Civinnovate

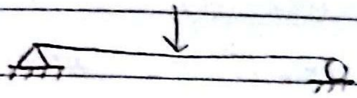
Discover, Learn, and Innovate in Civil Engineering

unit 6 Analysis of beam and frame

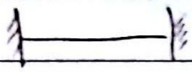
Beam:

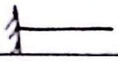
Beam is a structural member designed to support mainly transverse loads. In general beams are straight, long, horizontal.

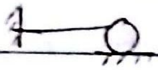
Types of beam:

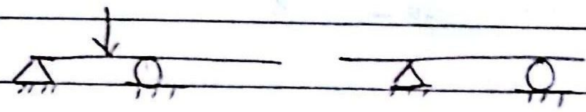
a) simply supported beam 

b) Continuous beam 

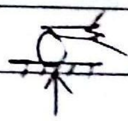
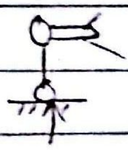
c) Fixed beam 

d) Cantilever beam 

e) Proped cantilever beam 

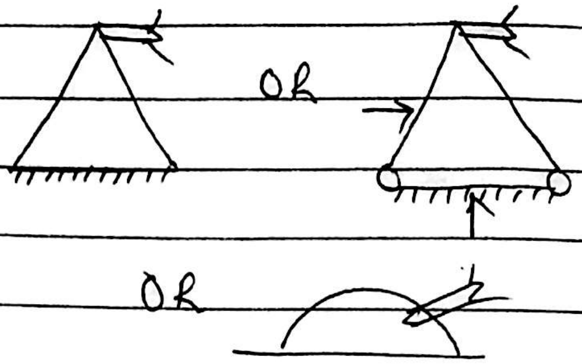
f) Overhanging beam 

Types of support

a) Roller support  or  

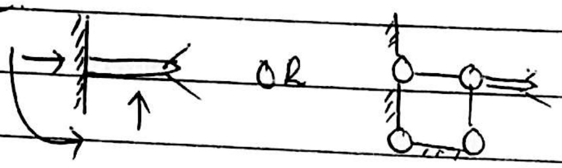
r Horizontal displacement (H.D) = ✓
Vertical displacement (V.D) = x
Rotation (θ) = ✓

ii) Hinge Support:



Horizontal displacement (H.D) = X
Vertical displacement (V.D) = X
Rotation (θ) = ✓

iii) Fixed support:

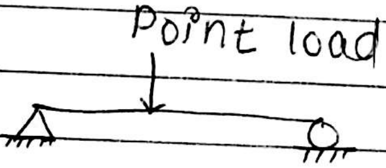


H.D = X
V.D = X
θ = X

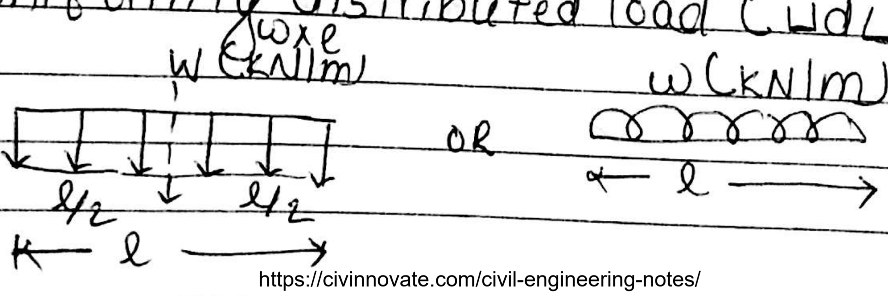
No of reactions = 3

Types of load:

i) Point load



ii) Uniformly distributed load (UDL)



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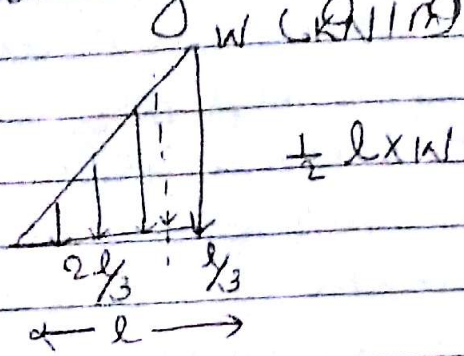
iii) Uniformly varying load (UVL)



iv) Hydrostatic load
v) Dead load and live load
vi) static and dynamic load
vii) wind load
viii) earthquake load / sesmic load
ix) Moment load
x) couple

Statically determinate and indeterminate structure.

statically determinate structure.

A statically determinate structure system is one for which the reaction and internal stresses developed in the plane member can be completely determined by using the three equations of static equilibrium ($\Sigma F_x=0$, $\Sigma F_y=0$, $\Sigma M=0$) and conditional equ^n if any.

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Statically indeterminate structure :

A statically indeterminate structure is one for which the reaction and internal stresses developed in the plane member can'tbe completely determined by using three equations of static equilibrium ($\Sigma Fx = 0$, $\Sigma Fy = 0$, $\Sigma FM = 0$) and conditional if any.

Degree of indeterminancy :- Number of unknown reaction - equn of equilibrium

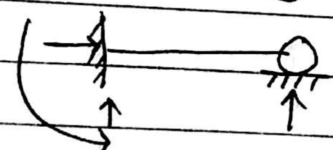
$$\boxed{ni = \text{unknown} - 3}$$

i)

$ni = 3 - 3$
$= 0$

Statically determinate

ii)



$ni = 4 - 3$
$= 1$

Statically indeterminate

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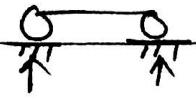


$n_i = 2 - 3$
$= -1$

statically determinate but unstable

* For beam and frame structure

i) Total degree of external indepedency
$(n_{ei}) = r - (3 + c)$

ii) Total degree of internal independency
$n_{ii} = 3(m + 1 - j)$
$= 3 \times$ no. of closed loop

iii) Total degree of indeterminancy
$(n_i) = (3m + r) - (3j + c)$

where
m = no. of member
r = no. of unknown reaction
j = no. of joint
c = special condition

Case I:
If $(3m + r) = (3j + c)$, then the structure is statically determinate structure

Case II :

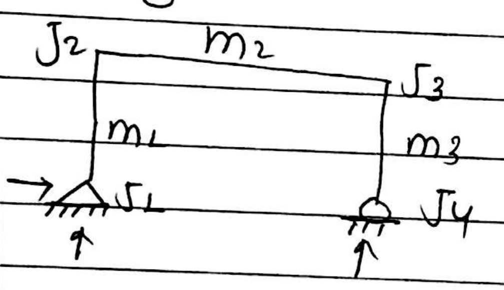
If $(3m + r) > (3j + c)$, then the structure is statically indeterminate. (n_i = +ve)

Case III :

If $(3m + r) < (3j + c)$, then the structure is determinate but unstable (n_i = −ve)

Q. Determine the total external indeterminancy, internal indeterminancy and total indeterminancy of given structure.

1)



$m = \cancel{4}\,3$

$J = 4$

$r = 3$

$c = 0$

$n_{ei} = r - (3 + c)$
$= 3 - (3 + 0)$
$= 0$

$n_{ii} = 3(m + 1 - j)$
$= 3(4 - 4)$
$= 0$

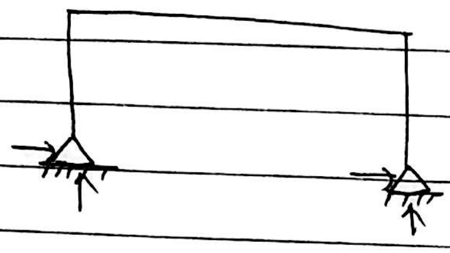
$n_i = n_{ei} + n_{ii}$

$= 0 + 0$

$= 0$

Given frame structure is statically determinate structure.

2)



$m = 3$

$r = 4$

$c = 0$

$j = 4$

$n_{ei} = r - (3 + c)$

$= 4 - (3 + 0)$

$= 1$

$n_{ii} = 3(m + 1 - j)$

$= 3(3 + 1 - 4)$

$= 0$

$n_i = n_{ei} + n_{ii}$

$= 1 + 0$

$= 1$

Given frame structure is statically indeterminate.

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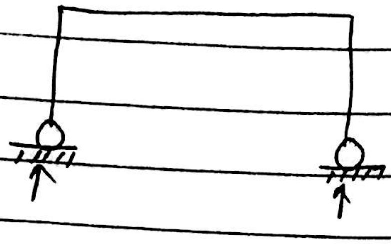
3)



$m = 3$
$j = 4$
$r = 2$
$c = 0$

$n_{ei} = \cancel{3} \cancel{c}$
$r - (3 + c)$
$= 2 - (3 + 0)$
$= -1$

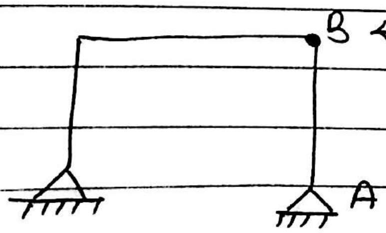
$n_{ii} = 3(m + 1 - j)$
$= 3(3 + 1 - 4)$
$= 0$

$n_i = n_{ii} + n_{ei}$
$= 0 - 1$
$= -1$

Given frame structure is statically determinate but unstable.

$C = \Sigma M' - 1$

10)



$B \leftarrow$ Internal hinge

(ΣM_B) left part $= 0$

OR

(ΣM_B) right part $= 0$

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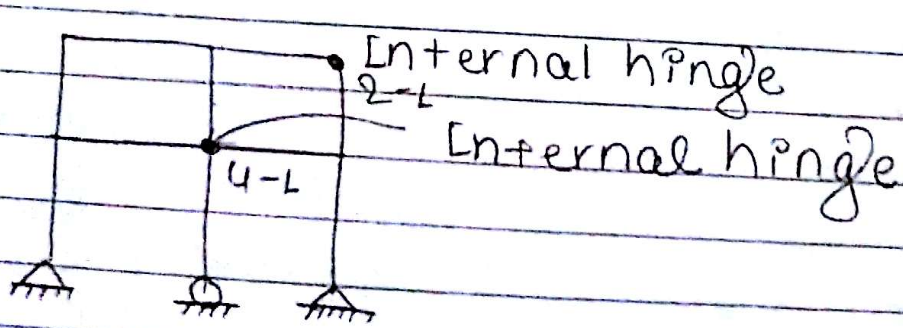
$c = \sum m' - 1$ (m' = no. of members connected at internal hinge)

$r = 4$
$m = 3$
$j = 4$
$c = 2 - 1 = 1$
$n_{ei} = r - (3 + c)$
$= 4 - (3 + 1)$
$= 0$
$n_{ii} = 3(m + 1 - j)$
$= 3(3 + 1 - 4)$
$= 0$
$n_i = n_{ei} + n_{ii}$
$= 0$
statically determinate structure.

*



$m = 10$
$c = 3 + 1 = 4$
$r = 5$
$j = 9$

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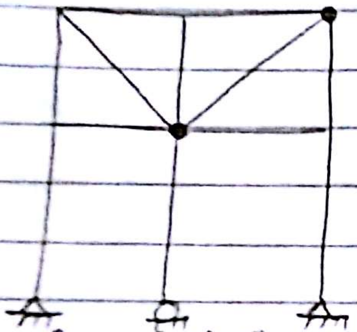
$n_{ei} = r-(3+c)$
$= 5-(3+4)$
$= -2$
$n_{ii} = 3(m+1-j)$
$= 3(10+1-9)$
$= 6$
$n_i = -2+6$
$= 4$
Statically indeterminate structure

(3)



$c = 2+5$
$= 7$
$m = 12$
$j = 9$
$r = 5$

$n_{ei} = r-(3+c)$
$= 5-(3+7)$
$= -5$
$n_{ii} = 3(m+1-j)$
$= 3(12+1-9)$
$= 12$

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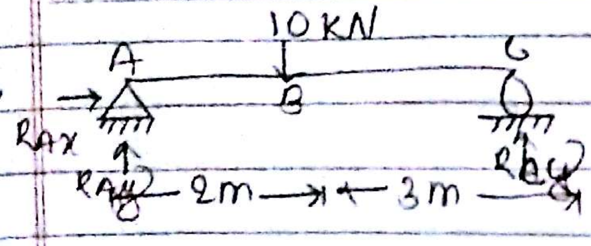
$n_i = n_{ei} + n_{ii}$
$= -5 + 12$
$= 7$

Q. Determination of support reaction

a) 

i) calculation of degree of indeterminancy
n_i = unknown − 3
$= 3 - 3$
$= 0$

Given beam is statically determinate structure

ii) calculation of support reaction by using equation of equilibrium

a) $\xrightarrow{+} \Sigma F_x = 0$
$R_{AX} = 0$

b) $+\circlearrowright \Sigma M_A = 0$
or, $(10 \times 2) - R_{cy} \times 5 = 0$
$R_{cy} = 4$

c) $+\circlearrowright \Sigma M_c = 0$
or, $3 \times (-10) + R_{Ay} \times 5 = 0$
$R_{Ay} = 6$ KN

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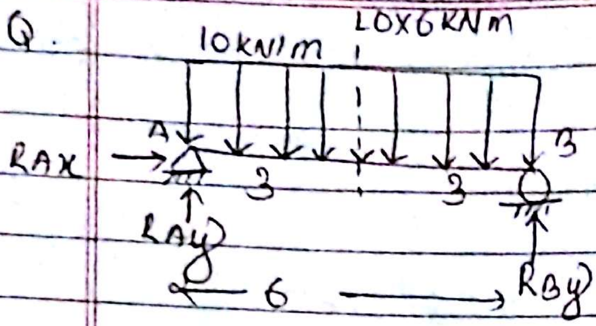
Q.



i) $\Sigma F_x = 0$

$R_{Ax} = 0$

ii) $+\circlearrowright \Sigma M_A = 0$

or, $(60 \times 3) - R_{By} \times 6 = 0$

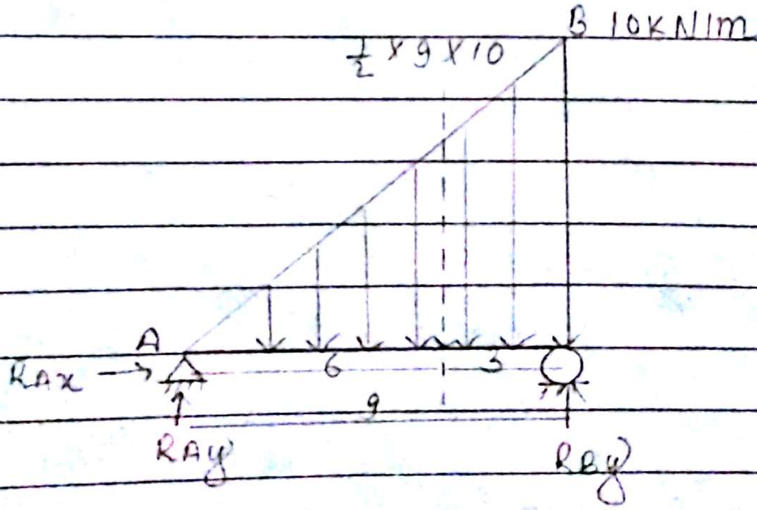
$R_{By} = 30$

$+\circlearrowright \Sigma M_B = 0$

$-60 \times 3 + 6 \times R_{Ay} = 0$

$R_{Ay} = 30$ kNm

Q.



