frac{1}{0.00785(1+0.5)^2}\frac{0.14-0.1}{2}$ = 1.132 m/s^2

Convective acceleration = $u\frac{\partial u}{\partial x} = \frac{Q}{0.00785(1+x)^2}\frac{-2Q}{0.00785(1+x)^3}$

At x = 0.5m

Convective acceleration $= \frac{-2x0.14^2}{0.00785^2(1+0.5)^5}$ = 83.77 m/s^2

5. The following cases represent the two velocity components, determine the third component of velocity such that they satisfy the continuity equation:

a) u = 3x^2, v = 4xyz

w = ?

According to continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial\,(3x^2)}{\partial x} + \frac{\partial(4xyz)}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$6x + 4xz + \frac{\partial w}{\partial z} = 0$$

$$w = \int -(6x + 4xz)\,dz$$

$$w = -6xz - 2xz^2 + f(x, y)$$

b) u = 5x^2+2xy, w = 2z^3-4xy-2yz

v = ?

According to continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial\,(5x^2+2xy)}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial(2z^3-4xy-2yz)}{\partial z} = 0$$

$$10x + 2y + \frac{\partial v}{\partial y} + 6z^2 - 0 - 2y = 0$$

$$v = \int -(10x + 6z^2)\,dy$$

$$v = -10xy - 6z^2y + f(x, z)$$

6. Which of the following velocity fields satisfies continuity equation?

a) u = $4xy+y^2$, v = 6xy+3x

Solution:
To satisfy the continuity equation,
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Computing $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}$
$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=\frac{\partial(4xy+y^2)}{\partial x}+\frac{\partial(6xy+3x)}{\partial y}$ = 4y +0+6x+0 = 4y+6x
Here, $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\neq 0$
Therefore, it does not satisfy continuity.

b) u =$2x^2+y^2$, v = -4xy

Solution:
To satisfy the continuity equation,
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Computing $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}$
$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=\frac{\partial(2x^2+y^2)}{\partial x}+\frac{\partial(-4xy)}{\partial y}$ = 4x +0-4x = 0
Here, $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$
Therefore, it does satisfy continuity.

c. u = $2x^2-xy+z^2$, v = $x^2-4xy+y^2$, w = $-2xy-yz+y^2$

To satisfy continuity equation,
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$
Computing $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=\frac{\partial\,(2x^2-xy+z^2)}{\partial x}+\frac{\partial(x^2-4xy+y^2)}{\partial y}+\frac{\partial(-2xy-yz+y^2)}{\partial z}$$
= 4x-y+0+0-4x+2y-0-y+0 = 0
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\neq 0$$
Therefore, it does satisfy continuity.

d. $u=-\frac{Ky}{x^2+y^2}, v=\frac{Kx}{x^2+y^2}$

Solution:

To satisfy the continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Computing $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial\left(-\frac{Ky}{x^2+y^2}\right)}{\partial x} + \frac{\partial\left(\frac{Kx}{x^2+y^2}\right)}{\partial y}$$

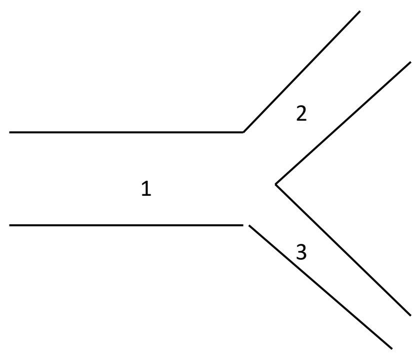
$$= (-Ky)\frac{-1}{(x^2+y^2)^2}2x + (Kx)\frac{-1}{(x^2+y^2)^2}2y$$

$$= \frac{2Kxy}{(x^2+y^2)^2} - \frac{2Kxy}{(x^2+y^2)^2} = 0$$

Here, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

Therefore, it does satisfy continuity.

7. A 40cm diameter pipe, conveying water, branches into two pipes of diameters 30cm and 20cm respectively. If the discharge in the 40cm diameter pipe is $0.38 m^3/s$, compute the average velocity in this pipe. If the average velocity in 30cm diameter pipe is 2m/s, find the discharge and average velocity in 20cm diameter pipe.



Solution:

Diameter of pipe 1 (d_1) = 40cm = 0.4m

C/S Area of pipe 1 (A_1) = $\frac{\pi}{4} x 0.4^2$ = 0.1256 m^2

Diameter of pipe 2 (d_2) = 30cm = 0.3m

C/S Area of pipe 2 (A_2) = $\frac{\pi}{4} x 0.3^2$ = 0.07068 m^2

Diameter of pipe 3 (d_3) = 20cm = 0.2m

C/S Area of pipe 3 (A_3) = $\frac{\pi}{4} x 0.2^2$ = 0.0314 m^2

Discharge through pipe 1(Q_1) = 0.38 m^3/s

Velocity at pipe 2 (V_2) = 2m/s

Velocity at pipe 1 (V_1) = ?

Discharge through pipe 3(Q_3) = ?

Velocity at pipe 3 (V_3) = ?

$V_1 = Q_1/A_1$ = 0.38/0.1256 = 3.025m/s

$Q_2 = A_2 V_2 = 0.07068x2 = 0.1413\ m^3/s$

From continuity,

$Q_1 = Q_2 + Q_3$

$Q_3 = Q_1 - Q_2 = 0.38 – 0.1413 = 0.2387\ m^3/s$

$V_3 = Q_3/A_3 = 0.1387/0.0314 = 7.6 m/s$

8. Is the continuity equation for steady, incompressible flow satisfied with the following velocity components in polar co-ordinate?

a. $v_\theta = Cr,\ v_z = K(R^2 - r^2),\ v_r = 0$

Solution:

To satisfy the continuity equation,

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{1}{r}\frac{\partial(v_\theta)}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

Computing $\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{1}{r}\frac{\partial(v_\theta)}{\partial \theta} + \frac{\partial v_z}{\partial z}$

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{1}{r}\frac{\partial(v_\theta)}{\partial \theta} + \frac{\partial v_z}{\partial z} = \frac{1}{r}\frac{\partial(rx0)}{\partial r} + \frac{1}{r}\frac{\partial(Cr)}{\partial \theta} + \frac{\partial\left(K(R^2-r^2)\right)}{\partial z}$$

$$= 0 + 0 + 0 = 0$$

Here, $\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{1}{r}\frac{\partial(v_\theta)}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$

Therefore, the continuity equation is satisfied.

b. $v_r = Kcos\theta(1 - \frac{b}{r^2}),\ v_\theta = -Ksin\theta(1 + \frac{b}{r^2})$

Solution:

To satisfy the continuity equation,

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{1}{r}\frac{\partial(v_\theta)}{\partial \theta} = 0$$

Computing $\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{1}{r}\frac{\partial(v_\theta)}{\partial \theta}$

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{1}{r}\frac{\partial(v_\theta)}{\partial \theta} = \frac{1}{r}\frac{\partial\left(rKcos\theta(1-\frac{b}{r^2})\right)}{\partial r} + \frac{1}{r}\frac{\partial\left(-Ksin\theta(1+\frac{b}{r^2})\right)}{\partial \theta}$$

$$= \frac{1}{r}\left[rKcos\theta\frac{\partial\left(1-\frac{b}{r^2}\right)}{\partial r} + \left(1 - \frac{b}{r^2}\right)\frac{\partial(rKcos\theta)}{\partial r}\right] + \frac{1}{r}\left[-Kcos\theta\left(1 + \frac{b}{r^2}\right)\right]$$

$$= \frac{1}{r}\left[rKcos\theta\left(0 + \frac{2b}{r^3}\right) + Kcos\theta\left(1 - \frac{b}{r^2}\right)\right] - \frac{1}{r}\left[Kcos\theta\left(1 + \frac{b}{r^2}\right)\right]$$

$$= \frac{1}{r}\left[\frac{2Kbcos\theta}{r^2} + Kcos\theta - \frac{Kbcos\theta}{r^2}\right] - \frac{1}{r}\left[Kcos\theta\left(1 + \frac{b}{r^2}\right)\right]$$

$$= \frac{Kbcos\theta}{r^3} + \frac{Kcos\theta}{r} - \frac{Kcos\theta}{r} - \frac{Kbcos\theta}{r^3} = 0$$

Here, $\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{1}{r}\frac{\partial(v_\theta)}{\partial \theta} = 0$

Therefore, the continuity equation is satisfied.

9. In polar co-ordinate, the two velocity components are given as $v_z = u_{max}(1 - r^2/R^2)$ and $v_\theta = 0$. Determine $v_r(r, z)$ from the incompressible relation if u_{max} is constant..

Solution:

From continuity equation,

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{1}{r}\frac{\partial(v_\theta)}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{1}{r}\frac{\partial(0)}{\partial \theta} + \frac{\partial[u_{max}(1-r^2/R^2)]}{\partial z} = 0$$

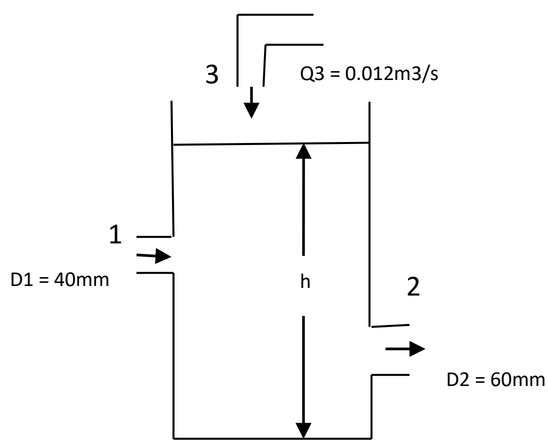
$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + 0 + 0 = 0$$

$$\int \partial(rv_r) = 0$$

$$rv_r = f(\theta, z)$$

$$v_r = \frac{f(\theta,z)}{r}$$

10. The water tank in the following figure is being filled through section 1 at v_1 = 5m/s and through section 3 at Q_3 = 0.012 m^3/s. If water level h is constant, determine the exit velocity v_2.



Solution:

Diameter of pipe 1 (d_1) = 40mm = 0.04m

C/S Area of pipe 1 (A_1) = $\frac{\pi}{4}x0.04^2$ = 0.001257 m^2

Diameter of pipe 2 (d_2) = 60mm = 0.06m

Area of pipe at section2 (A_2) = $\frac{\pi}{4}x0.06^2$ = 0.002827 m^2

Velocity of pipe at section1 (V_1) = 5m/s

Q_3 = 0.012 m^3/s

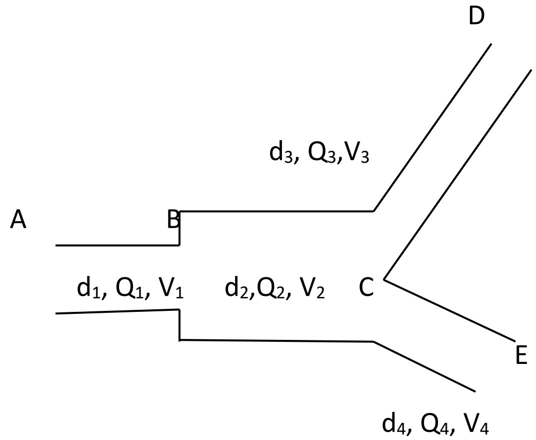
Exit velocity (v_2) = ?

Discharge through pipe1 (Q_1) = A_1 V_1 =0.001257x5 = 0.006285 m^3/s

Discharge through pipe 2 (Q_2) = Q_1 + Q_3 = 0.006285+0.012 = 0.018285 m^3/s

V_2 = Q_2/A_2 = 0.018285/0.002827 = 6.5m/s

11. Water flows from A to D and E through series pipelines shown in the figure.



Diameter of pipe AB = 50mm, Diameter of pipe BC = 75mm, Diameter of pipe CE = 30mm, velocity in pipe BC = 2m/s, velocity in pipe CD = 1.5m/s, $Q_3 = 2Q_4$
Compute Q_1, V_1, Q_2, d_3 and V_4.

Solution:
Diameter of pipe AB (d_1) = 50mm = 0.05m
C/S Area of pipe AB (A_1) = $\frac{\pi}{4} x 0.05^2$ = 0.001963 m^2
Diameter of pipe BC (d_2) = 75mm = 0.075m
C/S Area of pipe BC (A_2) = $\frac{\pi}{4} x 0.075^2$ = 0.004418 m^2
Diameter of pipe CE (d_4) = 30mm = 0.03m
C/S Area of pipe CE (A_4) = $\frac{\pi}{4} x 0.03^2$ = 0.000707 m^2
Velocity in pipe BC (V_2) = 2m/s
Velocity in pipe CD (V_3)= 1.5m/s
$Q_3 = 2Q_4$
Q_1, V_1, Q_2, d_3 and V_4 = ?

$Q_2 = A_2 V_2$ = 0.004418x2 = 0.008836 m^3/s
$Q_1 = Q_2$ = 0.008836 m^3/s
$V_1 = Q_1/A_1$ =0.008836/ 0.001963 = 4.5m/s

$Q_2 = Q_3+Q_4 = 2Q_4 + Q_4 = 3 Q_4$
$Q_4 = Q_2/3$ = 0.008836/3 = 0.002945 m^3/s
$V_4 = Q_4/A_4$ =0.002945/0.000707=4.17m/s

$Q_3 = 2Q_4$ = 2x0.002945 = 0.005891 m^3/s
$A_3 = Q_3/V_3$ = 0.005891/1.5 = 0.003927m^2

$A_3 = \frac{\pi}{4} x {d_3}^2$
$0.003927 = \frac{\pi}{4} x {d_3}^2$
d_3 = 0.07m = 70mm

12. The velocity potential (ϕ) is given by $\phi = x^2 - y^2$. Find the velocity components in x and y direction. Also show that ϕ represents a possible case of fluid flow.

Solution:
$\phi = x^2 - y^2$
u = ?, v= ?
$u = -\frac{\partial \phi}{\partial x} = -2x$
$v = -\frac{\partial \phi}{\partial y} = 2y$

The given value of ϕ represents a possible case of fluid flow if it satisfies Laplace equation.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

$\frac{\partial \phi}{\partial x} = 2x, \frac{\partial^2 \phi}{\partial x^2} = 2$
$\frac{\partial \phi}{\partial y} = -2y, \frac{\partial^2 \phi}{\partial x^2} = -2$
Substituting above values in Laplace equation, we get $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 2 - 2 = 0$
Hence, the given value of ϕ represents a possible case of fluid flow.

13. A stream function (ψ) is given by $\psi = 2x - 5y$. Calculate the velocity components, and magnitude and direction of resultant velocity.

Solution:
$\psi = 2x - 5y$
u = ?, v= ?
$u = -\frac{\partial \psi}{\partial y} = 5$
$v = \frac{\partial \psi}{\partial x} = 2$
Resultant velocity $(R) = \sqrt{u^2 + v^2} = \sqrt{5^2 + 2^2}$ = 5.38
Direction of resultant velocity $(\theta) = tan^{-1}\frac{v}{u} = tan^{-1}\frac{2}{5}$ = 21.8^0

14. If, for a two dimensional potential flow, the velocity potential is given by $\phi = 4x(3y - 4)$, determine the velocity at point (2, 3). Determine also the value of stream function ψ at point (2, 3).

Solution:
$\phi = 4x(3y - 4)$

Velocity at point (2, 3) = ?

$\psi = ?$

$u = -\frac{\partial \phi}{\partial x} = -12y + 16 = -12x3 + 16$ = -20

$v = -\frac{\partial \phi}{\partial y} = -12x = -12x2$ =-24

Resultant velocity $(R) = \sqrt{u^2 + v^2} = \sqrt{(-20)^2 + (-24)^2}$ = 31.2

$\frac{\partial \psi}{\partial x} = -\frac{\partial \phi}{\partial y} = -12x$ (a)

$\frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x} = 12y - 16$ (b)

Integrating a with respect to x

$\psi = -6x^2 + c$ (c)

c is a constant which is independent of x, but may be a function of y.

Differentiating c with respect to y

$\frac{\partial \psi}{\partial y} = \frac{\partial c}{\partial y}$ (d)

From b and d

$$\frac{\partial c}{\partial y} = 12y - 16$$

Integrating

$c = 6y^2 - 16y$

Substituting in c

$\psi = -6x^2 + 6y^2 - 16y = -6x2^2 + 6x3^2 - 16x3$ = -18

15. The stream function for a two dimensional flow is given by $\psi = 8xy$. Find the velocity potential function.

Solution:

$\psi = 8xy$

ϕ =?

$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} = 8x$ (a)

$\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} = -8y$ (b)

Integrating a with respect to x

$\phi = 4x^2 + c$ (c)

c is a constant which is independent of x, but may be a function of y.

Differentiating c with respect to y

$\frac{\partial \phi}{\partial y} = \frac{\partial c}{\partial y}$ (d)

From b and d

$$\frac{\partial c}{\partial y} = -8y$$

Integrating

$$c = -4y^2$$

Substituting in c

$$\phi = 4x^2 - 4y^2$$

16. For the velocity potential function given as: u = ay sinxy, v = ax sinxy, obtain an expression for velocity potential function.

Solution:
u = ay sinxy, v = ax sinxy
$\frac{\partial \phi}{\partial x} = -u = -ay\ sinxy$ (a)
$\frac{\partial \phi}{\partial y} = -v = -ax\ sinxy$ (b)
Integrating a with respect to x
$\phi = a\ cosxy + c$ (c)
c is a constant which is independent of x, but may be a function of y.
Differentiating c with respect to y
$\frac{\partial \phi}{\partial y} = -axsinxy + \frac{\partial c}{\partial y}$ (d)
From b and d
$-axsinxy + \frac{\partial c}{\partial y} = -axsinxy$
$\frac{\partial c}{\partial y} = 0$
c =0
Substituting in c
$\phi = a\ cosxy$

17. The velocity components in a two-dimensional flow are: $u = 8x^2y - \frac{8}{3}y^3$, $v = -8xy^2 + \frac{8}{3}x^3$. Show that these velocity components represent a possible case of an irrotational flow.

Solution:
$u = 8x^2y - \frac{8}{3}y^3, v = -8xy^2 + \frac{8}{3}x^3$
$\frac{\partial u}{\partial x} = 16xy, \frac{\partial v}{\partial y} = -16xy$

Continuity equation
$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 16xy - 16xy = 0$
As the continuity equation is satisfied, it is a possible case of fluid flow.
The rotation is given by

$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(-8y^2 + 8x^2 - 8x^2 + 8y^2) = 0$
As $\omega_z = 0$, the flow is irrotational.

18. Show that the following stream function represents an irrotational flow.

$$\psi = 6x - 4y + 7xy + 9$$

Solution:

$\psi = 6x - 4y + 7xy + 9$

$u = -\frac{\partial\psi}{\partial y} = -4 - 7x$

$v = \frac{\partial\psi}{\partial x} = 6 + 7y$

The rotation is given by

$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(0 - 0) = 0$

As $\omega_z = 0$, the flow is irrotational.

19. Given the velocity vector V = ax i + by I, where a, b = constant. Plot the streamlines of flow and explain whether stagnation point occurs.

Solution:

u = ax, v = by

From continuity,

$\frac{\partial u}{\partial x} + \frac{\partial y}{\partial y} = 0$

a+b = 0

a = -b

With this,

u = ax, v = -ay

Equation of streamline

$\frac{dx}{u} = \frac{dy}{v}$

$\frac{dx}{ax} = \frac{dy}{-ay}$

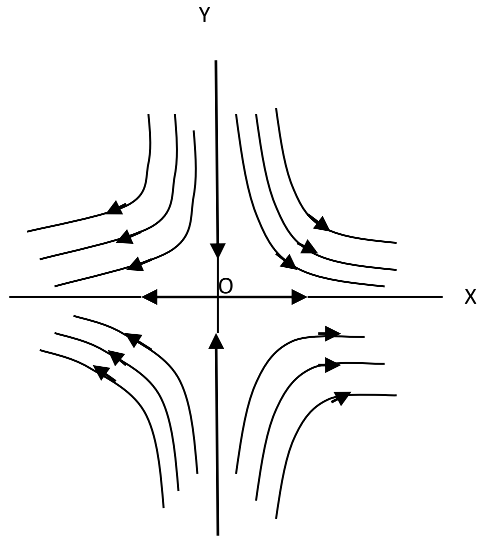
Integrating

$\int \frac{dx}{x} = -\int \frac{dy}{y}$

$lnx = -lny + c$

xy = c

This is the general expression for streamlines, which are hyperbolas.



Plotting
Let c= 0, xy = 0
x = 0, y can have any value
y = 0, x can have any value
Let c = 1
xy = 1
y = 1/x
For +ve x, y is also +ve
For –ve x, y is also –ve.
Hence there are 2 streamlines corresponding to c = 1 in first and third quadrant (for C>0).

For C<0, there are 2 streamlines in second and fourth quadrant.

For u = 0 and v = 0 (at stagnation point)
ax= 0 i.e. x=0
-ay = 0 i.e. y=0
At origin (0,0), the velocity is zero, which is stagnation point. At this point two streamlines have opposite directions and intersect each other.

Direction of streamlines
Looking at velocity, u = ax, v = -ay

First quadrant: down
Second quadrant: down
Third quadrant: up
Fourth quadrant: up

20. If u = ax, v = ay and w = -2az are the velocity components for a fluid flow, check whether they satisfy the continuity equation. If they do, is the flow rotational or irrotational? Also obtain equation of streamlines passing through the point (2, 2, 4).

Solution:
u = ax, v = ay and w = -2az
$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$ = a+a-2a = 0
The continuity equation is satisfied.
Rotational components
$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) = \frac{1}{2}(0-0) = 0$

$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = \frac{1}{2}(0-0) = 0$

$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(0-0) = 0$

As the rotational components are zero, the flow is irrotational.

Equation of streamline

$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$

$\frac{dx}{ax} = \frac{dy}{ay} = \frac{dz}{-2az}$

Considering first and second equations

$\frac{dx}{x} = \frac{dy}{y}$

Integrating

$lnx = lny + lnC1$

$\frac{x}{y} = c1$ (a)

Considering first and third equations

$\frac{dx}{x} = \frac{dz}{-2z}$

Integrating

$lnx = -\frac{1}{2}lnz + lnc2$

$lnx + lnz^{1/2} = lnc2$

$xz^{1/2} = c2$ (b)

At (2,2,4)

C1 =2/2= 1, C2 =2x$4^{1/2}$ = 4

Hence the equation of streamline

$x/y = 1$ and $xz^{1/2} = 4$

Tutorial 7

Fluid Dynamics

1. Oil with sp gr 0.75 is flowing through a 15cm diameter pipe under a pressure of 105KN/m^2. If the total energy relative to a datum plane 2.5m below the center of the pipe is 18m, determine the flow rate of oil.

Solution:
Pressure (P) = 105 Kpa
Diameter of pipe (d) = 15cm = 0.15m
C/S Area of pipe (A) = $\frac{\pi}{4} x 0.15^2$ = 0.01767 m^2
Datum head (Z) = 2.5m
Total energy (E) = 18m
Flow rate (Q) =?

According to Bernoulli's equation,
$E = \frac{P}{\gamma} + \frac{V^2}{2g} + Z$
$18 = \frac{105000}{0.75x9810} + \frac{V^2}{2g} + 2.5$
V = 4.91m/s
Q = AV = 0.01767x4.91 = 0.0867 m^3/s

2. A fluid is flowing in a 20cm diameter pipe at a pressure of 28 KN/m^2 with a velocity of 2.4m/s. The elevation of center of pipe above a given datum is 4m. Find the total energy head above the given datum if the fluid is (a) water, (b) oil of sp gr 0.82, and (c) gas with a specific weight of 6.4 N/m^3.

Solution:
Pressure (P) = 28 Kpa
Velocity (V) = 2.4m/s
Datum head (Z) = 4m
Energy head (E) = ?

a) For water, γ = 9810 N/m^3
$E = \frac{P}{\gamma} + \frac{V^2}{2g} + Z = \frac{28000}{9810} + \frac{2.4^2}{2g} + 4$ = 7.15m
b) For oil, γ = 0.82x9810 =8044.2N/m^3
$E = \frac{P}{\gamma} + \frac{V^2}{2g} + Z = \frac{28000}{8044.2} + \frac{2.4^2}{2g} + 4$ = 7.77m
b) For gas, γ = 6.4N/m^3

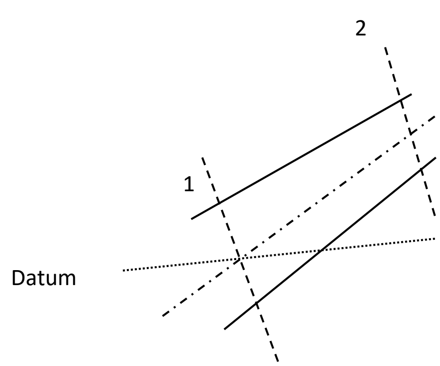
$E = \frac{P}{\gamma} + \frac{V^2}{2g} + Z = \frac{28000}{6.4} + \frac{2.4^2}{2g} + 4$ = 4379.3m

3. A pipe, through which water is flowing is having diameters 40cm and 20cm at sections 1 and 2 respectively. The velocity of water at section 1 is 5m/s. Find the velocity head at sections 1 and 2 and also compute discharge.

Solution:
Diameter of pipe at section1 (d_1) = 40cm = 0.4m
C/S Area of pipe at section1 (A_1) = $\frac{\pi}{4} x 0.4^2$ = 0.1256 m^2
Velocity of pipe at section1 (V_1) = 5m/s
Diameter of pipe at section2 (d_2) = 20cm = 0.2m
C/S Area of pipe at section2 (A_2) = $\frac{\pi}{4} x 0.2^2$ = 0.0314 m^2
Velocity heads = ?
Discharge (Q) =?
Q = A_1 V_1 = 0.1256x5 = 0.628 m^3/s
Q = A_1 V_1 = A_2 V_2
V_2 = Q/A_2 = 0.628/0.0314 = 20m/s
Velocity head at section 1 = $\frac{V_1^2}{2g} = \frac{5^2}{2g}$ = 1.274m

Velocity head at section 2 = $\frac{V_2^2}{2g} = \frac{20^2}{2g}$ = 20.38m

4. The water is flowing through a taper pipe of length 50m having diameters 40cm at the upper end and 20cm at the lower end, at the rate of 60 lps. The pipe has a slope of 1 in 40. Find the pressure at the lower end if the pressure at the higher level is 24.525 N/cm^2 (a) assuming no loss of energy (b) a loss of 0.2m.



Solution:
Diameter of pipe at section1 (d_1) = 20cm = 0.2m
C/S Area of pipe at section1 (A_1) = $\frac{\pi}{4} x 0.2^2$ = 0.0314 m^2

Diameter of pipe at section2 (d_2) = 40cm = 0.4m

C/S Area of pipe at section2 (A_2) = $\frac{\pi}{4}x0.4^2$ = 0.1256 m^2

Discharge (Q) = 60lps = 60x10^{-3} m^3/s =0.06 m^3/s

Velocity at section 1 (V_1) = Q/A_1 = 0.06/0.0314 = 1.91m/s

Velocity at section 2 (V_2) = Q/A_2 = 0.06/0.1256 = 0.47m/s

Pressure at section 2 (P_2) = 24.525 N/cm^2 =24.525x10^4 N/m^2

Pressure at section 1 (P_1) =?

Slope = tanθ = 1/40

θ = 1.43^0

Taking datum head at section 1 (Z_1) = 0

Z_2 = 50Sin1.43 = 1.25m

a. Applying Bernoulli's equation at section 1 and 2 (considering no loss)

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+Z_2$$

$$\frac{P_1}{9810}+\frac{1.91^2}{2g}+0=\frac{24.525x10^4}{9810}+\frac{0.47^2}{2g}+1.25$$

P_2 = 255799 N/m^2

b. Applying Bernoulli's equation at section 1 and 2

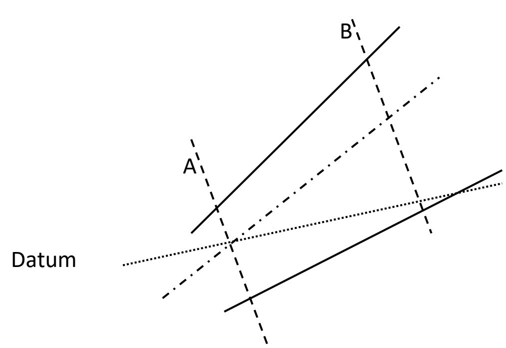
Loss of head (h_L) = 0.1m

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+Z_2+h_L$$

$$\frac{P_1}{9810}+\frac{1.91^2}{2g}+0=\frac{24.525x10^4}{9810}+\frac{0.47^2}{2g}+1.25+0.1$$

P_2 = 256780 N/m^2

5. A pipe line carrying oil of sp.gr. 0.8, changes in diameter from 300mm at a position A to 500mm at position B which is 5m at a higher level. If the pressures at A and B are 19.62 N/cm^2 and 14.91 N/cm^2 respectively, and the discharge is 150 lps, determine the loss of head and the direction of flow.