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$\overset{+}{\rightarrow} \sum F_x = 0$

$R_{Ax} = 0$

$+\circlearrowright \sum M_A = 0$

$45 \times 6 - R_{By} \times 9 = 0$

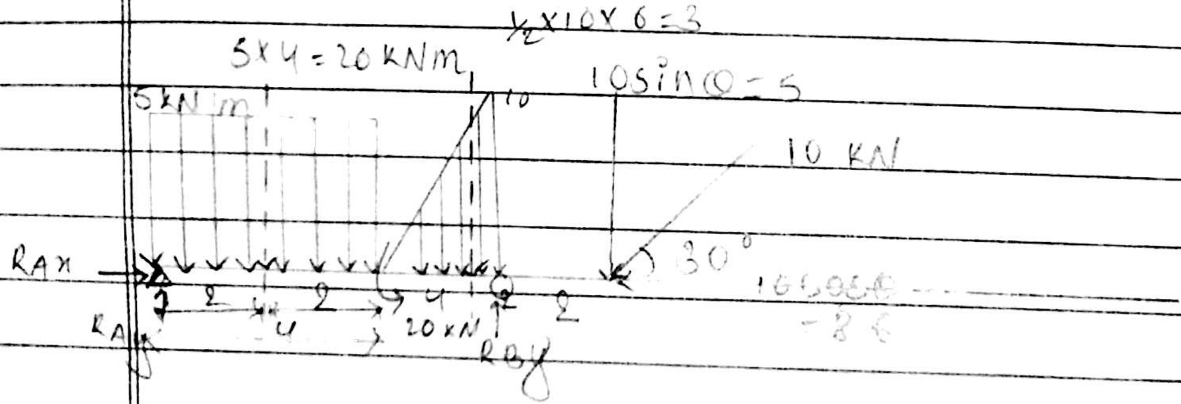
or, $R_{By} = 30$

$+\circlearrowright \sum M_B = 0$

$-45 \times 3 + R_{Ay} \times 3$

$R_{Ay} = 15$ KN/m

Q.



① $\overset{+}{\rightarrow} \sum F_x = 0$

$R_{Ax} - 8.66 = 0$

$R_{Ax} = 8.66$

⑪ $+\circlearrowright \sum M_A = 0$

$(20 \times 2) - 20 + (30 \times 8) - (R_{Cy} \times 10) + 5 \times 12 = 0$

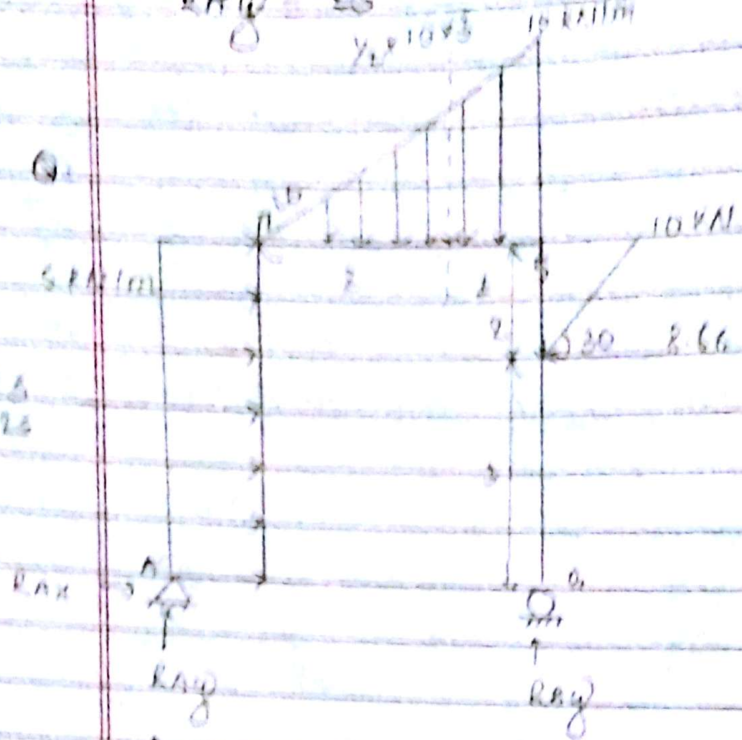
$R_{Cy} = 32$

iii) $\circlearrowright \sum M_B = 0$

$R_{Ay} \times 16 - 20 \times 8 - 20 - 30 \times 2 + 5 \times 2 = 0$

$R_{Ay} = 23$

Q.



$5 \times 5 = 25$

① $\overset{+}{\rightarrow} \sum F_X = 0$

$R_{Ax} + 25 - 8.66 = 0$

$R_{Ax} = -16.34$

$R_{Ax} = 16.34 \ (\leftarrow)$

ii) $\circlearrowright \sum M_A = 0$

$(25 \times 2.5) + 10 + (15 \times 2) - (8.66 \times 3) - R_{By} \times 3 = 0$

$R_{By} = 80.60$

iii) $\circlearrowright M_B = 0$

$R_{Ay} \times 3 + 25 \times 2.5 + 10 - 15 \times 1 - 8.66 \times 3 = 0$

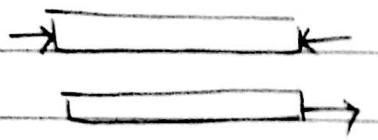
$R_{Ay} = -10.50 \text{ kNm } (\downarrow)$

Axial force, shearing force and bending moment

Axial force:

The axial force at any transverse cross section of a straight beam is algebric sum of the components parallel to the axis of the beam of all loads and reaction applied to the portion of the beam on either sides of cross section.

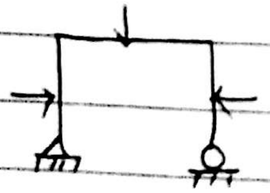
→ If the axial resisting force acts towards the cross section, it is called thrust. if away it is called axial tension.


thrust

axial tension

Shear force:

Shear force at any transverse cross section of straight beam is the algebric sum of component acting transverse (⊥r) to the axis of beam of all loads and reactions applied to the portion of the beam on either sides of cross section.


shear

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Bending moment

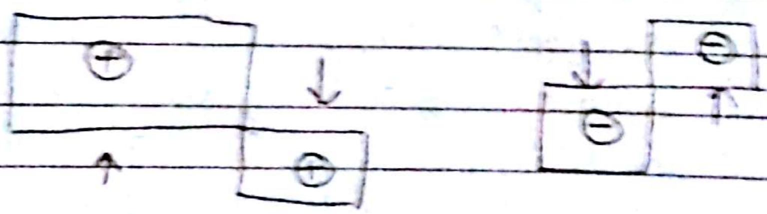
The bending moment at any transverse cross section of straight beam is algebric sum of the moment taken about the axis passing through centroid of cross section of all loads and reaction applied to the portion of the beam on either sides of cross section

* Sign convention
i) Axial force

Tension Compression

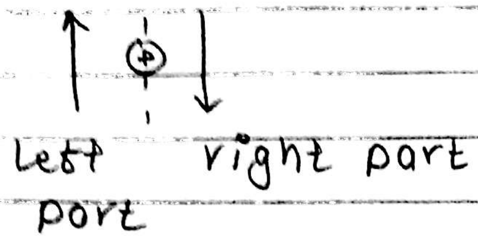
ii) Shear force:

Shear force, which tends to shear a member left up and right down is considered as +ve shear otherwise -ve shear



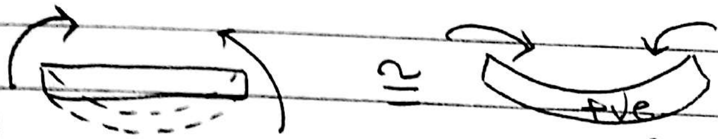
(+ve shear)
(clockwise couple)

(-ve shear)
(Anticlockwise couple)



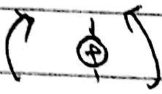
Bending moment:

Bending moment taken as +ve when it tends to produce tension in lower portion of the beam and compression in the upper portion. so that to bend the beam concave upward (sagging curve)



sagging curve (+ve)

Hogging curve (-ve)



Axial, shear force and bending moment diagram (AFD, SFD, BMD)

AFD o—— +ve / −ve ——o

SFD o—— +ve / −ve ——o

BMD o—— −ve / +ve ——o

Draw AFD, SFD, BMD of given loaded beam structure

$n_i = 3 - 3$
$= 0$

The given beam structure is statically determinate

Calculation of support reaction by using the equation of equilibrium

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$R_{Ax} = 8.66$

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ii) $+\circlearrowright \Sigma M_A = 0$

$5 \times 3 - R_{Cy} \times 5 = 0$

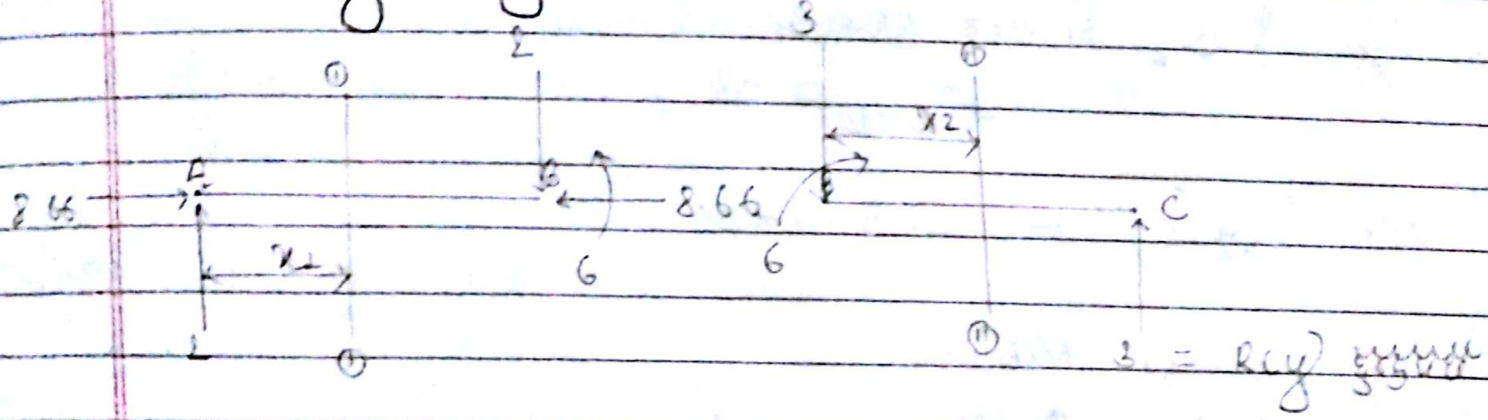
$R_{Cy} = 3$

iii) $+\circlearrowright \Sigma M_C = 0$

$-(5 \times 2) + R_{Ay} \times 5 = 0$

$R_{Ay} = 2$

Free body diagram



Taking portion AB

i) Axial force

$(A.F)_{AB} = -8.66$

ii) Shear force

$\uparrow \oplus \downarrow \quad (SF)_{1-1} = 2$

$(SF)_{AB} = 2$

iii) Bending moment

$(\oplus) \quad BM_{1-1} = 2x_1$ (st line point force मा हुँदा B.M st. line)

At $x_1 = 0$, $M_A = 0$

$x_1 = 1$, $M_1 = 2$

$x_1 = 2$, $M_2 = 4$

$x_1 = 3$, $M_B = 6$

Taking portion BC

i) Axial force

$(AF)_{BC} = 0$

ii) ↑ (+) Shear force

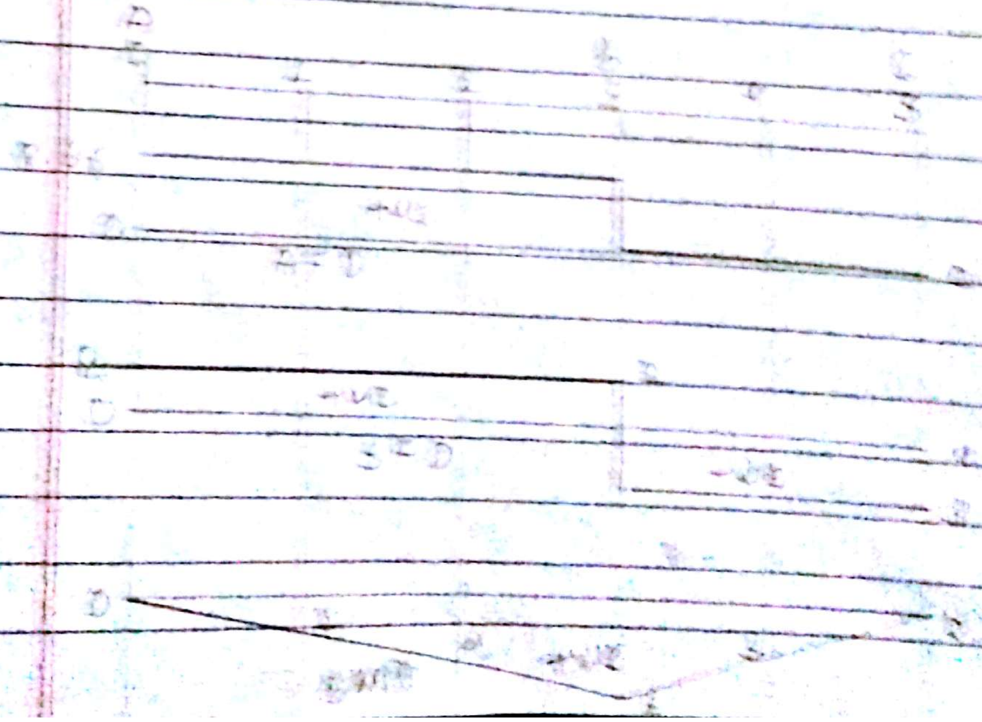
$SF_{②-②} = -3$

iii) ↻ (+) $M_{②-②} = 6 - 3x_2$

$x_2 = 0$, $M_B = 6$

$x_2 = 1$, $M_1 = 3$

$x_2 = 2$, $M_C = 0$



At $x_1 = 0$, $M_A = 0$

$x_2 = 1$, $M_1 = 2$

$x_3 = 2$, $M_2 = 4$

$x_4 = 3$, $M_B = 6$

Taking portion BC

i) Axial force

$(AF)_{BC} = 0$

ii) ↑ ⊕ ↓ shear force

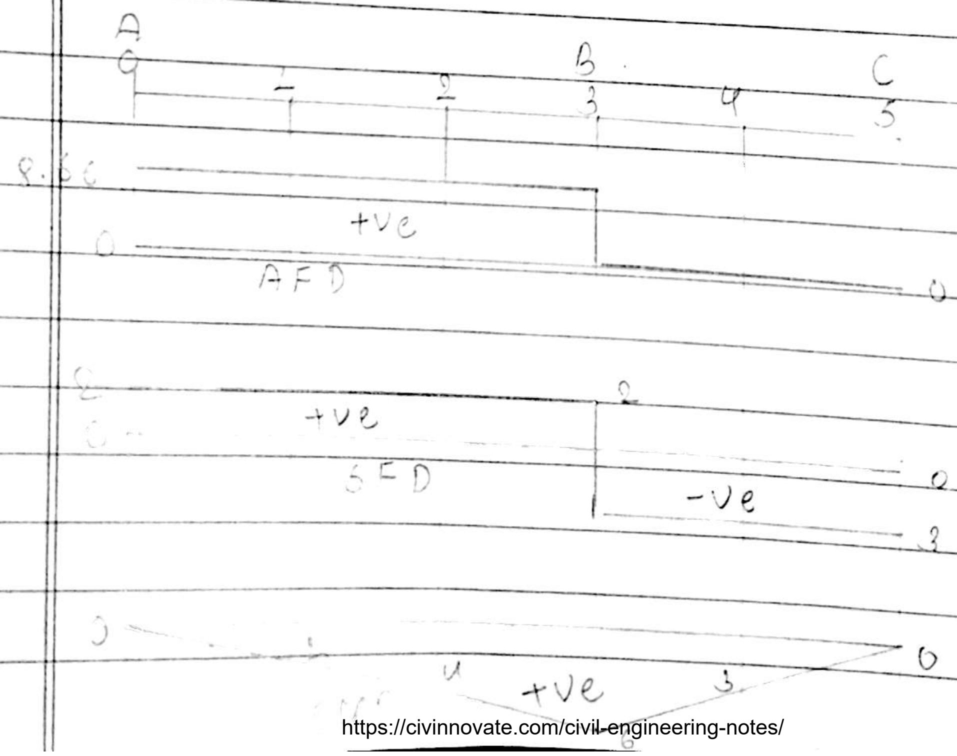
$S.F_{(II-II)} = -3$

iii) ↷ ⊕ ↶ $M_{(II-II)} = 6 - 3x_2$

$x_2 = 0$, $M_B = 6$

$x_2 = 1$, $M_1 = 3$

$x_2 = 2$, $M_C = 0$



Roller | Hinge - Moment

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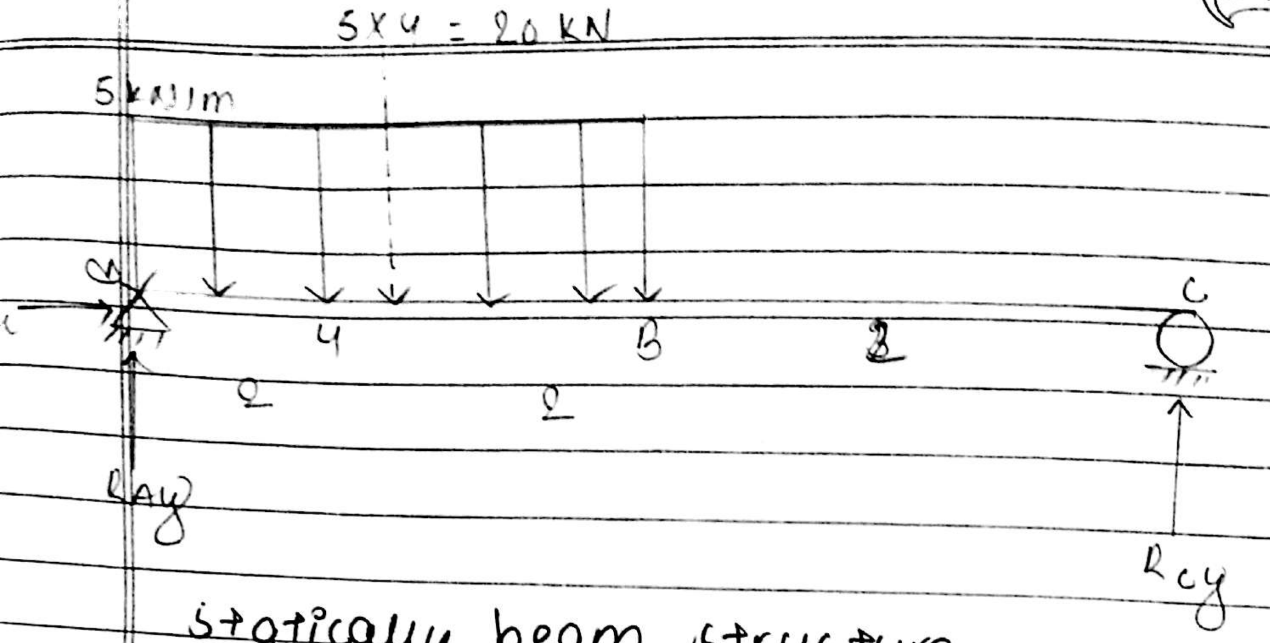


staticaly beam structure.

① $\sum F_x = 0$

$R_{Ax} = 0$

⑪ $\sum M_A = 0$

or, $20 \times 2 - 6 \times R_{cy} = 0$

or, $R_{cy} = 6.67$

20 - 13.32 = 6.67

6.67×4

moment = Force × distance

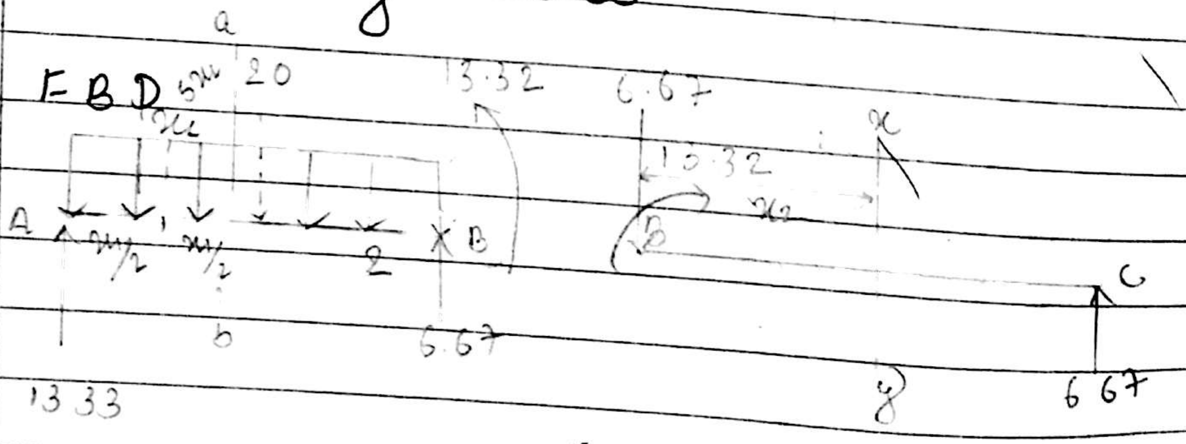
$= 5x \times x/2$

⑪ $\sum M_C = 0$

$-(20 \times 4) + R_{Ay} \times 6 = 0$

$R_{Ay} = 13.33$

FBD



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Taking portion AB

i) Axial force

$(A.F)_{AB} = 0$

ii) Shear force

$\uparrow \oplus \downarrow \quad (S.F)_{ab} = 13.33 - 5x_1$

UDL ~~ky~~ shear force diagram linear/ st...

At $x_1 = 0$ $(S.F)_A = 13.33$

$x_2 = 1$ $(SF)_1 = 8.33$

$x_3 = 2$ $(S.F)_2 = 3.33$

$x_4 = 3$ $(SF)_3 = -1.67$

$x_5 = 4$ $(SF)_B = -6.67$

iii) Bending moment

$B.M_{ab} = 13.33x_1 - \frac{5x_1^2}{2}$

$= 13.33x_1 - 2.5x_1^2$

At $x_1 = 0$ $M_A = 0$

$x_2 = 1$ $M_1 = 10.83$

$x_3 = 2$ $M_2 = 16.66$

$x_4 = 3$ $M_3 = 17.49$

$x_5 = 4$ $M_B = 13.32$

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Portion BC

(i) Axial force

$(A \cdot F)_{BC} = 0$

(ii) Shear force

$(S \cdot F)_{xy} = -6.67$

$(S \cdot F)_{BC} = 6.67$

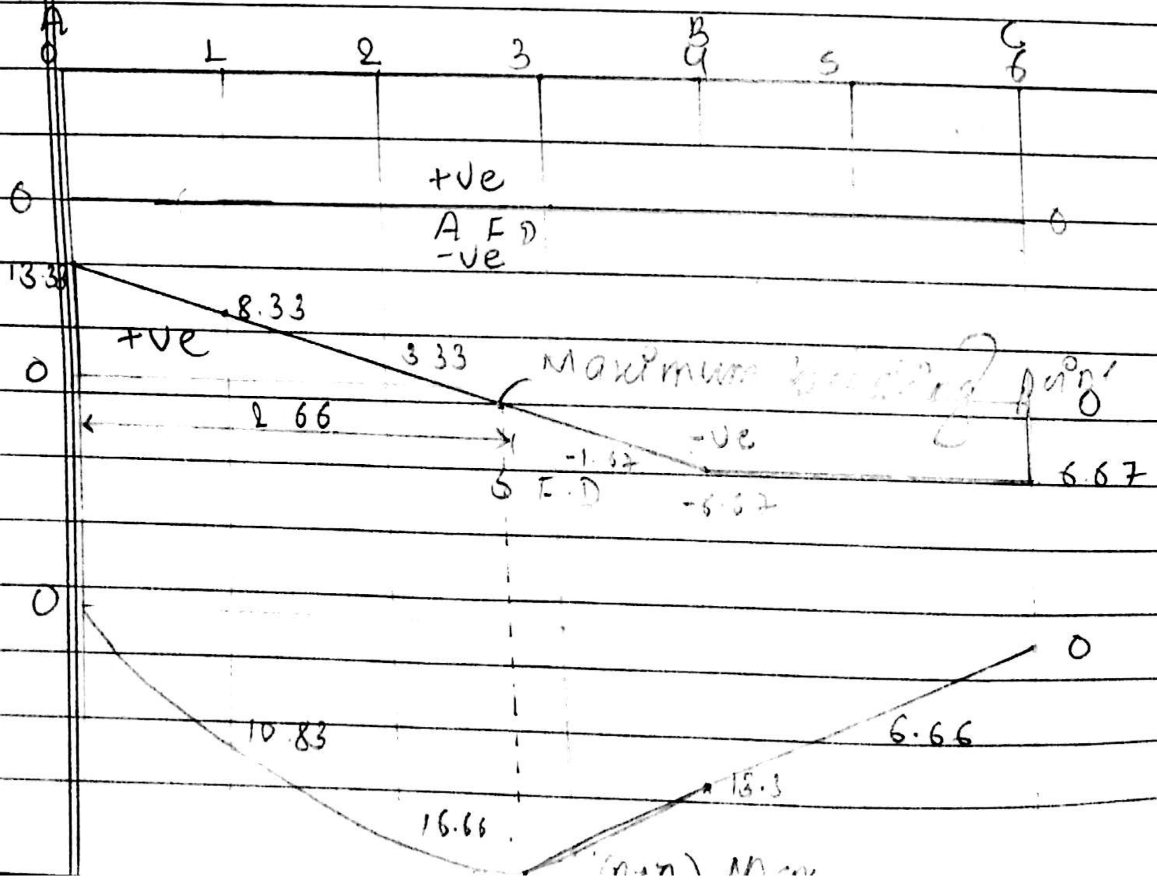
(iii) Bending moment

$M_{xy} = 13.32 - 6.67 x_2$

$x_1 = 0 \qquad M_B = 13.33$

$x_2 = 1 \qquad M_L = 6.66$

$x_3 = 2 \qquad M_C = 0$



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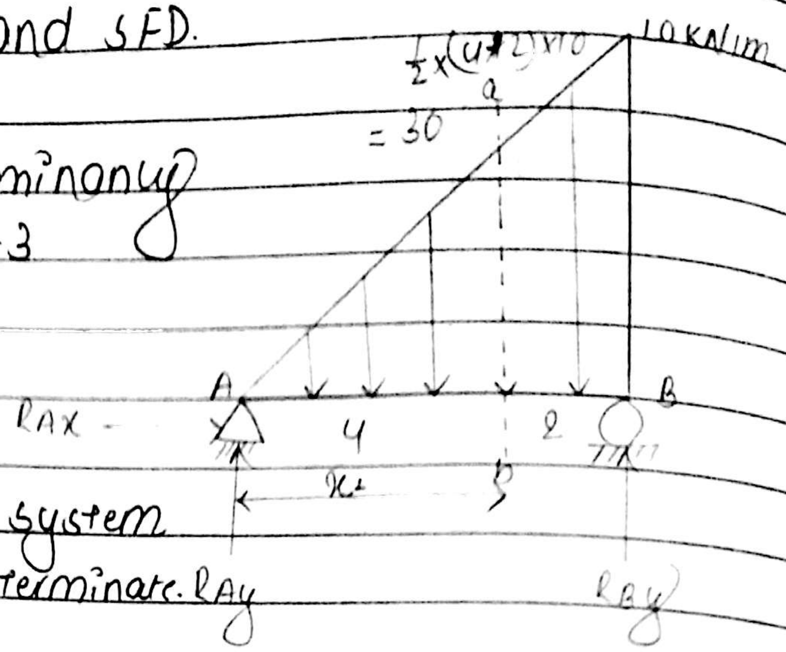
(*) Find FBD, AFD and SFD.



soln:

Degree of indeterminancy
= unknown − 3
= 3 − 3
= 0

The given beam system is statically determinate.

Calculation of support reaction by using equation equilibrium.

i) $\sum \vec{F_x} = 0$

$R_{Ax} = 0$

ii) $+\circlearrowright \sum M_A = 0$

or, $+30\times4 - R_{By}\times6 = 0$

or, $R_{By} = 20$ kN

iii) $+\circlearrowright \sum M_B = 0$

$-30\times2 + R_{Ay}\times6 = 0$

or, $R_{Ay} = 10$ kN

$$\frac{6}{x_1} = \frac{10}{h}$$

$$h = \frac{10x_1}{6} = \frac{5x_1}{3}$$

Shear force ~~(SF)~~ =

i) Axial force (A.F) AB = 0

ii) Shear force

$$\uparrow \oplus \downarrow (SF)_{ab} = 10 - \frac{1}{2} \times x_1 \times \frac{5x_1}{3}$$

$$= 10 - \frac{5x_1^2}{6}$$

$$10 - \frac{5x_1^2}{6} = 0$$

$$x_1 = 3.46$$

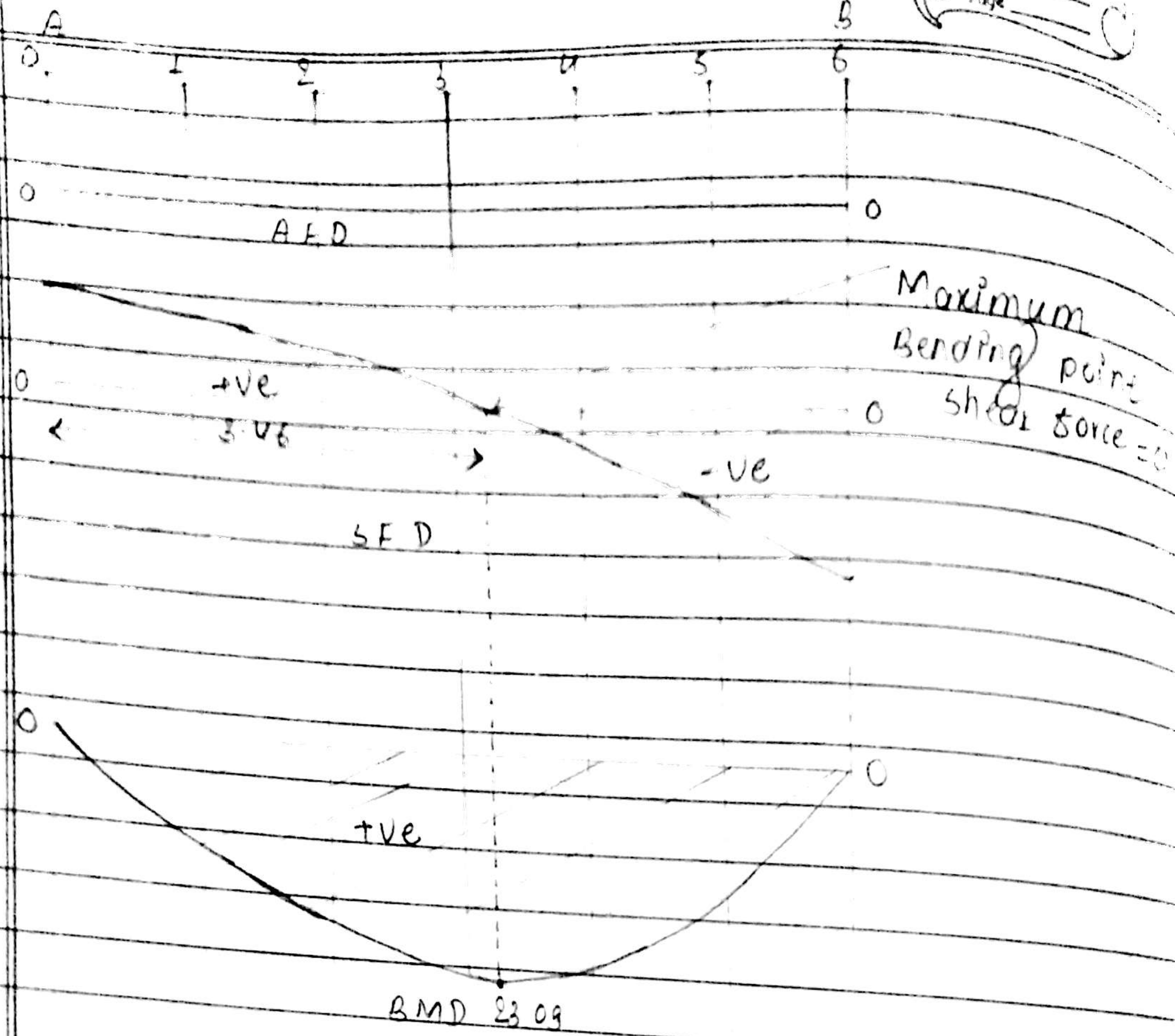
At $x_1 = 0$	$M_A = 10$
$x_2 = 1$	$M_1 = 9.166$
$x_3 = 2$	$M_2 = 6.67$
$x_4 = 3$	$M_3 = 2.5$
$x_5 = 4$	$M_4 = -3.33$
$x_6 = 5$	$M_5 = -10.83$
$x_7 = 6$	$M_B = -20$

iii) Bending moment

$$\curvearrowright \oplus \curvearrowleft (BM)_{ab} = 10x_1 - \frac{1}{2} x_1 \times \frac{5x_1}{3} \times \frac{x_1}{3}$$