Solution:

Diameter of pipe at section A (d_1) = 300mm = 0.3m

C/S Area of pipe at section A (A_1) = $\frac{\pi}{4}x0.3^2$ = 0.0707 m^2

Diameter of pipe at section B (d_2) = 500mm = 0.5m

C/S Area of pipe at section B (A_2) = $\frac{\pi}{4}x0.5^2$ = 0.1963 m^2

Discharge (Q) = 150lps = 150x10^{-3} m^3/s =0.15 m^3/s

Velocity at section A (V_1) = Q/A_1 = 0.15/0.0707 = 2.12m/s

Velocity at section B (V_2) = Q/A_2 = 0.15/0.1963 = 0.764m/s

Datum at section A (Z_1) = 0

Datum at section B (Z_2) = 5m

Pressure at section A (P_1) = 19.62 N/cm^2 = 19.62x10^{-4} N/m^2

Pressure at section B (P_2) = 14.91 N/cm^2 = 14.91x10^{-4} N/m^2

Loss of head (h_L) = ?

Direction of flow = ?

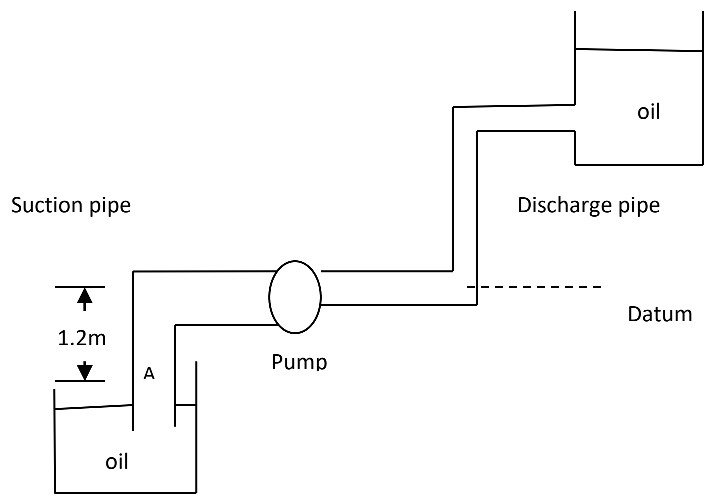
Energy head at A (E_A) = $\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{196200}{0.8x9810} + \frac{2.12^2}{2g} + 0$= 25.229m

Energy head at B (E_B) =$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 = \frac{149100}{0.8x9810} + \frac{0.764^2}{2g} + 5$ = 24.028

h_L = E_A-E_B = 25.229-24.028 = 1.201m

As E_A is greater than E_B, the flow takes from A to B.

6. A 100mm diameter suction pipe leading to a pump carries a discharge of 0.03 m^3/s of oil (sp gr = 0.85). If the pressure at point A in the suction pipe is a vacuum of 180mmHg, find the total energy head at point A w.r.t a datum at the pump.



Solution:

Diameter of pipe at section A (d) = 100mm = 0.1m

C/S Area of pipe at section A (A) = $\frac{\pi}{4}x0.1^2$ = 0.00785 m^2

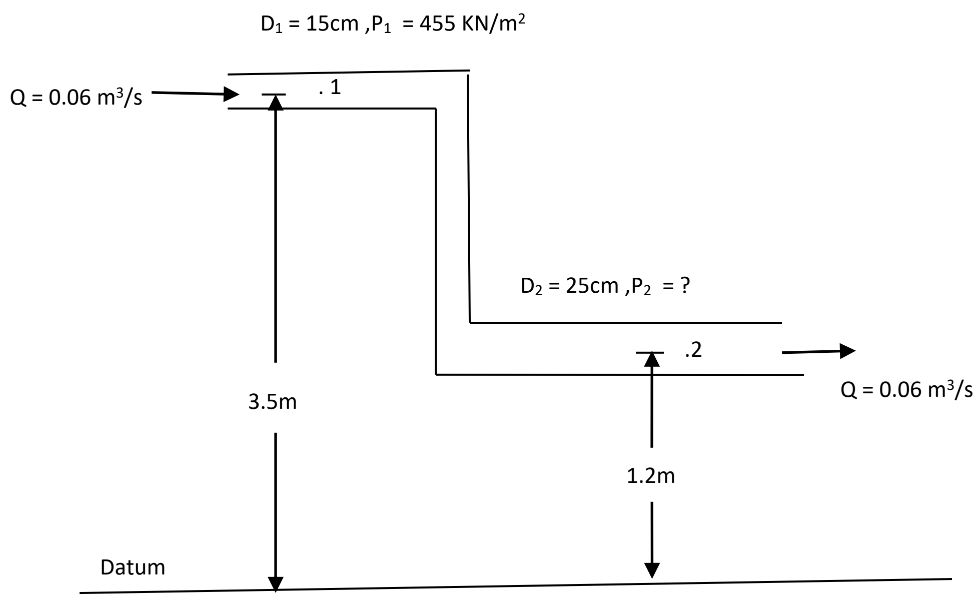
Discharge (Q) = 0.03 m^3/s

Velocity at point A (V) =Q/A = 0.03/0.00785 = 3.82m/s
Pressure head at A (h) = -180mm Hg =-0.18 m of Hg
Pressure at A (P) = $\gamma_{Hg}h$ = 13.6x9810x(-0.18) = -24015 Pa
Datum head (Z) = -1.2
Total energy head at A (E) = ?

According to Bernoulli's equation,

$$E = \frac{P}{\gamma} + \frac{V^2}{2g} + Z = \frac{-24015}{0.85x9810} + \frac{3.82^2}{2g} - 1.2 = -3.336\text{m}$$

7. Oil (sp gr = 0.84) is flowing in a pipe under the conditions shown in the fig. If the total head loss from point 1 to point 2 is 0.9m, find the pressure at point 2.



Solution:
Diameter of pipe at section 1 (d_1) = 15cm = 0.15m
C/S Area of pipe at section 1 (A_1) = $\frac{\pi}{4}x0.15^2$ = 0.0176 m^2
Diameter of pipe at section 2 (d_2) = 25cm = 0.25m
C/S Area of pipe at section 2 (A_2) = $\frac{\pi}{4}x0.25^2$ = 0.049 m^2
Discharge (Q) = 0.06 m^3/s
Velocity at point 1 (V_1) =Q/A_1 = 0.06/0.0176 = 3.4m/s
Velocity at point 2 (V_2) =Q/A_2 = 0.06/0.049 = 1.22m/s
Datum head at 1 (Z_1) = 3.5m
Datum head at 2 (Z_2) = 1.2m
Head loss (h_L) = 0.9m
Pressure at 1 (P_1) = 455 Kpa = 455000 Pa

Pressure at 2 (P_2) = ?

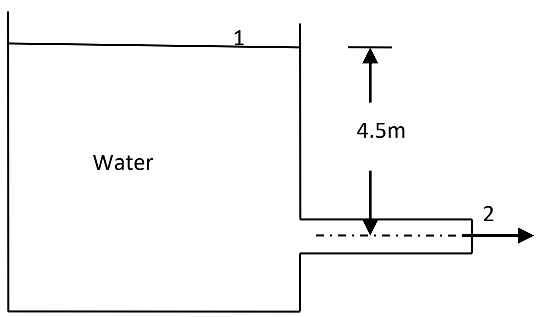
Applying Bernoulli's equation at 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{455000}{0.84x9810} + \frac{3.4^2}{2g} + 3.5 = \frac{P_2}{0.84x9810} + \frac{1.22^2}{2g} + 1.2 + 0.9$$

P_2 = 470767 Pa = 470.76 KPa

8. A 20cm diameter horizontal pipe is attached to a reservoir as shown in fig. If the total head loss between the water surface in the reservoir and the water jet at the end of the pipe is 1.8m, what are the velocity and flow rate of water being discharged from the pipe?



Solution:

Diameter of pipe at section 2 (d_2) = 20cm = 0.2m

C/S Area of pipe at section 2 (A_2) = $\frac{\pi}{4}x0.2^2$ = 0.0314 m^2

Velocity at point 1 (V_1) =0 m/s

Datum head at 1 (Z_1) = 4.5m

Datum head at 2 (Z_2) = 0

Head loss (h_L) = 1.8m

Pressure at 1 (P_1) = 0 (atmospheric)

Pressure at 2 (P_2) = 0 (atmospheric)

Velocity at point 2 (V_2) = ?

Discharge from point 1 (Q_2) = ?

Applying Bernoulli's equation at 1 and 2

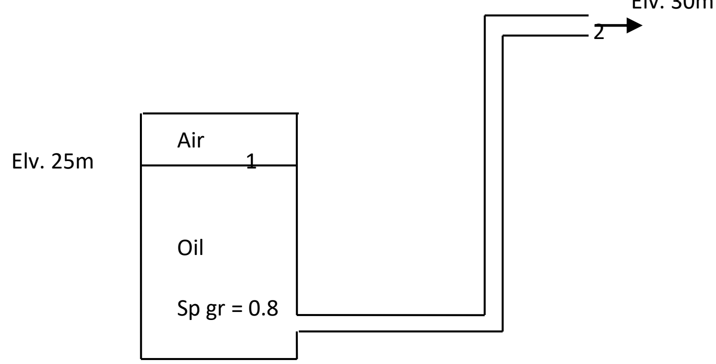
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$0 + 0 + 4.5 = 0 + \frac{V_2^2}{2g} + 0 + 1.8$$

V_2 = 7.28 m/s

Q_2 = A_2 V_2 =7.28x0.0314 = 0.228 m^3/s

9. Oil flows from a tank through 140m of 15cm diameter pipe and then discharge into air as shown in the fig. If the head loss from point 1 to point 2 is 0.55m of oil, determine the pressure needed at point 1 to cause 0.02 m^3/s of oil to flow.



Solution:

Diameter of pipe at section 2 (d_2) = 15cm = 0.15m

C/S Area of pipe at section 2 (A_2) = $\frac{\pi}{4} x 0.15^2$ = 0.0176 m^2

Velocity at point 1 (V_1) =0 m/s

Datum head at 1 (Z_1) = 25m

Datum head at 2 (Z_2) = 30m

Head loss (h_L) = 0.55m

Pressure at 2 (P_2) = 0 (atmospheric)

Discharge (Q) = 0.02 m^3/s

Velocity at point 2 (V_2) = Q/A_2 = 0.02/0.0176 = 1.14m/s

Pressure at 1 (P_1) = ?

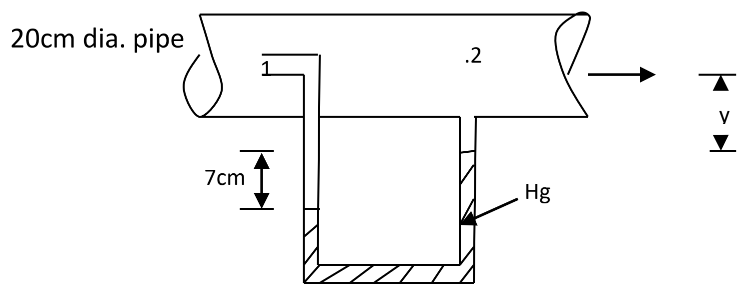
Applying Bernoulli's equation at 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{P_1}{0.8x9810} + 0 + 25 = 0 + \frac{1.14^2}{2g} + 30 + 0.55$$

P_1 = 43037 Pa = 43.037 Kpa

10. In the fig., one end of a U-tube is oriented directly into the flow so that the velocity of the stream is zero at this point (stagnation point). Neglecting friction, determine the flow of water in the pipe.



Solution:

Diameter of pipe (d) = 20cm = 0.2m

C/S Area of pipe (A) = $\frac{\pi}{4}x0.2^2$ = 0.0314 m^2

Velocity at point 1 (V_1) =0 m/s

Taking center of pipe as datum

Datum head at 1 (Z_1) = 0

Datum head at 2 (Z_2) = 0

Pressure at 1=P_1

Pressure at 2 = P_2

Velocity at point 2 = V_2

Flow rate (Q) = ?

Applying Bernoulli's equation at 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{P_1}{\gamma} + 0 + 0 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + 0$$

$$V_2 = \sqrt{\frac{2g}{\gamma}(P_1 - P_2)}$$

Writing manometric equation

$$P_1 + \gamma_{water}(y + 0.07) = P_2 + \gamma_{water}y + \gamma_{Hg}x0.07$$

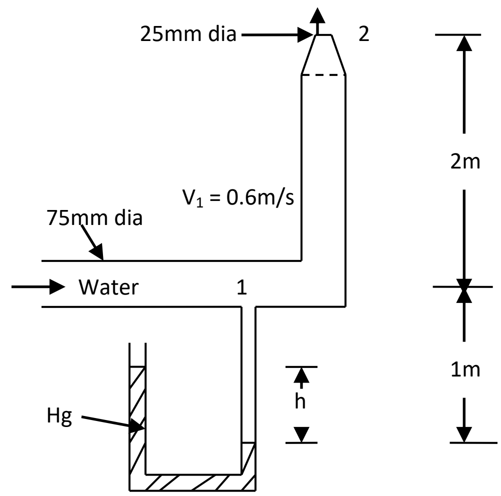
$$P_1 + 9810x0.07 = P_2 + 13.6x9810x0.07$$

P_1-P_2 = 8652.4 Pa

$V_2 = \sqrt{\frac{2g}{\gamma}(P_1 - P_2)} = \sqrt{\frac{2g}{9810}x8652.4}$ =4.15 m/s

Q = A V_2 = 0.0314x4.15 = 0.13 m^3/s

11. Find the manometer reading (h) in the lossless system of the fig.



Solution:

Diameter of pipe at section 1 (d_1) = 75mm = 0.075m

C/S Area of pipe at section 1 (A_1) = $\frac{\pi}{4}x0.075^2$ = 0.00442 m^2

Diameter of pipe at section 2 (d_2) = 25mm = 0.025m

C/S Area of pipe at section 2 (A_2) = $\frac{\pi}{4}x0.025^2$ = 0.00049 m^2

Velocity at point 1 (V_1) =0.6 m/s

Discharge (Q) = A_1 V_1 = 0.00442x0.6 = 0.00265 m^3/s

Velocity at point 2 (V_2) =Q/A_2 = 0.00265/0.00049 = 5.4 m/s

Pressure at 2 (P_2) = 0 (atmospheric)

Pressure at 1= P_1

Taking datum through point 1

Datum head at 1 (Z_1) = 0

Datum head at 2 (Z_2) = 2m

h = ?

Applying Bernoulli's equation at 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{P_1}{9810} + \frac{0.6^2}{2g} + 0 = 0 + \frac{5.4^2}{2g} + 2$$

P_1 = 34020 Pa

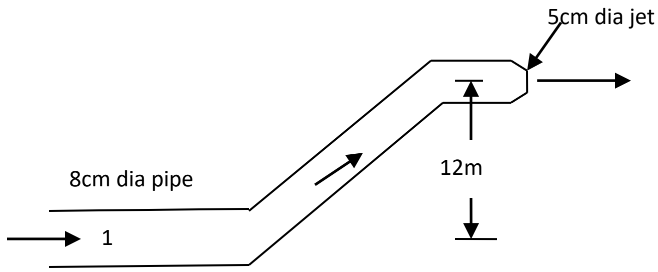
Writing manometric equation

$$P_1 + \gamma_{water}x1 = 0 + \gamma_{Hg}xh$$

$$34020 + 9810x1 = 13.6x9810xh$$

h = 0.328m

12. In the fig., the fluid is water and the pressure at point 1 is 180Kpa gage. If the mass flux is 15kg/s, what is the head loss between 1 and 2? (flux = flow rate)



Solution:

Diameter of pipe at section 1 (d_1) = 8cm = 0.08m

C/S Area of pipe at section 1 (A_1) = $\frac{\pi}{4}x0.08^2$ = 0.00503 m^2

Diameter of pipe at section 2 (d_2) = 5cm = 0.05m

C/S Area of pipe at section 2 (A_2) = $\frac{\pi}{4}x0.05^2$ = 0.00196 m^2

Mass flux (M) = 15kg/s

Velocity at point 1 (V_1) = $\frac{M}{\rho A_1} = \frac{15}{1000x0.00503}$ = 2.98m/s

Velocity at point 2 (V_2) = $\frac{M}{\rho A_2} = \frac{15}{1000x0.00196}$ = 7.65m/s

Pressure at 1 (P_1) = 180 Kpa

Pressure at 2 (P_2) = 0 (atmospheric)

Taking datum through point 1

Datum head at 1 (Z_1) = 0

Datum head at 2 (Z_2) = 12m

Head loss (h_L) =?

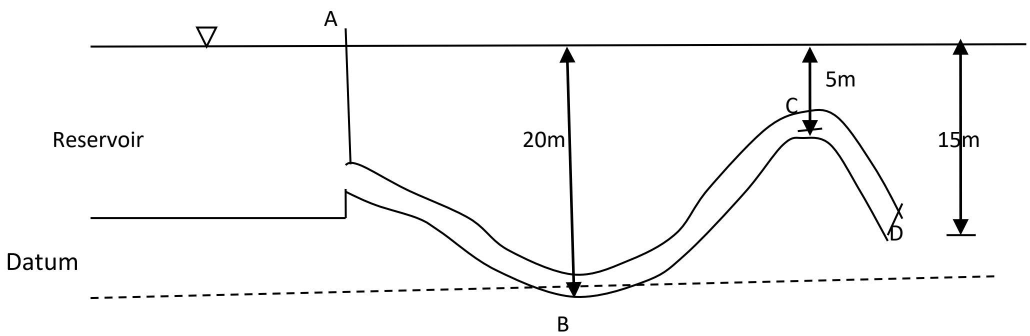
Applying Bernoulli's equation at 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{180000}{9810} + \frac{2.98^2}{2g} + 0 = 0 + \frac{7.65^2}{2g} + 12 + h_L$$

h_L = 3.82m

13. A pipeline connected to a reservoir discharges water to the atmosphere. The loss of head is 1 times velocity head from A to B, 1.5 times velocity head from B to C and 0.5 times velocity head from C to D. If the pipe is 150mm in diameter, calculate the pressure heads at B and C. Also compute discharge.



Solution:

Diameter of pipe (d) = 15cm = 0.15m

C/S Area of pipe at section (A) = $\frac{\pi}{4}x0.15^2$ = 0.0176 m^2

Velocity at A (V_A) = 0

V = Velocity of water through pipe = $V_B = V_C = V_D$

$hL_{AB} = \frac{V^2}{2g}, hL_{BC} = \frac{1.5V^2}{2g}, hL_{CD} = \frac{0.5V^2}{2g}$

Total head loss $(hL) = \frac{3V^2}{2g}$

Datum head at A (Z_A) = 20m

Datum head at B (Z_B) = 0

Datum head at C (Z_C) = 15m

Datum head at D (Z_D) = 5m

Pressure at A (P_A) = 0 (atmospheric)

Pressure at D (P_D) = 0 (atmospheric)

Pressure head at B (P_B/γ) =?

Pressure head at C (P_C/γ) =?

Discharge (Q) = ?

Applying Bernoulli's equation at A and D

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_D}{\gamma} + \frac{V_D^2}{2g} + Z_D + hL$$

$$0 + 0 + 20 = 0 + \frac{V^2}{2g} + 5 + \frac{3V^2}{2g}$$

V = 8.57 m/s

$V_B = V_C = V_D$ = 8.57 m/s

Applying Bernoulli's equation at A and B

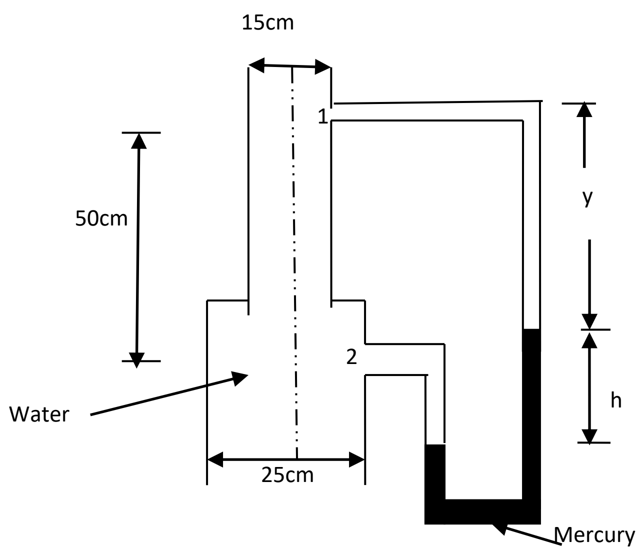
$$\frac{P_A}{\gamma}+\frac{V_A^2}{2g}+Z_A=\frac{P_B}{\gamma}+\frac{V_B^2}{2g}+Z_B+hL_{AB}$$
$$0+0+20=\frac{P_B}{\gamma}+\frac{V^2}{2g}+0+\frac{V^2}{2g}$$
$$20=\frac{P_B}{\gamma}+\frac{8.57^2}{2g}+\frac{8.57^2}{2g}$$
$$\frac{P_B}{\gamma}=12.5m$$

Applying Bernoulli's equation at B and C

$$\frac{P_B}{\gamma}+\frac{V_B^2}{2g}+Z_B=\frac{P_C}{\gamma}+\frac{V_C^2}{2g}+Z_C+hL_{BC}$$
$$12.5+\frac{V^2}{2g}+0=\frac{P_C}{\gamma}+\frac{V^2}{2g}+15+\frac{1.5V^2}{2g}$$
$$-2.5=\frac{P_C}{\gamma}+\frac{1.5*8.57^2}{2g}$$
$$\frac{P_C}{\gamma}=-8.11m$$

Q = AV = 0.0176x8.57 = 0.1508 m^3/s

14. A 15cm diameter pipe is expanded to 25cm diameter suddenly at a section. The head loss at a sudden expansion from section 1 to 2 is given by $h_L = (V_1-V_2)^2/2g$. For a discharge of 45 lps, calculate the manometer reading h.



Solution:
Diameter of pipe at section 1 (d_1) = 15cm = 0.15m
C/S Area of pipe at section 1 (A_1) = $\frac{\pi}{4}x0.15^2$ = 0.0176 m^2

Diameter of pipe at section 2 (d_2) = 25cm = 0.25m

Area of pipe at section 2 (A_2) = $\frac{\pi}{4}x0.25^2$ = 0.049 m^2

Discharge (Q) = 45 lps = 0.045 m^3/s

Velocity at point 1 (V_1) =Q/A_1 = 0.045/0.0176 = 2.55 m/s

Velocity at point 2 (V_2) =Q/A_2 = 0.045/0.049 = 0.92 m/s

Pressure at 1= P_1

Pressure at 2 = P_2

Taking datum through point 2

Datum head at 1 (Z_1) = 0.5 m

Datum head at 2 (Z_2) = 0

Head loss $(h_L) = \frac{(V_1-V_2)^2}{2g} = \frac{(2.55-0.92)^2}{2g}$ = 0.14m

h = ?

Applying Bernoulli's equation at 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{P_1}{9810} + \frac{2.55^2}{2g} + 0.5 = \frac{P_2}{9810} + \frac{0.92^2}{2g} + 0 + 0.14$$

$P_1 - P_2$ = -6359.6 Pa

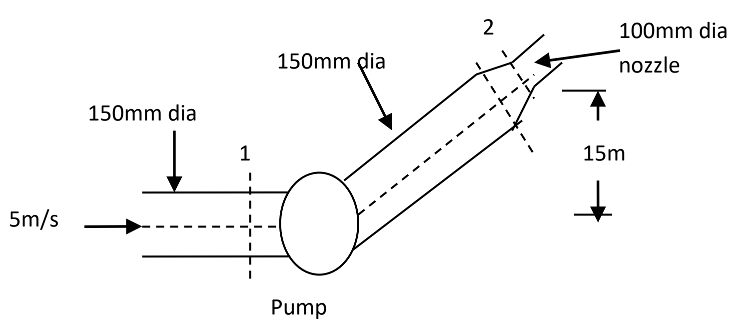
Writing manometric equation

$$P_1 + \gamma_{water}x\, y + \gamma_{Hg}xh = P_2 + \gamma_{water}xh$$

-6359.6+9810(0.5-h)+13.6x9810h = 0

h = 0.0117m

15. Water is pumped from a reservoir through 150mm diameter pipe and is delivered at a height of 15m from the centerline of pump through a 100mm nozzle connected to 150mm discharge line as shown in the figure. If the pressure at the pump inlet is 210 KN/m^2 absolute, inlet velocity of 5m/s and the jet is discharged into atmosphere, determine the energy supplied by the pump. Take atmospheric pressure = 101.3 KN/m^2 and assume no friction.



Solution:

Diameter of pipe at section 1 (d_1) = 150mm = 0.15m

C/S Area of pipe at section 1 (A_1) = $\frac{\pi}{4}x0.15^2$ = 0.01767m^2

Diameter of pipe at section 2 (d_2) = 100mm = 0.1m

C/S Area of pipe at section 2 (A_2) = $\frac{\pi}{4}x0.1^2$ = 0.007854 m^2

Velocity at point 1 (V_1) =5 m/s

Discharge (Q) = A_1 V_1 = 0.01767x5 = 0.08835 m^3/s

Velocity at point 2 (V_2) =Q/A_2 = 0.08835/0.007854 = 11.25 m/s

Pressure at 1 (P_1) = 210 KN/m^2

Pressure at 2 (P_2) = 101.3 KN/m^2 (atmospheric)

Taking datum through centerline of pump

Datum head at 1 (Z_1) = 0

Datum head at 2 (Z_2) = 15m

Energy supplied by pump = ?

h_p = Head supplied by the pump

Applying Bernoulli's equation at 1 and 2

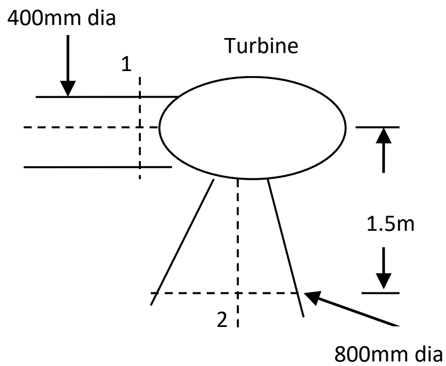
$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_1+h_p=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+Z_2$$

$$\frac{210000}{9810}+\frac{5^2}{2g}+0+h_p=\frac{101300}{9810}+\frac{11.25^2}{2g}+15$$

h_p = 9.09m

Energy supplied by the pump = $\gamma Q h_p$ = 9810x0.08835x9.09 = 7878W

16. For a turbine shown in the figure, P_1 = 200 Kpa, P_2= -35Kpa, Q = 0.3m^3/s. Determine the energy output of the machine if its efficiency is 80%.