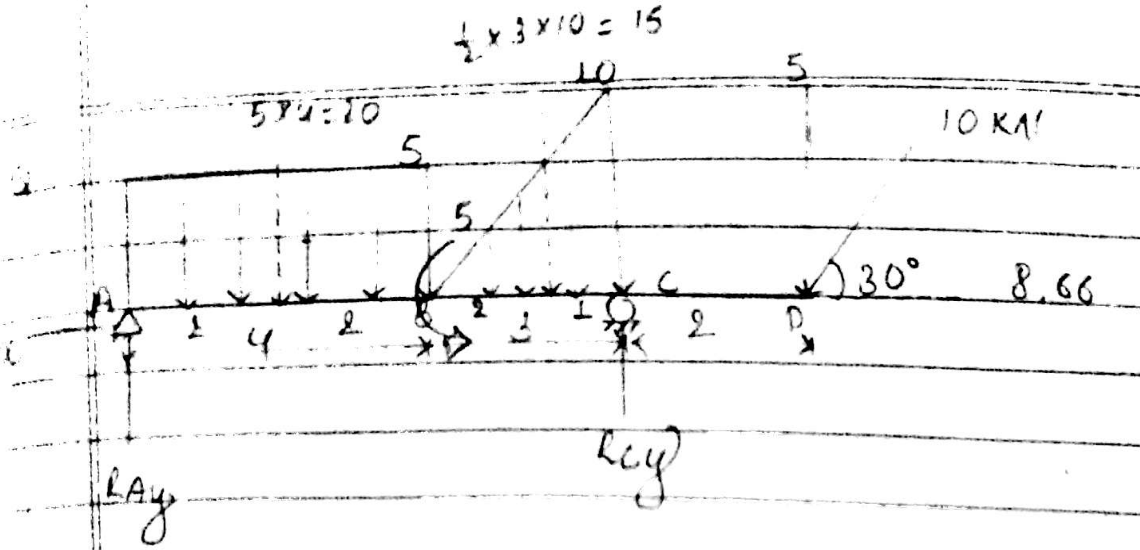
$$= 10x_1 - \frac{5x_1^3}{18}$$

At $x_1 = 0$	$M_A = 0$
$x_2 = 1$	$M_1 = 9.722$
$x_3 = 2$	$M_2 = 17.77$
$x_4 = 3$	$M_3 = 22.5$
$x_5 = 4$	$M_4 = 22.22$
$x_6 = 5$	$M_5 = 15.27$
$x_7 = 6$	$M_B = 0$



Loads	SFD	BMD
i) Point Load / External moment	rectangular (const.) const.	St line (linear)
ii) UdL	st. line	Square parabala
iii) UVL	sq. parabala	cubic parabola

concave upward



Degree of indeterminancy = unknown − 3
= 3 − 3
= 0

The given beam structure is statically determinate

calculation of support reaction by using equation of equilibrium.

i) $\overset{+}{\rightarrow} \sum Fx = 0$

$RAx - 8.66 = 0$

$\therefore RAx = 8.66$ KN

ii) $+\circlearrowright \sum MA = 0$

or, $20 \times 2 - 5 + 15 \times 6 - RCy \times 7 \pm 5 \times 9 = 0$

or, $RCy = 24.28$ KN

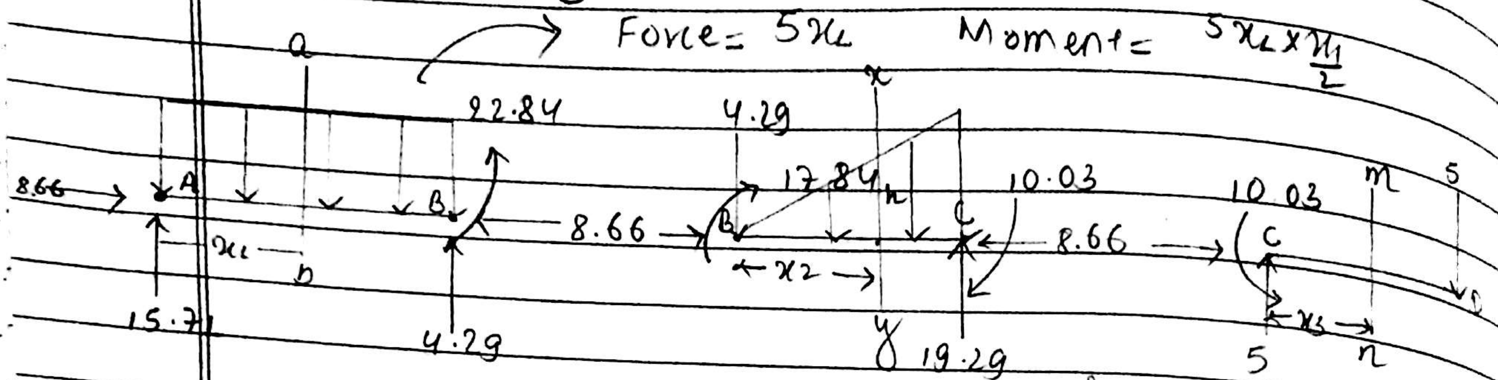
iii) $+\circlearrowright \sum MC = 0$

or, $(RAy \times 7) - (20 \times 5) - 5 - (15 \times 1) + 2 \times 5 = 0$

$RAy = 15.71$ KN

JMJM

Free Body diagram:



$\frac{3}{x_2} = \frac{10}{h}$

$h = \frac{10x_2}{3}$

Force $= \frac{1}{2} \times x_2 \times \frac{10x_2}{3}$

$= \frac{5x_2^2}{3}$

Moment $= \frac{5x_2^2}{3} \times \frac{x_2}{3}$

$= \frac{5x_2^3}{9}$

i) Portion AB

i) Axial Force (A.F) AB = −8.66

ii) Shear Force (S.F)

$\uparrow \oplus \downarrow$ $(SF)_{ab} = +15.71 - 5x_1$

At $x_1 = 0$, $SF_A = 15.71$

$x_1 = 1$, $SF_1 = 10.71$

$x_1 = 2$, $SF_2 = 5.71$

$x_1 = 3$, $SF_3 = 0.71$

$x_1 = 4$, $SF_B = -4.29$

iii) Bending Moment

$(BM)_{ab} = 15.71x_1 - 5x_1 \times \frac{x_1}{2}$

$= 15.71x_1 - 2.5x_1^2$

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$x_1 = 0 \quad M_A = 0$
$x_1 = 2 \quad M_2 = 21.42$
$x_1 = 4 \quad M_B = 22.84$

2) Portion BC

i) Axial Force

$(AF)_{BC} = -8.66$

ii) Shear Force

$\uparrow \oplus \downarrow \; (SF)_{xy} = -4.29 - \frac{5x_2^2}{3}$

At $x_2 = 0 \quad SF_B = -4.19$
$x_2 = 1.5 \quad SF_{1.5} = -8.04$
$x_2 = 3 \quad SF_C = -19.29$

iii) Bending Moment

$(\oplus) \; (BM)_{xy} = 17.84 - 4.29x_2 - \frac{5x_2^3}{9}$

At $x_2 = 0 \quad M_B = 17.84$
$x_2 = 1.5 \quad M_{1.5} = 9.53$
$x_2 = 3 \quad M_C = -10$

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3) Portion BCD

i) Axial Force (A·F) = −8.66

ii) Shear force = 5
(SF)

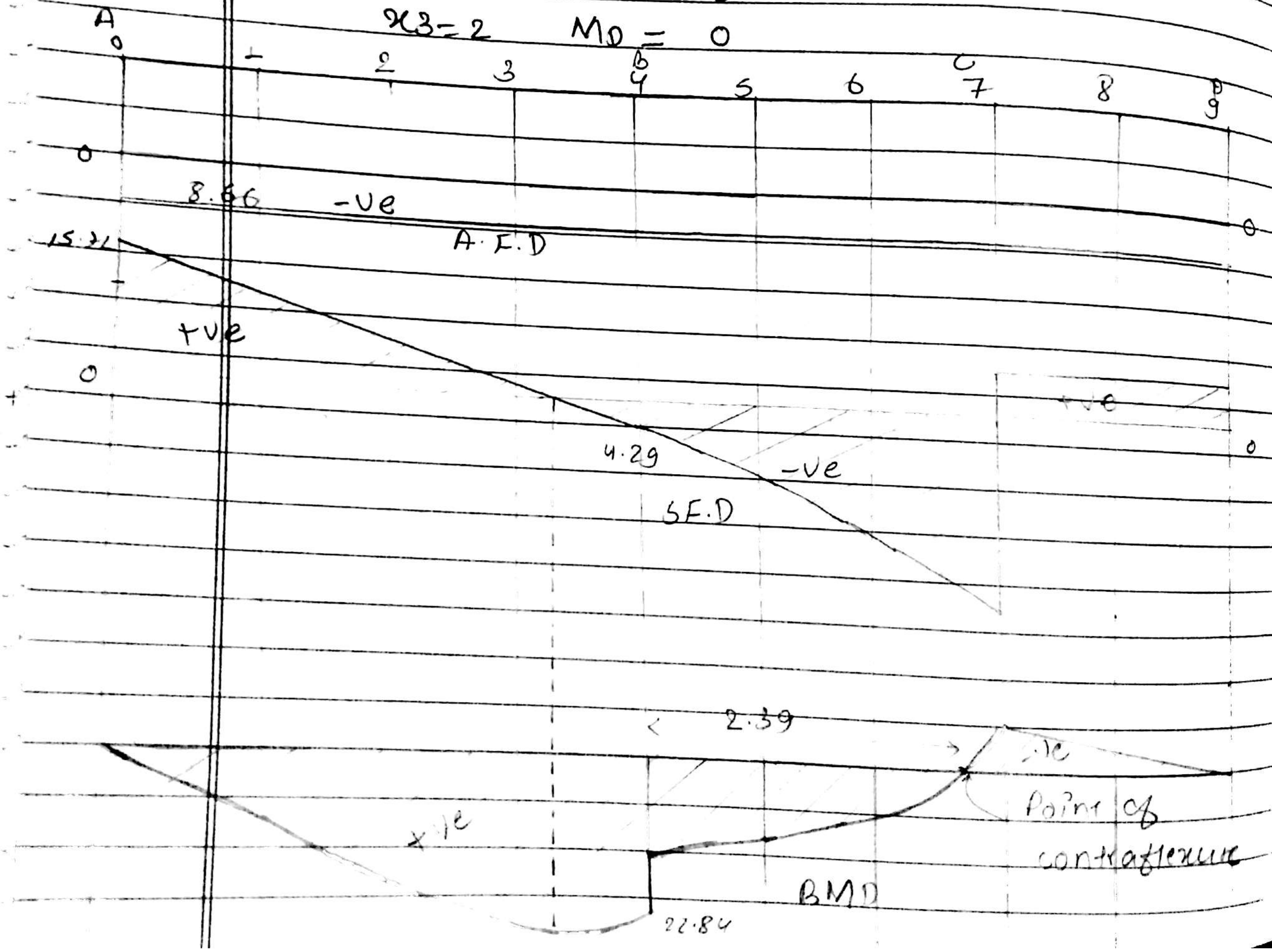
iii) Bending moment

$(BM) = -10 + 5x_3$

At $x_3 = 0$ $M_C = -10$

$x_3 = 1$ $M_L = -5$

$x_3 = 2$ $M_D = 0$



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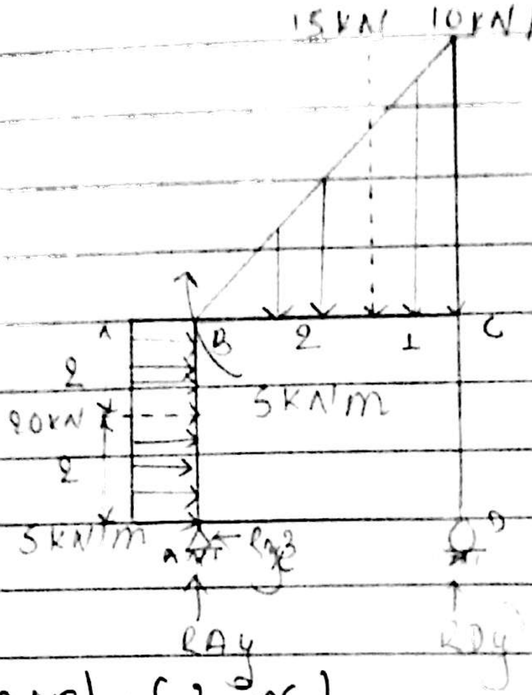
Draw AFD, SFD and bending moment diagram of given loaded frame structure. Also determine the degree of indeterminancy?



$n_i = (3m + r) - (3j + c)$
$= (3 \times 3 + 3) - (3 \times 4 + 0)$
$= 0$

The given frame structure is statically determinate.
calculation of support reaction by using equation of equilibrium.

1) $\overset{+}{\rightarrow} \Sigma F_x = 0$
$20 - R_{Ax} = 0$
$R_{Ax} = 20KN$

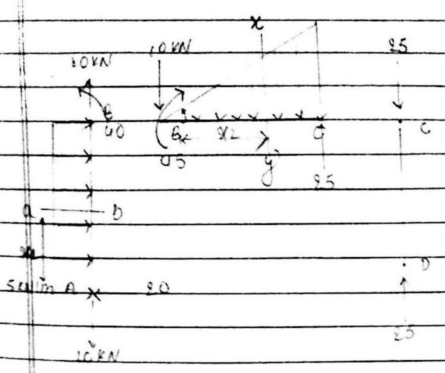
2) $\circlearrowright^{+} \Sigma M_A = 0$
or, $20 \times 2 + 5 + 15 \times 2 - R_{Dy} \times 3 = 0$
$R_{Dy} = 25$

$\circlearrowright \Sigma M_D = 0$

or, $R_{Ay} \times 3 + 20 \times 2 + 5 - 15 \times 1 = 0$

$R_{Ay} = -10\ kN$

Free Body diagram



→ Taking portion AB

Axial Force (A·F)

$(A \cdot F)_{AB} = 10\ kN$

Shear Force

$(S \cdot F)_{ab} = +20 - 5x_1$

At $x_1 = 0$	$(SF)_A = 20$
$x_1 = 2$	$(SF)_2 = 10$
$x_1 = 4$	$(SF)_B = 0$

Bending moment

$(BM)_{ab} = 20x_1 - 2.5x_1^2$

At $x_1 = 0$	$M_A = 0$
$x_1 = 2$	$M_2 = 30$
$x_2 = 4$	$M_B = 40$

2) Taking portion BC

Axial Force

$(A \cdot F)_{BC} = 0$

Shear Force

$\uparrow \oplus \downarrow = -10 - \frac{1}{2} \times x_2 \times \frac{10}{3} x_2$

$= -10 - \frac{5}{3} x_2^2$

At $x_2 = 0$	$(SF)_B = -10$
$x_2 = 1.5$	$(SF)_{1.5} = -13.375$
$x_2 = 3$	$(SF)_C = -25$

$\frac{10}{w_x} = \frac{3}{x_2}$

$w_x = \frac{10}{3} x_2$

Bending moment (B·M)

$(B \cdot M)_{xy} = -10x_2 - \frac{10x_2}{3} \times \frac{1}{2} \times x_2 \times \frac{x_2}{3}$

$= 45 - [illegible]$

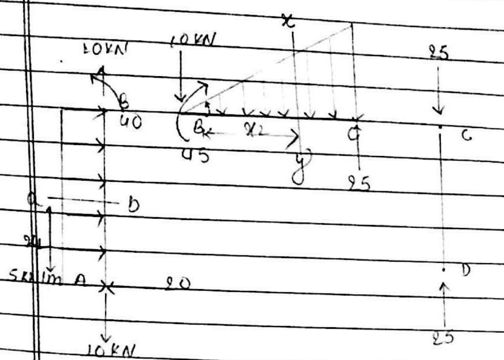
At $x_2 = 0$	$M_B = 45$
$x_2 = 1.5$	$M_{1.5} = 08.125$
$x_2 = 3$	$M_C = 0$

$\circlearrowleft \Sigma M_D = 0$

or, $R_{Ay} \times 3 + 20 \times 2 + 5 - 15 \times 1 = 0$

$R_{Ay} = -10\ kN$

Free Body diagram



1) Taking portion AB

Axial Force (A.F)

$(A.F)_{AB} = 10\ kN$

Shear Force

$(S.F)_{ab} = +20 - 5x_1$

At $x_1 = 0$ $(SF)_A = 20$

$x_1 = 2$ $(SF)_2 = 10$

$x_1 = 4$ $(SF)_B = 0$

Bending moment

$(BM)_{ab} = 20x_1 - 2.5x_1^2$

At $x_1 = 0$ $M_A = 0$

$x_1 = 2$ $M_2 = 30$

$x_2 = 4$ $M_B = 40$

2) Taking portion BC

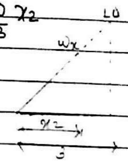
Axial Force

$(A.F)_{BC} = 0$

Shear Force

$\uparrow \oplus \downarrow = -10 - \frac{1}{2} \times x_2 \times \frac{10}{3} x_2$

$= -10 - \frac{5}{3} x_2^2$



At $x_2 = 0$ $(S.F)_B = -10$

$x_2 = 1.5$ $(SF)_{1.5} = -13.375$

$x_2 = 3$ $(SF)_C = -25$

$\frac{10}{w_x} = \frac{3}{x_2}$

$w_x = \frac{10x_2}{3}$

Bending moment (B.M)

$(B.M)_{xy} = -10x_2 - \frac{10x_2}{3} \times \frac{1}{2} \times x_2 \times \frac{x_2}{3} + 45$

$= 45 - 10x_2 - \frac{10x_2^3}{18}$

At $x_2 = 0$ $M_B = 45$

$x_2 = 1.5$ $M_{1.5} = 28.125$

$x_2 = 3$ $M_C = 0$

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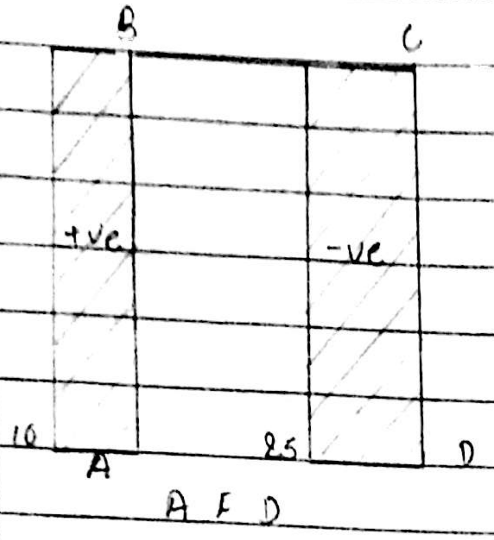
Taking portion CD

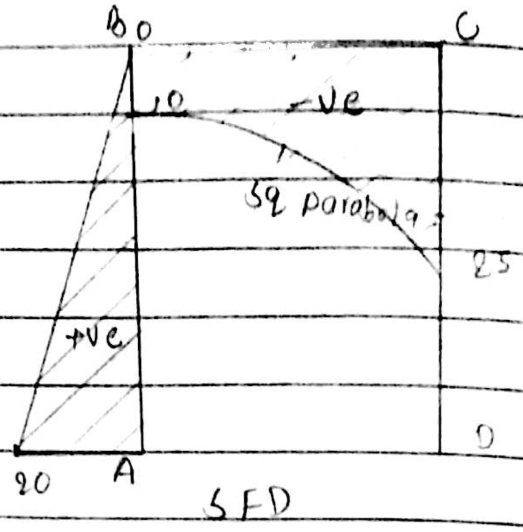
Axial Force (A.F) CD = −25

Shear Force (S.F) CD = 0

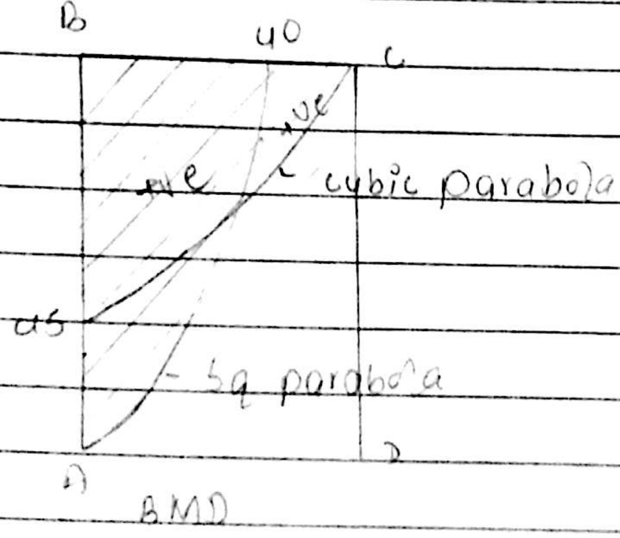
Bending moment (B.M) CD = 0



A F D

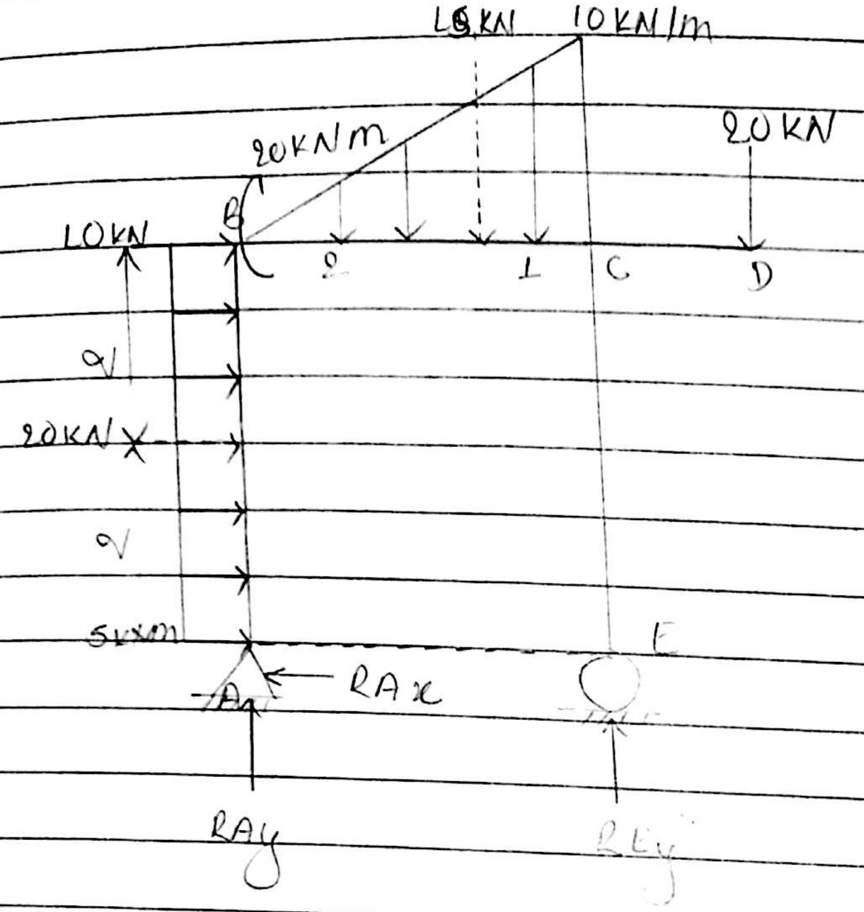


SFD



BMD

Q. Draw AFD, SFD and BMD.



Calculation of support reaction by using equation of equilibrium

1) $\overset{+}{\rightarrow} \sum Fx = 0$

Or, $-RAx + 20 + 10 = 0$

$RAx = 30\ kN$

2) $\circlearrowright^{+} \sum MA = 0$

Or, $(20\times2) + (10\times4) + 20 + (15\times2) + 20\times5 + 0 - RAy \times 3 =$

Or, $REy = 76.67\ kN$

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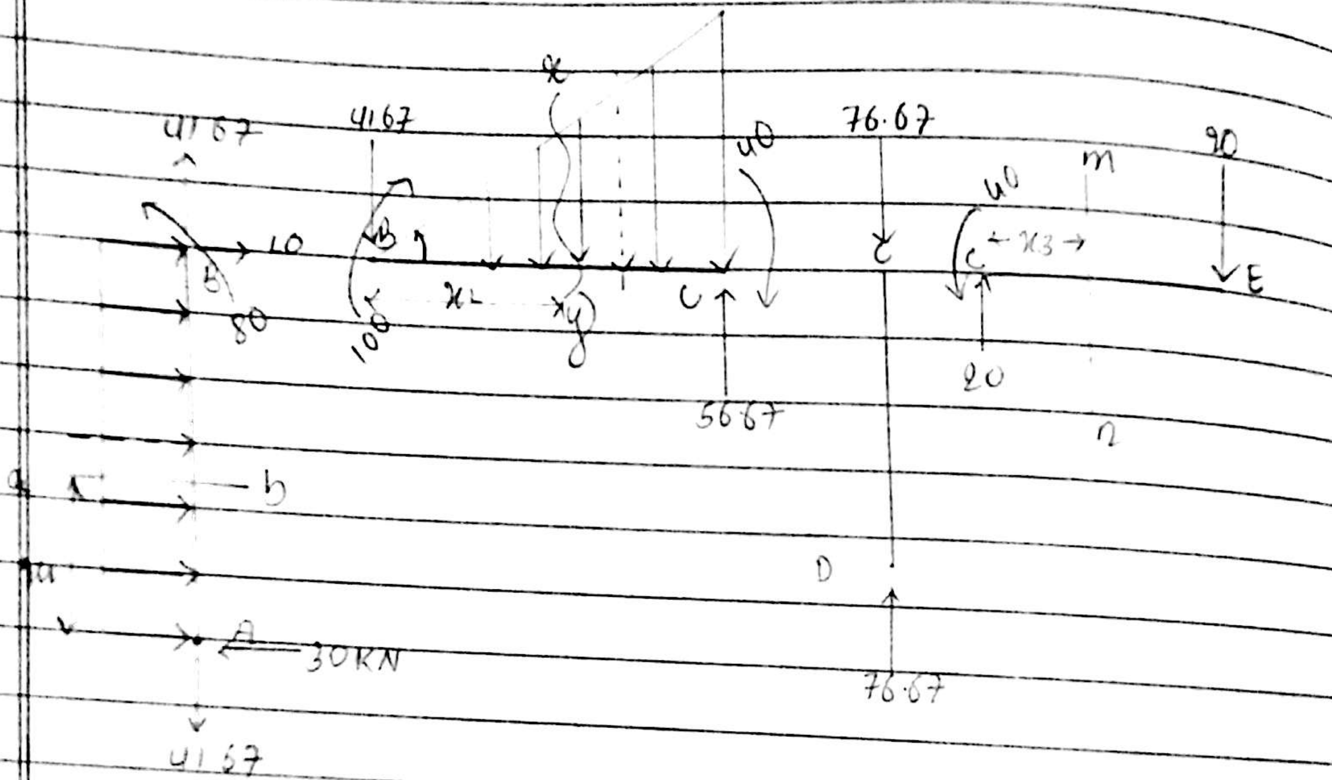
3) $\sum M_E = 0$

$(R_{Ay} \times 3) + (20\times2) + 20 - 15\times L + L0\times4 + 20\times2 = 0$

$R_{Ay} = -41.67$ KN

Free Body diagram:



Taking portion AB

i) $(A.F)_{AB} = 41.67$

ii) shear Force

$(S.F)_{ob} = 30 - 5x_1$

At $x_1 = 0$ $(SF)_A = 30$

$x_1 = 2$ $(SF)_2 = 20$

$x_1 = 4$ $(SF)_B$ 70

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iii) Bending moment

$$(B.M)_{ab} = 30x_1 - \frac{5x_1^2}{2}$$

$$= 30x_1 - 2.5x_1^2$$

At $x_1 = 0$ $(B.M)_A = 0$

$x_1 = 2$ $(B.M)_2 = 50$

$x_1 = 4$ $(B.M)_4 = 80$

2) Taking portion BC

i) Axial Force

$(AF)_{BC} = 0$

ii) Shear Force (S.F)

$$(SF)_{xy} = -41.67 - \frac{10}{6} x_2^2$$

At $x_2 = 0$ $(SF)_B = -41.67$

$x_2 = 1.5$ $(SF)_{1.5} = -45.42$

$x_2 = 3$ $(SF)_C = -56.67$

10

wx

$$\frac{10}{w_x} = \frac{3}{x_2}$$

$$w_x = \frac{10}{3} x_2$$

Force (F) $= \frac{1}{2} \times x_2 \times \frac{10}{3} x_2$

$$= \frac{10}{6} x_2^2$$

Moment $= \frac{10}{6} x_2^2 \times \frac{x_2}{3}$

$$= \frac{10x_2^3}{18}$$

iii) Bending moment

$$(B.M)_{xy} = -41.67x_2 - \frac{10x_2^3}{18} + 100$$

At $x_2 = 0$ $(M)_B = 100$

$x_2 = 1.5$ $M_{1.5} = 35.62$

$x_2 = 3$ $M_C = -40$

3) Taking portion CD

i) Axial Force

$(A \cdot F) = -76.67$

ii) Shear force $(S \cdot F)_{CD} = 0$

iii) Bending moment $(B \cdot M) = 0$

4) Taking portion CE

i) Axial Force $(A \cdot F)_{CE} = 0$

ii) Shear Force $(SF)_{CE} = 20$

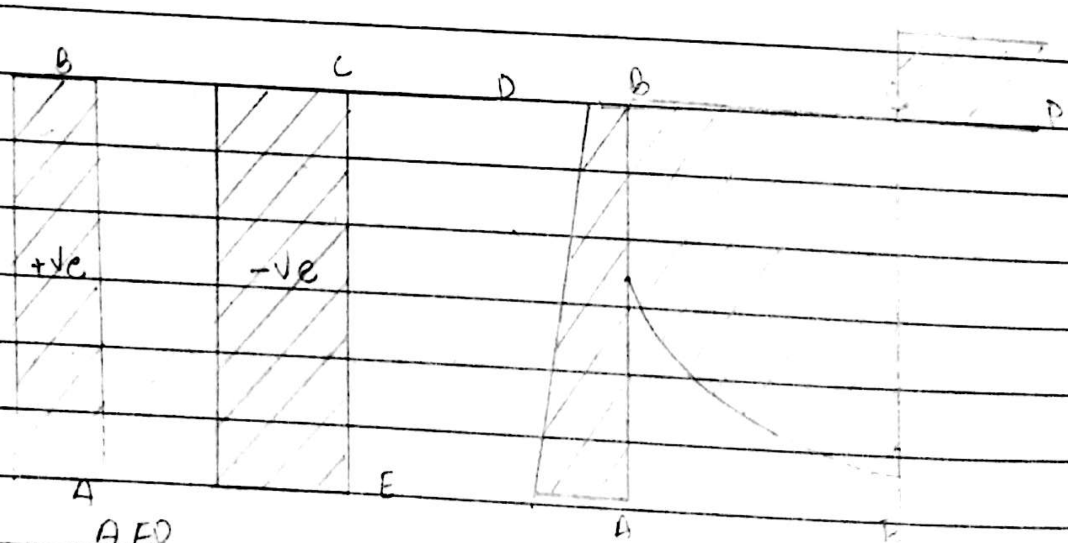
iii) Bending moment $(B \cdot M)$

$(B \cdot M)_{mn} = 20x$

At $x_3 = 0$ $M_C = 0$

$x_3 = L$ $M_L =$

$x_3 = 2$ M_E



AFD

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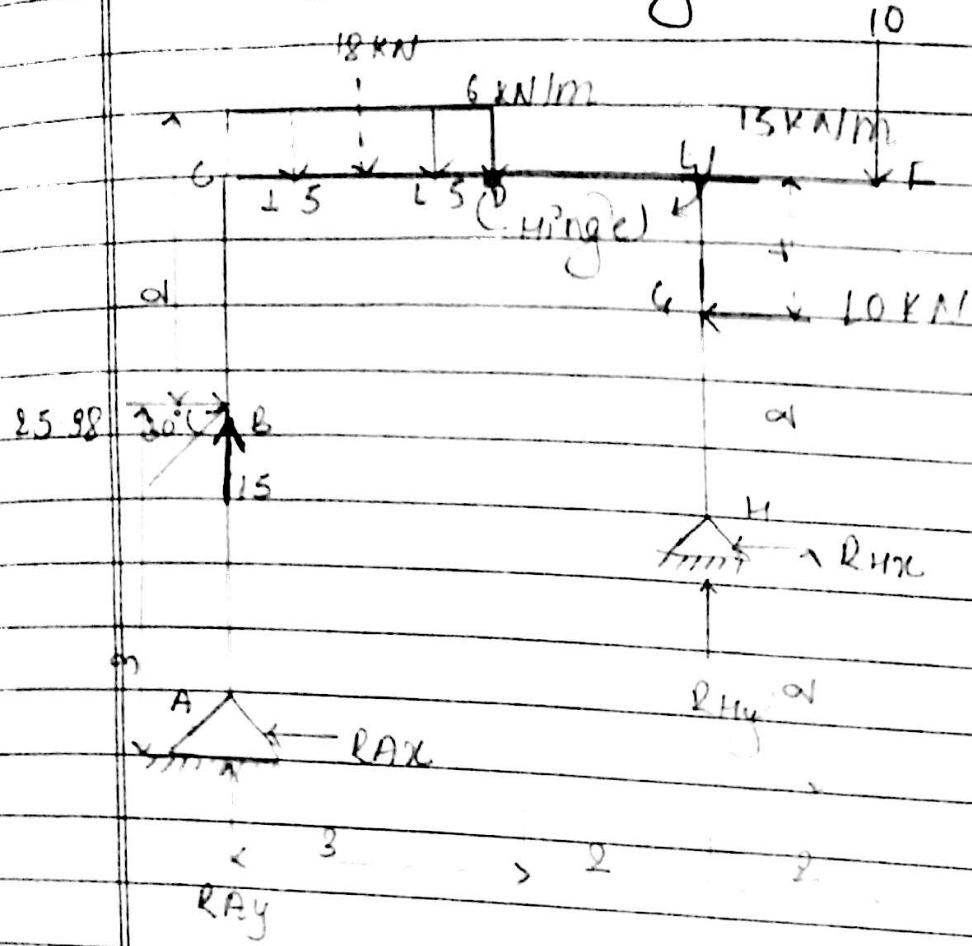
Q. Draw AFD, SFD, BMD of given frame and indicate silent feature if any.



Soln.

Calculation of support reaction by using equation of equilibrium

$\circlearrowright \Sigma M_A = 0$

or $(25.98 \times 3) + (18 \times 1.5) + 15 - (10 \times 4) + (10 \times 7) - (R_{Hx} \times 2) - (R_{Hy} \times 5)$

$2R_{Hx} + 5R_{Hy} = 149.94$ —— (1)

$\circlearrowright \Sigma M_{D\ \text{right part}} = 0$

or, $15 + (10 \times 4) + (10 \times 1) + (R_{Hx} \times 3) - R_{Hy} \times 2 = 0$

$3R_{Hx} - 2R_{Hy} = -65$ — (ii)

Solving equⁿ (i) and (ii) we get

$R_{Hx} = \cancel{-3.122}\ 1.322$

$R_{Hy} = 30.517$

$\circlearrowright \Sigma M_H = 0$

$(R_{Ay} \times 5) + (R_{Ax} \times 2) + (25.98 \times 1) - (18 \times 1.5) + 15 + 15 \times 5$
$- (10 \times 2) + (10 \times 2) = 0$

$5R_{Ay} \times 2R_{Ax} = 52.98$ — (iii)

$\circlearrowright \Sigma M_{D\ \text{(left part)}} = 0$

or, $-(18 \times 1.5) - (25.98 \times 2) + (15 \times 3) + (R_{Ay} \times 3) + (R_{Ax} \times 5) = 0$

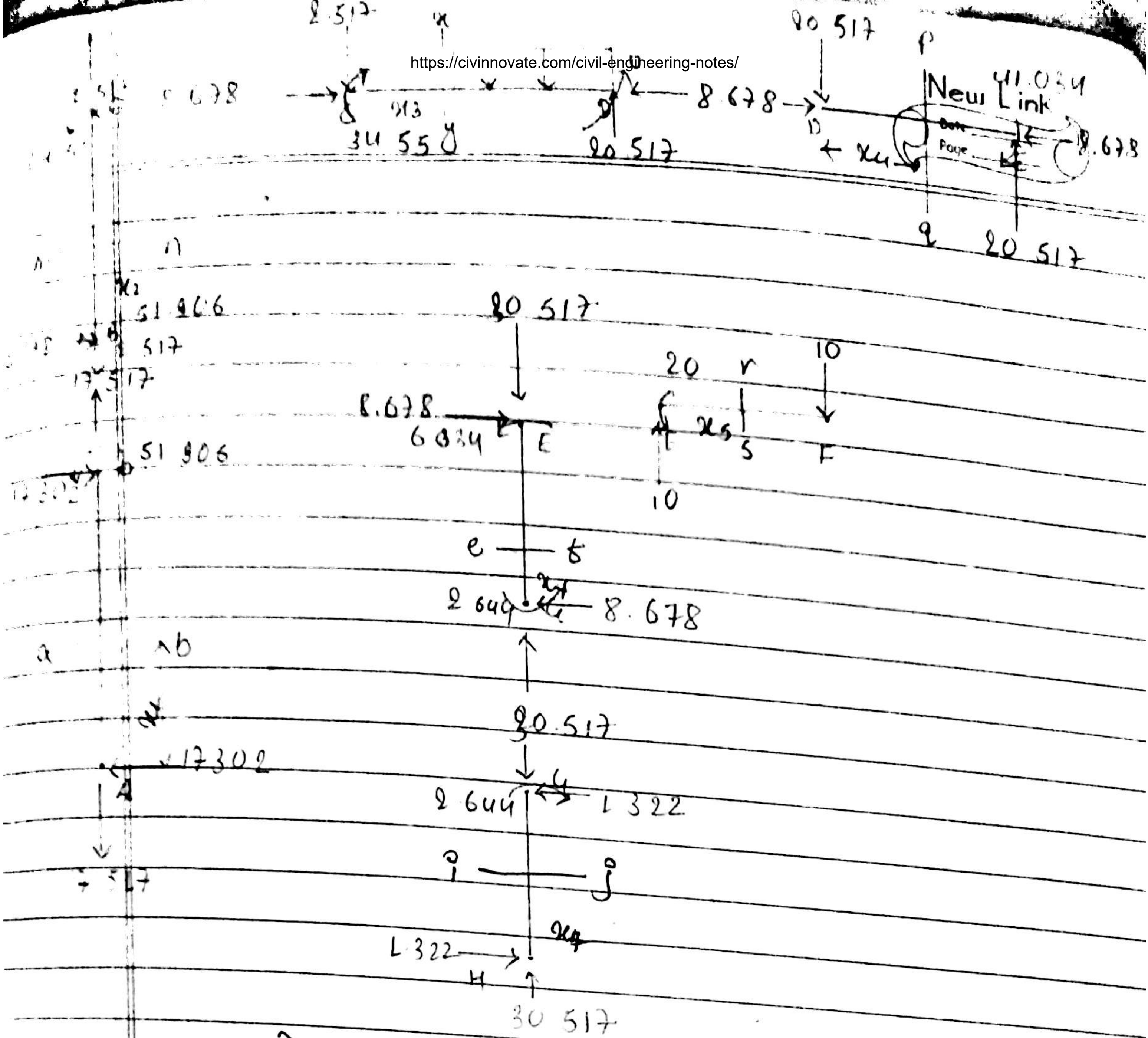
$5R_{Ax} + 3R_{Ay} = 33.96$ — (iv)

Solving equations (iii) and (iv) we get,

$R_{Ax} = 17.30$

$R_{Ay} = -17.517$

Free Body diagram:



Taking portion AB

Axial force $(A.F)_{AB} = 17.517$

Shear force

$(S.F)_{ab} = 17.302$

Bending moment

$(B.M)_{ab} = 17.302 \times x_1$

At $x_1 = 0$, $M_A = 0$

$x_1 = 3$, $M_B = 51.906$

2) Portion BC

Axial Force $(A.F)_{BC} = 2.517$

Shear Force (S.F)

$(S.F)_{mn} = -8.678$

Bending moment (B.M)

$(B.M)_{mn} = 51.906 - 8.678 x_2$

At $x_2 = 0$ $M_B = 51.906$

$x_2 =$ $M_C = 34.55$

3) Portion CD

Axial Force $(A.F)_{CD} = -8.678$

Shear force

$(S.F)_{xy} = -2.517 - 6x_3$

At $x_3 = 0$ $(S.F)_C = -2.517$

$x_3 = 3$ $(S.F)_D = -20.517$

Bending moment

$(BM)_{xy} = 34.55 - \frac{6x_3^2}{2} - 2.517x_3$

At $x_3 = 0$, $(B.M)_C = 34.55$

$x_3 = 3$ $(B.M)_D = 0$

4) Portion D.E

$A.F = -8.678$

$S.F_{PQ} = -20.517$

Bending moment

$\curvearrowright\oplus\curvearrowleft$ $(B.M)_{Pq} = -20.517 x_4$

At $x_4 = 0$ $\quad M_D = 0$
$x_4 =$ $\quad M_E = 41.034$

5) Portion EF
$(A.F)_{EF} = 0$

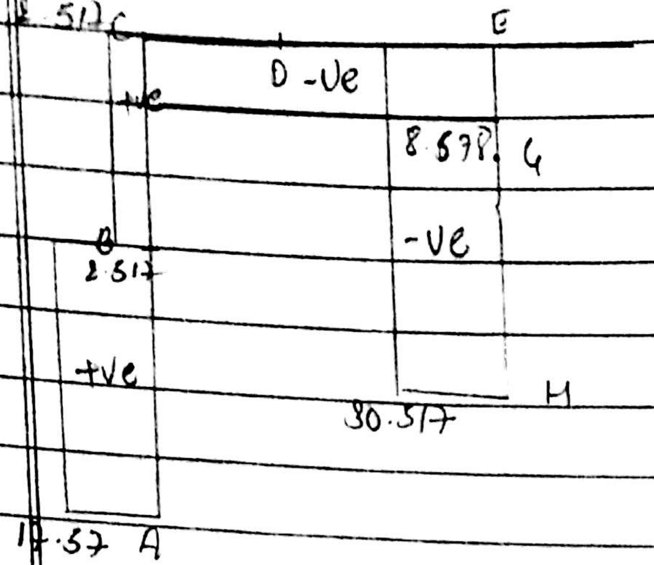
$\uparrow\oplus\downarrow$ $(S.F) = 10$
$\curvearrowright\oplus\curvearrowleft$ $(B.M)_5 = -20 + 10x_5$
At $x_5 = 0$ $\quad M_E = -20$
$x_5 = 2$ $\quad M_F = 0$

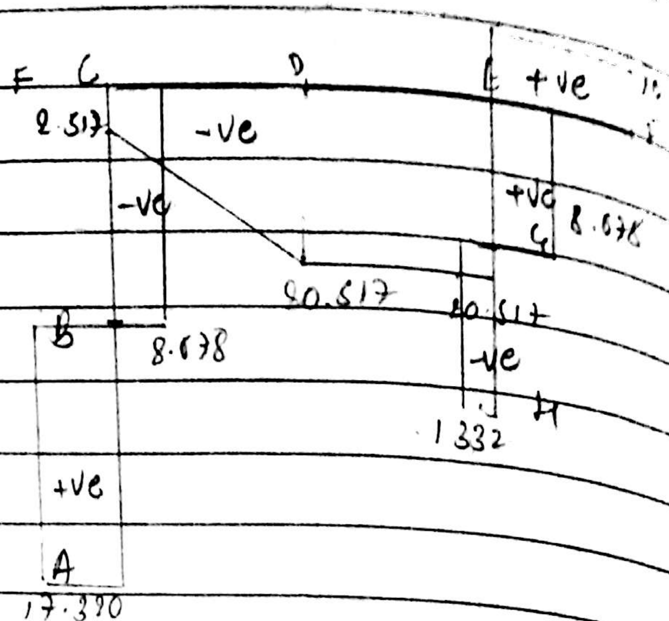
6) Portion EG
$(A.F)_{EG} = -30.517$
$(S.F)_{EG} = 8.678$
Bending moment
$(BM)_{eg} = +2.644 - 8.678x_6$