Solution:

Diameter of pipe at section 1 (d_1) = 400mm = 0.4m

C/S Area of pipe at section 1 (A_1) = $\frac{\pi}{4}x0.4^2$ = 0.1256m^2

Diameter of pipe at section 2 (d_2) = 800mm = 0.8m

C/S Area of pipe at section 2 (A_2) = $\frac{\pi}{4}x0.8^2$ = 0.5026 m^2

Discharge (Q) = 0.3 m^3/s

Velocity at point 1 (V_1) = Q/A_2 =0.3/0.1256 = 2.38 m/s

Velocity at point 2 (V_2) =Q/A_2 = 0.3/0.5026 = 0.6 m/s

Pressure at 1 (P_1) = 200 Kpa

Pressure at 2 (P_2) = -35Kpa

Taking datum at 2,

Datum head at 1 (Z_2) = 0

Datum head at 2 (Z_1) = 1.5m

Efficiency (η) = 0.8

Energy output of machine = ?

h_t = Head extracted by turbine

Applying Bernoulli's equation at 1 and 2

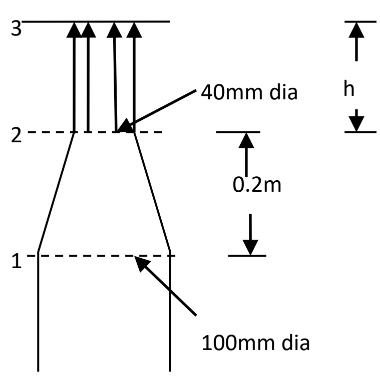
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 - h_t = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{200000}{9810} + \frac{2.38^2}{2g} + 1.5 - h_t = \frac{-35000}{9810} + \frac{0.6^2}{2g} + 0$$

h_t = 25.72m

Energy output = $\eta(\gamma Q h_t)$ = 0.8x(9810x0.3x25.72) = 60555W

17. A jet of water issues vertically upwards from 0.2m high nozzle whose inlet and outlet diameters are 100mm and 40mm respectively. If the pressure at the inlet is 20 Kpa above the atmospheric pressure, determine the discharge and the height to which the jet will rise. Assume no friction



Solution:

Inlet diameter (d_1) = 100mm = 0.1m

C/S Area at inlet (A_1) = $\frac{\pi}{4}x0.1^2$ = 0.007854 m^2

Outlet diameter (d_2) = 40mm = 0.04m

C/S Area at outlet (A_2) = $\frac{\pi}{4}x0.04^2$ = 0.001257m^2

Pressure at 1 (P_1) = 20 KN/m^2

Pressure at 2 and 3 (P_2, P_3) = 0

Discharge (Q) = ?

Height to which jet will rise (h) = ?

Applying Bernoulli's equation at 1 and 2 (Taking datum at 1)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{20000}{9810} + \frac{V_1^2}{2g} + 0 = 0 + \frac{V_2^2}{2g} + 0.2$$

$$V_1^2 - V_2^2 = -36.076 \quad \text{(a)}$$

From continuity

$$A_1V_1 = A_2V_2$$

$$0.007854V_1 = 0.001257V_2$$

$$V_1 = 0.16V_2 \quad \text{(b)}$$

From a and b

$$(0.16V_2)^2 - V_2^2 = -36.076$$

V_2 =6.08m/s

V_1 = 0.97m/s

$Q = A_1V_1$ = 0.007854x0.97 = 0.007618 m^3/s

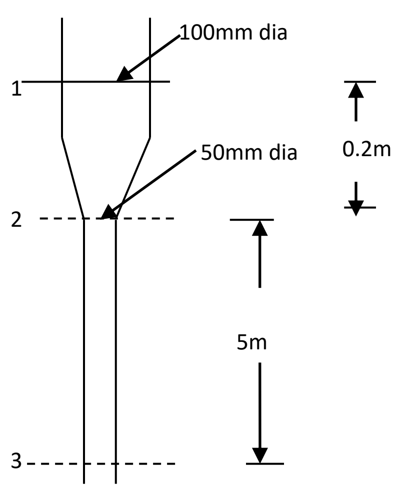
Applying Bernoulli's equation at 2 and 3 (Taking datum at 2)

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3$$

$$0 + \frac{6.08^2}{2g} + 0 = 0 + 0 + h$$

h = 1.88m

18. A jet of water coming out from 50mm diameter rounded nozzle attached to 100mm diameter pipe is directed vertically downwards. If the pressure in the 100mm diameter pipe 0.2m above the nozzle is 200 Kpa gauge, determine the diameter of jet 5m below the nozzle level.



Solution:

Diameter of pipe at section 1 (d_1) = 100mm = 0.1m

C/S Area of pipe at section 1 (A_1) = $\frac{\pi}{4}x0.1^2$ = 0.007854 m^2

Diameter of pipe at section 2 (d_2) = 50mm = 0.05m

C/S Area of pipe at section 2 (A_2) = $\frac{\pi}{4}x0.05^2$ = 0.001963m^2

Pressure at 1 (P_1) = 200 Kpa

Pressure at 2 and 3 (P_2, P_3) = 0

Applying Bernoulli's equation at 1 and 2 (Taking datum at 2)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{20000}{9810} + \frac{V_1^2}{2g} + 0.2 = 0 + \frac{V_2^2}{2g} + 0$$

$V_1^2 - V_2^2 = -404$ (a)

From continuity

$$A_1V_1 = A_2V_2$$

$$0.007854V_1 = 0.001963V_2$$

$V_1 = 0.25V_2$ (b)

From a and b

$$(0.25V_2)^2 - V_2^2 = -404$$

V_2 =20.75m/s

V_1 = 5.18m/s

$Q = A_1V_1$ = 0.007854x5.18 = 0.0407 m^3/s

Applying Bernoulli's equation at 2 and 3 (Taking datum at 3)

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3$$

$$0 + \frac{20.75^2}{2g} + 5 = 0 + \frac{V_3^2}{2g} + 0$$

V_3 = 23m/s

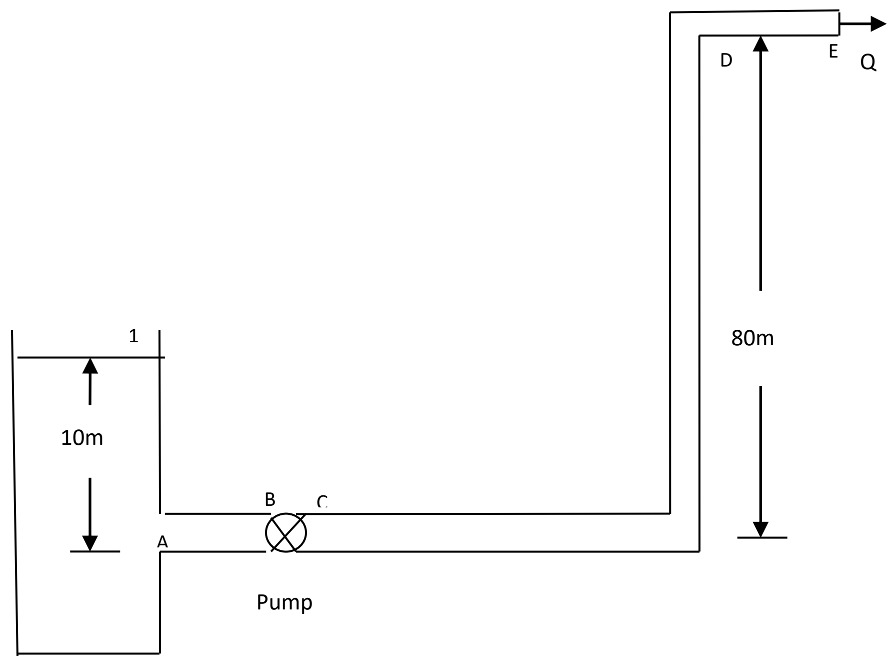
$A_3 = Q/V_3$ = 0.0407/23 = 0.00177 m^2

$$A_3 = \frac{\pi}{4}d_3^2$$

$$0.00177 = \frac{\pi}{4}d_3^2$$

d_3 = 0.047m = 47mm

19. The pipe flow in the figure is driven by the pump. What gauge pressure is needed to be supplied by the pump to provide water flow rate of Q = 60m^3/h? Neglect head loss from A to B. Head loss from C to D = $30\frac{V_{CD}^2}{2g}$; Head loss from D to E = $20\frac{V_{CD}^2}{2g}$; d_{AB} (diameter of pipe AB) = d_{CD} = 5cm; d_{DE} = 2cm. where V_{CD} = velocity in pipe CD and V_{DE} = velocity in pipe DE.



Solution:

Discharge (Q) = 60m^3/h = 60/3600 m^3/s = 0.0167 m^3/s

Diameter of pipe AB and CD = 5cm = 0.05m

C/s Area of pipe AB and CD (A_{AB}, A_{CD}) = $\frac{\pi}{4}x0.05^2$= 0.001963m^2

Diameter of pipe DE =2cm = 0.02m

C/s Area of pipe DE (A_{DE}) = $\frac{\pi}{4}x0.02^2$= 0.000314m^2

Velocity of flow through AB and CD ($V_{AB} = V_{CD}$) =Q/A_{AB} = 0.0167/0.001963 = 8.5m/s

Velocity of flow through DE (V_{DE}) = Q/A_{DE} = 0.0167/0.000314 = 53.2m/s

Head loss between A and B (hL_{AB}) = 0

Head loss from C to D (hL_{CD}) = $30\frac{V_{CD}^2}{2g} = 30\frac{8.5^2}{2g}$ = 110.5m

Head loss from D to E (hL_{DE}) = $20\frac{V_{CD}^2}{2g} = 20\frac{8.5^2}{2g}$ = 73.65m

Total head loss (h_L) = 0+110.5+73.65 = 184.15m

h_p = Head supplied by the pump

At point 1, V_1 = 0, P_1 = 0 (atm)

At point E, P_E = 0 (atm)

Applying Bernoulli's equation between 1 and E (Taking datum through A)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 + h_p = \frac{P_E}{\gamma} + \frac{V_E^2}{2g} + Z_E + h_L$$

$0 + 0 + 10 + h_p = 0 + \frac{53.2^2}{2g} + 80 + 184.15$

h_m =398.4m

Applying Bernoulli's equation between B and C

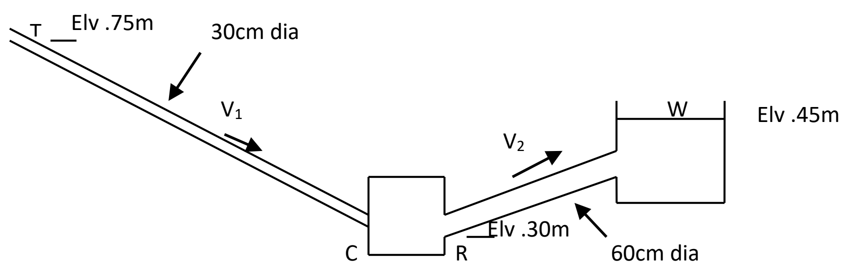
$\frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + h_p = \frac{P_C}{\gamma} + \frac{V_C^2}{2g} + Z_C$

$V_B = V_C$, $Z_B = Z_C$

$P_C - P_B = \gamma h_p$ = 9810x398.4 = 3908304 Pa = 3908.304 Kpa

Gauge pressure to be supplied by the pump = 3908.304 Kpa

20. The head extracted by turbine CR in the fig. is 60m and the pressure at T is 505 KN/m^2. For losses of $2V_2^2/2g$ between W and R and $3V_1^2/2g$ between C and T, determine (a) how much water is flowing and (b) the pressure head at R. Draw the energy gradient line and hydraulic gradient line.



Solution:

Diameter of pipe TC (d_1) = 30cm = 0.3m

C/S Area of pipe TC (A_1) = $\frac{\pi}{4}x0.3^2$ = 0.0707 m^2

Diameter of pipe RW (d_2) = 60cm = 0.6m

C/S Area of pipe RW (A_2) = $\frac{\pi}{4}x0.6^2$ = 0.2827 m^2

Pressure at T (P_T) = 505 KPa

Velocity at T (V_T) = V_1

$hL_{TC} = 3V_1^2/2g$

$hL_{RW} = 2V_2^2/2g$

h_t = 60m

Datum head at T (Z_T) = 75m

Datum head at R (Z_R) = 30m

Datum head at W (Z_W) = 45m

Pressure at W (P_W) = 0

Velocity at W (V_W) = 0

Flow rate (Q) = ?

Pressure head at R (P_R/γ) =?

a) Applying Bernoulli's equation at T and W

$\frac{P_T}{\gamma} + \frac{V_T^2}{2g} + Z_T - h_t = \frac{P_W}{\gamma} + \frac{V_W^2}{2g} + Z_W + Total\ head\ loss$

$\frac{505000}{9810} + \frac{V_1^2}{2g} + 75 - 60 = 0 + 0 + 45 + \frac{3V_1^2}{2g} + \frac{2V_2^2}{2g}$

$V_1^2 + V_2^2 = 210.7$ (a)

Continuity equation

$A_1 V_1 = A_2 V_2$

$0.0707 V_1 = 0.2827 V_2$

$V_1 = 4 V_2$ (b)

From a and b

$V_2 = 3.52$ m/s

$V_1 = 14.08$ m/s

$Q = A_1 V_1 = 0.0707x14.08 = 0.995$ m^3/s

b) Applying Bernoulli's equation at R and W

$\frac{P_R}{\gamma} + \frac{V_R^2}{2g} + Z_R = \frac{P_W}{\gamma} + \frac{V_W^2}{2g} + Z_W + hL_{RW}$

$\frac{P_R}{\gamma} + \frac{V_2^2}{2g} + 30 = 0 + 0 + 45 + \frac{2V_2^2}{2g}$

$\frac{P_R}{\gamma} = 15 + \frac{V_2^2}{2g} = 15 + \frac{3.52^2}{2g}$ = 15.63m

Finding the elevation of EGL and HGL

EGL at T = $\frac{P_T}{\gamma} + \frac{V_T^2}{2g} + Z_T = \frac{505000}{9810} + \frac{14.08^2}{2g} + 75$ = 136.6m

EGL at C = EGL at T- hL_{TC} = $136.6 - \frac{3x14.08^2}{2g}$ = 106.3m

EGL at R = EGL at C - h_{CR} = 106.3-60 = 46.3m

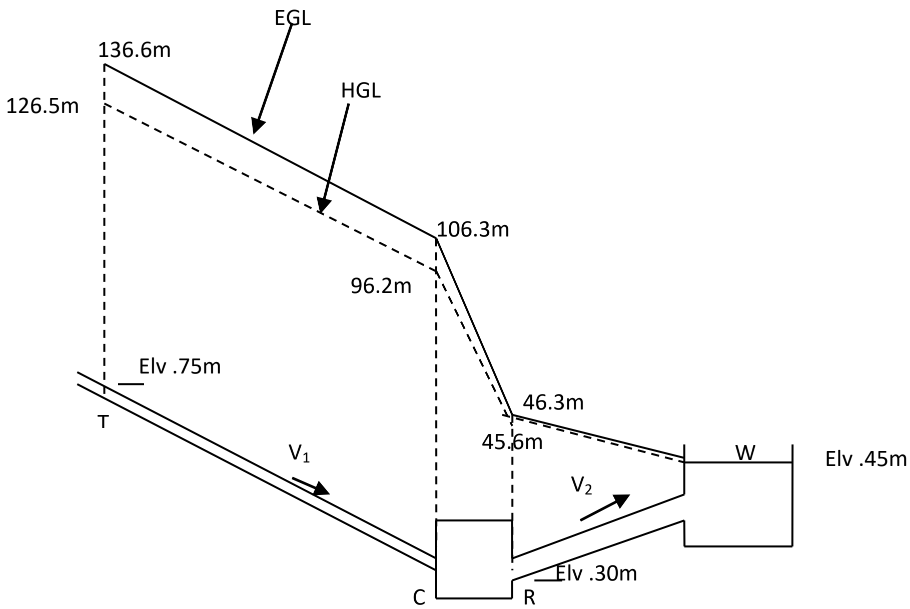
EGL at W = EGL at R – hL_{RW} = $46.3 - \frac{2x3.52^2}{2g}$ = 45m

HGL at T = $EGL\ at\ T - \frac{V_1^2}{2g} = 136.6 - \frac{14.08^2}{2g}$ = 126.5m

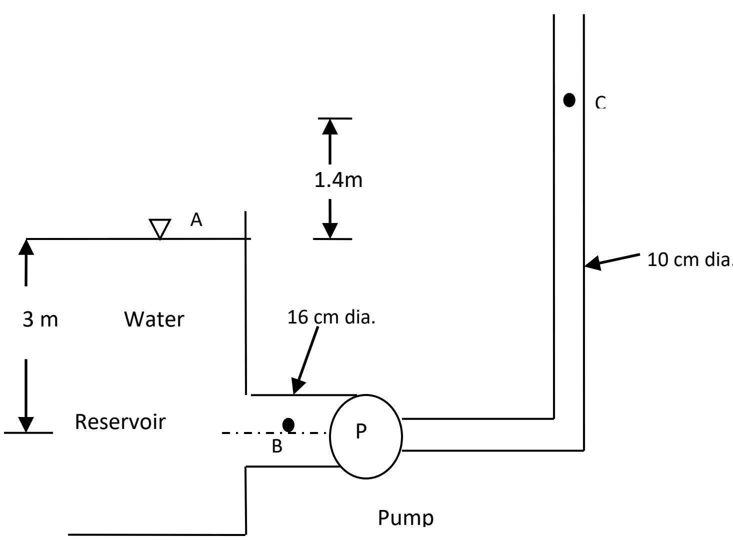
HGL at C = $EGL\ at\ C - \frac{V_1^2}{2g} = 106.3 - \frac{14.08^2}{2g}$ = 96.2m

HGL at R = $EGL\ at\ R - \frac{V_2^2}{2g} = 46.3 - \frac{3.52^2}{2g}$ = 45.6m

HGL at W = $EGL\ at\ W - \frac{V_W^2}{2g} = 45 - 0$ = 45m



21. The figure below shows a pump P pumping 90 lps of water from a tank.
(a) What will be the pressure at points B and C when the pump delivers 14.5KW of power to the flow? Assume the losses in the system to be negligible.
(b) What will be the pressure at C when the loss in the in the inlet to the pump is negligible and between the pump and the point C, a loss equal to 2 times the velocity head at B takes place.



Solution:
Discharge (Q) = 90 lps = 0.09 m^3/s
Diameter of large pipe (d_B) = 16cm = 0.16m
Diameter of smaller pipe (d_C) = 10cm = 0.1m

(a) Power (P) = 14.5 KW = 14500W
Pressure at point B (P_B) = ?
Pressure at point C (P_C) = ?
Head supplied by the pump = h_p

$$P = \gamma Q h_p$$
$$14500 = 9810x0.09xh_p$$

h_p = 16.42m

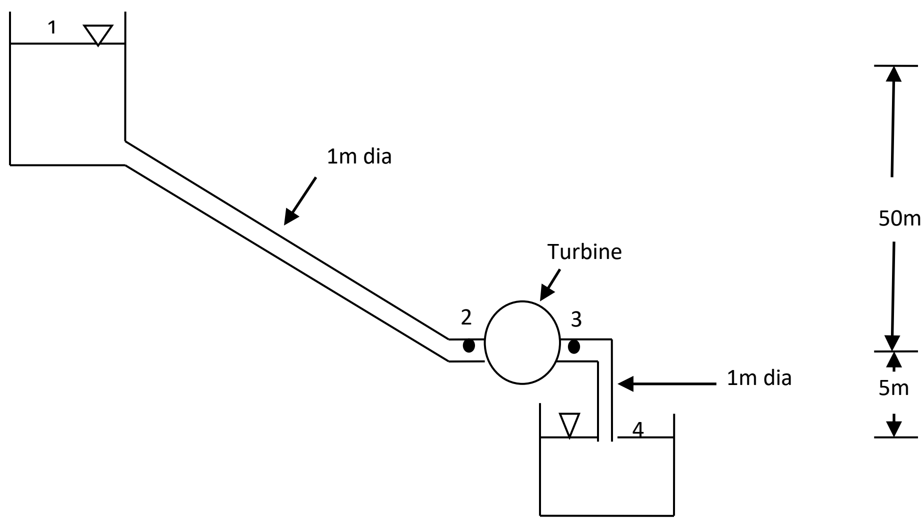
V_A = 0, P_A = 0 (atm. pr.)

Using continuity equation at B and C
$V_B = \frac{Q}{A_B} = \frac{0.09}{\frac{\pi}{4}0.16^2}$ = 4.47m/s
$V_C = \frac{Q}{A_C} = \frac{0.09}{\frac{\pi}{4}0.1^2}$ = 11.46m/s

Applying Bernoulli's equation between A and B (taking B as datum)
$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B$
$0 + 0 + 3 = \frac{P_B}{9810} + \frac{4.47^2}{2x9.81} + 0$
P_B = 19440 Pa
Applying Bernoulli's equation between A and C (taking B as datum)
$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A + h_p = \frac{P_C}{\gamma} + \frac{V_C^2}{2g} + Z_C$
$0 + 0 + 3 + 16.42 = \frac{P_C}{9810} + \frac{11.46^2}{2x9.81} + 4.4$
P_C = 81680 Pa

b) Loss between A and B = 0
Loss between B and C = $2\frac{V_B^2}{2g} = 2\frac{4.47^2}{2x9.81}$ = 2.04m
Total loss of head (h_L) = 2.04m
Pressure at point C (P_C) = ?
Applying Bernoulli's equation between A and C (taking B as datum)
$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A + h_p = \frac{P_C}{\gamma} + \frac{V_C^2}{2g} + Z_C + h_L$
$0 + 0 + 3 + 16.42 = \frac{P_C}{9810} + \frac{11.46^2}{2x9.81} + 4.4 + 2.04$
P_C = 61668 Pa

22. The figure below shows a pipe connecting a reservoir to a turbine which discharges water to the tailrace through another pipe. The head loss between the reservoir and the turbine is 8 times kinetic head in the pipe and that from the turbine to tailrace is 0.4 times the kinetic head in the pipe. The rate of flow is 1.36m^3/s and the pipe diameter in both cases is 1m. Determine (a) the pressure at inlet and exit of turbine, and (b) the power generated by the turbine.



Solution:

Diameter of pipe 1-2 and 3-4 (d) = 1m

Discharge (Q) = 1.36m^3/s

As d is same, velocity (V) is also same for pipe 1-2 and 3-4.

$V = \frac{Q}{A} = \frac{1.36}{\frac{\pi}{4}1^2}$ =1.73 m/s

Head loss between reservoir and turbine (h_{La})= $8\frac{V^2}{2g} = 8\frac{1.73^2}{2x9.81}$ = 1.22m

Head loss between turbine and tailrace (h_{Lb})= $0.4\frac{V^2}{2g} = 0.4\frac{1.73^2}{2x9.81}$ = 0.06m

Total loss of head (h_L) = 1.22+0.06 = 1.28m

P_1 = 0, P_4 = 0, V_1 = 0, V_4 = 0

(a) Pressure at 2 (P_2) = ?

Pressure at 3 (P_3) = ?

Applying Bernoulli's equation between 1 and 2 (taking datum through 2)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_{La}$$

$$0 + 0 + 50 = \frac{P_2}{9810} + \frac{1.73^2}{2x9.81} + 0 + 1.22$$

P_2 = 477035 Pa

Applying Bernoulli's equation between 3 and 4 (taking datum through 4)

$\frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3 = \frac{P_4}{\gamma} + \frac{V_4^2}{2g} + Z_4 + h_{Lb}$

$\frac{P_3}{9810} + \frac{1.73^2}{2x9.81} + 5 = 0 + 0 + 0 + 0.06$

P_3 = -49958 Pa

b) Power generated by the turbine (P) = ?

Head extracted by turbine = h_T

Applying Bernoulli's equation between 2 and 3 (taking datum through 2)

$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 - h_T = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3$

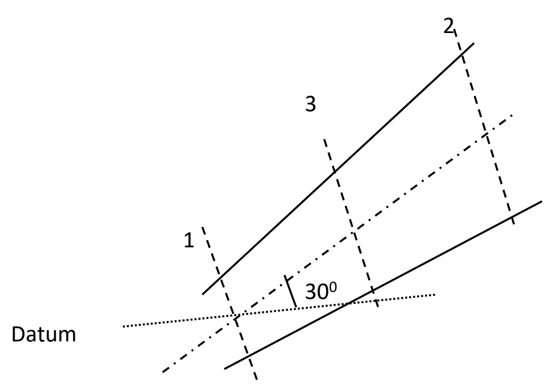
$\frac{477035}{9810\gamma} + \frac{1.73^2}{2x9.81} + 0 - h_T = \frac{-49958}{9810} + \frac{1.73^2}{2x9.81} + 0$

h_T = 53.7m

$P = \gamma Q h_T = 9810x1.36x53.7$ = 711176W = 711.176KW

A 2.5m long pipeline tapers uniformly from 10cm diameter to 20cm diameter at its upper end. The pipe centerline slopes upwards at an angle of 30^0 to the horizontal and the flow direction is from smaller to bigger cross-section. If the pressure at lower and upper ends of the pipe are 2bar and 2.4bar respectively, determine the flow rate and the pressure at the mid-length of the pipeline. Assume no energy losses.

Solution:



Solution:

Diameter of pipe at section1 (d_1) = 10cm = 0.1m

C/S Area of pipe at section1 (A_1) = $\frac{\pi}{4}x0.1^2$ = 0.00785 m^2

Diameter of pipe at section2 (d_2) = 20cm = 0.2m

C/S Area of pipe at section2 (A_2) = $\frac{\pi}{4}x0.2^2$ = 0.0314 m^2

Pressure at section 1 (P_1) =2 bar = $2x10^5$ N/m^2

Pressure at section 2 (P_2) = 2.4bar =$2.4x10^5$ N/m^2

Discharge (Q) = ?

Pressure at mid length (P_3) = ?

Taking datum head at section 1 (Z_1) = 0
Z_2 = 2.5Sin30= 1.25m
Z_3 = 1.25Sin30= 0.625m

a. From continuity equation
$A_1 V_1 = A_2 V_2$
$\frac{\pi}{4}x0.1^2V_1 = \frac{\pi}{4}x0.2^2V_2$
$V_1 = 4 V_2$

Applying Bernoulli's equation at section 1 and 2
$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$
$\frac{2x10^5}{9810} + \frac{(4V_2)^2}{2g} + 0 = \frac{2.4x10^5}{9810} + \frac{V_2^2}{2g} + 1.25$
V_2 = 2.64m/s
V_1 = 4x2.64 = 10.56m/s
Q = $A_1 V_1$ = 0.00785x10.56 = 0.083 m^3/s

b. At mid length d_3 = 15cm = 0.15m
C/S Area of pipe at section3 (A_3) = $\frac{\pi}{4}x0.15^2$ = 0.01767 m^2
$V_3 = Q/A_3$ = 0.083/0.01767 = 4.7m/s

Applying Bernoulli's equation at section 1 and 2
$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3$
$\frac{2x10^5}{9810} + \frac{10.56^2}{2g} + 0 = \frac{P_3}{9810} + \frac{4.7^2}{2g} + 0.625$
P_3 = 238581N/m^2 = 238.581 Kpa

Tutorial 8

Flow measurement

1. A jet of water issuing from 5mm diameter orifice working under a head of 2.0m, was found to travel horizontal and vertical distances of 2.772m and 1m respectively. If C_C = 0.61, determine discharge.

Solution:

Diameter of jet (d) = 5mm = 0.005m

C/s of jet (a) = $\frac{\pi}{4}x0.005^2$ = 1.96X10^{-5} m^2

Head (H) = 2m

x = 2.772m, y = 1m

C_C = 0.61

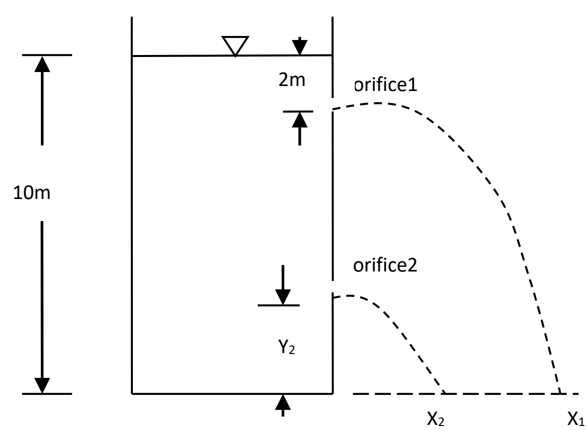
Discharge (Q) = ?

$C_v = \frac{x}{\sqrt{4yH}} = \frac{2.772}{\sqrt{4x1x2}}$ = 0.98

$C_d = C_C C_V$ = 0.61x0.98 = 0.5978

$Q = C_d a\sqrt{2gH} = 0.5978x1.96x10^{-5}\sqrt{2gx2}$ = 7.34X10^{-5} m^3/S = 0.0734 lps

2. For the two orifices shown in the figure below, determine Y_2 such that $X_2 = \frac{3X_1}{4}$.



Solution:

H_1 = 2m, Y_1 = 10-2 = 8m, H_2 = 10-Y_2

$X_2 = \frac{3X_1}{4}$

Y_2 = ?