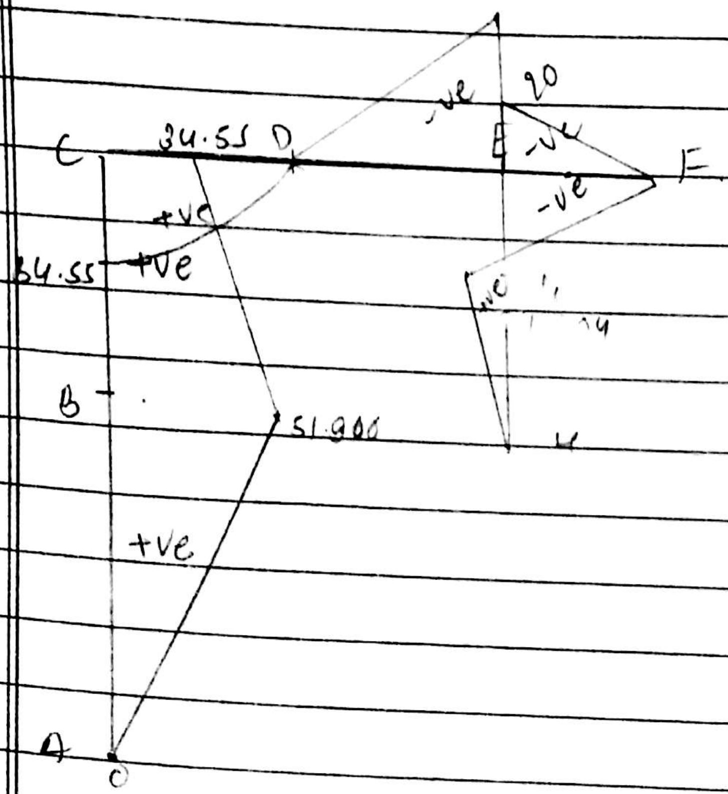
At $x_6 = 0$ $\quad (B.M)_G = +2.644$
$x_6 = 1$ $\quad (B.M)_E = -6.034$

7) Portion GH
$(A.F)_{GH} = -30.517$
$(S.F)_{ij} = -1.322$

$(B.M)_{ij} = 1.332x_7$
$M_A = 0$ $\quad 2.644$
$M_G =$


2.517
C
D -ve
E
+ve
8.578
G
-ve
B
2.517
+ve
30.517
H
17.37
A


F
C
D
E
+ve
2.517
-ve
-ve
+ve
G
8.678
20.517
20.517
B
8.678
-ve
H
1.332
+ve
A
17.370


C
34.55
D
+ve
20
E
-ve
F
-ve
+ve
34.55
+ve
B
51.900
+ve
A

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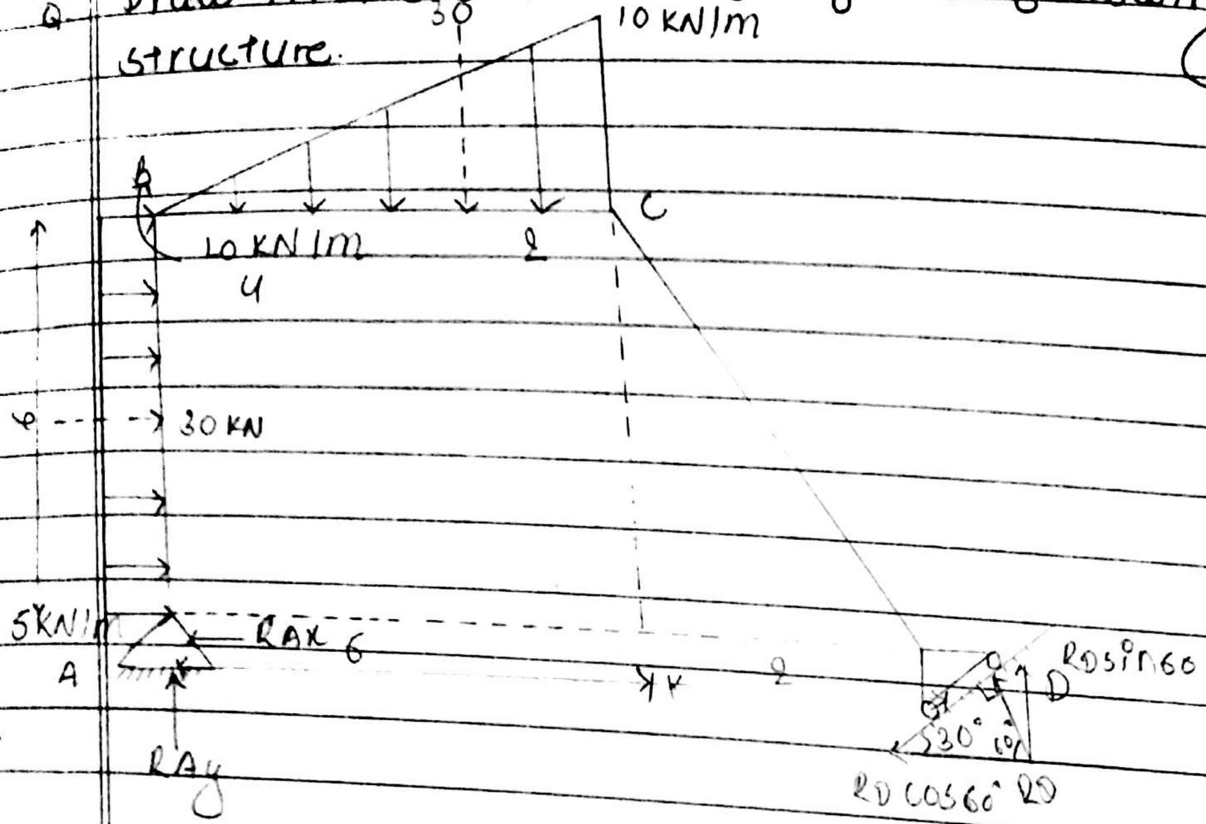
Q. Draw AFD, SFD and BMD of the following frame structure.



$\circlearrowright \Sigma M_A = 0$

or, $(30 \times 3) + 10 + (30 \times 4) - R_D \sin 60 \times 8 = 0$

$R_D = 31.754$

$\circlearrowright \Sigma M_D = 0$

or, $(R_{Ay} \times 8) + (30 \times 3) + 10 - (30 \times 4) = 0$

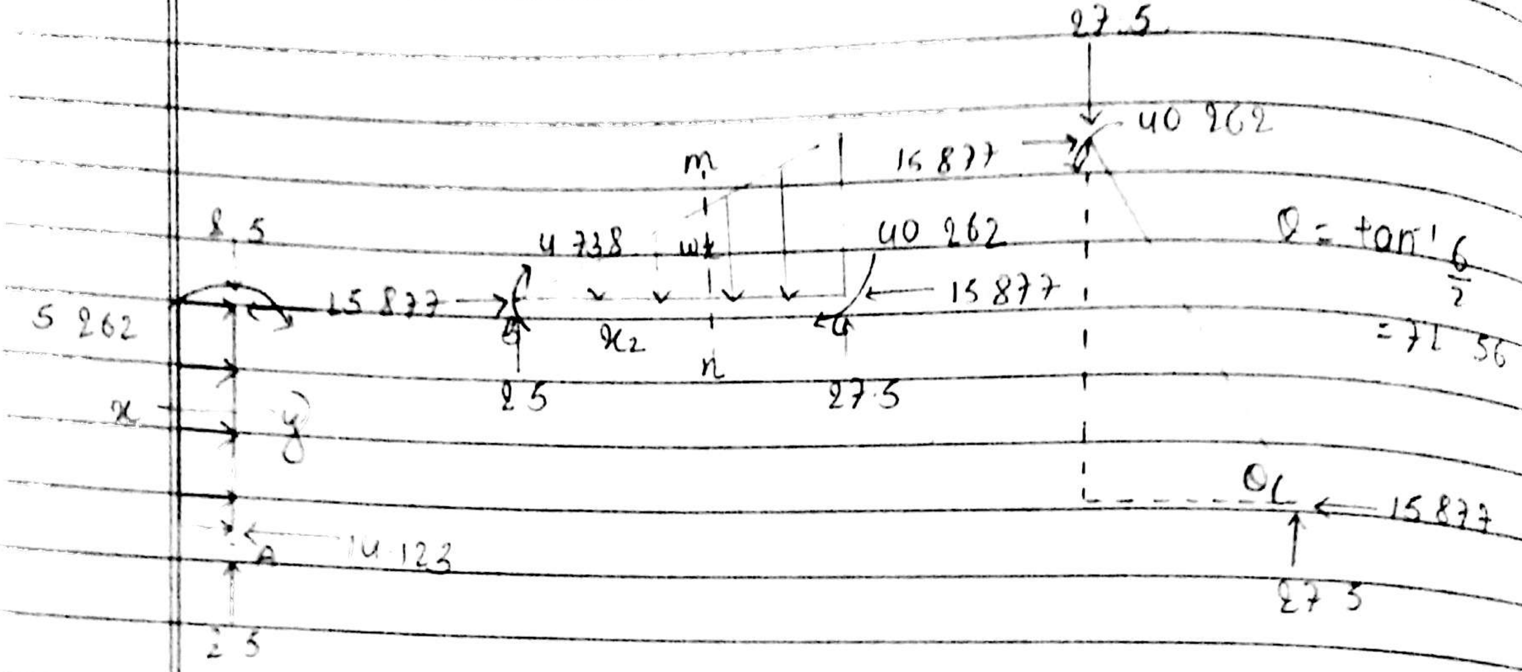
or, $R_{Ay} = 2.5$

$\overset{+}{\rightarrow} \Sigma F_x = 0$

or, $30 - R_{Ax} - R_D \cos 60° = 0$

$R_{Ax} = 14.123$

Free Body diagram



i. Taking portion AB

i) Axial Force $(A.F)_{AB} = -2.5$

ii) Shear force

$(S.F)_{xy} = 14.123 - 5x_1$

At $x_1 = 0$ $(S.F)_A = 14.123$

$x_1 = 6$ $(S.F)_B = -15.877$

iii) Bending moment

$(B.M)_{xy} = 14.123x_1 - 2.5x_1^2$

At $x_1 = 0$ $(B.M)_A = 0$

$x_1 = 6$ $(B.M)_B = -5.262$

Taking portion BC

i) Axial force $(A.F)_{BC} = -15.877$

ii) shear force

$(S.F)_{mn} = 2.5 - \frac{10}{6} x_2^2$

$$\frac{w}{10} = \frac{x_2}{3}$$

$$w = \frac{10}{3} x_2$$

$$S.F = \frac{1}{2} \cdot \frac{10}{6} x_2 \times x_2$$

$$B.M = \frac{10}{12} x_2^2 \times \frac{x_2}{3}$$

$(S.F)_A = 2.5$

$(S.F)_B = -27.5$

Bending moment

$= 2.5 x_2 - \frac{10}{12} \frac{x_2^3}{3}$

$(B.M)_B = 0$

$(B.M)_C = -40.262$

Unit 7

Analysis of Truss

Definition:

A truss is a structure made up of slender members pin connected at ends. They are capable of taking the loads the joints. They are also known as pin connected frames.

The truss in which all the members in a single plane and loads and reaction act in the plane of the truss are called plane truss (two dimensional) eg: bridge truss, roof truss.

Asumption of truss:

i) The ends of the members are perfect pin connection.

ii) The self weight of the truss is negligible.

iii) Load acts at joint only.

iv) Member are capable to resist the tensile or compressive force only.

Classification of Truss:

1) According to analysis

a) Plane truss

b) Space truss (3D truss)

eg: transmission tower.

2) According to support condition:

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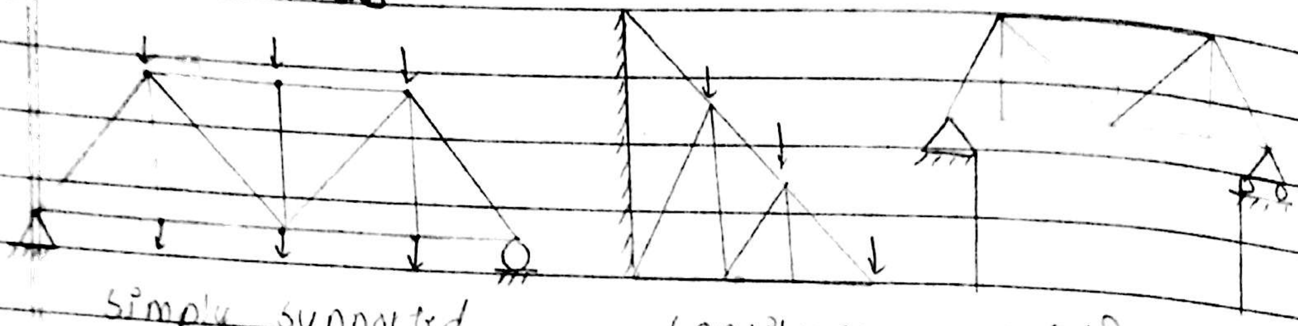
a) Simply supported
b) Cantilever

3) According to the purpose of utilization
a) roof truss
b) bridge truss
c) tower truss



statically determinate and indeterminate structure
i) Total degree of external indeterminancy
$n_{ei} = r - 3$
ii) Total degree of internal indeterminancy
$n_{ii} = m - 2j + 3$
iii) Total degree of indeterminancy
$n_i = (m+r) - 2j$

case i) If $(m+r) = 2j$ then truss is statically determinate
i.e $n_i = 0$
case ii) If $(m+r) > 2j$ then truss is statically indetermina[te]
i.e n_i = +ve
case iii) If $(m+r) < 2j$ then the truss is determinate but unstable.

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Types of truss:

i) Perfect truss:

A perfect frame is a pin jointed ~~truch~~ truss which can resist the load applied at joints without undergoing visible changes in its shape and has just sufficient members to keep its shape. Mathematically $m = 2j - 3$

ii) Deficient truss:

A truss is said to be deficient, if it has no. of members less than required for a perfect truss. Its shape undergoes visible change for the loads applied at joints. Mathematically $m < 2j - 3$.

iii) Redundant truss:

A redundant truss is a stable structure in which no. of members are more than required for a perfect truss. Redundant truss can't be analyzed by using the equation of equilibrium only. Mathematically,

$$m > 2j - 3$$

Analysis of Truss:

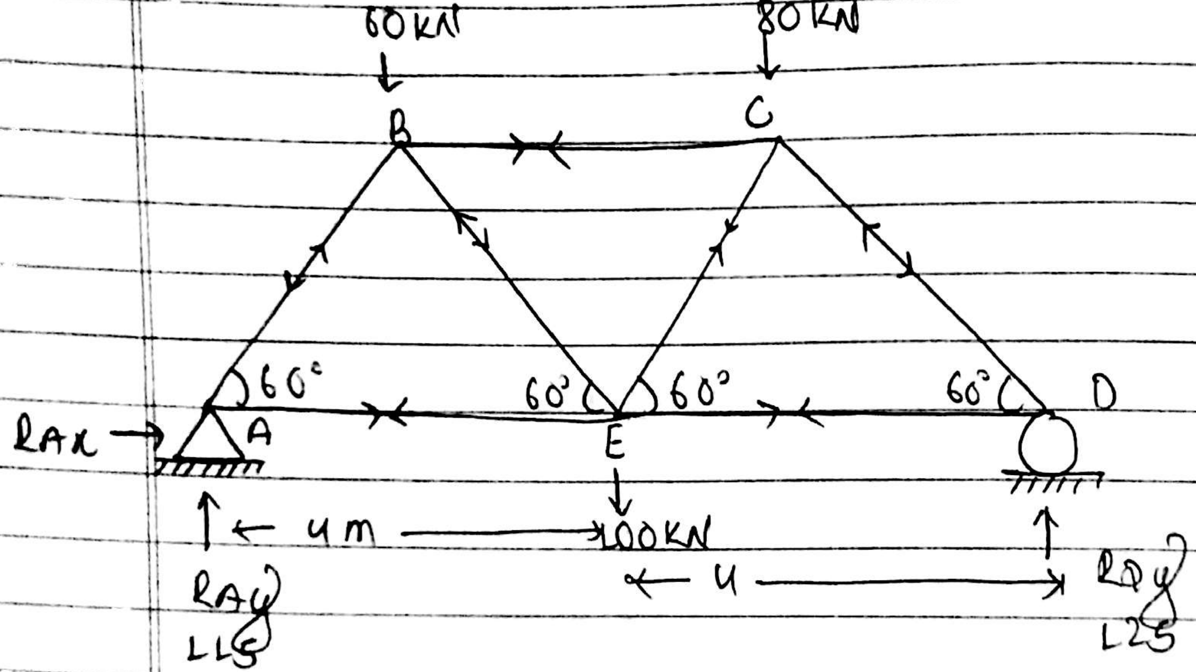
The following method are available for the analysis:

i) method of joints ($\Sigma F_x = 0$, $\Sigma F_y = 0$)

ii) method of section ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$)

iii) graphical method
iv) Tension coefficient method
v) Matrix method

Q. Determine the forces develop in the members of given truss by using method of joint. Also determine the total degree of internal indeterminancy, total degree of external indeterminancy and total degree of indeterminancy.