Coefficient of velocity for orifice 1 $(C_{v1}) = \frac{X_1}{\sqrt{4Y_1H_1}}$

Coefficient of velocity for orifice 2 $(C_{v2}) = \frac{X_2}{\sqrt{4Y_2H_2}}$

Since the two orifices are identical

$C_{v1} = C_{v2}$

$\frac{X_1}{\sqrt{4Y_1H_1}} = \frac{X_2}{\sqrt{4Y_2H_2}}$

$\frac{X_1^2}{4Y_1H_1} = \frac{X_2^2}{4Y_2H_2}$

$\frac{X_1^2}{4x2x8} = \frac{\left(\frac{3X_1}{4}\right)^2}{4Y_2(10-Y_2)}$

$Y_2^2 - 10Y_2 + 9 = 0$

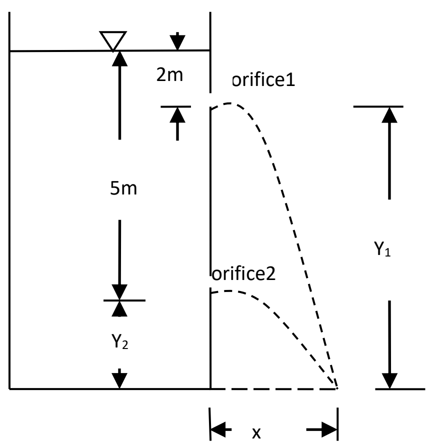
Solving for Y_2

Y_2 = 1, 9

As Y_1 = 8m, Y_2 = 9 (>Y_1) is not feasible.

Hence Y_2 = 1m

3. A vessel has two identical orifices provided in one of its sides as shown in the figure. Locate the point of intersection of two jets. Take C_v =0.98 for both orifices.



Solution:

$C_v = \frac{x}{\sqrt{4yH}}$

$C_{v1} = C_{v2}$

$\frac{x}{\sqrt{4Y_1H_1}} = \frac{x}{\sqrt{4Y_2H_2}}$

$Y_1H_1 = Y_2H_2$

$Y_1x2 = Y_2x5$

$Y_1 = 2.5Y_2$ (a)

Here,

$Y_1 - Y_2 = H_1 - H_2$ = 3

From a and b

Y_1 = 5m, Y_2 = 2m

$x = C_v\sqrt{4Y_1H_1} = 0.98\sqrt{4x5x2}$ = 6.2m

4. A rectangular orifice 1.0m wide and 1.5m deep is discharging water from a vessel. The top edge of the orifice is 0.8m below the water surface in the vessel. Calculate the discharge through the orifice if C_d = 0.6. Also calculate the percentage error if the orifice is treated as a small orifice.

Solution:
Width of orifice (b) = 1m
Depth of orifice (d) = 1.5m
H_1 = 0.8m, $H_2 = H_1$+d = 0.8+1.5 = 2.3m
C_d = 0.6
Discharge (Q) through large orifice = ?
error in discharge treating as small orifice = ?

$$Q = \frac{2}{3}C_d\, b\sqrt{2g}\left(H_2^{3/2} - H_1^{3/2}\right)$$

$$= \frac{2}{3}x0.62x\ 1\sqrt{2x9.81}\left(2.3^{3/2} - 0.8^{3/2}\right) = 5.076\ m^3/s$$

For small orifice, h = H_1 + d/2 = 0.8+1.5/2 = 1.55m
Discharge (Q1) through small orifice $(Q1) = C_d a\sqrt{2gH}$
$= 0.62x1x1.5\ \sqrt{2x9.81x1.55}$ = 5.13 m^3/s

% error in measuring discharge = $\frac{5.13-5.076}{5.076}x100$= 1.06%

5. A rectangular orifice of 1.5m wide and 1.2m deep is fitted in one side of a large tank. The water level on one side of the orifice is 2m above the top edge of the orifice, while on the other side of the orifice, the water level is 0.4m below its top edge. Calculate the discharge through the orifice if C_d = 0.62.

Solution:
Width of orifice (b) = 1.5m
Depth of orifice (d) = 1.2m
Height of water from top edge of orifice (H_1) = 2m
Difference of water level on both sides (H) =2+0.4 = 2.4m
Height of water from bottom edge of orifice (H_2) = H1+d = 2+1.2 = 3.2
C_d = 0.62
Discharge through partially submerged orifice (Q) = ?

$$Q = \frac{2}{3}C_d\, b\sqrt{2g}\left(H^{3/2} - H_1^{3/2}\right) + C_d b(H_2 - H)\sqrt{2gH}$$

$$= \frac{2}{3}x0.62x1.5\sqrt{2x9.81}\left(2.4^{3/2} - 2^{3/2}\right) + 0.62x1.5(3.2 - 2.4)\sqrt{2x9.81x2.4}$$

= 7.55 m^3/s

6. A horizontal venturimeter in a water main has a 20cm diameter at one end and tapers to 10cm at its throat. A piezometer installed at the inlet reads 30cm, while the one at the throat reads 18cm. Determine the discharge through the main, if C_d = 0.98.

Solution:

Diameter at inlet (d_1) = 20cm = 0.2m

C/s area of inlet (A_1) = $\frac{\pi}{4}x0.2^2$ = 0.0314m^2

Diameter at throat (d_2) = 10cm = 0.1m

C/s area of throat (A_2) =$\frac{\pi}{4}x0.1^2$ = 0.00785m^2

Pressure head at inlet ($P_1/\rho g$) =30cm =0.3m

Pressure head at throat ($P_2/\rho g$) =18cm =0.18m

Head (h) = $\frac{P_1}{\rho g} - \frac{P_2}{\rho g}$ = 0.12m

C_d = 0.98

Discharge (Q)=?

$$Q = C_d\sqrt{2gh}\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}$$

$$= 0.98\sqrt{2x9.81x0.12}\frac{0.0314x0.00785}{\sqrt{0.0314^2-0.00785^2}}$$

= 0.0122 m^3/s

7. A horizontal venturimeter is used to measure the flow of water in a 200mm diameter pipe. The throat diameter of the venturimeter is 80mm and the discharge coefficient is 0.85. If the pressure difference between the two measurement points is 10cm of mercury, calculate the average velocity in the pipe.

Solution:

Diameter at inlet (d_1) = 200mm = 0.2m

C/s area of inlet (A_1) = $\frac{\pi}{4}x0.2^2$ = 0.0314m^2

Diameter at throat (d_2) = 80mm = 0.08m

C/s area of throat (A_2) = $\frac{\pi}{4}x0.08^2$ = 0.00503m^2

Pressure difference (P_1-P_2) = 10cm of Hg = $\gamma_{Hg}h$ = 13.6x9810x0.1 Pa

$Z_1 = Z_2$

Head (h) = $\frac{P_1-P_2}{\gamma} = \frac{13.6x9810x0.1}{9810}$ = 1.36m

C_d = 0. 85

Average velocity in the pipe (V) = ?

$$Q = C_d\sqrt{2gh}\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}$$

$= 0.85\sqrt{2x9.81x1.36}\frac{0.0314x0.00503}{\sqrt{0.0314^2-0.00503^2}}$
= 0.0223 m^3/s
V = Q/A_1 = 0.0223/0.0314 = 0.71m/s

8. An orificemeter is provided in a vertical pipeline of 250mm diameter carrying oil of sp.gr. 0.9, the flow being upwards. The difference in elevation of the upstream and downstream ends of the manometer on the orificemeter is 350mm. The differential U-tube manometer shows a gauge deflection of 200mm. Calculate the discharge of oil. The diameter of the orifice is 150mm. Take C_d = 0.65.

Solution:
Diameter of pipe (d_1) = 250mm = 0.25m
C/s Area of pipe (A_1) = $\frac{\pi}{4}x0.25^2$ = 0.049 m^2
Diameter of orifice (d_2) = 150mm = 0.15m
C/s Area of pipe (A_2) = $\frac{\pi}{4}x0.15^2$ = 0.0176 m^2
Sp.gr. of oil (S_0) =0.9
Difference in elevation (Z_1-Z_2) = 350mm =0.35m
Reading of manometer (x) = 200mm = 0.2m
Sp.gr. of mercury (S) = 13.6
C_d =0.65
Discharge of oil (Q) =?

Head h is given by
$h = x\left(\frac{S}{S_0}-1\right)$= 0.2 $\left(\frac{13.6}{0.9}-1\right)$=2.82m

$Q = C_d\sqrt{2gh}\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}$
$= 0.65\sqrt{2x9.81x2.82}\frac{0.049x0.0176}{\sqrt{0.049^2-0.0176^2}}$
= 0.091 m^3/s

9. A 20cmx10cm venturimeter is mounted in a vertical pipeline carrying oil of sp.gr. 0.8 flowing upwards. The throat section is 20cm above the entrance section of the venturimeter. The differential U-tube manometer shows a gauge deflection of 25cm. Calculate the discharge of the oil and the pressure difference between the entrance and the throat section. Take C_d =0.96

Solution:
Diameter at inlet (d_1) = 20cm = 0.2m
C/s area of inlet (A_1) =$\frac{\pi}{4}x0.2^2$ = 0.0314m^2
Diameter at throat (d_2) = 10cm = 0.1m
C/s area of throat (A_2) = $\frac{\pi}{4}x0.1^2$ = 0.00785m^2

Sp.gr. of oil (S_0) =0.8
Density of oil (ρ) =0.8x1000 = 800 kg/m^8
Difference in elevation (Z_2-Z_1) = 20cm =0.2m
Reading of manometer (x) = 25mm = 0.25m
Sp.gr. of mercury (S) = 13.6
Discharge of oil (Q) =?
Pressure difference (P_1-P_2) =?

Head h is given by
$h = x\left(\frac{S}{S_0} - 1\right)$= $0.25\left(\frac{13.6}{0.8} - 1\right)$= 4m

$Q = C_d\sqrt{2gh}\frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}$
$= 0.96\sqrt{2x9.81x4}\frac{0.0314x0.00785}{\sqrt{0.0314^2 - 0.00785^2}}$
= 0.0689 m^3/s

h is also expressed as
$h = \frac{P_1 - P_2}{\rho g} + (Z_1 - Z_2)$
$\frac{P_1 - P_2}{\rho g} = 4 + 0.2$
P_1-P_2 = 4.2x800x9.81 = 32962 N/m^2

10. A venturimeter with a throat diameter of 100mm is fitted in a vertical pipeline of 200mm diameter with oil of sp.gr. 0.88 flowing upwards. The venturimeter coefficient is 0.98. The pressure gauges are fitted at tapping points, one at the throat and the other in the inlet pipe 320mm below the throat. The difference between two pressure gauge readings is 28 KN/m^2. Working from Bernoulli's equation, determine (a) the volume rate of oil through the pipe, (b) the difference in level in the two limbs of mercury if it is connected to the tapping points and connecting pipes are filled with same oil.

Solution:
Diameter at inlet (d_1) = 20cm = 0.2m
C/s area of inlet (A_1) =$\frac{\pi}{4}x0.2^2$ = 0.0314m^2
Diameter at throat (d_2) = 10cm = 0.1m
C/s area of throat (A_2) = $\frac{\pi}{4}x0.1^2$ = 0.00785m^2
Sp.gr. of oil (S_0) =0.88
Density of oil (ρ) =0.88x1000 = 880 kg/m^8
Difference in elevation (Z_2-Z_1) = 320cm =0.32m
Difference in pressure (P_1-P_2)= 28KN/m^2 = 28000N/m^2
Sp.gr. of mercury (S) =13.6

C_d =0.98
Discharge of oil (Q) =?
Manometer reading (x) =?

a. Applying Bernoulli's equation between inlet (1) and throat (2)

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

$$\frac{P_1-P_2}{\rho g}+(Z_1-Z_2)=\frac{V_2^2}{2g}-\frac{V_1^2}{2g}$$

$$\frac{28000}{880x9.81}+(-0.32)=\frac{V_2^2}{2g}-\frac{V_1^2}{2g}$$

$$\frac{V_2^2}{2g}-\frac{V_1^2}{2g}=2.92 \qquad \text{(a)}$$

According to continuity equation
$A_1 V_1=A_2V_2$

$$V_2=\frac{A_1V_1}{A_2}=\frac{0.0314V_1}{0.00785}=4V_1 \qquad \text{(b)}$$

Solving a and b

$$\frac{(4V_1)^2}{2g}-\frac{V_1^2}{2g}=2.92$$

V_1 = 1.95m/s
Discharge (Q) =A_1V_1 = 0.0314x1.95 = 0.612 m^3/s
Actual discharge = C_d Q = 0.98x0.612 = 0.599 m^3/s

b. $h=\frac{P_1-P_2}{\rho g}+(Z_1-Z_2)$=$\frac{28000}{880x9.81}+(-0.32)$ =2.92

$$h=x\left(\frac{S}{S_0}-1\right)$$

$$2.92=x\left(\frac{13.6}{0.88}-1\right)$$

x = 0.2m

11. A venturimeter is used for measurement of discharge of water in a horizontal pipeline. The ratio of the upstream pipe diameter and throat is 2:1 and upstream diameter is 300mm. Mercury manometer connected at the pipe and throat shows the reading of 0.24m and the loss of head through the meter is 1/8 of the throat velocity head. Calculate the discharge in the pipe using the continuity and energy equations.

Solution:
Pipe diameter (d_1): throat diameter (d_2) = 2:1
Pipe diameter (d_1) = 300mm = 0.3m
Throat diameter (d_2) =d_1/2 = 0.15m

C/s area of inlet (A_1) = $\frac{\pi}{4}x0.3^2$ = 0.0707m^2

C/s area of throat (A_2) = $\frac{\pi}{4}x0.15^2$= 0.01767m^2

Manometer reading (x) =0.24m

Sp.gr. of mercury (S) =13.6

Velocity at inlet =V_1

Velocity at throat =V_2

Head loss (h_L) =$\frac{1}{8}.\frac{V_2^2}{2g}$

Discharge in the pipe (Q) =?

Head h is given by

$h = x\left(\frac{S}{S_0} - 1\right)$= $0.24\left(\frac{13.6}{1} - 1\right)$=3.02m

Applying Bernoulli's equation between inlet (1) and throat (2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{P_1 - P_2}{\rho g} + (Z_1 - Z_2) = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + \frac{1}{8}.\frac{V_2^2}{2g} \qquad \text{(a)}$$

$$h = \frac{P_1 - P_2}{\rho g} + (Z_1 - Z_2) \qquad \text{(b)}$$

From a and b

$$h = \frac{9V_2^2}{16g} - \frac{V_1^2}{2g}$$

$$\frac{9V_2^2}{16g} - \frac{V_1^2}{2g} = 3.02 \qquad \text{(c)}$$

According to continuity equation

$A_1 V_1 = A_2 V_2$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.0707V_1}{0.01767} = 4V_1 \qquad \text{(d)}$$

Solving c and d

$$\frac{9(4V_1)^2}{16g} - \frac{V_1^2}{2g} = 3.02$$

V_1 = 1.86m/s

Discharge (Q) = A_1V_1 = 0.0707x1.86 = 0.131 m^3/s

12. A horizontal venturimeter with inlet and throat diameters of 400mm and 200mm respectively, is connected to a pipe. If the pressure in the inlet portion is 200KPa and the vacuum pressure (negative) on the throat is 400mm of mercury, find the rate of flow in the pipe taking C_d = 0.97.

Solution:

Diameter at inlet (d_1) = 400mm = 0.4m

C/s area of inlet (A_1) = $\frac{\pi}{4}x0.4^2$ = 0.1256m^2

Diameter at throat (d_2) = 200mm = 0.2m

C/s area of throat (A_2) = $\frac{\pi}{4}x0.2^2$= 0.0314m^2

Pressure at inlet (P_1) =200 Kpa = 200x10^3N/m^2

Pressure head at throat = -400 mm of mercury

C_d =0.97

Discharge (Q) =?

Pressure head at inlet $\left(\frac{P_1}{\rho g}\right)$ =$\frac{200000}{1000x9.81}$ =20.38m of water

Pressure head at throat $\left(\frac{P_2}{\rho g}\right)$ = -0.4 m of mercury =-0.4x13.6m of water =-5.44 m of water

Z_1=Z_2

Head h is given as

$h = \frac{P_1 - P_2}{\rho g} + (Z_1 - Z_2)$ =20.38-(-5.44)+0 = 25.82m

$$Q = C_d\sqrt{2gh}\frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}$$

$$= 0.97\sqrt{2x9.81x25.82}\frac{0.1256x0.0314}{\sqrt{0.1256^2 - 0.0314^2}}$$

= 0.708 m^3/s

13. A Venturimeter is to be fitted in a horizontal pipe of 0.15m diameter to measure a flow of water which may be anything up to 240m^3/hour. The pressure head at the inlet for this flow is 18m above atmospheric and the pressure head at the throat must not be lower than 7m below atmospheric. Between the inlet and the throat there is an estimated frictional loss of 10% of the difference in pressure head between these points. Calculate the minimum allowable diameter for the throat.

Solution:

Diameter of pipe (d_1) = 0.15m

C/s area of pipe (A_1) = $\frac{\pi}{4}x0.15^2$= 0.01767m^2

Discharge (Q) = 240m^3/hour = $\frac{240}{3600}$ = 0.0667m^3/s

Velocity at inlet (V_1) = Q/A_1 = 0.0667/0.01767 = 3.77m/s

Pressure head at inlet $\left(\frac{P_1}{\rho g}\right)$ =18m

Pressure head at throat $\left(\frac{P_2}{\rho g}\right)$ = -7 m

Z_1=Z_2

Head loss (h_L) = 0.1x$\left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right)$ = 0.1(18+7) = 2.5m

Diameter of throat (d_2) = ?

Applying Bernoulli's equation between inlet (1) and throat (2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + h_L$$

$18 + 7 = \frac{V_2^2}{2g} - \frac{3.77^2}{2g} + 2.5$

V_2 = 21.34 m/s

C/S area of inlet (A_2) = Q/V_2 = 0.0667/21.334 = 0.003126 m^2

$A_2 = \frac{\pi}{4} x d_2^2$

$0.003126 = \frac{\pi}{4} x d_2^2$

d_2 = 0.063m = 63mm

14. A Venturimeter of throat diameter 0.07m is fitted in a 0.15m diameter vertical pipe in which liquid of relative density 0.8 flows downwards. Pressure gauges are fitted to the inlet and to the throat sections. The throat being 0.9m below the inlet. Taking the coefficient of the meter as 0.96 find the discharge a) when the pressure gauges read the same b) when the inlet gauge reads 15170 N/m^2 higher than the throat gauge.

Solution:

Diameter of pipe (d_1) = 0.15m

C/s area of pipe (A_1) = $\frac{\pi}{4} x 0.15^2$ = 0.01767m^2

Diameter at throat (d_2) = 0.07m

C/s area of throat (A_2) = $\frac{\pi}{4} x 0.07^2$ = 0.00385m^2

$Z_1 - Z_2$ = 0.9m

C_d = 0.96

Discharge (Q) = ?

a) $P_1 = P_2$

$h = \frac{P_1 - P_2}{\rho g} + (Z_1 - Z_2) = 0 + 0.9$ =0.9m

$Q = C_d \sqrt{2gh} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$

$= 0.96\sqrt{2x9.81x0.9} \frac{0.01767x0.00385}{\sqrt{0.01767^2 - 0.00385^2}}$

= 0.016 m^3/s

b) $P_1 = P_2 + 15170$

$h = \frac{P_1 - P_2}{\rho g} + (Z_1 - Z_2) = \frac{15170}{0.8x1000x9.81} + 0.9$ = 2.83m

$Q = C_d \sqrt{2gh} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$

$= 0.96\sqrt{2x9.81x2.83} \frac{0.01766x0.00385}{\sqrt{0.01766^2 - 0.00385^2}}$

= 0.0285 m^3/s

15. Water is flowing over a sharp-crested rectangular weir of width 50cm into a tank with cross-sectional area $0.6m^2$. After a period of 30s the depth of water in the tank is 1.4m. Assuming a discharge coefficient of 0.9, determine the height of the water above the weir.
If the rectangular weir is replaced by a 90^0 notch weir with the same head and a discharge coefficient of 0.8, calculate the depth increase of the water in the tank after 30s.

Solution:
a) Width of rectangular weir (L) = 50cm = 0.5m
C/S area (A) = $0.6m^2$
Depth of water in tank (h) = 1.4m
Volume of water in 30 Sec = Ah = 0.6x1.4 = $0.84m^3$
Discharge (Q) = Volume/time = 0.84/30 = 0.028 m^3/s
C_d = 0.9
Height of water (H) =?

$Q = \frac{2}{3} C_d \sqrt{2g}\, L H^{3/2}$
$0.028 = \frac{2}{3} x 0.9 \sqrt{2x9.81}\, x 0.5 x H^{3/2}$
H = 0.076m

b) With 90^0 V-notch, C_d = 0.8
Head (H) = 0.076m
Increase in depth after 30s (h_1) = ?
$Q = \frac{8}{15} C_d \sqrt{2g} \tan(\theta/2)\, H^{5/2}$
$Q = \frac{8}{15} x 0.8 \sqrt{2x9.81} \tan(90/2)\, 0.076^{5/2}$ = 0.003 m^3/s

$Q = \frac{A h_1}{t}$
$0.003 = \frac{0.6 x h_1}{30}$
h_1 = 0.15m

16. A sharp edged weir is to be constructed across a stream in which the normal flow is 200 litres/sec. If the maximum flow likely to occur in the stream is 5 times the normal flow then determine the length of weir necessary to limit the rise in water level to 40cm above that for normal flow. C_d=0.61.

Solution:
Normal discharge (Q_1) = 200 lps = 0.2 m^3/s
Maximum discharge (Q_2) = 5x0.2 = 1 m^3/s
C_d=0.61
Water level for normal flow = y
For y+0.4, Length of weir (L) = ?

$Q = \frac{2}{3} C_d \sqrt{2g}\, L H^{3/2}$

$Q_1 = \frac{2}{3} x 0.61 \sqrt{2x9.81}\, L y^{3/2}$
$0.2 = 1.801\, L y^{3/2}$
$L = 0.111\, y^{-3/2}$ (a)

$Q_2 = \frac{2}{3} x 0.61 \sqrt{2x9.81}\, L(y+0.4)^{3/2}$
$1 = 1.801\, L(y+0.4)^{3/2}$
$0.555 = \ L(y+0.4)^{3/2}$ (b)

From a and b
$0.555 = \ 0.111\, y^{-3/2}(y+0.4)^{3/2}$
$\frac{(y+0.4)^{3/2}}{y^{3/2}} = 5$
$\frac{y+0.4}{y} = 2.924$
y = 0.208m
$L = 0.111\, y^{-3/2} = 0.111\ 0.208^{-3/2}$ = 1.17m

17. Compute the flow rate if the measured head above the bottom of the V-notch is 35cm, when θ= 60^0 and C_d=0.6. If the flow is wanted within an accuracy of 2%, what are the limiting values of the head.

Solution:
Head (H) = 35cm = 0.35m
θ= 45^0
C_d=0.6
Flow rate (Q) = ?

$Q = \frac{8}{15} C_d \sqrt{2g} \tan(\theta/2)\, H^{5/2}$
$Q = \frac{8}{15} x 0.6 \sqrt{2x9.81} \tan(60/2)\, H^{5/2} = 0.818 H^{5/2}$
$= 0.818 x 0.35^{5/2}$ = 0.059 m^3/s

With $\pm$2% error
Q_1 = 0.059+0.02X0.059= 0.0602 m^3/s
Q_2 = 0.059-0.02X0.059= 0.0578 m^3/s

$Q = 1.417 H^{5/2}$
$Q_1 = 1.417 H_1^{5/2}$
$0.0602 = 0.818 H_1^{5/2}$

H_1 = 0.352 m

$Q_2 = 1.417 H_2^{5/2}$
$0.0578 = 0.818 H_2^{5/2}$
H_2 = 0.346 m

18. Water is flowing in a rectangular channel of 1m wide and 0.8m deep. Find the discharge over a rectangular weir of crest length 60cm if the head of water over the crest of weir is 30cm and water from channel flows over the weir. Take C_d = 0.62. Take velocity of approach into consideration.

Solution:
Area of channel (A) = 1x0.8 = 0.8 m^2
Length of weir (L) = 0.6m
Head of water (H_1) = 0.3m
C_d = 0.62

Discharge over a rectangular weir without velocity of approach is

$$Q = \frac{2}{3} C_d\, L \sqrt{2g} H_1^{3/2}$$
$$= \frac{2}{3} x 0.62 x 0.6 \sqrt{2x9.81} x 0.3^{3/2} = 0.18 \text{ m}^3/\text{s}$$

Velocity of approach (V_a) = Q/A = 0.18/0.8 = 0.225 m/s
Velocity head $(h_a) = \frac{V_a^2}{2g} = \frac{0.225^2}{2x9.81}$ = 0.00258m

$$Q = \frac{2}{3} C_d\, L \sqrt{2g} \left[(H_1 + h_a)^{3/2} - h_a^{3/2}\right]$$
$$= \frac{2}{3} x 0.62 x 0.6 \sqrt{2x9.81} \left[(0.3 + 0.00258)^{3/2} - 0.00258^{3/2}\right] = 0.183 \text{ m}^3/\text{s}$$

19. A broad-crested weir of length 40m, has 400mm height of water above its crest. Find the maximum discharge neglecting velocity of approach. If the velocity of approach is taken into consideration, find the maximum discharge when the channel has a cross-sectional area of 40m^2 on the upstream side. Take C_d =0.6.

Solution:
Length of weir (L) = 40m
Head of water (H_1) = 0.4m
C_d = 0.6

Maximum discharge neglecting velocity of approach

$$Q_{max} = 1.705 C_d\, L H^{3/2}$$
$$= 1.705 x 0.6 x 40 x 0.4^{3/2} = 10.352 \text{ m}^3/\text{s}$$

Taking velocity of approach into consideration

Area of channel (A) = 40 m^2
Velocity of approach (V_a) = Q_{max}/A = 10.35/40 = 0.25875 m/s
Velocity head $(h_a) = \frac{V_a^2}{2g} = \frac{0.25875^2}{2x9.81} =$ 0.0034m
Maximum discharge considering velocity of approach

$$Q_{max} = 1.705C_d\,L\left[(H+h_a)^{3/2} - h_a^{3/2}\right]$$

$$= 1.705x0.6x40\left[(0.4+0.0034)^{3/2} - 0.0034^{3/2}\right] = 10.476 \text{ m}^3\text{/s}$$

20. The heights of water on the upstream and downstream side of a submerged weir of length 3.5m are 300mm and 150mm respectively. If C_d for free and drowned portion is 0.6 and 0.8 respectively, find the discharge over the weir.

Solution:
Height of water on upstream side (H) = 0.3m
Height of water on downstream side (h) = 0.15m
Length of weir (L) = 3.5m
C_{d1} = 0.6, C_{d2} = 0.8
Discharge through drowned orifice (Q)= ?

$$Q = \frac{2}{3}C_{d1}L\sqrt{2g}(H-h)^{3/2} + C_{d2}Lh\sqrt{2g(H-h)}$$

$$= \frac{2}{3}x0.6x3.5\sqrt{2x9.81}(0.3-0.15)^{3/2} + 0.8x3.5x0.15\sqrt{2x9.81(0.3-0.15)} = 1.08 \text{ m}^3\text{/s}$$

21. A 1.25m diameter circular tank contains water up to a height of 5m. At the bottom of the tank, an orifice of 50mm diameter is provided. Find the height of water above the orifice after 1.5 minutes. Take C_d = 0.62.

Solution:
Diameter of tank (D) =1.25m
Area of tank (A) = $\frac{\pi}{4}x1.25^2$ = 1.227 m^2
Diameter of orifice (d) =50mm = 0.05m
Area of orifice (a) = $\frac{\pi}{4}x0.05^2$ = 0.00196 m^2
Coeff. of discharge (C_d) = 0.62
Initial head (H_1) = 5m
Time (t) =1.5 minutes = 90 Sec
Final head (H_2) =?

From continuity
-Q dt = Adh
$t = -\int_{H_1}^{H_2}\frac{A}{Q}dh$

$= -\int_{H_1}^{H_2} \frac{A}{C_d a_0 \sqrt{2gh}} dh$

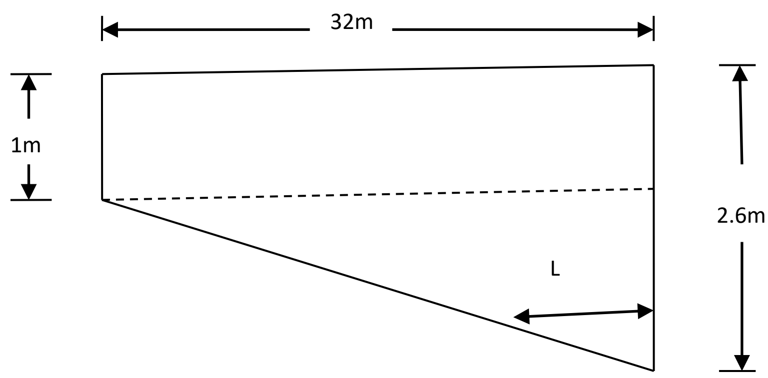
$= \frac{2A(\sqrt{H_1}-\sqrt{H_2})}{C_d.a\sqrt{2g}}$

$90 = \frac{2x1.227(\sqrt{5}-\sqrt{H_2})}{0.62x0.00196\sqrt{2x9.81}}$

$0.197 = (2.236 - \sqrt{H_2})$

H_2 = 4.15m

22. A rectangular swimming pool is 1m deep at one end and increases uniformly in depth to 2.6m at the other end. The pool is 8m wide and 32m long and is emptied through an orifice of area 0.224m^2, at the lowest point in the side of the deep end. Taking C_d for the orifice as 0.6, find a) the time for the depth to fall by 1m b) the time to empty the pool completely.



Solution:

Area of orifice (a_0) = 0.224m^2

Cd = 0.6

a) For 1m fall in depth

H_1= 2.6m, H_2 = 1.6m

Time to fall 1m depth (t1) = ?

A = 32x8 = 256m^2

From continuity

-Q dt = Adh

$t1 = -\int_{H_1}^{H_2} \frac{A}{Q} dh$

$= -\int_{H_1}^{H_2} \frac{A}{C_d a_0 \sqrt{2gh}} dh$

$= \frac{2A(\sqrt{H_1}-\sqrt{H_2})}{C_d.a\sqrt{2g}}$

$= \frac{2x256(\sqrt{2.6}-\sqrt{1.6})}{0.6x0.224\sqrt{2x9.81}}$
= 299 S

b) Time to completely empty the tank (t) = ?
Let us find out the time to empty the tank (t2) from H_1 = 1.6m to H_2 = 0m
We need A in terms of h.
A =8L
$\frac{L}{h} = \frac{32}{1.6}$
L = 20h
A = 8x20h = 160h

-Q dt = Adh
$t2 = -\int_{H_1}^{H_2} \frac{A}{Q} dh$

$= -\int_{H_1}^{H_2} \frac{160h}{C_d a_0 \sqrt{2gh}} dh = \frac{160}{C_d a_0 \sqrt{2g}} \int_{H_1}^{H_2} h^{1/2}\, dh$

$= \frac{160}{0.6x0.224\sqrt{2x9.81}} x \frac{2}{3}\left(H_1^{3/2} - H_2^{3/2}\right)$
$= 179.2\left(1.6^{3/2} - 0\right)$ = 363 S

Time to completely empty the tank (t1) = t1 + t2 = 299 + 363 = 662S

23. A cylindrical tank of internal diameter 0.6m, length 1.5m and axis vertical has a 5cm diameter sharp-edged orifice (C_d = 0.6) in the bottom, open to atmosphere. The tank is open at the top and empty. If water were admitted into the tank from above at a constant rate of 14lps, how long will it take to just fill the tank? How much water will escape through the orifice during that period?

Solution:
Diameter of tank (D) =0.6m
Area of tank (A) = $\frac{\pi}{4} x 0.6^2$ = 0.2827 m^2
Diameter of orifice (d) =5cm = 0.05m
Area of orifice (a) = $\frac{\pi}{4} x 0.05^2$ = 0.00196 m^2
Coeff. of discharge (C_d) = 0.6
$K = Cd.a\sqrt{2g} = 0.6x0.00196x\sqrt{2x9.81}$=0.0052
Inflow (Q_i) = 14 lps =0.014 m^3/s
Initial head (H_1) = 0m
Final head (H_2) = 1.5m
Time required to fill the tank (t) =?
Water escaped through orifice =?

$$t = \frac{2A}{K^2}\left[Q_i \ln\frac{Q_i - K\sqrt{H_2}}{Q_i - K\sqrt{H_1}} + K\left(\sqrt{H_2} - \sqrt{H_1}\right)\right]$$

$$t = \frac{2x0.2827}{0.0052^2}\left[0.014 \ln\frac{0.014 - 0.0052\sqrt{1.5}}{0.014 - 0.0052\sqrt{0}} + 0.0052\left(\sqrt{1.5} - \sqrt{0}\right)\right]$$

t = 44.46 Sec

Inflow volume = Q_i X t = 0.014x44.46 = 0.6224 m^3

Volume of water contained in the tank = A x H_2 = 0.2827x1.5 = 0.424 m^3

Water escaped through orifice = 0.6224 – 0.424 = 0.1984 m^3

24. A vertical cylindrical tank 2m diameter has, at the bottom, 0.05m diameter sharp-edged orifice (Cd = 0.6).

(I) If the water enters the tank at a constant rate of 0.0095 cumecs, find the depth of water above the orifice when the level in the tank becomes stable.
(II) Find the time for the level to fall from 3m to 1m above the orifice when the inflow is turned off.
(III) If water now runs into the tank at 0.02cumecs, the orifice remaining open, find the rate of rise in water level when the level has reached a depth of 1.7m above the orifice.

Solution:
Diameter of tank (D) =2m
Area of tank (A) = $\frac{\pi}{4}x2^2$ = 3.14 m^2
Diameter of orifice (d) =5cm = 0.05m
Area of orifice (a) = $\frac{\pi}{4}x0.05^2$ = 0.00196 m^2
Coeff. of discharge (Cd) = 0.6

General equation for tank with inflow (Q_i) and outflow (Q_o)

$$(Q_i - Q_o)dt = Adh$$

Where $Q_o = Cd.a\sqrt{2gh} = k\sqrt{h}$ =Discharge through orifice
$K = Cd.a\sqrt{2g} = 0.6x0.00196x\sqrt{2x9.81}$=0.0052

I) Q_i = 0.0095 m^3/s
Depth of water above orifice (h) =?
For stable condition, dh =0
$Q_i = Q_o = K\sqrt{h}$
$0.0095 = 0.0052\sqrt{h}$
h = 3.34 m

II) Initial Head (H_1) = 3m

Final Head (H_2) = 1m
Time required to lower the head from H_1 to H_2 (t) =?
When the inflow is turned off, Q_i = 0
$(-Q_o)dt = Adh$
$dt = -\frac{Adh}{Q_o} = -\frac{Adh}{K\sqrt{h}}$

$t = \int_{H_1}^{H_2} -\frac{Adh}{K\sqrt{h}} = \frac{2A}{K}\left(\sqrt{H_1} - \sqrt{H_2}\right)$
$= \frac{2x3.14}{0.0052}\left(\sqrt{3} - \sqrt{1}\right)$
= 885Sec

III) Q_i = 0.2 m^3/s
Head (h) =1.7m
Rate of rise in water level (dh/dt) =?

$(Q_i - Q_o)dt = Adh$

$\frac{dh}{dt} = \frac{Q_i - Q_o}{A} = \frac{Qi - K\sqrt{h}}{A}$
$\frac{dh}{dt} = \frac{0.2 - 0.0052\sqrt{1.7}}{3.14}$
= 0.0615 m/s

25. A cylindrical tank is placed with its axis vertical and is provided with a circular orifice of 4cm diameter at the bottom. A steady inflow and free discharge at the bottom of the orifice causes the depth of water in the tank to rise from 0.59m to 0.75m in 106 Sec. Further it is observed that the depth rises from 1.2m to 1.29m in 129 Sec. Determine the inflow rate and the diameter of the tank. Assume C_d = 0.62.

Solution:

Diameter of orifice (d) = 4cm = 0.04m
Area of orifice (a) = $\frac{\pi}{4}x0.04^2$ = 0.001257 m^2
Coeff. of discharge (C_d) = 0.62
$K = Cd.a\sqrt{2g} = 0.62x0.001257x\sqrt{2x9.81}$=0.00345
Inflow rate (Q_i) =?
Diameter of tank (D) =?

First case
dh = 0.75-0.59 = 0.16m
dt = 107S
Average head (h) = 0.67m
dh/dt = 0.001495

Second case

dh = 1.29-1.2 = 0.09m

dt = 129S

Average head (h) = 1.245m

dh/dt = 0.000698

$$(Q_i - Q_o)dt = Adh$$

$$\frac{dh}{dt} = \frac{Q_i - Q_o}{A} = \frac{Qi - K\sqrt{h}}{A}$$

Substituting values for both cases

$$0.001495 = \frac{Qi - 0.00345\sqrt{0.67}}{A}$$

Q_i=0.001495A+0.002824 (a)

$$0.000698 = \frac{Qi - 0.00345\sqrt{1.245}}{A}$$

Q_i=0.000698A+0.003849 (b)

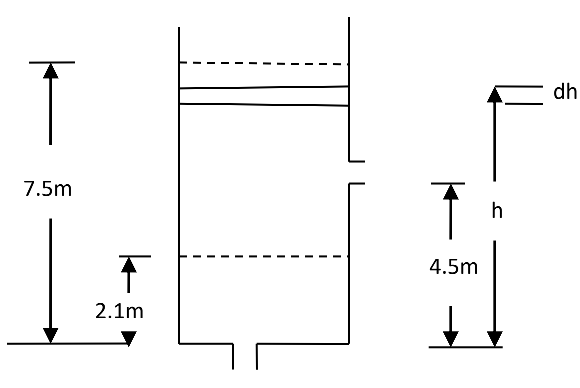
Solving a and b

A =1.285 m^2

Q_i = 0.0047 m^3/s

$$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4x1.285}{\pi}} = 1.28m$$

26. A tank of constant cross-sectional area of 2.8m^2 has two orifices each 9.3x10^{-4} m^2 in area as shown in the figure. Calculate the time taken to lower the water level from 7.5m to 2.1m above the bottom of the tank. Assume C_d = 0.62.



Solution:

C/s of tank (A) = 2.8m^2

C/s of orifice (a) = 9.3x10^{-4} m^2

Let the water level is at height h above the lower orifice at time t. dh is the decrease in water level in time dt.

Q = discharge

Qdt = -Adh

dt = -Adh/Q

Discharge from top orifice (Q1) = $C_d a\sqrt{2g(h-4.5)}$

Discharge from lower orifice (Q2) = $C_d a\sqrt{2gh}$

Q = Q1+Q2 = $C_d a\sqrt{2g(h-4.5)} + C_d a\sqrt{2gh}$

H1 = 7.5m, H2 = 4.5m

T1 = Time taken to lower the water from 7.5m to 4.5m when both orifices are discharging

T2 = Time taken to lower the water from 4.5m to 2.1m when only lower orifice is discharging

Finding T1

$$dt = -\frac{Adh}{Q} = -\frac{2.8dh}{C_d a\sqrt{2g(h-4.5)}+C_d a\sqrt{2gh}}$$

$$dt = -\frac{2.8dh}{C_d a\sqrt{2g}[\sqrt{(h-4.5)}+\sqrt{h}]} = -\frac{2.8dh}{0.62x9.3x10^{-4}\sqrt{2g}[\sqrt{(h-4.5)}+\sqrt{h}]}$$

$$dt = -1096.3\frac{dh}{[\sqrt{(h-4.5)}+\sqrt{h}]}$$

Integrating

$$T1 = -1096.3\int_{4.5}^{7.5}\frac{dh}{[\sqrt{(h-4.5)}+\sqrt{h}]}$$

$$T1 = -1096.3\int_{4.5}^{7.5}\frac{[\sqrt{(h-4.5)}-\sqrt{h}]dh}{-4.5}$$

= 942Sec

Finding T2 using direct formula

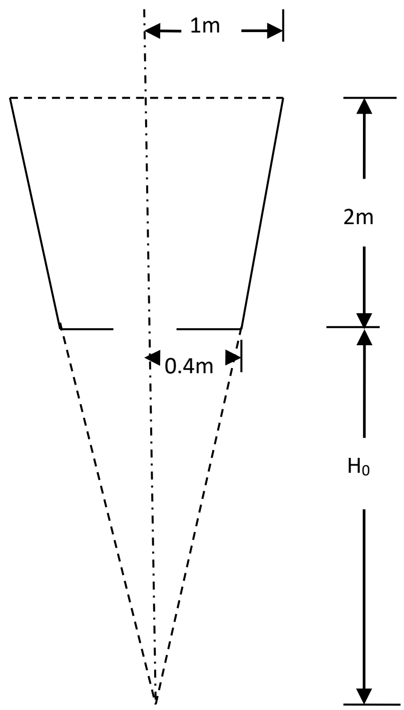
H1 = 4.5m, H2 = 2.1m

$$T2 = \frac{2A}{C_d a\sqrt{2g}}\left[\sqrt{H1}-\sqrt{H2}\right]$$

$T2 = \frac{2x2.8}{0.62x9.3x10^{-4}\sqrt{2g}}\left[\sqrt{4.5}-\sqrt{2.1}\right]$ = 1474 Sec

Total time taken to empty the tank from 7.5m to 2.1m = T1+T2 = 2416Sec = 40.26 min

27. A tank is in the form of frustum of a cone having top diameter of 2m, a bottom diameter of 0.8m and height 2m and is full of water. Find the time of emptying the tank through an orifice 100mm in diameter provided at the bottom. Take C_d = 0.625.



Solution:

C_d = 0.625

Area of orifice (a) = $\frac{\pi}{4} x 0.1^2$= 0.00785 m^2

H_1 = 2m, H_2 = 0m, R_1 = 1m, R_0 = 0.4m

From similar triangles,

$$\frac{1}{2 + H_0} = \frac{0.4}{H_0}$$

H_0 = 1.33m

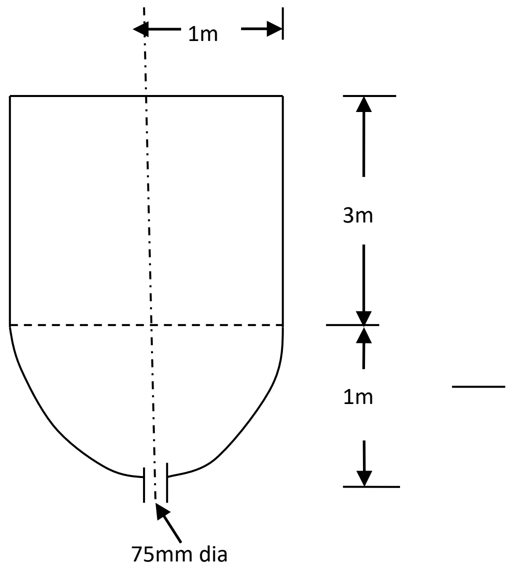
$K = \frac{1}{C_d a\sqrt{2g}} \frac{\pi R_1^2}{(H_1+H_0)^2} = \frac{1}{0.625x0.00785\sqrt{2x9.81}} \frac{\pi x 1^2}{(2+1.33)^2}$ = 13.03

Time of emptying the tank is

$$T = K\left[\frac{2}{5}\left(H_1^{5/2} - H_2^{5/2}\right) + \frac{4}{3}H_0\left(H_1^{3/2} - H_2^{3/2}\right) + 2H_0^2\left(H_1^{1/2} - H_2^{1/2}\right)\right]$$

$= 13.03\left[\frac{2}{5}\left(2^{5/2} - 0^{5/2}\right) + \frac{4}{3}x1.33\left(2^{3/2} - 0^{3/2}\right) + 2x1.33^2\left(2^{1/2} - 0^{3/2}\right)\right]$ = 160 Sec

28. A tank is in the form of hemisphere of 2m diameter and having a cylindrical upper part of 2m diameter and 3m height. Find the time of emptying the tank through an orifice of 75mm diameter at its bottom if the tank is initially full of water. Take C_d = 0.62.



Solution:

C_d = 0.625

Area of cylinder (A) = $\frac{\pi}{4} x 2^2$= 3.14 m^2

Area of orifice (a) = $\frac{\pi}{4} x 0.075^2$= 0.004418 m^2

T1 = time taken to lower water from 4m to 1m in the cylindrical part

T2 = time taken to empty the hemispherical part from 1m to 0m

Computing T1

H1 = 4m, H2 = 1m

$$T1 = \frac{2A}{C_d a\sqrt{2g}}\left[\sqrt{H1} - \sqrt{H2}\right]$$

$$T1 = \frac{2x3.14}{0.62x0.004418\sqrt{2g}}\left[\sqrt{4} - \sqrt{1}\right]$$ = 518Sec

Computing T2

H1 = 1m, H2 = 0m

$$T2 = \frac{\pi}{C_d a\sqrt{2g}}\left[\frac{4R}{3}\left(H_1^{3/2} - H_2^{3/2}\right) - \frac{2}{5}\left(H_1^{5/2} - H_2^{5/2}\right)\right]$$

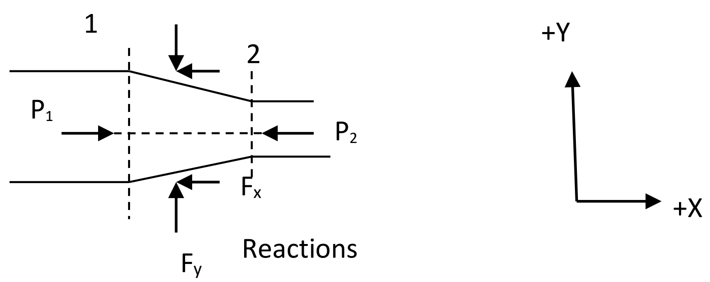
$$T2 = \frac{\pi}{0.62x0.004418\sqrt{2g}}\left[\frac{4x1}{3}\left(1^{3/2} - 0\right) - \frac{2}{5}\left(\left(1^{5/2} - 0\right) - 0\right)\right]$$ = 242 Sec

Total time taken to empty the tank = T1+T2 = 518+242 = 760 Sec

Tutorial 9

Application of Momentum Principle

1. A 60cm pipe is connected to a 30cm pipe by a standard reducer fitting. For the same flow of 0.9 m^3/s of water and a pressure of 200Kpa, what force is exerted by the water on the reducer, neglecting any lost head?



Solution:

Diameter at section 1 (d_1) = 600mm = 0.6m

Area at section 1 (A_1) = $\frac{\pi}{4}x0.6^2$= 0.2827m^2

Diameter at section 2(d_2) = 300mm = 0.3m

Area at section 2 (A_2) = $\frac{\pi}{4}x0.3^2$= 0.07068m^2

Discharge (Q) = 0.9 m^3/s

Velocity at section 1 (V_1) =Q/A_1 = =0.9/0.2827 = 3.18 m/s

Velocity at section 2 (V_2) =Q/A_2 =0.9/0.07068 = 12.73 m/s

Pressure at section 1 (P_1) = 200Kpa = 200000N/m^2

Force exerted by water on reducer =?

Applying Bernoulli's equation between 1 and 2 ($Z_1 = Z_2$)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{200000}{1000x9.81} + \frac{3.18^2}{2x9.81} = \frac{P_2}{1000x9.81} + \frac{12.73^2}{2x9.81}$$

P_2 = 124030 N/m^2

$$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ in\ X\ direction$$

$$(P_1\,A_1 - P_2\,A_2) - F_x = \rho Q(V_{2x} - V_{1x})$$

$$(P_1\,A_1 - P_2A_2) - F_x = \rho Q(V_2 - V_1)$$

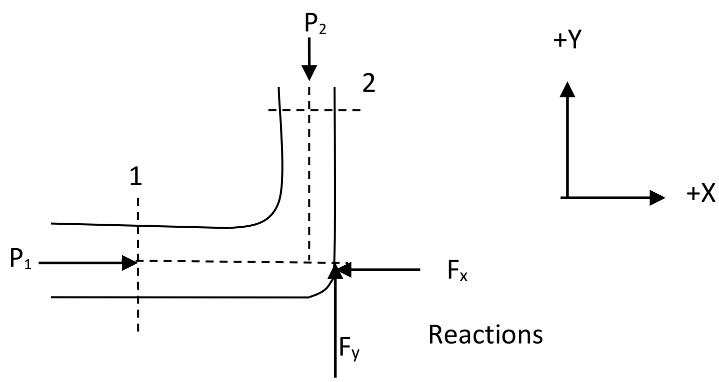
$$F_x = (P_1\,A_1 - P_2A_2) + \rho Q(V_1 - V_2)$$

$= (200000x\ 0.2827 - 124030\ x\ 0.07068) + 1000x0.9(3.18 - 12.73)$ = 39178 N

The forces in the Y-direction will balance each other and F_y = 0.

Hence the force exerted by water on reducer is 39178 N to the right.

2. A 500mm pipe carrying 0.8 m^3/s of oil (sp gr 0.85) has a 90^0 bend in a horizontal plane. The loss of head in the bend is 1.1m of oil, and the pressure at the entrance is 290KPa. Determine the resultant force exerted by the oil on the bend.



Solution:

Diameter at section 1 ad 2 (d) = 500mm = 0.5m

Area at section 1 and 2 ($A_1 = A_2$) = $\frac{\pi}{4} x 0.5^2$= 0.19635m^2

Discharge (Q) = 0.8 m^3/s

Velocity at 1 and 2 ($V_1 = V_2$) = Q/A_1 = 0.8/0.19635 = 4.07m/s

Pressure at 1 (P_1) = 290KPa = 290000 Pa

Loss of head (h_L) = 1.1m

Resultant force exerted by the oil on the bend = ?

Applying Bernoulli's equation between 1 and 2 ($Z_1 = Z_2$)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{290000}{0.85x1000x9.81} + \frac{4.07^2}{2x9.81} = \frac{P_2}{0.85x1000x9.81} + \frac{4.07^2}{2x9.81} + 1.1$$

P_2 = 280828 N/m^2

$$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ \ in\ X\ direction$$

$$P_1\ A_1 - F_x = \rho Q(V_{2x} - V_{1x})$$

$$P_1\ A_1 - F_x = \rho Q(0 - V_1)$$

$$F_x = P_1\ A_1 + \rho Q V_1$$

$$= 290000x0.19635 + 0.85x1000x0.8x4.07$$

= 59709 N

$$\sum Forces\ in\ Y\ direction = Rate\ of\ change\ of\ momentum\ \ in\ Y\ direction$$

$$F_y - P_2\ A_2 = \rho Q\left(V_{2y} - V_{1y}\right)$$

$$F_y - P_2\ A_2 = \rho Q(V_2 - 0)$$

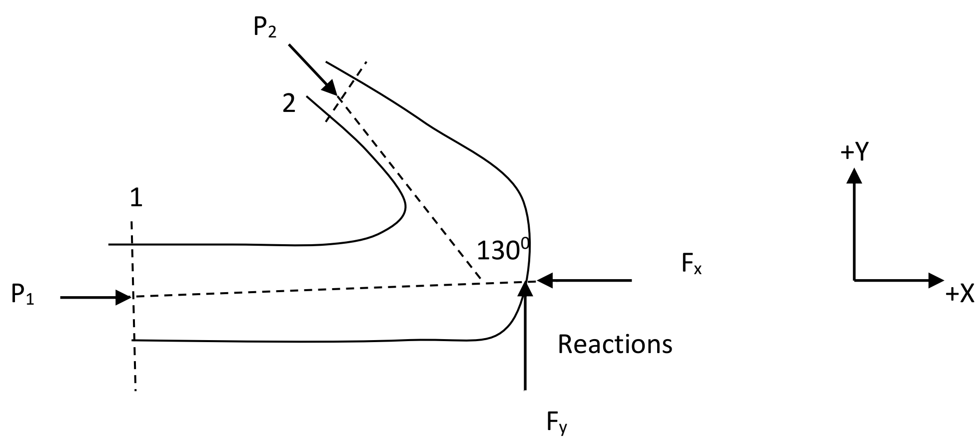
$F_y = P_2\, A_2 + \rho Q V_2$
$= 280828x0.19635 + 0.85x1000x0.8x4.07$
= 57908 N

Resultant force $(F_R) = \sqrt{F_{x+}^2 F_y^2}$ =83177N

Resultant force exerted by the water on the bend = 83177N (to the right and downward)

Direction of resultant force = $Tan^{-1}\frac{F_y}{F_x} = Tan^{-1}\frac{57908}{59709}$ = 44.1^0

3. The discharge of water through a 130^0 bend is 30 litres/s. The bend is lying in the horizontal plane and the diameters at the entrance and exit are 200mm and 100mm respectively. The pressure measured at the entrance is 100 kN/m^2, what is the magnitude and direction of the force exerted by the water on the bend?



Solution:
Diameter at section 1 (d_1) = 200mm = 0.2m
Area at section 1 (A_1) = $\frac{\pi}{4}x0.2^2$= 0.0314m^2
Diameter at section 2(d_2) = 100mm = 0.1m
Area at section 2 (A_2) = $\frac{\pi}{4}x0.1^2$= 0.00785m^2
Discharge (Q) = 30 lps = 0.03 m^3/s

Velocity at section 1 (V_1) =Q/A_1 = 0.03/0.0314 = 0.95 m/s
Velocity at section 2 (V_2) =Q/A_2 = 0.03/0.00785=3.82 m/s
θ= 180-130 = 50^0
Pressure at 1 (P_1) = 100KPa = 100000 Pa
Resultant force exerted by the oil on the bend = ?

Applying Bernoulli's equation between 1 and 2 ($Z_1 = Z_2$)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{100000}{1000x9.81} + \frac{0.95^2}{2x9.81} = \frac{P_2}{1000x9.81} + \frac{3.82^2}{2x9.81}$$

$P_2 = 93155\ N/m^2$

Resultant force (F_R) =?

Direction of resultant force =?

$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ in\ X\ direction$

$(P_1\ A_1 + P_2 Cos\theta\ A_2) - F_x = \rho Q(V_{2x} - V_{1x})$

$(P_1\ A_1 + P_2 A_2 Cos\theta) - F_x = \rho Q(-V_2 Cos\theta - V_1)$

$F_x = (P_1\ A_1 + P_2 A_2 Cos\theta) + \rho Q(V_1 + V_2 Cos\theta)$

$= (100000x0.0314 + 93155x0.00785 Cos50) + 1000x0.03(0.95 + 3.82 Cos50)$

= 3712 N

$\sum Forces\ in\ Y\ direction = Rate\ of\ change\ of\ momentum\ in\ Y\ direction$

$F_y - P_2 Sin\theta\ A_2 = \rho Q\left(V_{2y} - V_{1y}\right)$

$F_y - P_2\ A_2 Sin\theta = \rho Q(V_2 Sin\theta - 0)$

$F_y = P_2\ A_2 Sin\theta + \rho Q V_2 Sin\theta$

$= 93155x0.00785 Sin50 + 1000x0.03x3.82 Sin50$

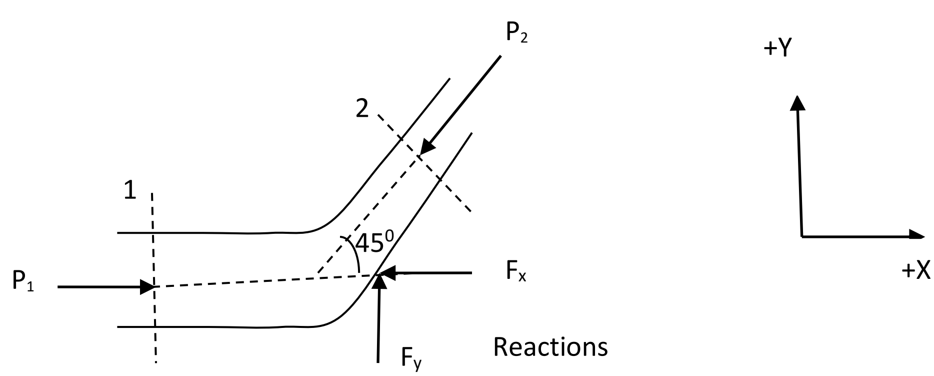
= 648N

Resultant force $(F_R) = \sqrt{F_x^2 + F_y^2}$ =3768N

Resultant force exerted by the water on the bend = 3768N (to the right and downward)

Direction of resultant force = $Tan^{-1}\frac{F_y}{F_x}$ = $Tan^{-1}\frac{648}{3712}$ =10^0

4. A 45^0 pipe bend tapers from 600mm diameter at inlet to 300mm diameter at outlet. The pressure at inlet is 140KN/m^2 and the rate of flow is 0.425m^3/s. At outlet the pressure is 123KN/m^2 gauge. Neglecting friction, calculate the resultant force exerted by the water on the bend in magnitude and direction. The bend lies in a horizontal plane.