Soln:

$m = 7$ $r = 3$

$j = 5$

Total degree of external indeterminancy (n_{ex}) = $r - 3$

$= 3 - 3$

$= 0$

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$n_{ii} = m - 2j + 3$
$= 7 - 2 \times 5 + 3$
$= 0$

$n_i = 0 + 0$
$= 0$

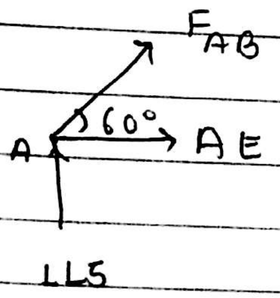
The given truss is statically determinate
calculation of support reaction by using the equation of equilibrium.

i) $\overset{+}{\rightarrow} \Sigma F_x = 0$
$R_{Ax} = 0$

ii) $\circlearrowright^{+} \Sigma M_A = 0$
or, $(60 \times 2) + (100 \times 4) + (80 \times 6) - (R_{Dy} \times 8) = 0$
$R_{Dy} = 125\ kN$

iii) $\circlearrowright^{+} \Sigma M_D = 0$
or, $(R_{Ay} \times 8) - (60 \times 6) - (100 \times 4) - (80 \times 2) = 0$
$R_{Ay} = 115$

FBD of joint A



$\overset{+}{\rightarrow} \cancel{\Sigma F_x = 0}$
or,

$+\uparrow \Sigma F_y = 0$
or, $115 + F_{AB} \sin 60° = 0$
$F_{AB} = -132.79$

$F_{AB} = 132.79$ (C)

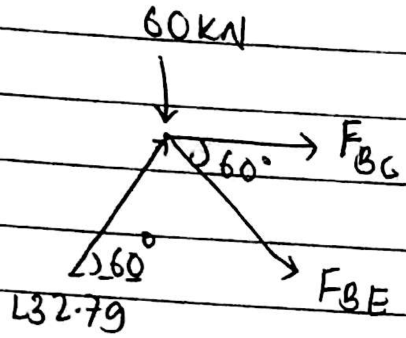
ii) $\xrightarrow{+}$ $\Sigma F_x = 0$

or, $F_{AB}\cos 60° + F_{AE} = 0$

$F_{AE} = -(-132.79\cos 60°)$

$= 66.396$

FBD of joint B



$+\uparrow \Sigma F_y = 0$

or, $132.79\sin 60 - 60 - F_{BE}\sin 60° = 0$

$\therefore F_{BE} = 63.307$ KN

$\xrightarrow{+}$

$\Sigma F_x = 0$

or, $132.79\cos 60 + F_{BC} + F_{BE}\cos 60° = 0$

or, $132.79\cos 60 + F_{BC} + 63.507\cos 60° = 0$

$F_{BC} = -98.485$

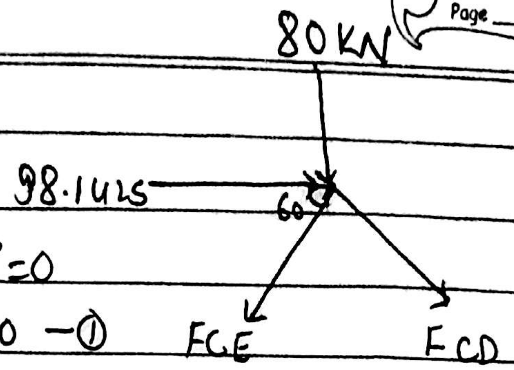
$F_{BC} = 98.485$ (C)

FBD of joint C

$+\uparrow \Sigma F_y = 0$

or, $-80 - F_{CE}\sin60^\circ - F_{CD}\sin60^\circ = 0$

or, $F_{CE}\sin60^\circ + F_{CD}\sin60^\circ = -80$ —(i)



$\overset{+}{\rightarrow} \Sigma F_x = 0$

or, $98.1425 - F_{CE}\cos60^\circ + F_{CD}\cos60^\circ = 0$

or, $-F_{CE}\cos60^\circ + F_{CD}\cos60^\circ = -98.1425$ —(ii)

on solving equns (i) and (ii)

$F_{CE} = 51.95$

$F_{CD} = -144.33$

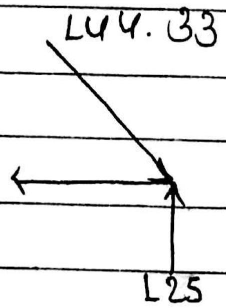
$\therefore F_{CD} = 144.33$ (C)

FBD of joint D

$\overset{+}{\rightarrow} \Sigma F_x = 0$

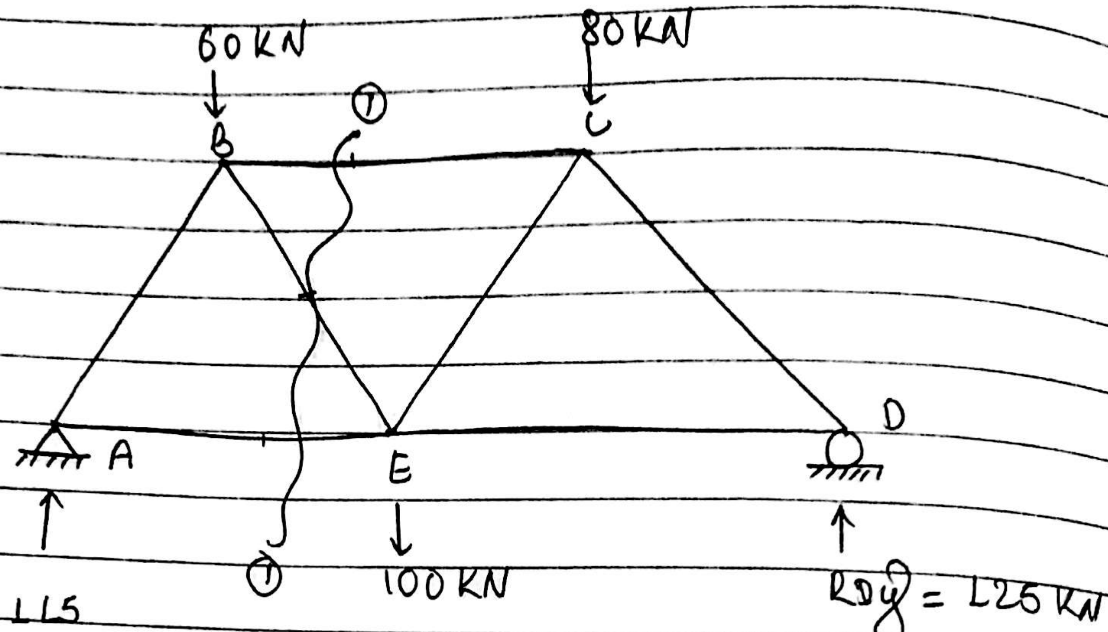
or, $-F_{DE} + 144.33\cos60 = 0$

$F_{DE} = 72.18$

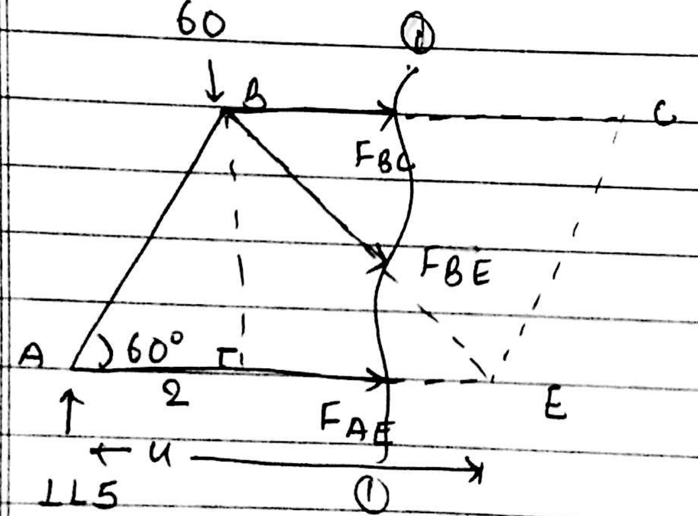


SN	Member	Magnitude	Nature of force
1	AB	132.79	C
2	AE	66.395	T
3	BC	98.148	C
4	BE	63.507	T
5	CE	51.96	T
6	CD	144.336	C
7	DE	72.18	T

Q. Determine the member force in member BC, BE and AE of given truss by using section method.



At section 1-1 considering left part only



$\tan 60^\circ = \dfrac{h}{2}$

$h = 3.46$

$+) \ \Sigma M_E = 0$

or, $(115 \times 4) - (60 \times 2) + F_{BC} \times 3.46 = 0$

$F_{BC} = -98.26$

$F_{BC} = 98.26$ (C)

For the Measurement of FBE

$+\uparrow \Sigma F_y = 0$

or $-60 + 115 - F_{BE} \sin 60^\circ = 0$

$F_{BE} = 63.50$ kN

$\circlearrowright \Sigma M_B = 0$

or, $(115 \times 2) - F_{AE} \times 3.46 = 0$

$F_{AE} = 66.47$

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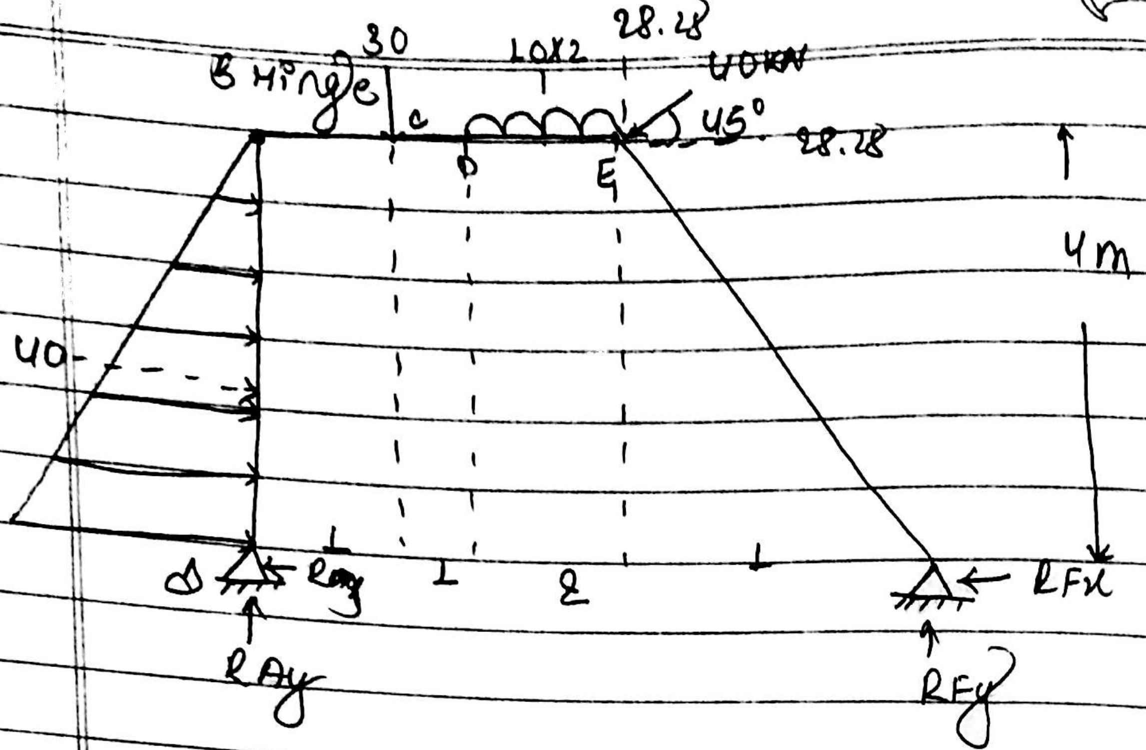


$\circlearrowleft + \sum M_A = 0$

or, $(40 \times 1.33) + (30 \times 1) + (20 \times 3) + (28.28 \times 4) - (28.28 \times 4)$
$- R_{Fy} \times 5 = 0$

or, $R_{Fy} = 28.64$

$\circlearrowleft + \sum M_F = 0$

or, $(R_{Ay} \times 5) + (40 \times 1.33) - (30 \times 4) - (20 \times 2) - (28.28 \times 4)$
$- (28.28 \times 1) = 0$

$R_{Ay} = 49.64$

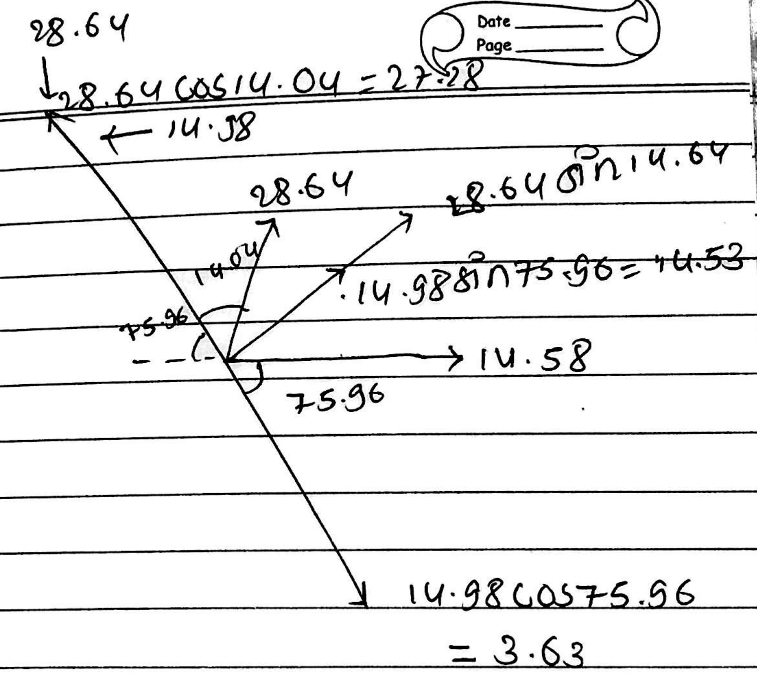
iii) $\circlearrowleft + \sum M_{B \text{ left part}} = 0$

or, $-(40 \times 2.67) + (R_{Ax} \times 4) = 0$
$R_{Ax} = 26.7$

iv) $\circlearrowleft + \sum M_B$ right part $= 0$

or, $(30 \times 1) + (20 \times 3) + (28.28 \times 4) + (R_{Fx} \times 4) - R_{Fy} \times 5 =$

$R_{Fx} = -14.98$


28.64
28.64 cos14.04 = 27.78
14.58
28.64
14.04
28.64 sin 14.64
14.98 sin 75.96 = 14.53
75.96
14.58
75.96
14.98 cos 75.96
= 3.63

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