Solution:

Diameter at section 1 (d_1) = 600mm = 0.6m

Area at section 1 (A_1) = $\frac{\pi}{4}x0.6^2$= 0.2827m^2

Diameter at section 2(d_2) = 300mm = 0.3m

Area at section 2 (A_2) = $\frac{\pi}{4}x0.3^2$= 0.07068m^2

Discharge (Q) = 0.425 m^3/s

Velocity at section 1 (V_1) =Q/A_1 = 1.5 m/s

Velocity at section 2 (V_2) =Q/A_2 = 6.01 m/s

Pressure at section 1 (P_1) = 140KN/m^2 = 140000N/m^2

Pressure at section 2 (P_2) =123KN/m^2 =123000N/m^2

Angle of bend (θ)= 45^0

Resultant force (F_R) =?

Direction of resultant force =?

$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ \ in\ X\ direction$

$(P_1\ A_1 - P_2 Cos\theta\ A_2) - F_x = \rho Q(V_{2x} - V_{1x})$

$(P_1\ A_1 - P_2 A_2 Cos\theta) - F_x = \rho Q(V_2 Cos\theta - V_1)$

$F_x = (P_1\ A_1 - P_2 A_2 Cos\theta) + \rho Q(V_1 - V_2 Cos\theta)$

$= (140000x0.2827 - 123000x0.07068Cos45) + 1000x0.425(1.5 - 6.01Cos45)$

= 32262 N

$\sum Forces\ in\ Y\ direction = Rate\ of\ change\ of\ momentum\ \ in\ Y\ direction$

$F_y - P_2 Sin\theta\ A_2 = \rho Q\left(V_{2y} - V_{1y}\right)$

$F_y - P_2\ A_2 Sin\theta = \rho Q(V_2 Sin\theta - 0)$

$F_y = P_2\ A_2 Sin\theta + \rho Q V_2 Sin\theta$

$= 123000x0.07068Sin45 + 1000x0.425x6.01Sin45$

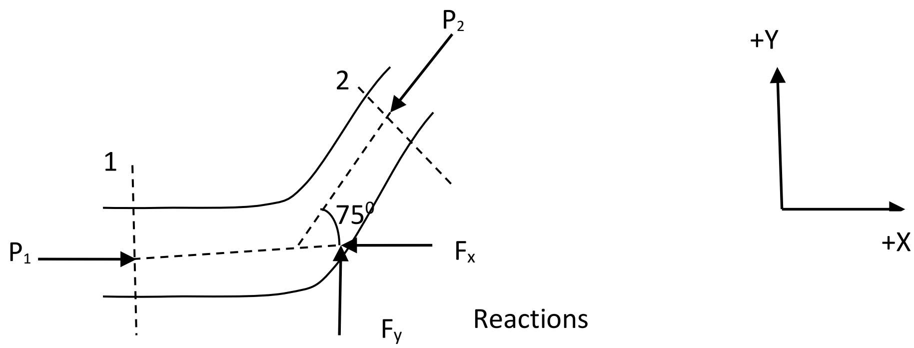
= 7953N

Resultant force $(F_R) = \sqrt{F_{x+}^2 F_y^2}$ =33228N

Resultant force exerted by the water on the bend = 33228N (to the right and downward)

Direction of resultant force = $Tan^{-1}\frac{F_y}{F_x}$ = $Tan^{-1}\frac{7953}{32262}$ =13.8^0

5. A 150mm diameter pipe on the horizontal plane carries water under the head of 16m of water with the velocity of 3.5 m/s. Find the direction and magnitude of the pipe bend, if the axis of the bend was turned with angle 75^0. Assume no loss of energy at the pipe bend.



Solution:

Diameter ($d_1 = d_2$) = 150mm = 0.15m

Area at section 1 and 2 (A_1=A_2) = $\frac{\pi}{4}x0.15^2$ = 0.01767m^2

Velocity (V_1=V_2) = 3.5 m/s

Discharge (Q) = A_1 V_1 = 0.01767x3.5 =0.0618 m3/s

Pressure head = $\frac{P}{\rho g}$ = 16 m

P =156960 N/m^2

Pressure (P=P_1=P_2) = 156960 N/m^2

Angle of bend (θ)= 75^0

Resultant force (F_R) =?

Direction of resultant force =?

$$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ in\ X\ direction$$
$$(P_1\ A_1 - P_2 Cos\theta\ A_2) - F_x = \rho Q(V_{2x} - V_{1x})$$
$$(P_1\ A_1 - P_2 A_2 Cos\theta) - F_x = \rho Q(V_2 Cos\theta - V_1)$$
$$F_x = (P_1\ A_1 - P_2 A_2 Cos\theta) + \rho Q(V_1 - V_2 Cos\theta)$$
$$= (156960x0.01767 - 156960x0.01767Cos75) + 1000x0.0618(3.5 - 3.5Cos75)$$

= 2216 N

$$\sum Forces\ in\ Y\ direction = Rate\ of\ change\ of\ momentum\ in\ Y\ direction$$
$$F_y - P_2 Sin\theta\ A_2 = \rho Q(V_{2y} - V_{1y})$$
$$F_y - P_2\ A_2 Sin\theta = \rho Q(V_2 Sin\theta - 0)$$
$$F_y = P_2\ A_2 Sin\theta + \rho Q V_2 Sin\theta$$
$$= 156960x0.01767Sin75 + 1000x0.0618x3.5Sin75$$

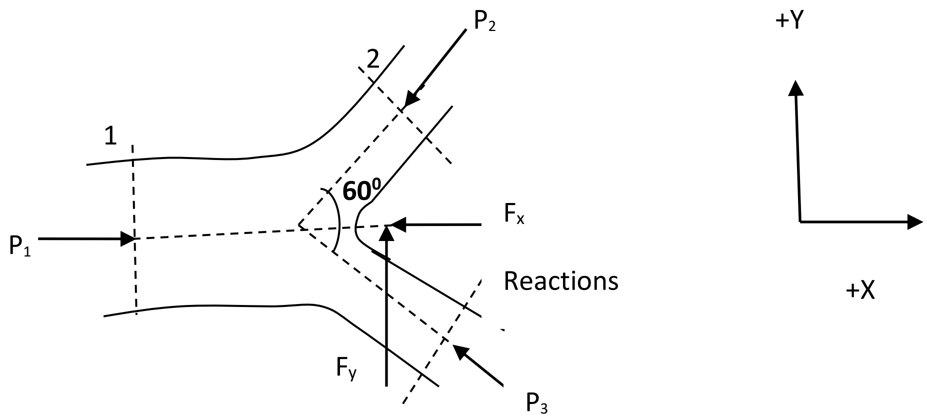
= 2888N

Resultant force $(F_R) = \sqrt{F_x^2 + F_y^2}$ =3640N

Resultant force exerted by the water on the bend = 3640N (to the right and downward)

Direction of resultant force = $Tan^{-1}\frac{F_y}{F_x}$ = $Tan^{-1}\frac{2888}{2216}$ = 52.5^0

6. A main pipe of diameter 500mm branches in two pipes of diameter 300mm each in the horizontal plane. Angle between the branches is 60^0, which is symmetrical with respect to the main pipe. Flow discharge through the main pipe is 1.0 m^3/s, which is equally divided into the branch pipes. If the pressure intensity at the main pipe is 400KPa, find the magnitude and direction of resultant force in the bend. Assume no loss of energy due to branch junction and in pipe sections.



Diameter at section 1 (d_1) = 500mm = 0.5m

Area at section 1 (A_1) = $\frac{\pi}{4}x0.5^2$= 0.19635m^2

Diameter at section 2 and 3 (d_2 =d_3) = 300mm = 0.3m

Area at section 2 and 3 (A2 =A3) = $\frac{\pi}{4}x0.3^2$ = 0.07068m^2

Angle of bend (θ)= 30^0 for pipe 2 and 3

Discharge through 1 (Q_1) = 1 m^3/s

Discharge through 2 and 3 (Q_2=Q_3) = $Q_1/2$ =0.5 m^3/s

Pressure at 1 (P1) = 400 KPa = 400000N/m^2

Resultant force (F_R) =?

Direction of resultant force =?

Velocity at 1 (V_1) = Q_1/A_1 = 5.09 m/s

Velocity at 2 (V_2) = Q_2/A_2 = 7.07 m/s

Velocity at 3 (V_3) = Q_3/A_3 = 7.07 m/s

Using Bernoulli's equation at 1 and 2 (Z_1=Z_2)

$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$

$\frac{400000}{1000x9.81} + \frac{5.09^2}{2x9.81} = \frac{P_2}{1000x9.81} + \frac{7.07^2}{2x9.81}$

P_2 = 387962 N/m^2

Using Bernoulli's equation at 1 and 3 (Z_1=Z_3)

$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + Z_3$

$\frac{400000}{1000x9.81} + \frac{5.09^2}{2x9.81} = \frac{P_3}{1000x9.81} + \frac{7.07^2}{2x9.81}$

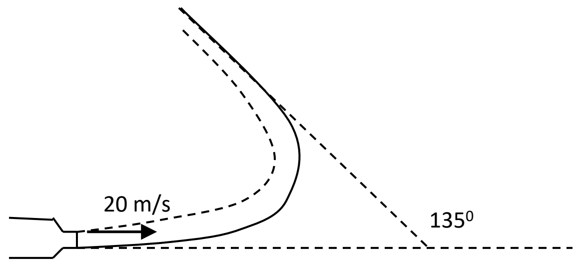
P_3 = 387962 N/m^2

$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ in\ X\ direction$
$(P_1\, A_1 - P_2 Cos\theta\, A_2 - P_3 Cos\theta\, A_3) - F_x = \rho\, [(Q_2\, V_{2x} + Q_3\, V_{3x}) - Q_1 V_{1x}]$
$(P_1\, A_1 - P_2 A_2 Cos\theta\ - P_3 A_3 Cos\theta\ ) - F_x = \rho\, [(Q_2\, V_2 Cos\theta + Q_3\, V_3 Cos\theta) - Q_1 V_1]$
$F_x = (P_1\, A_1 - P_2 A_2 Cos\theta\ - P_3 A_3 Cos\theta\ ) + \rho(Q_1 V_1 - Q_2\, V_2 Cos\theta - Q_3\, V_3 Cos\theta)$
$= (400000x0.19635 - 387962x0.07068Cos30 - 387962x0.07068Cos30) + 1000(1x5.09 - 0.5x7.07Cos30 - 0.5x7.07Cos30)$
= 30012N

$\sum Forces\ in\ Y\ direction = Rate\ of\ change\ of\ momentum\ in\ Y\ direction$
$F_y - P_2 Sin\theta\, A_2 + P_3 Sin\theta\, A_3 = \rho\, \left[\left(Q_2\, V_{2y} + Q_3\, V_{3y}\right) - Q_1 V_{1y}\right]$
$F_y - P_2 A_2 Sin\theta\ + P_3 A_3 Sin\theta\ = \rho\, [(Q_2\, V_2 Sin\theta + Q_3 x -\ V_3 Sin\theta) - 0]$
$F_y = (P_2 A_2 Sin\theta - P_3 A_3 Sin\theta) + \rho\, (Q_2\, V_2 Sin\theta - Q_3\, V_3 Sin\theta)$
$= 387962x0.07068Sin30 - 387962x0.07068Sin30 + 1000x0.5x7.07Sin30 - 1000x0.5x7.07Sin30$
= 0

Resultant force $(F_R) = \sqrt{F_{x+}^2 F_y^2}$ =30012N
Direction of resultant force = $Tan^{-1}\frac{F_y}{F_x}$ = $Tan^{-1}\frac{0}{30012}$ = 0^0

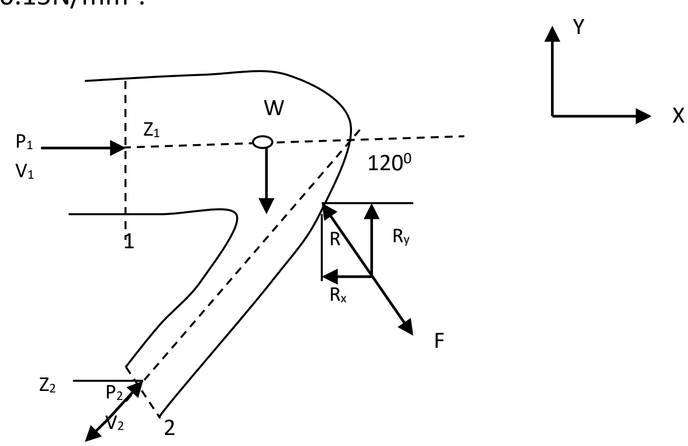
7. Determine the magnitude of resultant force and its direction on the vane shown in the figure below if a water of jet 50mm diameter and 20m/s velocity strikes the vane tangentially and deflects without friction.



Solution:
Velocity (V) = 20m/s
Diameter of jet (d) = 50mm = 0.05m
C/s of jet (A) = $\frac{\pi}{4}x0.05^2$ 0.001963 m^2
Discharge (Q) = AV = 0.001963x20 = 0.03927 m^3/s
No friction: No loss of head
Pressure is atmospheric: so no pressure force
V is constant throughout.

$F_x = \rho Q(V_{1x} - V_{2x}) = 1000x0.03927(20 - (-20cos45))$ = 1340.8 N
$F_y = \rho Q(V_{1y} - V_{2y}) = 1000x0.03927(0 - 20sin45)$ = 555.4 N
Resultant force = $\sqrt{F_x^2 + F_y^2} = \sqrt{1340.8^2 + 555.4^2}$ = 1451N
Direction = $tan^{-1}\frac{F_y}{F_x} = tan^{-1}\frac{555.4}{1451}$ = 22.5^0

8. The diameter of a pipe bend is 30cm at inlet and 15cm at outlet and the flow is turned through 120^0 in a vertical plane. The axis at inlet is horizontal and the center of the outlet section is 1.5m below the center of the inlet section. Total volume of water in the bend is 0.9m^3. Neglecting friction, calculate the magnitude and direction of the force exerted on the bend by water flowing through it at 250lps and when the inlet pressure is 0.15N/mm^2.



Solution:

Diameter at section 1 (d_1) = 30cm = 0.3m

Area at section 1 (A_1) = $\frac{\pi}{4}x0.3^2$= 0.07068m^2

Diameter at section 2(d_2) = 15cm = 0.15m

Area at section 2 (A_2) = $\frac{\pi}{4}x0.15^2$= 0.01767m^2

Discharge (Q) = 250 lps = 0.25 m^3/s

Volume of water within control volume (Vol) = 0.9m^3

Weight of water within control volume (W) = $\gamma_{water}Vol$ = 9810x0.9 = 8829N

Velocity at section 1 (V_1) =Q/A_1 = 0.25/0.07068 = 3.54 m/s

Velocity at section 2 (V_2) =Q/A_2 = 0.25/0.01767=14.15 m/s

Z_2 = 0, Z_1 = 1.5m

θ= 180-120 = 60^0

Pressure at 1 (P_1) = 0.15N/mm^2 = 0.15x10^6 N/m^2

Resultant force exerted by the water on the bend = ?

Applying Bernoulli's equation between 1 and 2

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

$$\frac{0.15x10^6}{1000x9.81}+\frac{3.54^2}{2x9.81}+1.5=\frac{P_2}{1000x9.81}+\frac{14.15^2}{2x9.81}+0$$

P_2 = 70870 N/m^2

$$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ \ in\ X\ direction$$

$$(P_1\,A_1+P_2 Cos\theta\,A_2)-R_x=\rho Q(V_{2x}-V_{1x})$$

$$(P_1\,A_1+P_2A_2Cos\theta)-R_x=\rho Q(-V_2Cos\theta-V_1)$$

$$R_x=(P_1\,A_1+P_2A_2Cos\theta)+\rho Q(V_1+V_2Cos\theta)$$

$$=(0.15x10^6x0.07068+70870x0.01767Cos60)+1000x0.25(3.54+14.15Cos60)$$

= 13882 N

$$\sum Forces\ in\ Y\ direction = Rate\ of\ change\ of\ momentum\ \ in\ Y\ direction$$

$$R_y-P_2Sin\theta\,A_2-W=\rho Q\left(V_{2y}-V_{1y}\right)$$

$$R_y-P_2\,A_2Sin\theta-W=\rho Q(-V_2Sin\theta-0)$$

$$R_y=P_2\,A_2Sin\theta-\rho QV_2Sin\theta+W$$

$$=70870x0.01767Sin60-1000x0.25x14.15Sin60+8829$$

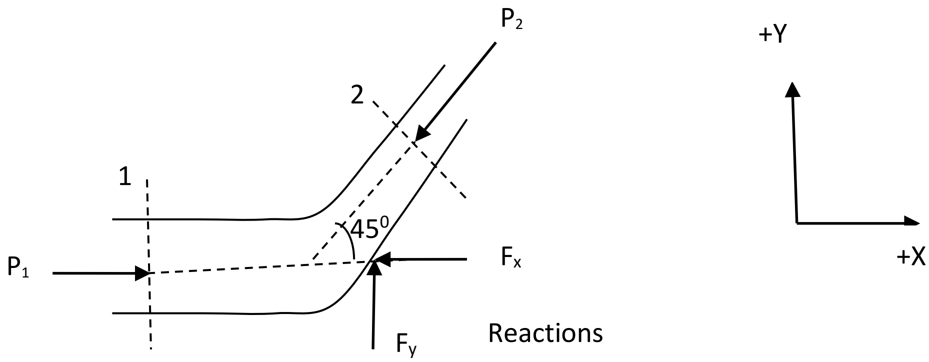
= 6850N

Resultant force $(R)=\sqrt{R_{x+}^2R_y^2}$ =15481N

Resultant force exerted by the water on the bend = 15481N (to the right and downward)

Direction of resultant force = $Tan^{-1}\frac{R_y}{R_x}$ = $Tan^{-1}\frac{6850}{13882}$ =26.3^0

9. A 45^0 reducing bend is connected in a pipe line carrying water. The diameter at inlet and outlet of the bend is 400mm and 200mm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet of the bend is 215.8KN/m^2. The rate of flow of water is 0.5m^3/s. The loss of head in the bend is 1.25m of oil of sp.gr. 0.85.



Solution:
Diameter at section 1 (d_1) = 400mm = 0.6m
Area at section 1 (A_1) = $\frac{\pi}{4}x0.4^2$= 0.1256m^2
Diameter at section 2(d_2) = 200mm = 0.2m
Area at section 2 (A_2) = $\frac{\pi}{4}x0.2^2$= 0.0314m^2
Discharge (Q) = 0.5 m^3/s

Velocity at section 1 (V_1) =Q/A_1 = 3.98 m/s
Velocity at section 2 (V_2) =Q/A_2 = 15.92 m/s
Pressure at section 1 (P_1) = 215.8KN/m^2 = 215800N/m^2
Loss of head = 1.25m of oil of sp gr 0.85, P= 0.85x9810x1.25 N/m^2
Loss of head in terms of water (h_L) = $\frac{P}{\gamma} = \frac{0.85x9810x1.25}{9810}$ =1.0625m
Angle of bend (θ)= 45^0
Resultant force (F_R) =?
Direction of resultant force =?

Applying Bernoulli's equation between 1 and 2 ($Z_1 = Z_2$)

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2+h_L$$

$$\frac{215800}{1000x9.81}+\frac{3.98^2}{2x9.81}=\frac{P_2}{1000x9.81}+\frac{15.92^2}{2x9.81}+1.0625$$

P_2 = 86574N/m^2

$$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ in\ X\ direction$$

$$(P_1\,A_1-P_2Cos\theta\,A_2)-F_x=\rho Q(V_{2x}-V_{1x})$$

$(P_1 A_1 - P_2 A_2 Cos\theta) - F_x = \rho Q(V_2 Cos\theta - V_1)$
$F_x = (P_1 A_1 - P_2 A_2 Cos\theta) + \rho Q(V_1 - V_2 Cos\theta)$
$= (215800x0.1256 - 86574x0.0314Cos45) + 1000x0.5(3.98 - 15.92Cos45)$
= 21544N

$\sum Forces\ in\ Y\ direction = Rate\ of\ change\ of\ momentum\ in\ Y\ direction$
$F_y - P_2 Sin\theta\ A_2 = \rho Q(V_{2y} - V_{1y})$
$F_y - P_2\ A_2 Sin\theta = \rho Q(V_2 Sin\theta - 0)$
$F_y = P_2\ A_2 Sin\theta + \rho Q V_2 Sin\theta$
$= 86574x0.0314Sin45 + 1000x0.5x15.92Sin45$
= 7551N

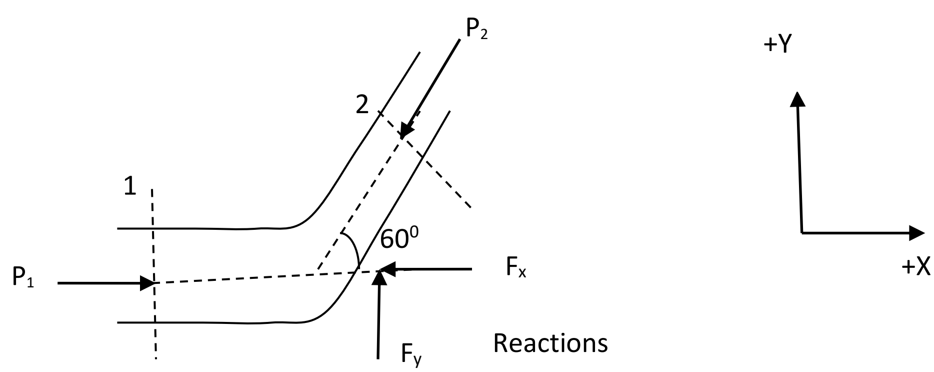
Resultant force $(F_R) = \sqrt{F_x^2 + F_y^2}$ =22829N

Resultant force exerted by the water on the bend = 22829N (to the right and downward)

Direction of resultant force = $Tan^{-1}\frac{F_y}{F_x} = Tan^{-1}\frac{7551}{21544}$ =19.3^0

10. The angle of a reducing bend is 60^0. Its initial diameter is 300mm and final diameter is 150mm and is lifted in a pipeline carrying water at a rate of 330 lps. The pressure at the commencement of the bend is 3.1 bar. The friction loss in the pipe may be assumed as 10% of kinetic energy at the exit of the bend. Determine the force exerted by the reducing bend.

Solution:



Solution:

Diameter at section 1 (d_1) = 300mm = 0.3m

Area at section 1 (A_1) = $\frac{\pi}{4}x0.3^2$= 0.07068m^2

Diameter at section 2(d_2) = 150mm = 0.15m

Area at section 2 (A_2) = $\frac{\pi}{4}x0.15^2$= 0.01767m^2

Discharge (Q) = 330lps = 0.33 m^3/s

Velocity at section 1 (V_1) =Q/A_1 = 4.67 m/s
Velocity at section 2 (V_2) =Q/A_2 = 18.67 m/s
Pressure at section 1 (P_1) = 3.1 bar = $3.1x10^5$ N/m^2
Loss of head (h_L) = 10% of velocity head at 2 = $0.1\frac{V_2^2}{2g} = 0.1\frac{18.67^2}{2x9.81}$ = 1.77m
Angle of bend (θ)= 50^0
Resultant force (F_R) =?
Direction of resultant force =?
Finding pressure at section 2 (P_2)
Using Bernoulli's equation at 1 and 2 (Z_1=Z_2)

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2+h_L$$

$$\frac{3.1x10^5}{1000x9.81}+\frac{4.67^2}{2x9.81}=\frac{P_2}{1000x9.81}+\frac{18.67^2}{2x9.81}+1.77$$

P_2 = 129256 N/m^2

$$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ in\ X\ direction$$
$$(P_1\ A_1 - P_2 Cos\theta\ A_2) - F_x = \rho Q(V_{2x} - V_{1x})$$
$$(P_1\ A_1 - P_2 A_2 Cos\theta) - F_x = \rho Q(V_2 Cos\theta - V_1)$$
$$F_x = (P_1\ A_1 - P_2 A_2 Cos\theta) + \rho Q(V_1 - V_2 Cos\theta)$$
$$= (310000x0.07068 - 129256x0.01767Cos60) + 1000x0.33(4.67 - 18.67Cos60)$$
= 19229 N

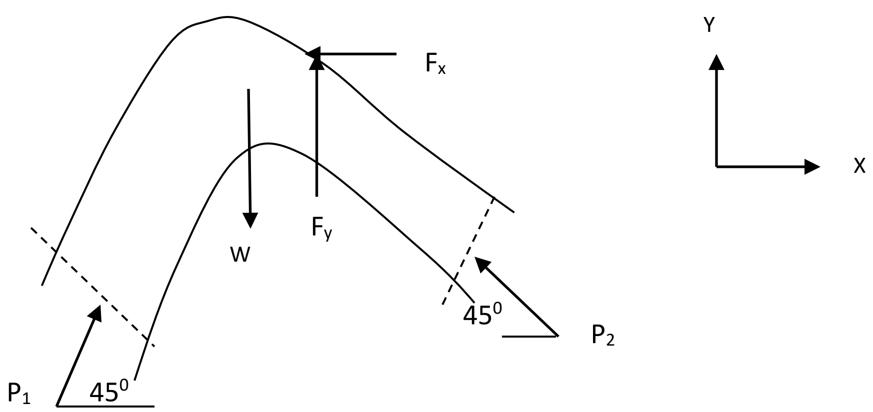
$$\sum Forces\ in\ Y\ direction = Rate\ of\ change\ of\ momentum\ in\ Y\ direction$$
$$F_y - P_2 Sin\theta\ A_2 = \rho Q\left(V_{2y} - V_{1y}\right)$$
$$F_y - P_2\ A_2 Sin\theta = \rho Q(V_2 Sin\theta - 0)$$
$$F_y = P_2\ A_2 Sin\theta + \rho Q V_2 Sin\theta$$
$$= 129256x0.01767Sin60 + 1000x0.33x18.67Sin60$$
= 7314N

Resultant force (F_R) = $\sqrt{F_x^2 + F_y^2}$ =20573N

Resultant force exerted by the water on the bend = 20573N (to the right and downward)

Direction of resultant force = $Tan^{-1}\frac{F_y}{F_x}$ = $Tan^{-1}\frac{7314}{19229}$ =20.8^0

11. A 0.4mx0.3m, 90^0 vertical bend carries $0.6m^3/s$ oil of sp gr 0.8 with a pressure of 120 Kpa at inlet to the bend. The volume of the bend is 0.1 m3. Find the magnitude and direction of the force on the bend. Neglect friction and assume both inlet and outlet sections to be at same horizontal level. Also assume that water enters the bend at 45^0 to the horizontal.



Solution:

Diameter at section 2(d_1) = 0.4m

Area at section 2 (A_1) = $\frac{\pi}{4}x0.4^2$= $0.1256m^2$

Diameter at section 1 (d_2) = 0.3m

Area at section 1 (A_2) = $\frac{\pi}{4}x0.3^2$= $0.07068m^2$

Discharge (Q) = $0.6\ m^3/s$

Weight of oil (W) = $\gamma_{oil}\ Vol$ = 0.8x9810x0.1 = 784.8N

Velocity at section 1 (V_1) =Q/A_1 = 4.8 m/s

Velocity at section 2 (V_2) =Q/A_2 = 8.5 m/s

Pressure at section 1 (P_1) = 120 Kpa

θ = 45^0

Resultant force (F_R) =?

Direction of resultant force =?

Finding pressure at section 2 (P_2)

Using Bernoulli's equation at 1 and 2 (Z_1=Z_2)

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

$$\frac{120000}{0.8x1000x9.81}+\frac{4.8^2}{2x9.81}=\frac{P_2}{0.8x1000x9.81}+\frac{8.5^2}{2x9.81}$$

P_2 = 100316 N/m^2

$$\sum Forces\ in\ X\ direction = Rate\ of\ change\ of\ momentum\ in\ X\ direction$$

$(P_1 Cos\theta\ A_1 - P_2 Cos\theta\ A_2) - F_x = \rho Q(V_{2x} - V_{1x})$
$(P_1\ A_1 - P_2 A_2 Cos\theta) - F_x = \rho Q(V_2 Cos\theta - V_1 Cos\theta)$
$F_x = (P_1\ A_1 Cos\theta - P_2 A_2 Cos\theta) + \rho Q(V_1 Cos\theta - V_2 Cos\theta)$
$= (120000x0.1256Cos45 - 100316x0.07068Cos45) + 0.8x1000x0.6(4.8Cos45 - 8.5Cos45)$
= 4388 N

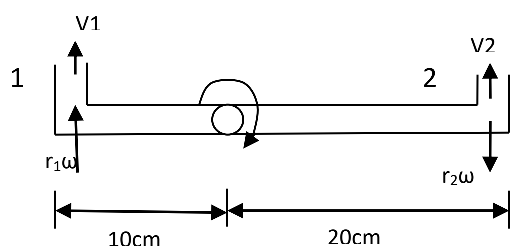
$\sum Forces\ in\ Y\ direction = Rate\ of\ change\ of\ momentum\ in\ Y\ direction$
$F_y + (P_1 Sin\theta\ A_1 + P_2 Sin\theta\ A_2) - W = \rho Q\left(V_{2y} - V_{1y}\right)$
$F_y + (P_1 A_1 Sin\theta\ + P_2 A_2 Sin\theta\ ) - W = \rho Q(V_2 Sin\theta - V_1 Sin\theta)$
$F_y = -(P_1 A_1 Sin\theta\ + P_2 A_2 Sin\theta\ ) + W + \rho Q(V_2 Sin\theta - V_1 Sin\theta)$
$= -(120000x0.1256Sin45 + 100316x0.07068Sin45) + 784.8 + 0.8x1000x0.6(4.8Sin45 - 8.5Sin45)$
= -6115N

Resultant force $(F_R) = \sqrt{F_x^2 + F_y^2}$ =7526N

Resultant force exerted by the water on the bend = 7526N (to the right and upward)

Direction of resultant force = $Tan^{-1}\frac{F_y}{F_x}$ = $Tan^{-1}\frac{6115}{4388}$ =54^0

12. The lawn sprinkler shown below has nozzles of 5mm diameter and carries a total discharge of 0.20 lps. Determine the angular speed of rotation of the sprinkler and torque required to hold the sprinkler stationary. Assume no friction at the pivot.



Solution:
Diameter of nozzle (d) = 5mm =0.005m
Area of nozzle (A) = $\frac{\pi}{4}x0.005^2$ = $1.963x10^{-5}$ m^2
r_1 = 10cm =0.1m
r_2 = 20cm =0.2m
Total discharge = 0.2 lps
Discharge through each nozzle (Q) = 0.2/2 = 0.1 lps = 0.0001 m^3/s
Relative velocity at outlet of each nozzle ($V_1 = V_2$) = Q/A = 5.09 m/s
For torque(T) = 0, Angular speed of rotation (ω)=?
For ω =0, Torque (T) =?

a. Initial moment of momentum of fluid entering the sprinkler is zero. So torque exerted is equal to the final moment of momentum. As no external torque acts (no friction), final moment of momentum should also be zero.

Jet exerts force in opposite direction at nozzle 1 and 2 (downward direction).
Torque at 1: anticlockwise, torque at 2: clockwise.
As the torque arm for 2 is greater, the sprinkler will rotate clockwise if free to rotate.

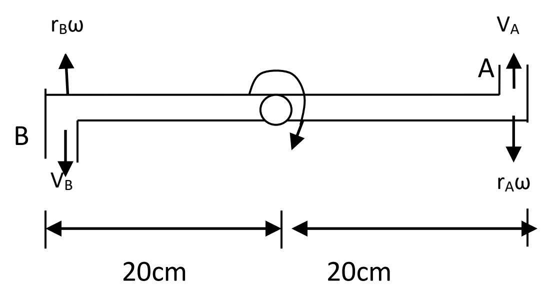
Absolute velocity at 1 (V_{1a}) = 5.09 + $r_1\,\omega$ = = 5.09 + 0.1 ω (tangential velocity and relative velocity in the same direction)
Absolute velocity at 1 (V_{2a}) = 5.09 - $r_2\,\omega$ = 5.09 - 0.2 ω (tangential velocity and relative velocity in opposite direction)

Final moment of momentum = $\rho Q V_{2a}\, r_2 - \rho Q V_{1a}\, r_1 = 0$
(Two torques in opposite direction, net torque = greater torque-smaller torque)
$V_{1a}\, r_1 = V_{2a}\, r_2$
(5.09 + 0.1 ω)0.1=(5.09 - 0.2 ω)0.2
ω = 10.18rad/s

$$\omega = \frac{2N\pi}{60}$$
N = 98 rpm

b. For ω =0, velocities are V_1 and V_2.
Torque exerted by the water on sprinkler = $\rho Q V_2 r_2 - \rho Q V_1 r_1$
= 1000x0.0001x5.09x0.2-1000x0.0001x5.09x0.1
= 0.0509 Nm
Torque required to hold the sprinkler stationary = 0.0509 Nm

13. A lawn sprinkler shown in the figure has 0.8cm diameter nozzle at the end of a rotating arm and discharges water at the rate of 12m/s. Determine the torque required to hold the rotating arm stationary. Also determine the constant speed of rotation of the arm, if free to rotate.



Solution:
Diameter of nozzle (d) = 0.8cm =0.008m
Area of nozzle (A) = $\frac{\pi}{4}x0.008^2$= 5.026x10^{-5} m2
$r_a = r_b$= 20cm =0.2m
Relative velocity at A and B ($V=V_A = V_B$) = 12 m/s
Discharge through each nozzle (Q) = A V = 0.000603 m^3/s
For angular velocity (ω) =0, Torque required to hold the rotating arm stationary (T) =?
For torque (T) = 0, constant speed of rotation of the arm (N) =?

a. Jet exerts force in opposite direction at nozzle A and B (upward at A and downward at B).
Torque at A and B: both clockwise
(Two torques in same direction, net torque = sum of two torques)
For ω =0, velocities are V_A and V_B.

Torque exerted by the water on sprinkler = $\rho Q V_A\, r_A + \rho Q V_B\, r_B$
= 1000x0.000603x12x0.2+1000x0.000603x12x0.2
= 2.89 Nm
Torque required to hold the rotating arm stationary = 2.89 Nm

b. Initial moment of momentum of fluid entering the sprinkler is zero. So torque exerted is equal to the final moment of momentum. As no external torque acts (no friction), final moment of momentum should also be zero.

Absolute velocity at A (V_{1a}) = 12- $r_a\,\omega$ = 12 - 0.2 ω (tangential velocity and relative velocity in opposite direction)
Absolute velocity at B (V_{2a}) = 12- $r_b\,\omega$ = 12 - 0.2 ω (tangential velocity and relative velocity in opposite direction)
Final moment of momentum = $\rho Q V_{1a}\, r_a + \rho Q V_{2a}\, r_b = 0$

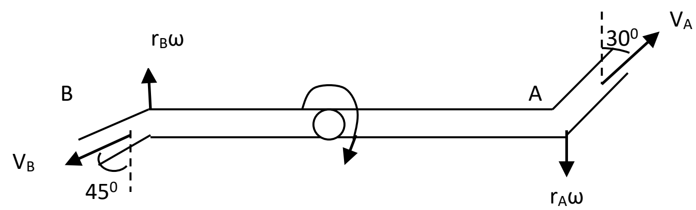
$$V_{1a}\, r_a = -V_{2a}\, r_b$$

(12 - 0.2 ω)0.2 = -(12 - 0.2 ω)0.2
ω = 60 rad/s

$$\omega = \frac{2N\pi}{60}$$

N = 573 rpm

14. A lawn sprinkler with two nozzles 5mm in diameter each at 0.2m and 0.15m radii is connected across a tap capable of discharging 6 litres/min. The nozzles discharge water upwards and outwards from the plane of rotation. What torque will sprinkler exert on the hand if held stationary, and at what angular velocity will it rotate free?



Solution:
Diameter of nozzle (d) = 5mm =0.005m
Area of nozzle (A) = $\frac{\pi}{4}x0.005^2$= $1.9635\text{x}10^{-5}$ m2
r_A= 0.2m, r_B= 0.15m
Assuming discharge to be equally divided,
Discharge ($Q_A = Q_B$) = 6/2 litres/min = 3/(1000x60) m^3/s = 0.00005 m^3/s
Relative velocity at A (V_A) = Q_A/A = 0.00005/$1.9635\text{x}10^{-5}$ = 2.54m/s
Relative velocity at B (V_B) = Q_B/A = 0.00005/$1.9635\text{x}10^{-5}$ = 2.54m/s

Vertical component of relative velocity at A (V_{YA}) = 2.54Cos30 = 2.2m/s
Vertical component of relative velocity at B (V_{YB}) = 2.54Cos45 = 1.8m/s

For angular velocity (ω) =0, Torque required to hold the rotating arm stationary (T) =?
For torque (T) = 0, constant speed of rotation of the arm (N) =?

a. Jet exerts force in opposite direction at nozzle A and B (downward at A and upward at B).
Torque at A and B: both clockwise
(Two torques in same direction, net torque = sum of two torques)
For ω =0, velocities are V_{YA} and V_{YB}.

Torque exerted by the water on sprinkler = $\rho Q V_{YA}\, r_A + \rho Q V_{YB}\, r_B$
= 1000x0.00005x2.2x0.2+1000x0.00005x1.8x0.15
= 0.0355 Nm
Torque required to hold the rotating arm stationary = 0.0355 Nm

b. Initial moment of momentum of fluid entering the sprinkler is zero. So torque exerted is equal to the final moment of momentum. As no external torque acts (no friction), final moment of momentum should also be zero.

Absolute velocity at A (V_{1a}) = V_{YA}- $r_A\omega$ = 2.2 - 0.2 ω (tangential velocity and relative velocity in opposite direction)
Absolute velocity at B (V_{2a}) = V_{YB}- $r_A\omega$ = 1.8 - 0.15 ω (tangential velocity and relative velocity in opposite direction)
Final moment of momentum = $\rho Q V_{1a}\, r_A + \rho Q V_{2a}\, r_B = 0$

$$V_{1a}\, r_a = -V_{2a}\, r_b$$

(2.2 - 0.2 ω)0.2 = -(1.8 - 0.15 ω)0.15

ω = 11.36 rad/s

$$\omega = \frac{2N\pi}{60}$$

N =109rpm

15. A flat plate is struck normally by a jet of water 50mm in diameter with a velocity of 18m/s. Calculate: a) the force on the plate when it is stationary, b) the force on the plate when it moves in the same direction as the jet with a velocity of 6m/s, and c) the work done per sec and the efficiency in case (b).

Solution:
Diameter of jet (d) = 50mm =0.05m
Area of jet (A) = $\frac{\pi}{4}x0.05^2$ = 0.001963m^2
Velocity of jet (V) = 18 m/s

a. Force exerted by the jet on the plate $(F) = \rho AV^2$
$= 1000x0.001963x18^2$ = 636N

b. Velocity of plate (u) = 6m/s
Force exerted by the jet on the plate when the plate is moving $(Fp) = \rho A(V-u)^2$
$= 1000x0.001963x(18-6)^2$ = 283N

c. Work done per/sec (W) = Fp x distance/time = Fp x u = 283 x 6 = 1698J
Kinetic energy of jet (KE)/sec = $\frac{1}{2}(\rho AV)V^2 = \frac{1}{2}\rho AV^3$=$\frac{1}{2}x1000x0.001963x18^3$ = 5724 J
Efficiency = W/KE = 1698/5724= 0.3 = 30%

16. A jet of water 60 mm in diameter with a velocity of 15m/s strikes a flat plate inclined at an angle of 25^0 to the axis of the jet. Calculate the normal force exerted on the plate (a) when the plate is stationary, (b) when the plate is moving at 4.5 m/s in the direction of jet and (c) the work done per sec and the efficiency for case b.

Solution:
Diameter of jet (d) = 60mm =0.06m
Area of jet (A) = $\frac{\pi}{4}x0.06^2$ = 0.00283 m^2

Velocity of jet (V) = 15 m/s
Angle of inclination of the plate with the axis of jet (θ) =25^0

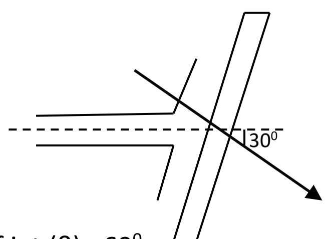
a. Normal force exerted on the plate $(F) = \rho AV^2 Sin\theta$
$= 1000x0.00283x15^2 Sin25$ = 269N

b. Velocity of plate (u) = 4.5m/s
Normal force exerted on the plate when the plate is moving $(Fp) = \rho A(V-u)^2 Sin\theta$
$= 1000x0.00283x(15-4.5)^2 Sin25$ = 132N

c. Work done per/sec (W) = Fp x distance/time = Fp x u = 132 x 4.5 = 594J
Kinetic energy of jet (KE)/sec = $\frac{1}{2}(\rho AV)V^2 = \frac{1}{2}\rho AV^3$=$\frac{1}{2}x1000x0.00283x15^3$ = 4776J
Efficiency = W/KE = 594/4776 = 0.12 = 12%

17. A 75mm diameter jet of water having a velocity of 25m/s strikes a flat plate, the normal of which is inclined at 30^0 to the jet. Find the force normal to the surface of the plate and in the direction of the jet.

Solution:
Diameter of jet (d) = 75mm =0.075m
Area of jet (A) = $\frac{\pi}{4}x0.075^2$ = 0.00442 m^2
Velocity of jet (V) = 25 m/s

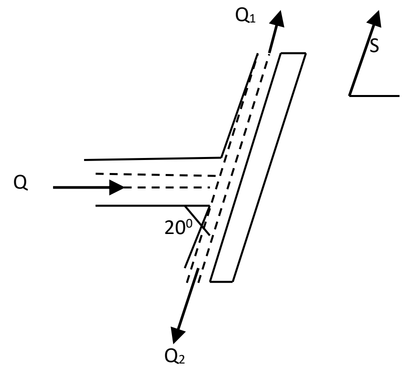


Angle made by normal with horizontal = 30^0

Angle of inclination of the plate with the axis of jet (θ) =60^0
Normal force (F_n) =?
Force in the direction of jet (F_x) = ?

$F_n = \rho AV^2 Sin\theta = 1000x0.00442x25^2 Sin60$ = 2392N
$F_x = F_n Cos30 = 2392 Cos30$ = 2071N

18. A jet of 20mm in diameter moving with a velocity of 5m/s strikes a smooth plate, which is inclined at an angle of 20^0 to the horizontal. Compute the amount of flow on each side of the plate and the force exerted on the plate.



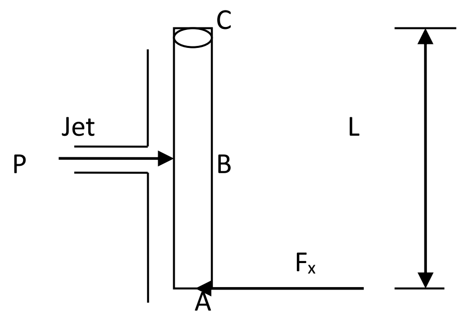
Solution:
Diameter of jet (d) = 20mm =0.02m
Area of jet (A) = $\frac{\pi}{4}x0.02^2$ = 0.000314 m^2
Velocity of jet (V) = 5 m/s

Angle of inclination of jet with horizontal (θ) = 20^0
Q = Flow through jet before striking plate
Q_1 and Q_2 = Flow on upper and lower side of plate
A_1 and A_2 = Area of jet on upper and lower side of plate
Q = A V = 0.000314x5 = 0.00157 m^3/s
Force exerted on plate(F_X) =?

Writing momentum equation in direction of S
$\rho A_1 V^2 - \rho A_2 V^2 - \rho A V^2 Cos\theta = 0$
With Q = A V, Q_1 =A_1 V, Q2 =A_2 V
$Q_1 - Q_2 = Qcos\theta$
From continuity, $Q_1 + Q_2 = Q$
Solving above two eq.,
$Q_1 = \frac{Q}{2}(1 + Cos\theta)$ and $Q_2 = \frac{Q}{2}(1 - Cos\theta)$
$Q_1 = \frac{0.00157}{2}(1 + Cos20)$ = 0.00152 m^3/s
$Q_2 = \frac{0.00157}{2}(1 - Cos20)$ = 0.00005 m^3/s

$F_n = \rho AV^2 Sin\theta = 1000x0.000314x5^2 Sin20$ = 2.7N
$F_x = F_n Cos(90 - 20) = 2.7\ Cos70$ = 0.92N

19. A flat plate hinged about its top edge, is suspended vertically. It weighs 8KN. A jet of water, 50mm in diameter strikes the plate normally at its mid-point with a velocity of 50m/s. (a) Determine the horizontal force that should be applied to it at the bottom edge to keep it vertical. (b) Determine the angle of deflection where it stays in equilibrium, if it is allowed to rotate about the hinge.



Solution:

Weight (W) =8KN = 8000N

Diameter of nozzle (d) = 50mm =0.05m

Area of nozzle (A) = $\frac{\pi}{4}x0.05^2$ = 0.001963m^2

Velocity of jet (V) = 50m/s

AB =BC =L/2

a. Horizontal force at bottom (F_x) =?

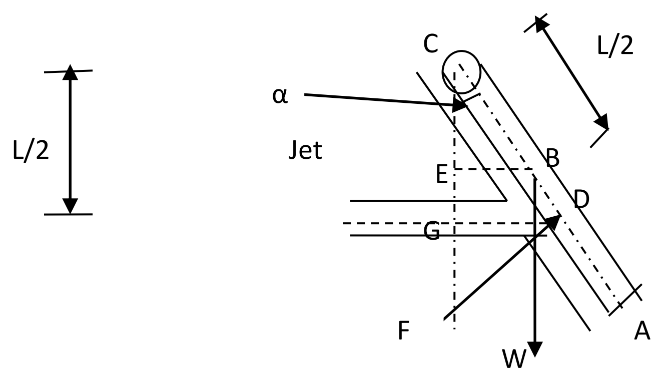
Force exerted by the jet on the plate $(P) = \rho AV^2$= $1000x0.001963x50^2$ = 4907N

Taking moment about hinge (C)

P L/2 =F_x L

F_x = P/2 = 2453.5 N

b. α = Angle of inclination of the plate with vertical



CB =L/2

As the jet strikes the plate at mid point and the plate deflects about hinge C, the vertical height from the center of the jet remains same as L/2 i.e. GC = L/2.

From ΔCGD, Perpendicular distance from C to force F = CD = CG/Cosα = L/2 $Sec\alpha$

From ΔCEB, Perpendicular distance from C to weight W = EB = CB $Sin\alpha$ = L/2 $Sin\alpha$

The axis of jet jet makes an angle (90-α) with the plate.
Force exerted by the jet normal to the plate $(F) = \rho AV^2 Sin(90-\alpha)$ = $\rho AV^2 Cos\alpha$
$= 1000x0.001963x50^2 Cos\alpha$ =$4907 Cos\alpha$

Taking moments about hinge C
$W\frac{L}{2}Sin\alpha = F\frac{L}{2}Sec\alpha$
$8000\ Sin\alpha = 4907\ Cos\alpha\ Sec\alpha$
α=37.8^0

Tutorial 10

Boundary layer theory

1. If the velocity distribution law in a laminar boundary layer over a flat plate is assumes to be of the form $u = ay + by^3$, determine the velocity distribution law.

Solution:
At y = 0, u= 0
At $y = \delta$, u = U
$U = a\delta + b\delta^3$ (a)
At $y = \delta, \frac{du}{dy} = 0$
$\frac{du}{dy} = a + 3by^2$
$a + 3b\delta^2 = 0$
$a = -3b\delta^2$ (b)
From a and b
$U = -3b\delta^3 + b\delta^3$
$b = -\frac{U}{2\delta^3}$
$a = \frac{3U}{2\delta}$
Hence the velocity distribution equation is
$u = \frac{3U}{2\delta}y - \frac{U}{2\delta^3}y^3$
$\frac{u}{U} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

2. A flat plate of 2m width and 4m length is kept parallel to air flowing at 5m/s. Determine the length of plate over which the boundary layer is laminar and shear stress at the location where boundary layer ceases to be laminar. Take ρ of air = 1.208 kg/m^3 and υ of air = 1.47x10^{-5} m^2/s.

Solution:
Width (b) = 2m
Length (L) = 4m
Velocity (U) = 5m/s
Reynold no. $(Re) = \frac{UL}{\upsilon} = \frac{5x4}{1.47x10^{-5}} = 1.361x10^6$
Re>5x10^5
On the front portion, the boundary layer is laminar and on the rear, it is turbulent.
$Re_x = \frac{Ux}{\upsilon} = 5x10^5$

$\frac{5x}{1.47x10^{-5}} = 5x10^5$
x = 1.47m
Up to 1.47m from the leading edge, the boundary layer is laminar.

$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5x1.47}{\sqrt{5x10^5}}$ = 0.01039
$C_f = \frac{0.664}{\sqrt{Re_x}} = \frac{0.664}{\sqrt{5x10^5}}$ = 0.000939
Shear stress $(\tau) = \frac{1}{2}C_f\rho U^2 = \frac{1}{2}0.000939x1.208x5^2$ = 0.01418 N/m²

3. For the velocity profile given below, compute the displacement thickness and momentum thickness:

$$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^2$$

Where U = free stream velocity, u = velocity in boundary layer at a distance y and $\delta =$ boundary layer thickness.

Solution:
$\frac{u}{U} = \frac{3y}{2\delta} - \frac{y^2}{2\delta^2}$
Displacement thickness (δ^*) = ?
Momentum thickness (θ) = ?

$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(1 - \frac{3y}{2\delta} + \frac{y^2}{2\delta^2}\right) dy$
$= \left| y - \frac{3y^2}{4\delta} + \frac{y^3}{6\delta^2} \right|_0^\delta$
$= \delta - \frac{3\delta}{4} + \frac{\delta}{6}$
$= \frac{5\delta}{12}$

$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(\frac{3y}{2\delta} - \frac{y^2}{2\delta^2}\right)\left(1 - \frac{3y}{2\delta} + \frac{y^2}{2\delta^2}\right) dy$
$= \int_0^\delta \left(\frac{3y}{2\delta} - \frac{9y^2}{4\delta^2} + \frac{3y^3}{4\delta^3} - \frac{y^2}{2\delta^2} + \frac{3y^3}{4\delta^3} - \frac{y^4}{4\delta^4}\right) dy$
$= \int_0^\delta \left(\frac{3y}{2\delta} - \frac{11y^2}{4\delta^2} + \frac{3y^3}{2\delta^3} - \frac{y^4}{4\delta^4}\right) dy$
$= \left| \frac{3y^2}{4\delta^2} - \frac{11y^3}{12\delta^3} + \frac{3y^4}{8\delta^3} - \frac{y^5}{20\delta^4} \right|_0^\delta$
$= \frac{3\delta}{4} - \frac{11\delta}{12} + \frac{3\delta}{8} - \frac{\delta}{20}$
$= \frac{19\delta}{120}$

Drag and lift

4. A flat plate 2mx2m moves at 40km/hr in stationary air of density 1.25kg/m^3. If the coefficient of drag and lift are 0.2 and 0.8 respectively, find the lift force, the drag force, the resultant force and the power required to keep the plate in motion.

Solution:

Area of plate (A) = 4m^2

Velocity (V) = 40 km/hr = $\frac{40x1000}{60x60}$ =11.11m/s

Density of air (ρ) = 1.25kg/m^3

Coefficient of drag (C_D) = 0.2

Coefficient of lift (C_L) = 0.8

Lift force (F_L) =?

Drag force (F_D) = ?

Resultant force (R) = ?

Power (P) =?

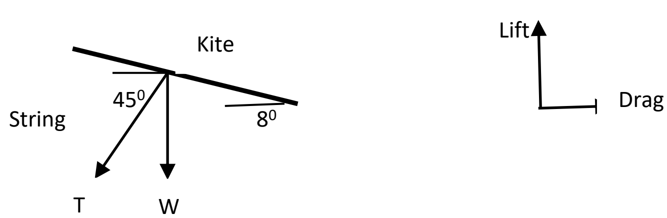
$F_L = \frac{1}{2} C_L \rho A V^2 = \frac{1}{2} x0.8x1.25x4x11.11^2$ = 246.86 N

$F_D = \frac{1}{2} C_D \rho A V^2 = \frac{1}{2} x0.2x1.25x4x11.11^2$ = 61.71N

$R = \sqrt{F_L^2 + F_D^2} = \sqrt{246.86^2 + 61.71^2}$ = 254.45N

P = F_D V = 61.71x11.11 = 685W

5. A kite weighs 0.9N and has an area of 7400 cm^2. The tension in the kite string is 3.3 N when the string makes an angle of 45^0 with the horizontal. For a wind of 30km/hr, what are the coefficients of lift and drag if the kite assumes an angle of 8^0 with the horizontal? Consider the kite essentially a flat plate and density of air = 1.2kg/m^3.



Solution:

Weight of kite (W) = 0.9N

Area of kite (A) = 7400 cm^2 = 0.74 m^2

Velocity (V) = 30 km/hr = $\frac{30x1000}{60x60}$ =8.33m/s

Density of air (ρ) = 1.2kg/m^3

Tension (T) = 3.3N

Coefficient of lift (C_L) = ?

Coefficient of drag (C_D) = ?

Forces in X-dir
F_D = 3.3cos45 = 2.33N
Force in Y-dir
F_L = 3.3sin45+0.9 = 3.23N

$F_L = \frac{1}{2} C_L \rho A V^2$
$3.23 = \frac{1}{2} x C_L x 1.2 x 0.74 x 8.33^2$
C_L = 0.104

$F_D = \frac{1}{2} C_D \rho A V^2$
$2.33 = \frac{1}{2} x C_D x 1.2 x 0.74 x 8.33^2$
C_D = 0.075

6. Calculate the weight of a ball of diameter 50mm which is just supported in a vertical air stream which is flowing at a velocity of 10 m/s. Take density of air = 1.25kg/m^3 and kinematic viscosity = 15 stokes.

Solution:
Diameter of ball (D) = 50mm = 0.05m
Area of ball (A) = $\frac{\pi}{4} x 0.05^2$ = 0.001963 m^2
Density of air (ρ) = 1.25kg/m^3
kinematic viscosity (ν) = 15 stokes = 15x10^{-4} m^2/s
Velocity (V) = 10m/s
Weight of ball = ?
Reynold no. $(Re) = \frac{VD}{\upsilon} = \frac{10x0.05}{15x10^{-4}}$ = 333
For Re between 5 to 1000, coefficient of drag (C_D) = 0.4
Weight of ball = Drag force
$= \frac{1}{2} C_D \rho A V^2 = \frac{1}{2} x 0.4 x 1.25 x 0.001963 x 10^2$ = 0.049N

7. A metallic sphere of sp.gr. 8.0 falls in an oil of density 800 kg/m^3. The diameter of the sphere is 10mm and it attains a terminal velocity of 50mm/s. Find the viscosity of the oil in Poise.

Solution:
Density of sphere (ρ_s) = 8x1000 = 8000 kg/m^3
Density of oil (ρ_o) = 800 kg/m^3
Diameter of sphere (D) = 0.01m
Terminal velocity (V) = 0.05m/s
Viscosity of oil (μ) = ?

Weight of sphere = Buoyant force on sphere + Drag force

$$\rho_s g x \frac{1}{6}\pi D^3 = \rho_0 g x \frac{1}{6}\pi D^3 + 3\pi\mu DV$$

$$8000x9.81x\frac{1}{6}\pi x0.01^3 = 800x9.81x\frac{1}{6}\pi x0.01^3 + 3\pi\mu x0.01x0.05$$

μ = 7.848 PaS = 78.48 Poise

The expression for drag force is valid for Re<0.2.

$Re = \frac{\rho VD}{\mu} = \frac{800x0.05x0.01}{7.848}$ = 0.05

8. A metallic ball of diameter 5mm drops in a fluid of sp.gr. 0.8 and viscosity 30 poise. The sp.gr. of metallic ball is 9.0. Find (a) the force exerted by the fluid on the ball, (b) the pressure drag and skin friction drag, and (c) the terminal velocity of the ball in the fluid.

Solution:

Diameter of ball (D) = 0.005m

Density of fluid (ρ) = 0.8x1000 = 800 kg/m^3

Viscosity of fluid (μ) = 30 poise = 30/10 = 3 PaS

Density of ball (ρ_b) = 9x1000 = 9000 kg/m^3

Drag force (F_D) = ?

Pressure drag and friction drag = ?

Terminal velocity (V) = ?

Weight of ball (W) = Drag force (F_D) + Buoyant force on ball (F_B)

$F_D = W - F_B$

$= \rho_b g x \frac{1}{6}\pi D^3 - \rho g x \frac{1}{6}\pi D^3 = 9000x9.81x\frac{1}{6}\pi x0.005^3 - 800x9.81x\frac{1}{6}\pi x0.005^3$ = 0.005265N

Pressure drag = $\frac{1}{3}F_D = \frac{1}{3}x0.005265$= 0.001755 N

Friction drag = $\frac{2}{3}F_D = \frac{2}{3}x0.005265$= 0.00351N

$$F_D = 3\pi\mu DV$$

$$0.005265 = 3\pi x3x0.005V$$

V = 0.0372 m/s

Checking the Reynold's no.

$Re = \frac{\rho VD}{\mu} = \frac{800x0.0372x0.005}{3}$ = 0.05

As Re<0.2, above expression for F_D is valid.

9. A jet plane which weighs 19620N has a wing area of 25m^2. It is flying at a speed of 200km/hr. When the engine develops 588.6KW, 70% of this power is used to overcome the drag resistance of the wing. Calculate the coefficient of lift and coefficient of drag for the wing. Take density of air = 1.25 kg/m^3.

Solution:

Weight of plane (W) = 19620N

Wing area (A) = 25m^2

Speed (V) = 200 km/hr = $\frac{200x1000}{3600}$ = 55.55m/s

Power = 588.6KW = 588600W

Power used to overcome drag resistance (P) = 0.7x588600 =412020W

Density of air (ρ) = 1.25 kg/m^3

Coefficient of drag (C_D) = ?

Coefficient of lift (C_L) = ?

P = F_D x V

412020 = F_D x 55.55

F_D = 7417.1 N

$F_D = \frac{1}{2} C_D \rho A V^2$

$7417.1 = \frac{1}{2} x C_D x 1.25 x 25 x 55.55^2$

C_D = 0.154

Lift force (F_L) = Weight of plane = 19620N

$F_L = \frac{1}{2} C_L \rho A V^2$

$19620 = \frac{1}{2} x C_L x 1.25 x 25 x 55.55^2$

C_L = 0.407

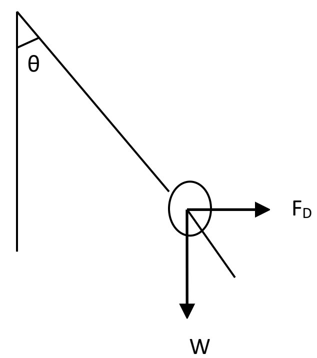
10. A 10mm ball of relative density 1.2 is suspended from a string, in air flowing at a velocity of 10m/s. Determine the angle which the string will make with the vertical. Take ρ of air = 1.208 kg/m^3 and viscosity of air = 1.85x10^{-5} Pa-S. Also compute the tension in the string.

Solution:

Diameter of ball (d) = 10mm

Velocity (V) = 10m/s

Angle(θ) = ?



Area of sphere (A) = $\frac{\pi}{4} x 0.01^2$ = 7.85x10^{-5} m^2

Weight of sphere (W) = $\gamma Vol = 1.2x9810x\frac{1}{6}x\pi x0.01^3$ = 0.006163 N

Reynold no. $(Re) = \frac{\rho V d}{\mu} = \frac{1.208x10x0.01}{1.85x10^{-5}} = 6530$

For Re between 1000-100000, C_D = 0.5

$F_D = \frac{1}{2}C_D\rho AV^2 = \frac{1}{2}0.5x1.208x7.85x10^{-5}x10^2$ = 0.002371

$tan\theta = \frac{F_D}{W} = \frac{0.002371}{0.006163}$

θ = 21^0

Tension in the string = $\sqrt{F_D^2 + W^2} = \sqrt{0.002371^2 + 0.006163^2}$ = 0.0066N

11. Determine the rate of deceleration that will be experienced by a blunt nosed projectile of drag coefficient 1.22 when it is moving horizontally at 1600 km/hr. The projectile has a diameter of 0.5m and weighs 3000N. Take ρ of air = 1.208 kg/m^3.

Solution:

Velocity (V) = 1600 km/hr = $\frac{1600x1000}{3600}$ = 444.44 m/s

Drag coeff. (C_D) = 1.22

Diameter of projectile (d) = 0.5m

Area of sphere (A) = $\frac{\pi}{4}x0.5^2$ = 0.1963 m^2

Weight (W) = 3000N

Deceleration (a) = ?

$F_D = \frac{1}{2}C_D\rho AV^2$

$F_D = -ma = -\frac{W}{g}a$

$-\frac{W}{g}a = \frac{1}{2}C_D\rho AV^2$

$-\frac{3000}{9.81}a = \frac{1}{2}x1.22x1.208x0.1963x444.44^2$

a = -93.4 m/s^2

12. An aeroplane weighing 22500N has a wing area of 22.5m^2 and span of 12m. What is the lift coefficient if it travels at 320 km/hr in the horizontal direction? Also compute the value of circulation and angle of attack measured from zero lift axis.

Solution:

Velocity (V) = 320 km/hr = $\frac{320x1000}{3600}$ = 88.89 m/s

Wing area (A) = 22.5m^2

Weight = 22500 N = Lift force (F_L)

Lift coefficient (C_L) =?

Angle of attack (θ) =?

Circulation(Γ) = ?

$F_L = \frac{1}{2}C_L\rho AV^2$

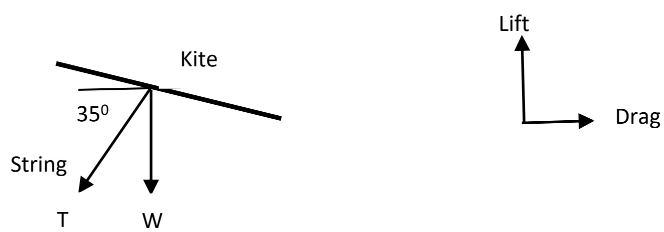
$22500 = \frac{1}{2}C_Lx1.208x22.5x88.89^2$

C_L = 0.2095

$C_L = 2\pi sin\theta$
$0.2095 = 2\pi sin\theta$
θ = 1.911^0
$\Gamma = \pi CV sin\theta = \pi x \frac{22.5}{12} x 88.89 sin 1.911$ = 17.46 m^2/s

13. A kite, which may be assumed to be a flat plate and mass 1kg, soars at an angle to the horizontal. The tension in the string holding the kite is 60N when the wind velocity is 50 km/h horizontally and the angle of string to the horizontal direction is 35^0. The density of air is 1.2 kg/m^3. Calculate the drag coefficient for the kite in the given position if the lift coefficient in the same position is 0.45. Both coefficients have been based on the full area of the kite.

Solution:



Solution:
Mass of kite = 1kg
Weight of kite (W) = 1x9.81 N = 9.81N
Area of kite = A
Velocity (V) = 50 km/hr = $\frac{50x1000}{60x60}$ =13.88/s
Density of air (ρ) = 1.2kg/m^3
Tension (T) = 60N
Coefficient of lift (C_L) = 0.45
Coefficient of drag (C_D) = ?

Forces in X-dir (Drag)
F_D = Tcos35 = 60cos35 =49.14N
Force in Y-dir (Lift)
F_L = Tsin35+9.81 = 60sin35+9.81=44.22N

$F_L = \frac{1}{2} C_L \rho A V^2$
$44.22 = \frac{1}{2} x 0.45 x 1.2 x A x 13.88^2$
A = 0.85 m^2

$F_D = \frac{1}{2} C_D \rho A V^2$

$49.14 = \frac{1}{2} x C_D x 1.2 x 0.85 x 13.88^2$

C_D = 0.5

14. A steel sphere of 5mm diameter falls in a glycerin at a terminal velocity of 0.05m/s. Assume Stoke's law is applicable, determine (a) dynamic viscosity of glycerin, (b) drag force and (c) coefficient of drag. Take sp wt of steel and glycerin as 75 KN/m^3 and 12.5 KN/m^3 respectively.

Solution:

Solution:

Diameter of ball (D) = 5mm = 0.005m

Sp wt of steel (γ_{steel}) = 75 KN/m^3

Sp wt of glycerin (γ_{gly}) = 12.5 KN/m^3

Terminal velocity (V)= 0.05m/s

Viscosity of fluid (μ) = ?

Drag force (F_D) = ?

Coefficient of drag (C_D) = ?

Weight of sphere (W) = Drag force (F_D) + Buoyant force on the sphere(F_B)

$F_D = W - F_B$

$= \gamma_{steel} x \frac{1}{6} \pi D^3 - \gamma_{gly} x \frac{1}{6} \pi D^3$

$= 75000 x \frac{1}{6} \pi x 0.005^3 - 12500 x \frac{1}{6} \pi x 0.005^3$ = 0.00409N

From Stoke's law

$$F_D = 3\pi\mu DV$$

$$0.00409 = 3\pi x \mu x 0.005 x 0.05$$

μ= 1.7NS/m^2

Reynold's no.

$Re = \frac{\rho VD}{\mu} = \frac{\left(\frac{12500}{9.81}\right) x 0.05 x 0.005}{1.7}$ = 0.18

As Re<0.2, Stoke's is valid.

$C_D = \frac{24}{Re} = \frac{24}{0.18}$ = 133

Dimensional and model analysis

15. Using Rayleigh's method, derive an expression for flow through orifice (Q) in terms of density of liquid (ρ), diameter of the orifice (D) and the pressure difference (P).

Solution:
$Q = f(\rho, D, P)$
$Q = K\rho^a D^b P^c$

Writing dimensions
$L^3T^{-1} = K(ML^{-3})^a(L)^b(ML^{-1}T^{-2})^c$
$L^3T^{-1} = KM^{a+c}L^{-3a+b-c}T^{-2c}$
Equating the powers of M, L and T
a+c = 0
-3a+b-c = 3
-2c = -1
Solving above equations
c = 1/2
a = -c= -1/2
b = 3a+c+3 = $-\frac{3}{2} + \frac{1}{2} + 3$ =2

Substituting values of a, b and c
$Q = K\rho^{-1/2}D^2P^{1/2} = KD^2\sqrt{P/\rho}$

16. Assuming the drag force exerted by a flowing fluid (F) is a function of the density (ρ), viscosity (μ), velocity of fluid (V) and a characteristics length of body (L), show by using Rayleigh's method that $F = C\rho A\frac{V^2}{2}$ where A is area and C is constant.

Solution:
$F = f(\rho, \mu, V, L)$
$F = K\,\rho^a\mu^bV^cL^d$
Writing dimensions
$MLT^{-2} = K(ML^{-3})^a(ML^{-1}T^{-1})^b(LT^{-1})^c(L)^d$
$MLT^{-2} = KM^{a+b}L^{-3a-b+c+d}T^{-b-c}$

Equating the powers of M, L and T
a+b = 1
-3a-b+c+d = 1
-b-c = -2

There are 3 equations and 4 unknowns. As we have to get expression in terms of ρ, V and L (Area), we can express the powers of these three variables i.e. a, c and d in terms of b.

a = 1-b
c = 2-b
Substituting a and c in second eqaution
-3+3b-b+2-b+d = 1
or d = 2-b

Substituting the values of a, c and d

$$F = K\,\rho^{1-b}\mu^{b}V^{2-b}L^{2-b}$$
$$= K\rho V^{2}L^{2}\rho^{-b}\mu^{b}V^{-b}L^{-b}$$
$$= K\rho V^{2}L^{2}\left(\frac{\rho VL}{\mu}\right)^{-b}$$
$$= [K(Re)^{-b}]\rho AV^{2}$$
$$= [2K(Re)^{-b}]\rho A\frac{V^{2}}{2}$$
$$= C\rho A\frac{V^{2}}{2}$$

17. Power input to a propeller (P) is expressed in terms of density of air (ρ), diameter (D), velocity of the air stream (V), rotational speed (ω), viscosity (μ) and speed of sound (C). Show that
$P = c\rho\omega^{3}D^{5}$ where c = constant. Use Rayleigh's method.

Solution:
$$P = f(\rho, D, V, \omega, \mu, C)$$
$$P = K\rho^{a}D^{b}V^{c}\omega^{d}\mu^{e}C^{f}$$

Writing dimensions
$$ML^{2}T^{-3} = K(ML^{-3})^{a}(L)^{b}(LT^{-1})^{c}(T^{-1})^{d}(ML^{-1}T^{-1})^{e}(LT^{-1})^{f}$$
$$ML^{2}T^{-3} = KM^{a+e}L^{-3a+b+c-e+f}T^{-c-d-e-f}$$

Equating the powers of M, L and T
a+e = 1
-3a+b+c-e+f = 2
-c-d-e-f = -3

There are 3 equations and 6 unknowns. As we have to get expression in terms of ρ, ω and D, we can express the powers of these three variables i.e. a, b and d in terms of remaining variables.

Solving above equations,

a = 1-e
d = 3-c-e-f
substituting a and d in second eqaution
-3+3e+b+c-e+f = 2
Or, b = 5-c-2e-f

Substituting the values of a, b and c

$P = K\rho^a D^b V^c \omega^d \mu^e C^f = K\rho^{1-e} D^{5-c-2e-f} V^c \omega^{3-c-e-f} \mu^e C^f$

$= K\rho D^5 \omega^3 \left(\rho^{-e} D^{-2e} \omega^{-e} \mu^e\right)\left(D^{-c} V^c \omega^{-c}\right)\left(D^{-f} \omega^{-f} C^f\right)$

$= K\rho D^5 \omega^3 \left(\frac{\rho D^2 \omega}{\mu}\right)^{-e} \left(\frac{V}{D\omega}\right)^c \left(\frac{C}{D\omega}\right)^f$

Representing $= K\left(\frac{\rho D^2 \omega}{\mu}\right)^{-e} \left(\frac{V}{D\omega}\right)^c \left(\frac{C}{D\omega}\right)^f$ by c

$P = c\rho\omega^3 D^5$

18. If the resistance to motion of a sphere through a fluid (R) is a function of the density (ρ), viscosity (μ) of the fluid, and the radius (r) and velocity (u) of the sphere, develop a relationship of R using Buckingham's π theorem.
(Take u, r and ρ as repeating variables and take the dimension of shear stress for R)

Solution:

$f1(R, \rho, \mu, r, u) = 0$

Total number of variables = 5
No. of fundamental dimensions = 3
No. of π terms = 5-3= 2

$f(\pi_1, \pi_2) = 0$ (I)

Choose u, r and ρ as repeating variables.

First π term

$\pi_1 = u^{a1} r^{b1} \rho^{c1} R$ (II)

Writing dimensions

$M^0 L^0 T^0 = (LT^{-1})^{a1} (L)^{b1} (ML^{-3})^{c1} ML^{-1}T^{-2}$

Equating the powers of M, L and T
c1+1 =0
a1+b1-3c1-1 = 0
-a1-2 = 0

c1 = -1, a1 = -2
Substituting a1 and c1 in second equation
-2+b1+3-1=0

b1 = 0

Substituting the values of a1, b1 and c1 in II

$\pi_1 = u^{-2} r^0 \rho^{-1} R$

$\pi_1 = \frac{R}{\rho u^2}$

Second π term

$\pi_2 = u^{a2} r^{b2} \rho^{c2} \mu$ (III)

Writing dimensions

$M^0 L^0 T^0 = (LT^{-1})^{a2} (L)^{b2} (ML^{-3})^{c2} ML^{-1}T^{-1}$

Equating the powers of M, L and T

c2+1 =0

a2+b2-3c2-1 = 0

-a2-1 = 0

a2 = -1, c2 = -1

Substituting a2 and c2 in second equation

-1+b2+3-1 =0

b2 =-1

Substituting the values of a2, b2 and c2 in III

$\pi_2 = u^{-1} r^{-1} \rho^{-1} \mu$

$\pi_2 = \frac{\mu}{ur\rho}$

Substituting values of π_1 and π_2 in I

$f\left(\frac{R}{\rho u^2}, \frac{\mu}{ur\rho}\right) = 0$

$\frac{R}{\rho u^2} = f\left(\frac{\mu}{ur\rho}\right)$

$R = \rho u^2 f\left(\frac{\mu}{ur\rho}\right)$

19. The pressure difference (ΔP) in a pipe of diameter (D) and length (L) due to viscous flow depends on the velocity of fluid (V), viscosity (μ) and density (ρ). Using Buckingham's π theorem, show that $\Delta P = \frac{\mu VL}{D^2} f(Re)$ where $Re = \frac{\rho DV}{\mu}$ is Reynold's number.

(Take D, V and ρ as repeating variables)

Solution:

$f1(\Delta P, D, L, V, \mu, \rho) = 0$

Total number of variables = 6

No. of fundamental dimensions = 3

No. of π terms = 6-3= 3

$f(\pi_1, \pi_2, \pi_3) = 0$ (I)

Choose D, V and μ as repeating variables.

First π term

$\pi_1 = D^{a1} V^{b1} \rho^{c1} \Delta P$ (II)

Writing dimensions

$M^0 L^0 T^0 = (L)^{a1} (LT^{-1})^{b1} (ML^{-3})^{c1} ML^{-1}T^{-2}$

Equating the powers of M, L and T

c1+1 =0

a1+b1-3c1-1 = 0

-b1- 2 = 0

c1 = -1, b1 = -2

a1-2+3-1=0

a1 = 0

Substituting the values of a1, b1 and c1 in II

$\pi_1 = D^0 V^{-2} \rho^{-1} \Delta P$

$\pi_1 = \frac{\Delta P}{\rho V^2}$

Second π term

$\pi_2 = D^{a2} V^{b2} \rho^{c2} L$ (III)

Writing dimensions

$M^0 L^0 T^0 = (L)^{a2} (LT^{-1})^{b2} (ML^{-3})^{c2} L$

Equating the powers of M, L and T

c2 = 0

a2+b2-3c2+1 = 0

b2 = 0

a2 = -1

Substituting the values of a2, b2 and c2 in III

$\pi_2 = D^{-1} V^0 \rho^0 L$

$\pi_2 = \frac{L}{D}$

Third π term

$\pi_3 = D^{a3}V^{b3}\rho^{c3}\mu$ (IV)

Writing dimensions
$M^0L^0T^0 = (L)^{a3}(LT^{-1})^{b3}(ML^{-3})^{c3}ML^{-1}T^{-1}$
Equating the powers of M, L and T
c3+1 =0
a3+b3-3c3-1 =0
-b3-1 = 0
c3 = -1
b3 = -1
a3-1+3-1 = 0
a3 =-1
Substituting the values of a3, b3 and c3 in (IV)
$\pi_3 = D^{-1}V^{-1}\rho^{-1}\mu$
$\pi_3 = \frac{\mu}{\rho DV}$

Substituting the values of π_1, π_2 and π_3 in (I)
$f\left(\frac{\Delta P}{\rho V^2}, \frac{L}{D}, \frac{\mu}{\rho DV}\right) = 0$
$\frac{\Delta P}{\rho V^2} = f\left(\frac{L}{D}, \frac{\mu}{\rho DV}\right)$
Multiplying first π term by 1/ π_2 and 1/ π_3 and expressing the product as a function of 1/π_3

$\frac{\Delta P}{\rho V^2}\frac{D}{L}\frac{\rho DV}{\mu} = f\left(\frac{\rho DV}{\mu}\right)$
$\Delta P = \frac{\mu VL}{D^2} f(Re)$

20. Show by dimensional analysis that the power P required to operate a test tunnel is given by

$$P = \rho L^2 V^3 \phi\left(\frac{\mu}{\rho LV}\right)$$

where ρ is density of fluid, μ is viscosity, V is fluid mean velocity, P is the power required and L is the characteristics tunnel length.

Solution:
$f1(P, \rho, L, V, \mu) = 0$
Total number of variables = 5
No. of fundamental dimensions = 3
No. of π terms = 5-3= 2
$f(\pi_1, \pi_2) = 0$ (I)

Choose ρ, L and V as repeating variables.

First π term

$\pi_1 = \rho^{a1} L^{b1} V^{c1} P$ (II)

Writing dimensions

$M^0 L^0 T^0 = (ML^{-3})^{a1} (L)^{b1} (LT^{-1})^{c1} ML^2 T^{-3}$

Equating the powers of M, L and T

a1+1 =0

-3a1+b1+c1+2 = 0

-c1-3 = 0

Solving

a1 = -1, c1 = -3

-3x-1+b1-3+2 = 0

b1 = -2

Substituting the values of a1, b1 and c1 in II

$$\pi_1 = \rho^{-1} L^{-2} V^{-3} P$$

$$\pi_1 = \frac{P}{\rho L^2 V^3}$$

Second π term

$\pi_2 = \rho^{a2} L^{b2} V^{c2} \mu$ (III)

Writing dimensions

$M^0 L^0 T^0 = (ML^{-3})^{a2} (L)^{b2} (LT^{-1})^{c2} ML^{-1} T^{-1}$

Equating the powers of M, L and T

a2+1 = 0

-3a2+b2+c2-1=0

-c2-1 = 0

Solving

a2 = -1, c2 = -1

-3x-1+b2-1-1 = 0

b2 = -1

Substituting the values of a2, b2 and c2 in III

$$\pi_2 = \rho^{-1} L^{-1} V^{-1} \mu$$

$$\pi_2 = \frac{\mu}{\rho L V}$$

Substituting the values of π_1 and π_2 in I

$$f\left(\frac{P}{\rho L^2 V^3}, \frac{\mu}{\rho L V}\right) = 0$$

$$\frac{P}{\rho L^2 V^3} = \phi\left(\frac{\mu}{\rho L V}\right)$$

$$P = \rho L^2 V^3 \phi\left(\frac{\mu}{\rho L V}\right)$$

21. A pipe of diameter 1.8m is required to transport oil of sp.gr. 0.8 and viscosity 0.04 poise at the rate of 4 m^3/s. Tests were conducted on a 20cm diameter pipe using water at 20^0C. Find the velocity and rate of flow in the model. Viscosity of water at 20^0C = 0.01 poise.

Solution:
Diameter of prototype (D_p) = 1.8m
Density of oil (ρ_p) = 0.8x1000 = 800kg/m^3
Viscosity of prototype (μ_p) = 0.04 poise = 0.004 PaS
Discharge for prototype (Q_p) = 4 m^3/s
Velocity of prototype (V_p) = $\frac{Q_p}{A_p} = \frac{4}{\frac{\pi}{4}x1.8^2}$ = 1.572 m/s
Diameter of model (D_m) = 0.2m
Density of water (ρ_m) = 1000kg/m^3
Viscosity of water (μ_m) = 0.01 poise = 0.001 PaS
Velocity of model (V_m) = ?
Rate of flow in the model (Q_m) = ?

From Reynolds' model law
Re)model = Re) Prototype

$$\frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho_p V_p D_p}{\mu_p}$$

$$\frac{1000xV_m x0.2}{0.001} = \frac{800x1.572x1.8}{0.004}$$

V_m = 2.83 m/s
Q_m = V_m A_m = $2.83x\frac{\pi}{4}x0.2^2$= 0.0889 m^3/s

22. A ship 250m long moves in seawater, whose density is 1030 kg/m^3. A 1:125 model of this ship is to be tested in wind tunnel. The velocity of air in the wind tunnel around the model is 20m/s and the resistance of the ship is 50N. Determine the velocity and resistance of the ship in seawater. The density of air is 1.24 kg/m^3. Take the kinematic viscosity of seawater and air as 0.012 stokes and 0.018 stokes respectively.

Solution:
Length of prototype (L_p) = 250m
Density of seawater (ρ_p) = 1030 kg/m^3
Kinematic viscosity of seawater (υ_p) = 0.012 stokes = 0.012x10^{-4} m^2/s
Length of model (L_m) = $\frac{1}{125}x250$ =2m
Density of air (ρ_m) = 1.24 kg/m^3
Kinematic viscosity of air (υ_m) = 0.018 stokes = 0.018x10^{-4} m^2/s
Velocity of model (V_m) = 20m/s
Resistance of model (F_m) = 50N
Velocity of prototype (V_p) = ?

Resistance of prototype (F_p) = ?

From Reynolds' model law
Re)model = Re) Prototype

$$\frac{V_m L_m}{\upsilon_m} = \frac{V_p L_p}{\upsilon_p}$$

$$\frac{20x2}{0.018x10^{-4}} = \frac{V_p x250}{0.012x10^{-4}}$$

V_p = 0.1066 m/s

Resistance = mass x acceleration = $\rho L^3 \frac{V}{t} = \rho L^2 \frac{L}{t} V = \rho L^2 V^2$

$$\frac{F_p}{F_m} = \frac{\rho_p\, L_p^2\, V_p^2}{\rho_m\, L_m^2\, V_m^2}$$

$$\frac{F_p}{50} = \frac{1030x250^2x0.1066^2}{1.24x2^2x20^2}$$

F_p = 18436 N

23. In 1:30 model of a spillway, the velocity and discharge are 1.5m/s and 2 m^3/s. Find the corresponding velocity and discharge in the prototype.

Solution:
Linear scale ratio (L_r) = 1/30
Velocity of model (V_m) = 1.5m/s
Discharge of model (Q_m) = 2 m^3/s
Velocity of prototype (V_p) = ?
Discharge of prototype (Q_p) = ?

From Froude model law,
Fr)model = Fr)prototype

$$\frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}}$$

$$\frac{1.5}{\sqrt{9.81x\frac{1}{30}L_p}} = \frac{V_p}{\sqrt{9.81L_p}}$$

V_p = 8.216 m/s

$$\frac{Q_p}{Q_m} = \frac{A_p V_p}{A_m V_m} = \frac{L_p^2 V_p}{L_m^2 V_m}$$

$$\frac{Q_p}{2} = \frac{(30L_m)^2 8.216}{L_m^2 x1.5}$$

Q_p = 9859 m^3/s

24. A spillway model is to be built geometrically similar scale of 1/40 across a flume of 50cm width. The prototype is 20m high and the maximum head on it is expected to be 2m. (a) What height of the model and what head on the model should be used? (b) If the flow over the model at a particular head is 10 lps, what flow per m length of the prototype is expected? (c) If the negative pressure in the model is 150mm, what is the negative pressure in the prototype?

Solution:
Scale ratio for length (L_r) = 1/40
Width of model (B_m) = 0.5m
Height of prototype (H_p) = 20m
Head on prototype (Hd_p) = 2m
a) Height of model (H_m) = ?
Head on model (Hd_m) = ?

$$L_r = \frac{H_m}{H_p}$$

$$\frac{1}{40} = \frac{H_m}{20}$$

H_m = 0.5m

$$L_r = \frac{Hd_m}{Hd_p}$$

$$\frac{1}{40} = \frac{Hd_m}{2}$$

Hd_m = 0.05m

b) Flow through model (Q_m) = 10 lps = 0.01 m^3/s
Flow through prototype (Q_p) = ?

$$\frac{Q_m}{Q_p} = L_r^{2.5}$$

$$\frac{0.01}{Q_p} = \left(\frac{1}{40}\right)^{2.5}$$

Q_p = 101.2 m^3/s
Width of prototype (B_p)

$$L_r = \frac{B_m}{B_p}$$

$$\frac{1}{40} = \frac{0.5}{B_p}$$

B_p= 20m

Discharge per unit width = 101.2/20 = 5.06 m^3/s

c) Negative pressure head in model (P_m) = -0.15m

Negative pressure head in prototype (P_p) = ?

$$L_r = \frac{P_m}{P_p}$$

$$\frac{1}{40} = \frac{-0.15}{P_p}$$

P_p = -6m

25. The pressure drop in an aeroplane model of size 1/50 of its prototype is 4 N/cm^2. The model is tested in water. Find the corresponding pressure drop in prototype. Take density of air = 1.24 kg/m^3. The viscosity of water is 0.01 poise while the viscosity of air is 0.00018 poise.

Solution:
Linear scale ratio (L_r) = 1/50
Pressure drop in model (P_m) = 4 N/cm^2 = 4x10^4 N/m^2
Density of air (ρ_p) = 1.24 kg/m^3
Density of water (ρ_m) = 1000 kg/m^3
Viscosity of water (μ_m) = 0.01 poise = 0.001 PaS
Viscosity of air (μ_p) = 0.00018 poise = 0.000018 PaS
Pressure drop in prototype (P_p) = ?

As the problem involves both viscous and pressure force, we have to use both Reynolds and Euler model law.
From Reynolds' model law
Re)model = Re) Prototype

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

$$\frac{1000 x V_m x \frac{1}{50} L_p}{0.001} = \frac{1.24 x V_p L_p}{0.000018}$$

V_m = 3.44 V_p

From Euler model law
Eu) model = Eu) prototype

$$\frac{V_m}{\sqrt{P_m/\rho_m}} = \frac{V_p}{\sqrt{P_p/\rho_p}}$$

$$\frac{3.44 V_p}{\sqrt{\frac{4x10^4}{1000}}} = \frac{V_p}{\sqrt{\frac{P_p}{1.24}}}$$

P_p = 4.2 N/m^2

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